Review

For cases when $Z_1 >> N$ condition is not met:

- Fermi-Dirac Distribution (fermions) ← state occupied or unoccupied

$$\bar{n}_{FD} = \frac{1}{e^{(\varepsilon-\mu)/kT}+1}$$

- Bose-Einstein Distribution (bosons) ← any number of particles from $0 \to N$

$$\bar{n}_{BE} = \frac{1}{e^{(\varepsilon-\mu)/kT}-1}$$

$\mu < \varepsilon$

using Gibbs factor to find these

Fermi-Dirac Distribution

- Consider cases looking at ε and μ

$$\bar{n}_{FD} = \frac{1}{e^{(\varepsilon-\mu)/kT}+1}$$

When

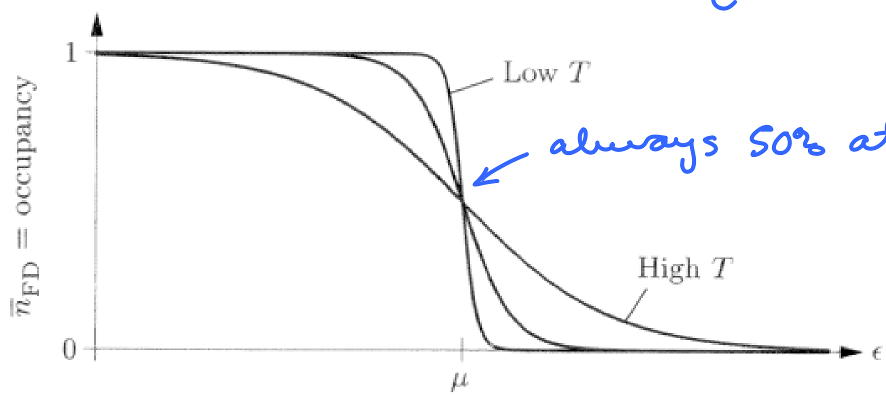
$\varepsilon \gg \mu \qquad \bar{n}_{FD} \longrightarrow 0$

$\mu \gg \varepsilon \qquad \bar{n}_{FD} \longrightarrow 1$

Fermi-Dirac Distribution

- Consider cases looking at ε and μ

$$\bar{n}_{FD} = \frac{1}{e^{(\varepsilon-\mu)/kT}+1}$$



always 50% at $\mu=\varepsilon$

Relation to Boltzmann Distribution

- Calculate occupancy $\bar{n}_{Boltzmann} = NP(s)$

$$\bar{n}_{Boltz} = N\cdot\frac{1}{Z_1}e^{-\varepsilon/kT}$$

$\mu = -kT\ln\frac{Z_1}{N}$ → comes from Helmholtz free energy $F=-kT\ln Z$ & $\mu = \left(\frac{\partial F}{\partial N}\right)_{T,V}$

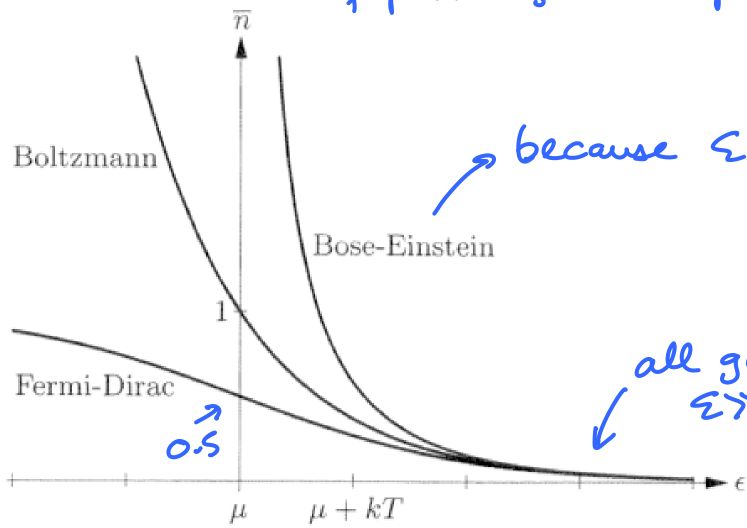
$$\bar{n}_{Boltzmann} = e^{\mu/kT}e^{-\varepsilon/kT}$$
$$= e^{-(\varepsilon-\mu)/kT}$$

for ε greater μ

$$\left.\begin{aligned}\bar{n}_{BE} &= \frac{1}{e^{(\varepsilon-\mu)/kT}-1}\\ \bar{n}_{FD} &= \frac{1}{e^{(\varepsilon-\mu)/kT}+1}\end{aligned}\right\} e^{-(\varepsilon-\mu)/kT}$$

