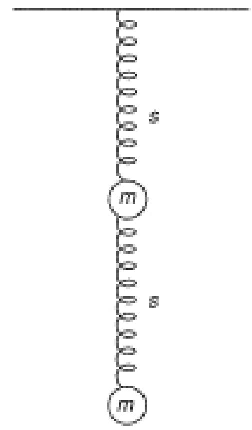
PHY293 Practice Problems #3

October 5, 2010



1) Hanging Oscillators: (simpler than King pbm. 4.7) Two equal masses, m, are connected as shown in the figure (right) with two identical, massless springs, with spring constant s.

(a) Considering only motion in the vertical direction find the angular frequencies of the two normal modes for this system.

(b) Show that the ratio of the two natural frequencies is $\frac{\sqrt{5}+1}{\sqrt{5}-1}$.

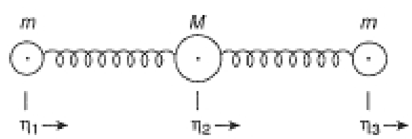
(c) Find the ratio of amplitudes for the two masses in each of the two modes.

Note: You do not need to consider gravitational forces acting on the masses because they are independent of the height of the masses as they execute simple harmonic oscillations. Gravitational forces merely shift the equilibrium position of the masses downwards. You are **not** asked to find these equilibrium positions.

2) Triatomic Molecule: (like King pbm. 4.6) The figure below represents a a triatomic molecule (eg. a simplified version of the water molecule) with a heavy atom of mass M bound to equal atoms of smaller mass m on either side. The binding is represented by springs of stiffness k and in equilibrium the atom centres are equally spaced along a straight line. Simple harmonic oscillations are considered only along the linear axis, that joins the three atoms, and are given by:

$$\eta_j = \eta_j^0 \cos(\omega t)$$

where η_j is the displacement from equilibrium of the jth atom.



(a) Setup the equations of motion for each atom and use the matrix method for coupled oscillators, discussed in class to show that the normal frequencies have $\omega_1^2 = 0$, $\omega_2^2 = k/m$ and $\omega_3^2 = \frac{k(M+2m)}{mM}$.

(b) Find the normal modes and describe the motion of the atoms in each of them.

3) Wave Equation: In class we found solutions to the wave equation that involved separate functions of x and t. Show that $y = f(ct \pm x)$ is also a solution to the wave-equation:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial x^2}$$