Physics
UNIVERSITY OF TORONTO

AMI
Advanced Undergraduate Laboratory

Applied Modern Interferometry

Revisions

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1 Introduction

1.1 Overview

Interferometers are an important instrument of applied physics, useful for measuring changes in length on the wavelength scale or lower. These devices have broad applications in both physics and engineering, notably recently being used for the detection of gravitational waves [1]. In this experiment you will use a simple variation of such a device, known as a Michelson Interferometer, to measure the wavelength of the laser, the thermal expansion of various metals, and the Néel temperature of chromium. Various interferometers (e.g. Sagnac, Mach-Zehnder, electro-optic effects) can be constructed and other measurements (e.g. refraction index, magnetostriction) made with the apparatus [2].

1.2 Principles of Interferometry

Interferometers operate by relying on the interference properties between two coherent beams of light. In principle, an interferometer device splits an input coherent beam into two paths, with electric fields E_1 and E_2, with

$$E_n \propto \ sin(\omega t \ - \beta z \ + \ \phi_n) \tag{1}$$

where ϕ_n is a constant phase shift in a beam path, ω is the angular frequency, t the time, $\beta = \frac{2\pi}{\lambda}$ the wavenumber and z the distance traveled by the beam [3]. These two beams are then recombined by the interferometer, so that the intensity of the final detected intensity is a summation of the two beams, with

$$I \propto \ |sin(\omega t \ - \beta z \ + \ \phi_1) \ + sin(\omega t \ - \beta z \ + \ \phi_2)|^2 \tag{2}$$

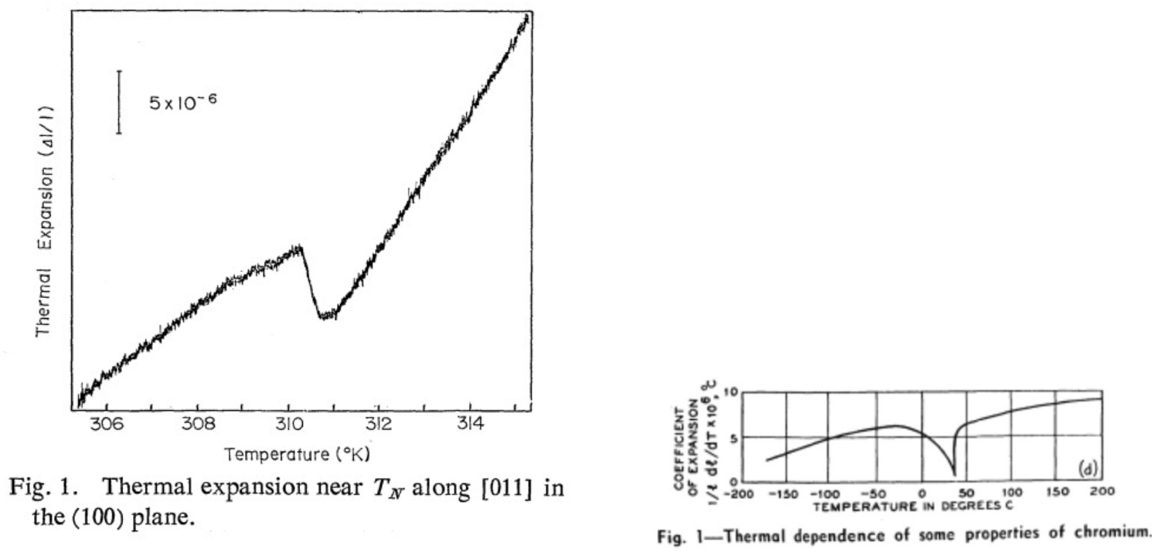
which has the same time average as $I \ \alpha \ |sin(\omega t \ - \beta z \ + \ \Delta\phi) \ + sin(\omega t \ - \beta z \ )|^2$, where $\Delta\phi = \phi_1 - \phi_2$ [3]. When $\Delta\phi = 2\pi n$, for some integer n, the average intensity reaches a maximum, whereas when $\Delta\phi = \ (2n+1)\pi$, a minimum is reached [3]. The phase difference traveled by a wave is:

$$\Delta\phi = n\beta\Delta d \tag{3}$$

where Δd is the difference in length of the two beam paths (which is twice the difference in interferometer arm length) and n is the refractive index of the medium [3]. Thus, when the difference in pathlength between the two beams changes, so does $\Delta\phi$ and the average measured intensity as detected by the photodetector.