Comparison of Distributions

← average number of particles → occupancy



because ε always $> \mu$

0.5

all go to zero at $\varepsilon \gg \mu$

energy of state

when all agree

$(\varepsilon - \mu)/kT \gg 1$

$\rightarrow |\mu| \ll kT$

$kT \gg \mu$ → many more accessible states than particles

Quantum Gas

- For ideal gas, single-particle partition function

$$Z_1 = \frac{VZ_{\text{int}}}{\upsilon_Q}$$

$$\upsilon_Q = \ell_Q^3 = \left(\frac{h}{\sqrt{2\pi mkT}}\right)^3$$

- For $Z = Z_1^N / N!$ to hold, $Z_1 >> N$

$$\frac{VZ_{int}}{\upsilon_Q} \gg N$$

if Z_{int} is small

$$\frac{V}{N} \gg \upsilon_Q$$

wavefunction of particles can overlap

when gas is sufficiently dense
or
when v_Q is sufficiently large

Application of Fermi-Dirac Distribution

- Modelling behavior of conduction electrons in metal
  - Gas of fermions at very low temperature

- Condition for Boltzmann statistics is not met

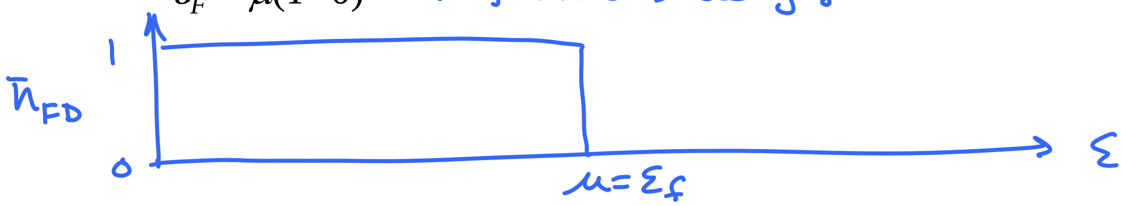
$$\frac{V}{N} << \upsilon_Q$$

$$v_Q = \left(\frac{h}{\sqrt{2\pi m k T}}\right)^3$$

consider system starting at $T=0$

At Zero Temperature

- At T=0, Fermi-Dirac distribution becomes step function
  - $\varepsilon_F \equiv \mu(T{=}0)$ ← fermi energy



- When nearly all states below ε_F are occupied and those above are empty
  - Called degenerate gas

- ε_F depends on total number of electrons in system

last electron in energy is ε_f $(=\mu)$

$$\mu = \left(\frac{\partial U}{\partial N}\right)_{S,V} \qquad dN = 1$$

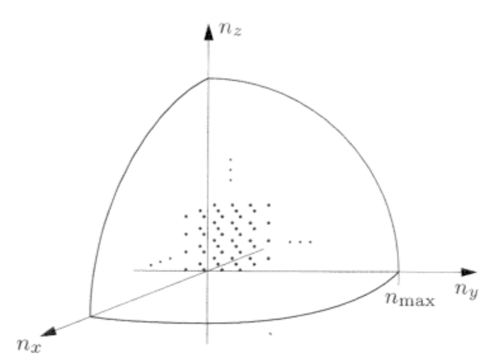
Degenerate Fermi Gas

- Electrons in the system are free particles
  - Ignoring attractive forces from ions in the crystal lattice

- Wavefunctions for free electron in metal block
  - Represented by standing wave modes

$$\lambda_n = \frac{2L}{n}; \qquad p_n = \frac{h}{\lambda_n} = \frac{hn}{2L}$$

L = length of block
n = mode index



Degenerate Fermi Gas

- Wavefunctions for free electron in metal block
  - Represented by standing wave in a box wavefunctions

$$\varepsilon = \frac{|\vec{p}|^2}{2m} = \frac{h^2}{8mL^2}\left(n_x^2 + n_y^2 + n_z^2\right)$$

$n \rightarrow n_x, n_y, n_z$

  - Each "lattice point" (n_x, n_y, n_z) is pair of electron states
    - One for each spin orientation
  - Surface of sphere => radius n_{max}