Family name, Given name (Please print) **Student Number** **Tutorial Leader's Name**

PHY293F – STATISTICAL MECHANICS
DEPARTMENT OF PHYSICS, UNIVERSITY OF TORONTO

EXAMPLE FOR MIDTERM TEST #2
10 November 2009

PLEASE read carefully the following instructions.

AIDS ALLOWED: Equation sheet provided with test paper and a non-programmable calculator without text storage (Type 2 calculator).

Before starting, please **print** your name, tutorial group and student number **at the top of this page and on the cover of your answer booklet**.

This test has one cover sheet, two question pages and one equation sheet.

There are three questions on this midterm test. The value of each question is indicated next to the question part. The total number of points for the midterm is 35 points.

The test questions can be answered in any order. It is your responsibility to clearly indicate the question number (and part, where appropriate) for each of your answers.

Partial credit will be given for partially correct answers, so show any intermediate calculations that you do and write down, **in a clear fashion**, any relevant assumptions you are making along the way.

Do not separate the two stapled sheets of the question paper. Hand in the question sheet with your exam booklet at the end of the test.

1. Please answer the following questions showing your reasoning as well as your final answer. To receive full marks for your answer, both your reasoning and your answer must be correct.

   a. The heat capacity of a diatomic ideal gas, such as H_2, decreases as the temperature of the gas decreases. Briefly explain how and why this happens using words and a diagram. Explicitly describe how this relates to the spacing between energy levels in different degrees of freedom. [4 points]

   b. Consider two macrostates of a system of two Einstein solids. The system is 10^6 times more likely to be in the first macrostate than in the second macrostate. Based on this information, what is the difference in entropy between these two macrostates? Express your answer in units of Boltzmann's constant, *k*. [4 points]

   c. Define the terms microstate and macrostate as they have been used in class and explain how they are related. [3 points]

   d. In the "low temperature" limit, the multiplicity of the Einstein solid and two-state paramagnet models are given by:

   $$\Omega = \left(\frac{eN}{q}\right)^{q} \text{ and } \Omega = \left(\frac{Ne}{N_{\downarrow}}\right)^{N_{\downarrow}}, \text{ respectively.}$$

   For each model system, define what is meant by "low temperature" in terms of system parameters (e.g. number of oscillators). Explain why, in the low temperature limit, the multiplicities of the Einstein solid and the two-state paramagnet systems are essentially the same. [4 points]

2. Consider an ideal monatomic gas that lives in a two-dimensional universe, occupying an area *A* instead of volume *V*. There are *N* molecules in this isolated two-dimensional gas with total energy *U*. The multiplicity of this system is

   $$\Omega(N,U,A) = \frac{1}{N!}\frac{A^N}{h^{2N}}\frac{\pi^N}{N!}(2mU)^N$$

   where *h* is Planck's constant and *m* is the molecular mass.

   a. Determine the entropy of this two-dimensional system. Explicitly indicate the terms that depend on area, *A*, and total energy, *U*. [3 points]

   b. Find an expression for the total energy, *U*, as a function of temperature, *T*. [3 points]

   c. What is the heat capacity C_V at constant volume for this system? Comment on how your expression compares with the equipartition theorem. [4 points]

3. As an experiment in statistical physics, you decide undertake a one-dimensional random walk. Each step you take is of equal size and each is chosen randomly to be either in the forward or backward direction. Using concepts we have discussed in PHY293, answer the following questions.

   a. Where are you most likely to find yourself at the end of a long random walk of *N* steps? [1 point]

   b. Find an expression for the net distance you have traveled, *L*, in terms of *l*, the length of your step, and *x*, the excess number of forward steps over *N*/2. [3 points]

   c. Using the relation for multiplicity of a large number of coin tosses ($\Omega \propto e^{-2x^2/N}$, where *x* is the excess number of heads) and your result from part (b), express the dependence of the distribution of ending positions for your random walk in terms of *l*, the length of your step, *N*, the number of steps you have taken, and *L*, the net distance you have traveled. [2 points]

   d. Suppose you take a random walk of 10,000 steps (each step, *l*, is 1 m long). About how far from your starting position could you expect to be at the end? Give an answer in terms of order of magnitude and provide justification for your answer using what you derived in part (c). [4 points]

A list of formulae, perhaps useful and certainly in no particular order

$$k = 1.38 \times 10^{-23}\ \mathrm{J/K} = 8.6 \times 10^{-5}\ \mathrm{eV/K} \qquad N_A = 6.02 \times 10^{23}$$

$$h = 6.626 \times 10^{-34}\ \mathrm{Js} \qquad c = 3 \times 10^{8}\ \mathrm{m/s}$$

$$e = 1.6 \times 10^{19}\ \mathrm{C} \qquad R = 8.31\ \mathrm{J/mol/K}$$

Physics Formulae:

$$PV = NkT = nRT \qquad \ln N! \approx N \ln N - N$$

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V \qquad N! \approx N^N e^{-N}\sqrt{2\pi N}$$

$$T^{-1} = \left(\frac{\partial S}{\partial U}\right)_{N,V} \qquad S = Nk\left[\ln\left(\frac{V}{N v_Q}\right) + \frac{5}{2}\right]$$

$$dU = TdS - PdV + \mu dN \qquad v_Q = \left(\frac{h^2}{2\pi mkT}\right)^{3/2}$$

$$dF = -SdT - PdV + \mu dN \qquad F = U - TS$$

$$\bar{E} = -\frac{1}{Z}\frac{\partial Z}{\partial \beta} \qquad \beta = \frac{1}{kT}$$

$$\lambda_{deBroglie} = h/p \qquad E_{K.E.} = mv^2/2 = p^2/2m$$

$$\bar{n}_{Planck} = \frac{1}{e^{hf/kT} - 1} \qquad u(\epsilon) = \frac{8\pi}{(hc)^3}\frac{\epsilon^3}{e^{\epsilon/kT} - 1}$$

Math formulae:

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2} \qquad \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi} \qquad \int_{-\infty}^{\infty} x^2 e^{-x^2} dx = \sqrt{\pi}/2 \qquad \int_0^{\infty} \frac{x^3}{e^x - 1} dx = \frac{\pi^4}{15}$$

$$\int_0^{\infty} \frac{x}{e^x - 1} dx = \frac{\pi^2}{6} \qquad \int_0^{\infty} \frac{x}{e^x + 1} dx = \frac{\pi^2}{12} \qquad \binom{N}{n} = \frac{N!}{n!(N-n)!}$$

For $x \ll 1$, $e^x \approx 1 + x$, $\ln(1+x) \approx x$, $\sin x \approx x$, $\cos x \approx 1 - x^2/2$, $\sinh x \approx x$, $\cosh x \approx 1 + x^2/2$, $\tanh x \approx x$. For $x < 1$, $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$.