Advanced Undergraduate Laboratory
Experiment 10, FTS

Fourier Transform Spectroscopy

December 25, 2013

1 Introduction: The Wiener-Khinchin Theorem

Consider two beams of light $\vec{E}_1$ and $\vec{E}_2$ produced by a source at S, which travel along two different paths before reaching a detector situated at O (see Fig **??**). Upon reaching O, the beams will intefere with each other, and superposition tell us that the time-averaged intensity[1] is:

$$\begin{aligned} I &= \langle (\vec{E}_1 + \vec{E}_2) \cdot (\vec{E}_1 + \vec{E}_2)^* \rangle \\ &= \langle |E_1|^2 \rangle + \langle |E_2|^2 \rangle + 2\mathrm{Re}\langle \vec{E}_1^* \cdot \vec{E}_2 \rangle \end{aligned}$$

If the times required to traverse each path differ by τ, then the fields E_1 and E_2 should have the same form, though one will be time-displaced with respect to the other. Accordingly, it is possible to write this last term in the above as $2\mathrm{Re}(\Gamma_{11}(\tau))$, where

$$\Gamma_{11}(\tau) = \langle \vec{E}_1(t)^* \cdot \vec{E}_1(t+\tau) \rangle \tag{1}$$

is the autocorrelation function, or self-coherence function. Given (1), it is possible to determine the temporal characteristics of the light source by making measurements in the time domain. Of course, if we were to make measurements in the frequency domain, the function containing the same information would be given by the Fourier transform of $\Gamma_{11}(\tau)$:

$$\begin{aligned} \int_{-\infty}^{\infty} \Gamma_{11}(\tau) \mathrm{e}^{i\omega\tau} d\tau &= \lim_{T\to\infty} \frac{1}{T} \int_{-\infty}^{\infty} \int_{-\frac{T}{2}}^{\frac{T}{2}} (\vec{E}(t)^* \cdot \vec{E}(t+\tau)) \mathrm{e}^{i\omega\tau} d\tau dt \\ &= \lim_{T\to\infty} \frac{1}{T} \int_{-\infty}^{\infty} \int_{-\frac{T}{2}}^{\frac{T}{2}} (\vec{E}(t)^* \mathrm{e}^{-i\omega t}) \cdot (\vec{E}(t+\tau) \mathrm{e}^{i\omega(t+\tau)}) d\tau dt \\ &= \lim_{T\to\infty} \frac{1}{T} \tilde{E}(\omega)^* \tilde{E}(\omega) \\ &= P(\omega) d\omega \end{aligned}$$

In this way, the Fourier transform of the autocorrelation function yields the power spectrum, namely, the energy contributed by a particular frequency band of the source. This is the *Wienner-Khinchin* (WK) *theorem*. In this experiment, you will be performing time-domain measurements of the coherence function using a Michelson inteforometer, and using the WK theorem to extract the frequency characteristics of the source.

[1] While such fields are typically treated as being constant in amplitude and completely coherent, for all known sources the amplitudes, frequencies, and phases of light oscillate in a random fashion. Accordingly, it makes more sense to work in terms of time averages.

