PHY293 Oscillations Lecture #1

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Last time saw:

1. Introduction/WebSite
2. Why study waves and oscillations? (see slides attached)
3. EKG demonstration (http://faraday.physics.utoronto.ca/IYearLab/EKG.pdf)
4. Tacoma Narrows bridge video (http://www.youtube.com/watch?v=P0Fi1VcbpAI)
5. Saw this time: http://www.upscale.utoronto.ca/PVB/Harrison/Flash/ClassMechanics/SHM/TwoSHM.html

Start of material

1. Simple Harmonic Oscillations
   - Linear resorting force
     - Displace mass, m, spring will pull it **back**
     - Assume $F(x = 0) = 0$, in other words $x = 0$ is equilibrium position
     - Consider:
       $$F(x) = -(kx + k_2x^2 + k_3x^3 + ...)$$
     - Minus sign is there to ensure the force pushes mass back to equilibrium
     - For small enough displacements, eg: $x \ll k/k_2$; $\sqrt{k/k_3}$; etc.
   - We can ignore all but the linear term (this defines "small x")
     $$F(x) = -kx \qquad \mathrm{Hooke's Law, 1660, England}$$
2. Equation of Motion
   - Newton's Law: $F = ma$ (1686!, England)
   - Gives us the following:
     $$F = ma = -kx$$
   - Notation $\dot{x} = dx/dt$; $\ddot{x} = d^2x/dt^2$
   - Gives:
     $$m\ddot{x} = -kx \quad \text{or} \quad m\ddot{x} + kx = 0$$
   - Will guess the solution for this second order differential equation
     $$x = A\cos(\omega t + \phi)$$
   - A (Amplitude) and ϕ (Phase) are determined from initial conditions (must be two for a second order differential equation).
   - ω is the oscillation frequency that is uniquely determined by the equations of motion.
   - If this is solution then: $\dot{x} = -A\omega\sin(\omega t + \phi)$ and $\ddot{x} = -A\omega^2\cos(\omega t + \phi)$
   - Plugging this into the equation of motion find:
     $$m[-A\omega^2\cos(\omega t + \phi)] + kA\cos(\omega t + \phi) = 0$$
   - This is a solution iff $\omega = \sqrt{k/m}$
   - Often we'll write: $\ddot{x} + \omega_0^2x = 0$
   - Where ω_0 is the natural frequency of the system $\sqrt{k/m}$
3. Energy Juggling
   - The mass and spring system consist of:

- A spring: capable of storing elastic **potential** energy
- A mass: capable of storing inertial **kinetic** energy

- This is a general feature of oscillating systems
  - Potential energy: $\frac{1}{2}kx^2$. Get this by integrating the force $\int_0^x k \cdot x ds$

$$\frac{1}{2}kx^2 = \frac{1}{2}m\omega_0^2 A^2 \cos^2(\omega t + \phi)$$

  - Kinetic energy $\frac{1}{2}m\dot{x}^2$

$$\frac{1}{2}m\dot{x}^2 = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi)$$

  - But since $\sin^2\theta + \cos^2\theta = 1$ for any θ

$$\text{Energy} \quad = P.E. + K.E. = \frac{1}{2}m\omega^2 A^2 = \frac{1}{2}kA^2$$

- The total energy of a spring/mass system is a constant.
- Energy is **juggled** between tension in the spring (potential) and kinetic energy in the mass.
- We will find this storage and exchange of energy in all oscillating systems.

4. Phase Space Diagram

- Mechanical systems completely characterised by knowing two variables: position x and velocity (momentum) $\dot{x}$ (or $p = m\dot{x}$)
- They are independent because **both** must be specified as initial conditions to solve $F = m\ddot{x}$.
- For SHO we have $x = A\cos(\omega_0 t + \phi)$ and $\dot{x} = -A\omega_0 \sin(\omega_0 t + \phi)$
- For $\phi = 0$ these are ellipses in $x, \dot{x}$ space, with $t = 0$ starting along the x-axis at $(A, 0)$
- Changing ϕ doesn't change the phase-space ellipse, just the starting point
- Changing A does change the size of the ellipse, but not the aspect ratio
- Changing ω_0 changes the aspect ratio (semi-major to semi-minor axes)
- Careful: The mass is executing a back-and-forth motion (in one dimension: x), but the phase space in $(x; \dot{x})$ is an ellipse.
- The equation of the ellipse is:

$$x^2/A^2 + \dot{x}^2/A^2\omega_0^2 = 1$$

- But the total energy is $\frac{1}{2}kA^2 = \frac{1}{2}m\omega_0^2 A^2$ so can re-write this as:

$$x^2/(2E/k) + \dot{x}^2/(2E/m) = 1$$

  or

$$PE(t)/E + KE(t)/E = 1$$

  or

$$PE(t) + KE(t) = E$$

- The phase space trajectory is an ellipse of constant energy

5. Other Simple Harmonic Oscillation Systems

- An electrical circuit with an inductor and capacitor is another example
- Consider the movement of charge around the circuit
- Inductor has a magnetic field due to the current in the coil: $V_L = LdI/dt = Ld^2q/dt^2 = L\ddot{q}$
- Increasing the current in the coil increases the magnetic field, inducing a voltage in the coil
- The voltage induced **opposes** the increasing current (Lenz, Estonia, 1833)
- Capacitor has an electric field created by the charge built up on the top/bottom plates (equal/opposite)
- The voltage drop is proportional to the charge: $V = q/C$
- Sum of the voltages must be 0 (Kirchoff, Germany 1845): $L\ddot{q} + q/C = 0$
- The solution of $\ddot{q} + \omega^2 q = 0$ is well known to us, but in this case $\omega = 1/\sqrt{LC}$
- Total energy: $\frac{1}{2}LI^2 + \frac{1}{2}q^2/C$ where the first term is magnetic inertia (kinetic) and the second term is electric potential (spring-like)

Mass/Spring	LC circuit
x	q
F	V
m	L
k	$1/C$