UNIVERSITY OF TORONTO
Faculty of Applied Science and Engineering

Final Examination December 2010

PHY293F (Oscillations and Modern Physics)
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Duration: 2.5 hours

Exam Type: B (Closed book)
Two formula sheets are provided as part of this exam.

Calculator Type: 2 (non-programmable calculator)

This examination paper consists of **7** pages and 5 questions. Please bring any discrepancy to the attention of an invigilator. Answer all 5 questions.

- The next two pages include some formulae and constants you may find useful. You may use these without proof in any of your solutions.
- The exam consists of five questions that are found on **pages 4 through 7**.
- Each question is worth **20 %** of the total exam grade.
- For questions 2 through 5 part marks will be given for partially correct answers, so show any intermediate calculations that you do and write down, **in a clear fashion**, any relevant assumptions you are making along the way.

Oscillations

	Amplitude	Velocity	Power
Peak Frequency	$\omega = \omega_0\sqrt{1-1/2Q^2}$	$\omega = \omega_0$	$\omega = \omega_0$
Peak Value	$A_m = \frac{a_0 Q}{\sqrt{1-\frac{1}{4Q^2}}}$	$V_m = a_0\omega_0 Q$	$P_m = \frac{1}{2}ma_0^2\omega_0^3 Q$
General	$A(\omega) = \frac{a_0\omega_0^2}{\sqrt{(\omega_0^2-\omega^2)^2+(\gamma\omega)^2}}$	$V(\omega) = \frac{a_0\omega_0^2}{\sqrt{(\omega_0^2-\omega^2)^2/\omega^2+\gamma^2}}$	$< P(\omega) >= P_{\text{m}}\frac{\gamma^2}{(\omega_0^2-\omega^2)^2/\omega^2+\gamma^2}$
	$\tan\delta = \frac{\omega\gamma}{(\omega_0^2-\omega^2)}$		$< P >= P_{\text{m}}\frac{\gamma^2/4}{(\omega_0-\omega)^2+\gamma^2/4} \quad Q \gg 1$

$$[M]\ddot{\vec{x}} + [S]\vec{x} = 0; \qquad \det|-\omega^2[M]+[S]| = 0; \qquad \vec{\xi}_i[M]\vec{\xi}_j = 0 \quad \text{for } i \neq j$$

$$\vec{x}(t) = \Sigma_{n=1}^{N} A_n\vec{\xi}_n\sin(\omega_n t) + B_n\vec{\xi}_n\cos(\omega_n t); \quad \text{with} \quad A_n = \frac{1}{\omega_n}\vec{\xi}_n\cdot\vec{v}_0/||\vec{\xi}_n||^2 \quad \text{and} \quad B_n = \vec{\xi}_n\cdot\vec{x}_0/||\vec{\xi}_n||^2 = \vec{\xi}_n[M]\vec{x}_0/||\vec{\xi}_n||^2$$

$$\det\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33}\end{vmatrix} = a_{11}(a_{22}a_{33}-a_{32}a_{23}) - a_{12}(a_{21}a_{33}-a_{31}a_{23}) + a_{13}(a_{21}a_{32}-a_{31}a_{22})$$

$$ax^2+bx+c=0 \qquad \Rightarrow \qquad x = \frac{1}{2a}(-b\pm\sqrt{b^2-4ac})$$

$$\frac{\partial^2}{\partial t^2}y(x,t) - c^2\frac{\partial^2}{\partial x^2}y(x,t) = 0 \qquad \text{where} \qquad c = \sqrt{T/\mu}$$

$$y(x,t) = \Sigma_{n=1}^{\infty}[C_n\sin(k_n x)\cos(\omega_n t) + D_n\sin(k_n x)\sin(\omega_n t)]$$

$$\text{with} \quad C_n = \frac{\int_0^L \sin(k_n x)s(x)dx}{\int_0^L \sin^2(k_n x)dx} \quad \text{and} \quad D_n = \frac{1}{\omega_n}\frac{\int_0^L \sin(k_n x)v(x)dx}{\int_0^L \sin^2(k_n x)dx}$$

$$\begin{aligned} y(x,t) &= A\sin\left(\frac{2\pi}{\lambda}(x-ct)\right) \\ &= A\sin\Big(k(x-ct)\Big) \qquad \text{since } k = 2\pi/\lambda \\ &= A\sin\Big(kx-\omega t\Big) \qquad \text{since } \omega = ck \\ &= A\sin\left(2\pi(\frac{x}{\lambda}-\nu t)\right) \qquad \text{since } k = 2\pi/\lambda,\ \omega = 2\pi\nu \end{aligned}$$