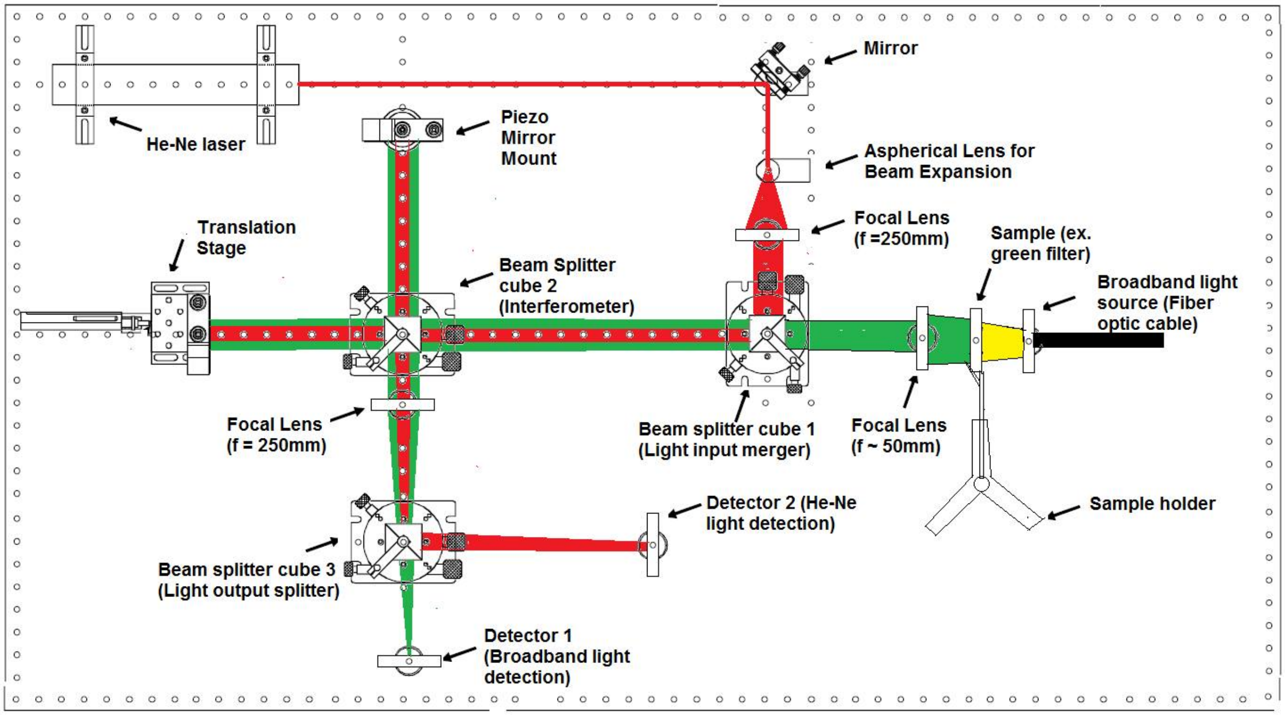


Figure 1: A schematic of the experimental apparatus.

2 Apparatus

2.1 Description of setup

The introduction was motivated by considering the intensity of two beams of light that were made to interfere after traversing different paths, and it was shown that this can reveal a good deal of information about the source employed. An instrument which performs this division and recombination of the source light is called an inteferometer, and the interference pattern is recorded in an inteferogram. In this experiment you will be using a Michelson Interformeter (see fig 1) (the most common configuration), which uses a beam splitter to create the optical paths. In the first, the light propagates across a fixed length L_{fix}, and bounced by mirrors into a detector[2]. For the other beam, the length of the path (L_{mob}) is controlled by a mirror mounted on a movable stage, which is translated on the order of microns by a stepper motor. Accordingly, there is a time difference between the two paths:

$$\tau = \frac{2(L_{mob} - L_{fix})}{c} \tag{2}$$

[2]For the light sources, mirrors, and scales employed, the process of reflection does not skew the amplitude or frequency characteristics of the source, to a good approximation.

So that, again, by superposition, and by the fact that the two beams are produced by the same source, the Intensity measured at the detector is:

$$I(\tau) = 2\langle I\rangle + \mathrm{Re}\langle\Gamma_{11}(\tau)\rangle \tag{3}$$

As the path is adjusted, the intensity changes, though the time-averaged intensities of the light beams $\langle I\rangle$ are not altered. In such a way, by determining the intensity at $\tau = 0$, this value can be subtracted from the full $I(\tau)$ to isolate the contribution of $\Gamma_{12}(\tau)$. For different values of τ, this information is collected in an inteferogram, which, via the Wiener-Khinchin theorem, may then be used to compute the power spectrum of the incident light sample.