1.3 Principles of The Néel Temperature

In 1970, Louis Néel won the Nobel Prize in Physics for discovering antiferromagnetism. He observed a temperature (now known as the "Néel temperature") at which metals undergo a structural phase shift from antiferromagnetic (low temperature) to paramagnetic (high temperature) [4]. In addition to a change in a sample's magnetic properties, this temperature also results in a shift in a sample's thermal expansion coefficient ((ΔL/L)/°C) and specific heat ((J/°C)/kg). The Néel transition is also accompanied by a latent heat (ΔQ) and a



(a) (b)

Figure 1: (a) The factor ΔL/L for a chromium single crystal as it is heated [6]. (b) The coefficient of thermal expansion ΔL/L/ΔT (units 1/Kelvin) for a chromium polycrystal as it is heated [7].

volume change (ΔV) during the transition. For more information on the physics of the Néel temperature and the antiferromagnetic and paramagnetic phases, read Introduction to Solid State Physics Chapter 15: Antiferromagnetic Order [5].

The Néel temperature for chromium ($T_{Néel}$ = *310.5 K = 37.35 ^{o}C* [4] can easily be reached using a simple heated coil. Figure 1a from Matsumoto and Mitsui illustrates the change in thermal expansion as a single crystal chromium sample undergoes uniform heating [6]. Similarly, a change in the thermal expansion rate would be expected in a polycrystal sample [7] as shown in Figure 1b.

The slope of this plot is proportional to the coefficient of thermal expansion. At the Néel temperature for a single crystal, the coefficient of thermal expansion changes sign, meaning the chromium contracts briefly. Above the Néel temperature, the coefficient of thermal expansion increases.

Safety Reminder

- This experiment has a class 3R/IIIa laser with under 5mW power that may be operated without additional training or personal protective equipment [8].
- Never look directly into the beam and ensure that any beams are constrained to the optical table (block any stray beams) to protect all lab users.
- Before starting the experiment, read the TeachSpin Modern Interferometry Manual's Appendices B (Laser Safety) and C (Laser Parameters).
- Piezoelectric warnings from Section 14 of TeachSpin Modern Interferometry manual.
  - You may receive a nasty shock if you compress the piezoelectric translator.
  - Be sure to keep the applies piezo voltage within the asymmetric safe range of -10 to +110 V.
- Only use electrical components that have Electrical Safety Authority approval
- When extensions cord are required, be sure they are not "daisy chained" or overcrowded, and that surge protectors are in use.
- The chromium and niobium samples are heated with a heating coil and wrapped in heat shrink. Keep the current supplied to the coil below 0.5A to reduce fire risk; the current supply's orange overload light should never be on.
- Unplug the heating coil after each session.
- Be sure to pay close attention to all Cautions in the AFM operating instructions.

Let the APL Coordinator know if any of the links above are broken.

NOTE: *This is not a complete list of all hazards; we cannot warn against every possible dangerous stupidity, e.g. opening plugged-in electrical equipment, juggling cryostats, etc. Experimenters must constantly use common sense to assess and avoid risks, e.g. if you spill liquid on the floor it will become slippery, sharp edges may cut you, etc. If you are unsure whether something is safe, ask the supervising professor, the lab technologist, or the lab coordinator. If an accident or incident happens, you must let us know. More safety information is available at* `http://www.ehs.utoronto.ca/resources.htm`.

2 Experiment

2.1 Setting up the Interferometer

2.1.1 The Michelson Interferometer

The particular interferometer design you will be using for these experiments is known as the Michelson Interferometer. The design uses a single beam splitter to separate a coherent light source into two beams, each reflected at a separate end mirror. These beams are then recombined into a single beam at the same beamsplitter, before this final combined beam is directed to the photodetector. The photodetector voltage then outputs an analog voltage, which is digitized by a keysight multimeter and fed into the lab computer, in the “NEEL” VI. A diagram of the interferometer setup is shown in Figure 2.

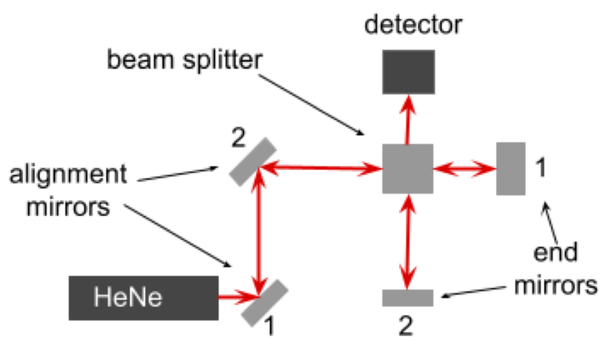


Figure 2: Michelson Interferometer Setup.