		Units
$\omega = 2\pi\nu$	ω = angular frequency	1/s
	ν = frequency	Hz
	c = wave speed	m/s
$k = 2\pi/\lambda$	λ = wavelength	m
	k = wave number	1/m

$$\text{Energy Flux} = 1/2\,\mu_i c\,\omega^2 A^2 = 1/2\,Z_i\,\omega^2 A^2$$

$$Z_i = \sqrt{T\mu_i}$$

$$r = \frac{Z_1 - Z_2}{Z_1 + Z_2} \qquad t = \frac{2Z_1}{Z_1+Z_2}$$

$$R = \frac{(Z_1-Z_2)^2}{(Z_1+Z_2)^2} \qquad T = \frac{4Z_1Z_2}{(Z_1+Z_2)^2}$$

Modern Physics

Speed of light c	$= 3.00 \times 10^{8}$ m/s	Mass of electron m_{e}	$= 9.11 \times 10^{-31}$ kg $= 511$ keV/c^2
Elementary charge e	$= 1.602 \times 10^{-19}$ C	Mass of proton m_{p}	$= 1.67 \times 10^{-27}$ kg $= 939$ MeV/c^2
Coulomb constant k_{e}	$= 8.99 \times 10^{9}$ J m/C^2	Planck's constant h	$= 6.626 \times 10^{-34}$ J s $= 4.14 \times 10^{-15}$ eV s
hc	$= 1.240$ keV nm	1 eV	$= 1.602 \times 10^{-19}$ J

$$\mathrm{d}s^2 = c^2\mathrm{d}t^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2 \qquad \gamma = (1-\beta^2)^{-1/2} \qquad \beta = u/c$$

$$x' = \gamma\,(x - \beta\,ct) \qquad ct' = \gamma\,(ct - \beta\,x) \qquad y' = y \qquad z' = z$$

$$\beta_{u'_{\parallel}} = \frac{\beta_{u_{\parallel}} - \beta}{1 - \boldsymbol{\beta}\cdot\boldsymbol{\beta}_u} \qquad \boldsymbol{\beta}_{u'_{\perp}} = \frac{\boldsymbol{\beta}_{u_{\perp}}}{\gamma\left(1 - \boldsymbol{\beta}\cdot\boldsymbol{\beta}_u\right)}$$

$$\mathbf{p} = m\,\mathbf{u} \qquad \mathbf{p} = \gamma\,m\,\mathbf{u}$$

$$T = \frac{1}{2}m\,u^2 \quad E = \gamma\,mc^2 = \sqrt{(pc)^2 + (mc^2)^2} \quad \left(\frac{T}{mc^2}\right)^2 + 2\left(\frac{T}{mc^2}\right) = \left(\frac{p}{mc}\right)^2 = \gamma^2\beta^2$$

In what follows, $\Phi(\mathbf{r},t)$ = general wave-function, $\Psi(\mathbf{r},t)$ = eigenstate.

$$E = h\nu = \hbar\omega \qquad \lambda_{\mathrm{dB}} = \frac{h}{p} \qquad \mathbf{p} = \frac{h}{\lambda} = \hbar\mathbf{k} \qquad \hbar = \frac{h}{2\pi}$$

$$T_{\mathrm{max}} = eV_0 = h\nu - W \qquad \Delta\lambda = \lambda_{\mathrm{f}} - \lambda_{\mathrm{i}} = \frac{h}{mc}\,(1 - \cos\theta)$$

$$\frac{1}{\lambda} = R_{\mathrm{H}}\left(\frac{1}{m^2} - \frac{1}{n^2}\right) \qquad E_n = -\frac{hc}{n^2}R_{\mathrm{H}} = -\frac{1}{2n^2}m_{\mathrm{e}}\frac{(k_{\mathrm{e}}e^2)^2}{\hbar^2} = \frac{E_1}{n^2} = -\frac{13.56\,\mathrm{eV}}{n^2}$$

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r},t) + V(\mathbf{r},t)\,\Psi(\mathbf{r},t) = H\,\Psi(\mathbf{r},t) = \mathrm{i}\hbar\,\partial_t\Psi(\mathbf{r},t) \qquad \nabla^2\psi(\mathbf{r}) = \frac{2m}{\hbar^2}\big(V(\mathbf{r}) - E\big)\,\psi(\mathbf{r})$$

$$\Psi(\mathbf{r},t)_{\mathrm{free}} = \cos(\mathbf{k}\cdot\mathbf{r} - \omega t) + \mathrm{i}\,\sin(\mathbf{k}\cdot\mathbf{r} - \omega t) = A\,\mathrm{e}^{\mathrm{i}(\mathbf{p}\cdot\mathbf{r} - Et)/\hbar}$$

$$\int_{\text{all space}} \left|\Phi(\mathbf{r},t)\right|^2 \mathrm{d}^3r = 1 \qquad \Delta p\,\Delta x \gtrsim \hbar \qquad \Delta E\,\Delta t \gtrsim \hbar$$

$$V(\mathbf{r},t) = V(\mathbf{r}) \implies \Psi(\mathbf{r},t) = \psi(\mathbf{r})\,\mathrm{e}^{-\mathrm{i}Et/\hbar}$$

$$z = R\,\mathrm{e}^{\mathrm{i}\theta} = R\,\cos\theta + \mathrm{i}\,R\,\sin\theta = a + \mathrm{i}b \quad \begin{aligned} a &= R\,\cos\theta \\ b &= R\,\sin\theta \end{aligned} \qquad \begin{aligned} R &= a^2 + b^2 \\ \theta &= \tan^{-1}(b/a) \end{aligned}$$

$$\cos\theta = \frac{\mathrm{e}^{\mathrm{i}\theta} + \mathrm{e}^{-\mathrm{i}\theta}}{2} \qquad \sin\theta = \frac{\mathrm{e}^{\mathrm{i}\theta} - \mathrm{e}^{-\mathrm{i}\theta}}{2\mathrm{i}} \qquad \begin{Bmatrix} \cos^2\theta \\ \sin^2\theta \end{Bmatrix} = \frac{1}{2}\,(1 \pm \cos 2\theta)$$