PHY293 Practice Problem Set #2

September 21, 2010

1) Forced Oscillation: In class we went through a derivation of the behaviour of a forced oscillator using a driving force $x_{\rm F} = a_0 \cos(\omega t)$. Consider instead a mass driven by $A_0 \sin(\omega t)$:

(a) Re-derive the expressions for $A(\omega)$ and $\delta(\omega)$ for the steady state response $x(t) = A\sin(\omega t - \delta)$ to see how/if they differ from those we derived in class

(b) For $\gamma = 0.1$ s^1, $\omega = 10$s^1, $\omega_0 = 5$s^1, and $a_0 = 2$ cm, write the full solution $x(t)$. Give all numbers to three significant figures. What are your units? [note: there may be undetermined coefficients.]

2) Oscillator Q: For many oscillatory systems, there is an exponential decay in stored average energy, even though the dissipation is not through viscous damping. For instance, if you can hear a tuning fork, then energy must be leaving the tuning fork and traveling to your ear. In that case, the quality factor Q may still be defined in reference to the rate of decay, i.e., if $< E >= E_0 e^{-\gamma t}$, then $Q = \omega_0/\gamma$, where ω_0 is the natural angular frequency.

(a) When the note middle C on a piano is struck, its energy of oscillation decreases to one half its initial value in about 1 s. The frequency of middle C is $\nu = 256$ Hz. What is the Q of the system?

(b) If the note an octave higher ($\nu = 512$ Hz) takes about the same time for its energy to decay, what is its Q?

3) RLC: Consider an RLC circuit drive with a voltage $V_0 \cos \omega t$:

(a) Write the power dissipated versus frequency, in terms of R, L, C, V_0, and ω.

(b) Now assume L = 1 mH, R = 20 Ω, and C = 1 μF; and a drive amplitude $V_0 = 5$ V. How much power is dissipated at $\omega = 3 \times 10^4$ s $^{-1}$?

(c) Sketch the power dissipation in Watts versus frequency in Hertz. Besides the drive frequency, use all the same values as in part (b) (remember $\omega = 2\pi\nu$, if ω is in s^{-1} and ν is in Hz.)