Changes in the beam-path are introduced by moving one of the two mirrors. In this case, end mirror 1 should be mounted on a flexure stage (Figure 3) . This stage accommodates a single pushrod, that upon an applied force will push the mounted mirror forwards, thus adjusting the beam length. It should be noted that pushing the mirror forward by distance Δd will change the optical path by length $2\Delta d$, as both the path to and from the mirror will change by distance Δd.

Construct the interferometer according to the diagram above. Ensure that there are two alignment mirrors located in the beam path prior to the beam splitter. These appear to be small circular mirrors mounted on a black mirror mount. Next, ensure that two rectangular vibration-resistant mirrors are used as the end mirrors. Note that these only have one adjustment knob each, and so are only able to adjust either the vertical axis, or the horizontal axis. You may also need to adjust the gain settings on the photodetector.

2.2 Alignment

Once the interferometer is constructed as shown in the diagram above, alignment can begin. Alignment of an interferometer is a fairly tricky task! However, a few things have been included here to help you with the task.

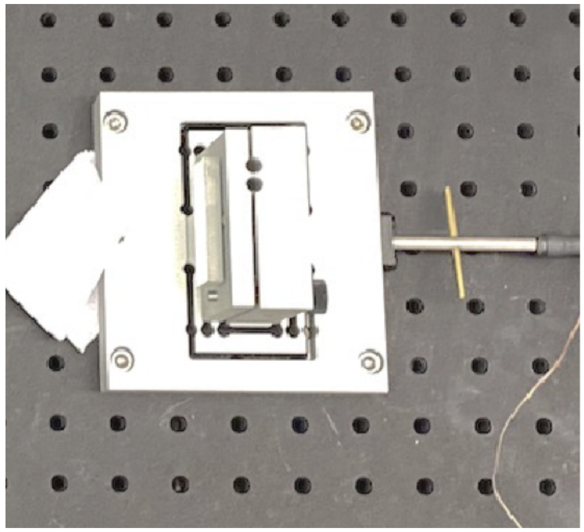


Figure 3: Flexure Stage

Pictured in Figure 4 are white aluminum guide blocks included with your lab setup. These blocks include small holes that can be used to guide the laser beam. While the taller ones can be placed directly on the optical board surface, the smaller ones are designed to be mounted on the bases of the beam splitter and the end mirrors.

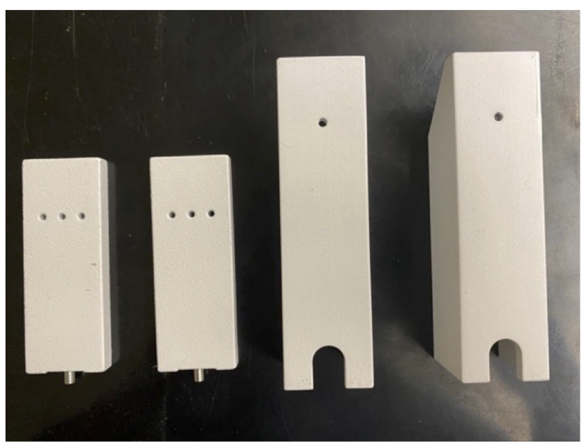


Figure 4: Aluminum Guide blocks

2.2.1 Creating a level beam

The first task that you should accomplish is creating a level beam. This can be done by placing any two of the white aluminum guide blocks along the path of the laser. Using the knobs located on the back of the alignment mirrors, adjust the beam path until it is level throughout the system. It should thus pass through the small holes located at the both alignment guides you've inserted. You can think of the first alignment mirror as adjusting the "position" of the beam, while the second alignment mirror adjusts the "angle" of the beam. Generally, it is useful to adjust the bottom knobs of both mirrors simultaneously, and then repeat this process for the top. Note that one of the two end mirrors also allows you to adjust the beam height! If further adjustment is needed on the horizontal end mirror, it is possible to slightly adjust its height by means such as inserting a small piece of paper beneath its base.