Component	Description
HeNe Laser 	• Source of red lased light, with a well defined peak at 630 nm • Used to callibrate data and correct noise from non-constant stepper velocity
Piezo Controller 	• Allows extremely fine control over placement of movable mirror

Table 1: Component list

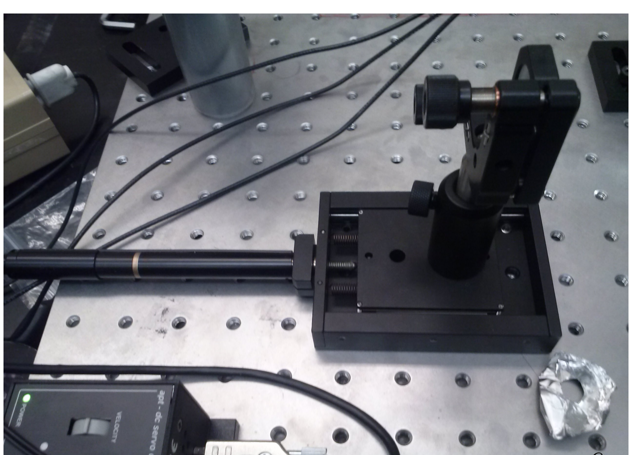
Component	Description
OSL1 White Light Source 	• Superposition of all wavelengths in visible spectrum • Connected to setup with fibre optic cable • Focused through samples to perform optical spectroscopy
Silicon Detector 	• Silicon detector used to measure intensity of incident light • This experiment contains two: one to acquire inteferograms (with white light), and the other to correct for deviations from non-constant stepper motor.
Translation Stage + Actuator 	• Responsible for holding and controlling motion of movable mirror • Motion can be controlled with provided software, or manually with onboard controller • Stepper motor exhibits non-constant velocity, which complicates time-domain measurements. Data Analysis software employs algorithms to correct this deviation based on HeNe calibration data (see section 4).

Table 2: Component list, cont.

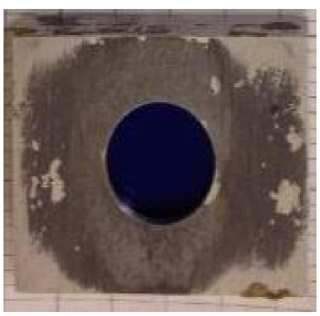
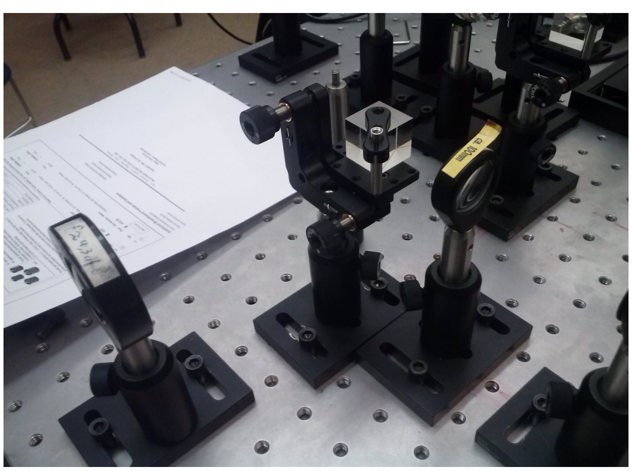
Component	Description
Actuator Control Box 	• Used to manually adjust position of movable mirror
Sample Holder	• Tripod with attached clamp used to hold sample filters
Interference filters 	• Placed in sample holder to filter white light source in experimental investigations. • Green filter used to find zero point, as the light filtered with it is brightest among the filters.
Relay lenses, beam spliiters, and mirrors 	• Steer beams to determine optical path • It is essential that the laser/-light in this experiment is aligned properly, otherwise fringes cannot be obtained. Given this, we highly advise against the position of these pieces on the table. If adjustment is needed, please contact the lab technicians.

Table 3: Component list, cont.

3 Experimental Procedure

3.1 The HeNe Laser

1. Use the alignment procedures (section 4.1) to align the setup for the HeNe light. Make sure you align at when the arm distance difference is zero.

2. Use the Data Acquisition procedures (section 4.2) to set up data acquisition for one of the detectors. For the HeNe part of the experiment, you only need one detector.

3. Make sure the output of the interferometer hits one of the detector through the pin hole in the aluminum foil.

4. Before recording the actual data, set up the motor control using the Motor Control Procedures (see secton 4.3).

5. Set the actuator to move about 1mm or more (However, if you move the actuator too far, your alignment will deviate since it is difficult to align the setup for all positions). Do not move the detector just yet by pressing Enter.

