PHY 293F – WAVES AND PARTICLES
DEPARTMENT OF PHYSICS, UNIVERSITY OF TORONTO

PROBLEM SET #5 - SOLUTIONS

Marked Q1 and Q3 each out of 5 marks for a total of 10.

1. During a hailstorm, you have the misfortune to be stuck in your car. The hailstones have an average mass of 1 g and are falling with a vertical speed of 20 m/s. Every second, 150 hailstones are hitting the roof of your car (approximately 1 m x 1.5 m). What is the average pressure exerted on the roof of your car by the hailstones? How does this value compare to the pressure exerted by the atmosphere?

Using same logic as we did in class for the "molecules in the cylinder" pressure calculation, we start with determining the average force on the roof of your car.

$$\overline{F}_{on\ roof} = -\overline{F}_{on\ stone} = -m\frac{\Delta v}{\Delta t}$$

The change in velocity of the hailstone is $v_{final} - v_{initial} = -2v$. *Here we use the same assumption as before, that we have elastic collisions.*

The hailstones hit the roof of your car every interval of Δt *= 1/(150 per second) = 0.0067 s.*
The area of your car's roof is 1.5 m^2.

Therefore, the average pressure exerted is the average force on your car's roof divided by the area of the roof:

$$\overline{P} = \frac{\overline{F}}{A} = -\frac{m\Delta v}{A\Delta t} = -\frac{(0.001\,\mathrm{kg})(-2\cdot 20\,\mathrm{m/s})}{(1.5\,\mathrm{m}^2)(0.0067\,\mathrm{s})} = 4\,\mathrm{N/m}^2 = 4\,\mathrm{Pa}$$

This average pressure is much less than the atmospheric pressure of about 100 kPa by a factor of about 25,000.

2. Calculate the total thermal energy in a litre of helium at room temperature and atmospheric pressure. Repeat the calculation for a litre of air (considering only the two most abundant species). At room temperature, are any degrees of freedom "frozen out" in any of these gas samples (helium or air)?

Using the equipartition theorem, the total thermal energy is:

$$U = f\cdot N\cdot\frac{1}{2}kT$$

Helium has three degrees of freedom (for translations only). So, per mole and by using the ideal gas law:

$$U = \frac{3}{2}N\cdot kT = \frac{3}{2}PV$$

At atmospheric pressure ($P = 10^5$ N/m^2*), a litre of helium (*$V = 10^{-3}$ m^3*),*

$$U = \frac{3}{2}(10^5\,\mathrm{N/m}^2)(10^{-3}\,\mathrm{m}^3) = 150\,\mathrm{J}.$$

The two most abundant components of air (nitrogen and oxygen) are both diatomic molecules, so there are seven degrees of freedom (three translational, two rotational and two vibrational). This assumes that all degrees of freedom are active at room temperature.

In this case, the total thermal energy is:

$$U = \frac{7}{2} N \cdot kT = \frac{7}{2}(100\,\text{J}) = 350\,\text{J} \text{ per mole.}$$

To verify that all degrees of freedom are active (not frozen out) for these gases, you can use values of the heat capacity at constant pressure, C_P, from the Reference Data Table in your textbook.

For an ideal gas, $C_P = C_V + nR = \frac{f}{2} nR + nR$ *per mole (Eq. 1.48).*

For helium, the reference data value for C_P is 20.79 J/K, which is very close to what is calculated with the equation above. Therefore, there are no frozen degrees of freedom for this gas at room temperature.

For oxygen and nitrogen, if all seven degrees of freedom are active at room temperature, then C_P is equal to 37.4 J/K. The reference data values of C_P for O_2 and N_2 gas are approximately 29 J/K. So, the vibrational degrees of freedom are frozen out at room temperature (as is true for a number of diatomic gases) and the total thermal energy of air is:

$$U = \frac{5}{2} N \cdot kT = 250\,\text{J} \text{ per mole.}$$

Note: This calculation involved using reference data. If required for an exam or midterm, this data would be given in a table as part of the exam paper.

3. Problem 2.3 on page 52 of Schroeder. (You can use your favorite computer program for this problem. Computer help can be found here: http://physics.weber.edu/thermal/computer.html.)

   Flipping 50 fair coins:

   a. *For each coin, there are two possible outcomes. Now accounting for all fifty coins, for each of the two possible states of the first coin, there are two possible states for the second coin and two possible states for the third coin and so on. So, the total number of microstates is* $2^{50} = 1.13 \times 10^{15}$.

   b. *The number of ways of getting exactly 25 heads and 25 tails is the multiplicity of the "25 heads" macrostate:*

   $$\Omega(25) = \binom{50}{25} = \frac{50!}{(25!)^2} = 1.26 \times 10^{14}.$$

   c. *The probability of getting exactly 25 heads and 25 tails is equal to the fraction of all states that have 25 heads:*

   $$P(25) = \frac{\Omega(25)}{2^{50}} = 0.112.$$

   d. *The probability of getting exactly 30 heads would be*

   $$P(30) = \frac{\Omega(30)}{2^{50}} = \frac{1}{2^{50}} \frac{50!}{30!20!} = 0.042.$$

   e. *The probability of getting exactly 40 heads would be*

   $$P(40) = \frac{\Omega(40)}{2^{50}} = \frac{1}{2^{50}} \frac{50!}{40!10!} = 0.0000091.$$

f. *The probability of getting 50 heads and no tails (one way to get this) is*

$$P(50) = \frac{1}{2^{50}} = 8.88 \times 10^{-16}.$$

g. *Using a spreadsheet (also handy for calculating the values above), the graph of the probability of getting n heads as a function of n is:*