2.2.2 Creating a Right Angled Beam

The horizontal alignment task is very similar to the process of creating a level beam. First, use the alignment mirrors to ensure that the beam passes squarely through the beam splitter. This can be done by mounting the small aluminum guides into the base of the beam splitter, and ensuring that the beam passes through the middle hole at all 4 sites. Next, for further alignment, move one of the guide blocks to the base of one of the end mirrors. Again, the beam should pass cleanly through the middle hole of both the guide block mounted on the beam splitter and the guide block mounted on the end mirror. Repeat until the beam passes cleanly through guide blocks located at all sites.

2.2.3 Creating the Final Beam

Finally, adjustment the knobs of the end mirrors so that the final combined beam forms a single dot on any vertical test surface. If the beams are correctly aligned, this final image should remain as one dot as this test surface is moved towards and away from the beamsplitter. If they are not correctly aligned, repeat the process starting from Section 2.2.1. Beam Alignment is tricky business! Iteration and repetition is an expected part of the process. For further hints, refer to the Modern Interferometry Instructor Handbook [2].

Vibration Isolation: Note that in the current setup, the optical table is mounted on a series of metal bars and rubber feet, to environmentally isolate the table. Why might this be important?

2.3 Measurements to be Made:

2.3.1 Measuring the Laser Wavelength

Now that the optical system is aligned, you're ready to measure the wavelength! Modify the interferometer to include a micrometer and motor, as shown in Figure 5 above. First insert the pushrod and micrometer into the flexure stage. Note that for this exercise, the flexure stage must start "in tension," and thus a forward force must be applied from the pushrod and micrometer before the micrometer is fixed to the table. Next, add the DC motor to the drive train. The completed drive train is shown in Figure 6. The photodetector voltage is fed into the lab computer, and data is collected using a VI "NEEL" located on the Desktop. The motor will slowly unwind the micrometer at a rate of 1 RPM[1] [2], thus releasing the flexure stage. How much does one rotation move the mirror by? What is the corresponding change

[1] Revolutions Per Minute

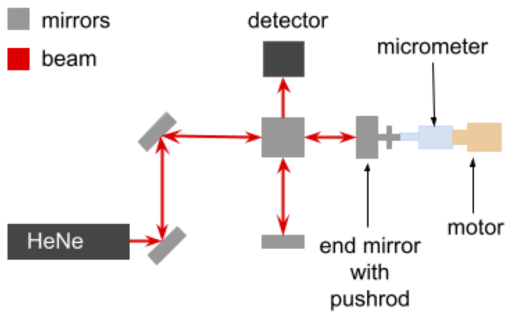


Figure 5: Setup for measuring laser wavelength

in optical path length? Using this, and measurements from the photodetector, calculate the wavelength of the laser.

Hint: The change in phase of the laser $\Delta\Phi = 2nk\Delta d$, where Δd is the change in pathlength of one arm of the interferometer, n is the index of refraction of the propagation medium, and $k = 2\pi/\lambda$ is the wavenumber. Note that the phase difference between 2 maxima should be equal to a phase shift of $\Delta\Phi = 2\pi$.

The measured signal should form a reliable sinusoidal pattern. If no pattern is seen, double check the alignment.

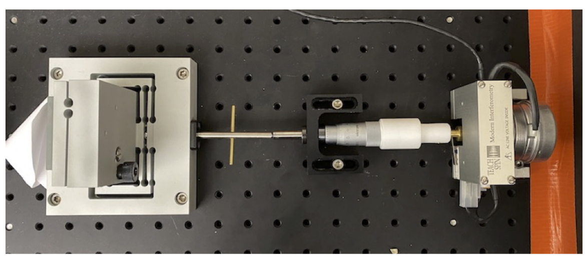
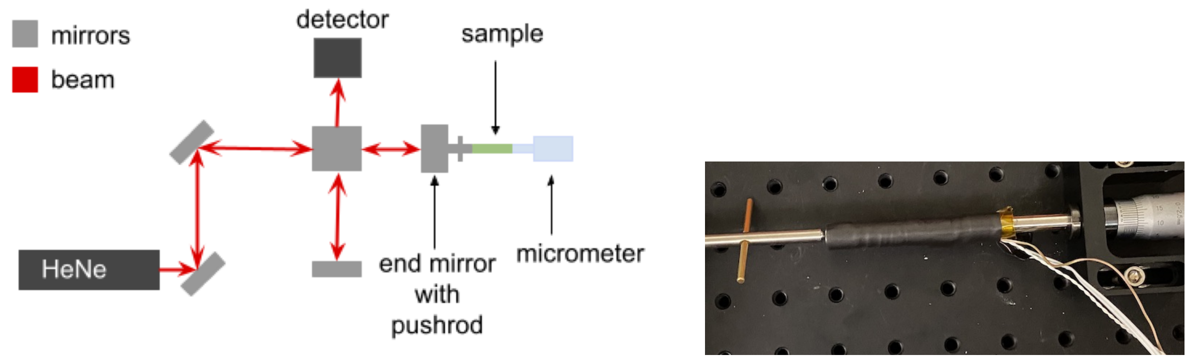


