PHY293 Problems #2

September 21, 2010

1) Damped Oscillator: A free, damped harmonic oscillator, consisting of a mass $m = 0.25$kg moving in a viscous liquid of damping coefficient b ($F_{\text{viscous}} = b\dot{x}$), and attached to a spring of spring constant $k = 1.5$ N/m, is observed as it performs damped oscillations. Its average energy decays to 10% of its initial value in 8 s.

(a) What is the Q of this oscillator?

(b) What is the value of b?

2) Driven Oscillator: Consider a damped oscillator with $m = 0.2$ kg, $b = 4$ Nm^{-1}s and $k = 80$N/m. Suppose that this oscillator is driven by a force $F = F_0 \cos(\omega t)$ where $F_0 = 2$ N and $\omega = 30\text{s}^{-1}$.

(a) What are the values for $a(\omega)$ and δ in the steady state response solution we've studied in class: $x = a_0 \cos(\omega t - \delta)$?

(b) What is the mean power that must be input by the driver?

(c) What is the Q of this oscillator?

(d) How much energy is dissipated against the damping force in one cycle?

3) Driven Pendulum: A simple pendulum has a length (l) of 1 m. If left to swing freely the amplitude of its swings falls off by a factor of e in 50 swings. The pendulum is set into forced vibration by moving its point of suspension horizontally in SHO with an amplitude of 1 mm:

(a) Show that if the horizontal displacement of the pendulum bob is x, and the horizontal displacement of the support point is ξ, the equation of motion of the bob for small oscillations is:

$$\ddot{x} + \gamma\dot{x} + \frac{g}{l}x = \frac{g}{l}\xi(t)$$

Use what we've learned about this type of equation in class to find $x(t)$ if $\xi(t) = \xi_0 \cos(\omega t)$. Be sure to evaluate constants in your solution (like ω_0 and γ) in terms of the parameters given in the introduction to the question).

(b) At resonance, what is the amplitude of the motion of the bob?

(c) At what angular frequencies is the amplitude half of its resonant value?