Figure 6: Drive train: (left to right) end mirror with pushrod, micrometer, DC motor.

2.3.2 Thermal Expansion of Niobium

Once the wavelength is established, we can now use this data to measure the thermal expansion of niobium. Remove the DC motor from the drive train, and between the micrometer and the flexure stage push rod, insert the 80 mm long niobium sample as seen in Figure 7a and Figure 7b above. Thermal measurements are done using a 4 point measurement technique. Connect the niobium sample heating wires to the DC power source, and the thermistor to the digital multimeter as illustrated in Figure 8. You may need to change the sampling rate in order to see a well-defined signal.



(a) (b)

Figure 7: (a) Setup for determining thermal expansion of sample. (b) A photo of the sample inserted into the setup.

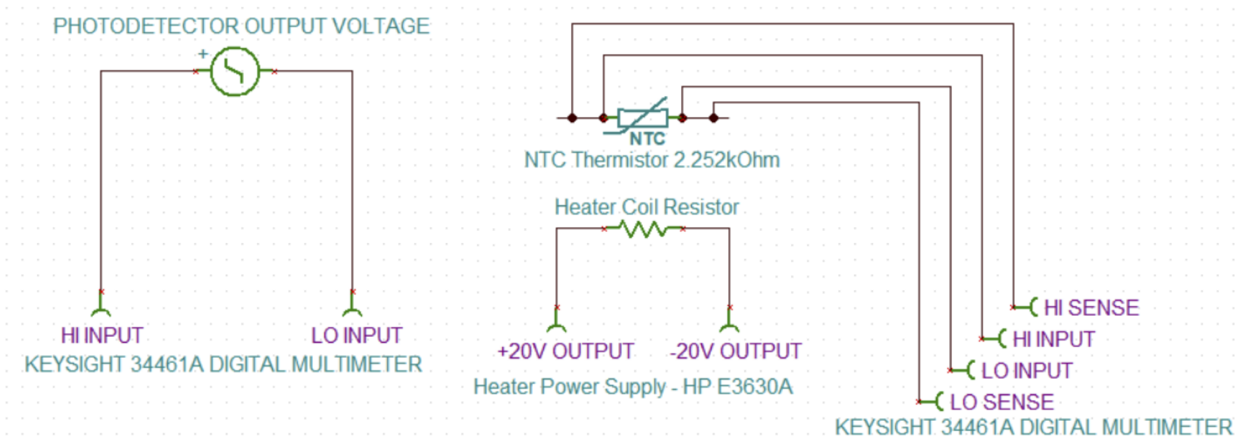


Figure 8: Circuit diagram for connecting thermistor wires to digital multimeter.

To calculate the thermal coefficient, C,

1. From the plot of temperature vs time, determine the rate of heating (Δ°C/s)

2. From the plot of detector signal vs time, determine the rate of expansion of the sample ((ΔL/L)/s)

   Hint: The change in phase of the laser $\Delta\phi = 2nk\Delta d$*, where* Δd *is the change in pathlength of one arm of the laser,* n *is the index of refraction of the propagation medium, and* $k = \frac{2\pi}{\lambda}$ *is the wavenumber.*

3. From these values, estimate the coefficient of thermal expansion (ΔL/L)/°C. Note that this assumes the sample is heated at a constant rate.

4. For a more accurate calculation of the coefficient of thermal expansion, plot (ΔL/L) against Δ°C and perform a linear fit (Section 2.3.3).

2.3.3 Measuring the Neel Temperature of Chromium

We can now repeat the previous section with an 80 mm long chromium sample in place of the niobium sample. For chromium, we expect to see a change in the coefficient of thermal expansion before and after the Néel temperature as described in Section 1.3.

Recreate the interferometer setup for measuring the thermal coefficient of niobium, as shown in Figure 7a. Remove the niobium sample, and instead prepare and wire the chromium setup in its place. The chromium sample should be wired identically to the niobium sample as shown in Figure 8. Using the DC voltage source, heat the chromium sample while measuring the amplitude of the detected photodetector signal. Ensure that both Temperature and photo-intensity are directly measured.

While the coefficient for thermal expansion of the niobium sample was expected to remain relatively constant, the thermal expansion of chromium is expected to vary with temperature, Thus, rather than following the steps in Section 2.3.2 to calculate the coefficient of thermal expansion, students are encouraged to review, use, and build on the pre-written Neel library. The program takes a windowed fourier transform over the dataset, allowing us to establish the dominant frequency of oscillation (in peaks/Kelvin) in a window around each temperature value. These frequencies are directly proportional to the thermal expansion coefficients.

To use this library, follow these steps:

1. Right click on your plot to export data to clipboard. Save to a text file in the NEEL folder
2. Open PyCharm and set environment (in the bottom right) to “NEEL Experiment Environment”
3. In the main function of the program, change the fileName variable to the file you saved in Step 1; this line can be found by CTRL-F “User-set parameters” in the main file
4. Run the main.py program

The program will write a PNG figure with the same filename as the input text file. For more information on the construction of the data analysis code, how to interpret the outputs of this program, and which parameters can be changed in it, it is strongly recommended to view details in Appendix A: . In what follows, a brief summary is included. The data analysis scripts operate by inferring the local oscillation frequency from the raw photodetector data. This is done via two distinct steps, which are plotted separately:

1. First, interpolation is required to estimate the non-uniformly spaced input (temperature, voltage) data points on a regular grid of temperature values; this is required in order to evaluate the FFT. The top plot is a plot of the photodetector voltage against the temperature. This plot should be sinusoidal. Both the data and the interpolated fit are displayed.

2. Secondly, the bottom plot shows the coefficient of thermal expansion for each temperature value. The coefficient of thermal expansion is directly proportional to the dominant frequency of oscillation in the data, which was computed over a window around each temperature value, using a sliding-windowed FFT approach.

The program will write a PNG figure to the same input filename, displaying the thermal expansion as a function of temperature; an example of this plot is included in Figure 9.

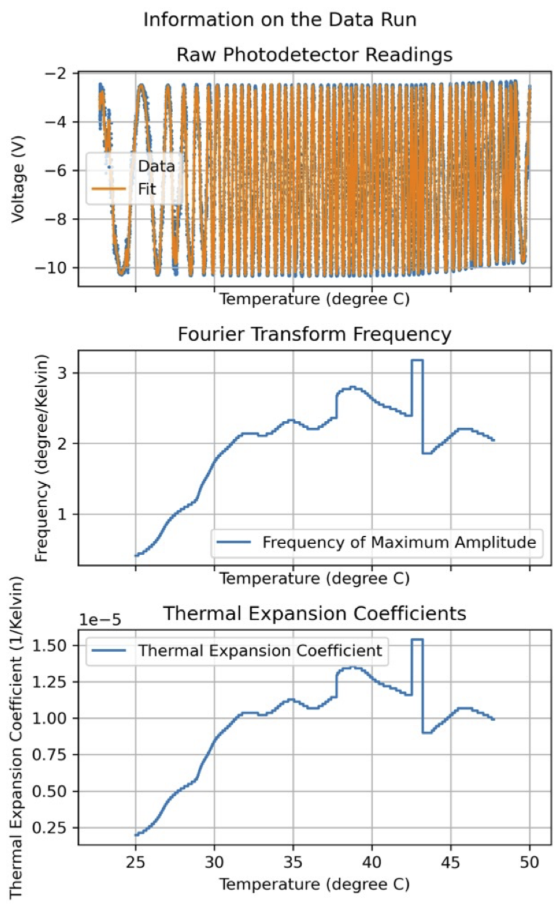


Figure 9: Example plots of photodetector readings and thermal expansion coefficient as a function of temperature.

2.3.4 Calculating the Neel Temperature point.

As discussed in Section 1.3, the Neel temperature point is associated with two phenomena: First, at the Neel Temperature, there is a sudden and instantaneous shift in volume, as the chromium atoms shift from paramagnetic to antiferromagnetic. The second phenomena is a sudden change in thermal expansion coefficient. We can measure the change in thermal expansion coefficient by noting that from equation (3),

$$\frac{d\phi}{dT} = 2nk\frac{d(d)}{dT} \tag{4}$$

Note that the thermal expansion coefficient can then be found as $\frac{d(d)}{dT}\frac{1}{L}$, where L is the length of the sample.

For a more thorough investigation, we can note that the time average for the intensity of the interferometer can be expressed as

$$I_{Time\ Averaged} \propto C_1 + C_2 sin(C_3 + \omega_{temperature}T) \tag{5}$$

where $\omega_{temperature} = \frac{d\phi}{dT}$.

> *Deriving Equation 5:*
> The change in phase of the laser $\Delta\Phi = 2nk\Delta d$, where Δd is the change in pathlength of one arm of the interferometer, n is the index of refraction of the propagation medium, and $k = 2\pi/\lambda$ is the wavenumber. Note that the phase difference between 2 maxima should be equal to a phase shift of $\Delta\Phi = 2\pi$.

2.4 Further Investigations in Error Analysis

- What are the sources of error in the data? List out the error sources, and explain whether each error source is systematic or statistical (Hint: there is at least one of each)
- Consider how uncertainty propagates through the procedure of finding the thermal expansion from the data. How might uncertainty propagate through a Fourier Transform as in the code provided?
  - The software provided provides very little error analysis. What are some estimates for the error on the obtained thermal coefficients, and the error on the Neel Temperature?
- The program provided finds the frequency (in interferometry peaks/Kelvin) using a moving Fourier Transform. However, Fourier transforms' accuracies are limited by the temperature range that we can measure over. Try measuring the frequencies over the range of temperatures using other methods, such as fitting sine waves to the data or finding the width of each peak (HINT: Sine wave fitting works very poorly if the frequency is constantly drifting).

Appendix A: Data Analysis Program

The data analysis scripts operate by inferring the local oscillation frequency from the raw photodetector data. Since the independent variable of interest is temperature, rather than time, data points are not evenly spaced and this process must be done via two steps:

1. Interpolating between the input (temperature, voltage) data points onto a regular grid of temperature values
2. Computing the dominant frequency of oscillation in a given window around each temperature value

The first step is performed via Radial Basis Function interpolation, as it offers a simple and stable routine to construct interpolants, with minimal extra parameters (aside from a user-selected smoothing value). The outcome of the interpolation is plotted upon termination, and is described in more detail below.

The second step is performed by iterating over the data in a sliding-window approach. In a neighbourhood around each temperature value, a user-selected number of points is extracted. The voltage values around this neighbourhood are standardized to zero mean, windowed via a Hann window, and zeropadded by a user-selected amount. The amplitude spectrum is then computed, and the frequency of the maximum amplitude is taken as the frequency value of oscillation. Given the interferometer setup, these frequencies are directly proportional to the thermal expansion coefficient.

The program will return two plots:

- The top plot is a plot of the photodetector voltage against the temperature, and should be sinusoidal, as well as a smoother plot which interpolates the original dataset onto a regularly-spaced temperature grid (this smoother plot is used for calculating the Fourier transform). The interpolation ("Fit") must always be inspected prior to interpreting the thermal expansion coefficient results, as poor fits will degrade the thermal coefficient computation.
- The bottom plot shows the coefficient of thermal expansion for each period of time. This plot was obtained by taking a moving fourier transform over the entire dataset, allowing us to establish the frequency of oscillation in a window around each temperature value.

Parameters that can be changed in the code:

- fileName: String containing the path to the output data file
- numPoints (default: 4000): The number of points in the sliding window used to determine the oscillation frequency. More points gives greater frequency accuracy at the expense of less localized information in real space.
- pad_frac (default: 0.5): The fraction of numPoints that will be zeropadded to either end of the sliding window fourier transform. Larger padding values improves resolution in frequency space.
- smoothing (default: 0.01): The smoothing value used during the interpolation routine. Noisier data (caused by poor alignment, vibrations, etc.) may benefit from larger values of smoothing. The fit should always be inspected in the topmost plot prior to interpreting the frequency or thermal expansion coefficient data.

- wavelength (default: 6.33e-7): The wavelength of the laser, in meters
- sampleLength (default: 8.0e-2): Length of the sample, in meters

References

[1] LIGO. "What is an Interferometer?" (2022). URL https://www.ligo.caltech.edu/page/what-is-interferometer.

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