6. Record the data and while it is recording, quickly go back to the actuator control and move the actuator by pressing Enter.

7. After the stop recording the data when the actuator is stopped.

8. The recorded data can be found in the Logs section in the Signal Express software. Right click the individual data streams (usually named filtered data, filtered data1) to open their file location in Windows. The files can be opened only with Excel, so remember to save them as .xlsx or any compatible format if planning on analyzing the data later. Isolate the points corresponding to the interference pattern and create two files with the data points from both detectors. Note: It is important that there is a one-to-one correspondence between the HeNe data and the data for the light under examination in order for the phase correction software to work properly.

9. Use the matlab program to take the FFT of the data to assess the quality of the data.

3.2 White Light

1. Finding the zero point

    - To observe interference fringes for broadband light such as the white light, you need to set the two arm length distance to be exactly equal within few micron of tolerance. (Why ?)
    - To find the position of the actuator where the broadband interference fringes can be observed, first, carefully measure the distance of each arm length using an electronic caliber. (Warning: Make sure you do not touch any surface of the mirrors or the beam splitter cubes with your fingers. Also make sure the calibers do not scrape any surfaces)

- After recoding down the positions, move the actuator so that the arm distances are roughly the same.
- Using the HeNe light only, adjust the setup at this position using the Alignment procedures. The alignment for the HeNe light should also align the setup for the broadband light source.
- Turn off the HeNe light and turn on the Broadband light source. To see the interference fringes for this light source, use the J-Jog feature of the actuator controller, or the controller software and move the actuator around the expected zero difference position. The actuator provides upto 50nm step size for each J-Jog button press, depending on your specified J-Jog distance.
- As you do this, you should be able to see interference fringes for the broadband light source if you put a piece of white paper like you did in the alignment procedure section.
- Now, use the Alignment Procedures again to align for the broadband light this time. Use the procedure with the broadband light only.
- Make sure the alignment is good across all positions where the interference can be observed. (Note, the position where the interference fringes are the brightest is where the difference in the two arm of the interferometer is exactly zero. However this is not always true, why ?).

2. Detector setup

    - Now you have finished alignment of the setup, its time to set up the detectors.
    - Turn on the HeNe laser light and the Broadband light.
    - The output of the interferometer should split into two beams of light at the last beamsplitter cube after the focal lens.
    - For the HeNe light detection, put the detector on the right side of the beamsplitter cube such that the ONLY HeNe light dot hits the detector at the pinhole.
    - For the Broadband light, put the detector on the bottom of the beamsplitter cube such that Only the broadband light dot hits the detector at the pinhole.
    - This ensures that each detector reads light from individual light source independently.
    - (Note: If the interference signal is weak when you take data, try narrowing the light dots by moving the detectors backwards and forwards. This should focus more light into the pinhole for improved intensity)

3. Running and measuring data

    - About 0.5mm away from your measured position where the arm length difference is zero.

- Go to the actuator controller software, set the actuator to move about 1mm. This should scan all range of the interferogram. However, do not move the actuator just yet.
- Use the procedures for the data acquisition to acquire data for both detectors (Note, record the data that is filtered using the recommended digital filters)
- Press record and afterwards quickly move the actuator to your specified position.
- Once the actuator stops moving, stop recording the data.

4. Pre-analysis

   - In the acquired data, you should crop the regions where there is actual interference for both detector signals (Why ? Hint: Alignment). (When you crop the data, make sure you crop both HeNe and broadband light data at the same places in time, and make sure that they have the same number of data points)
   - Analyze the interferograms using the Matlab code and obtain the spectrum of the light under examination.

4 Using the equipment

4.1 Alignment

A NOTE TO THE STUDENT: *The equipment used here is extremely sensitive to the alignment (in that it has been built to sense sub-micron changes in position). Please do not remove anything that is bolted to the bench. If you suspect that there is an issue with alignment that is beyond small tweaks,* ***do not fix it yourself****. If disturbed, the process of realignment is both arduous and highly time-consuming. Please contact the lab technicians instead.*

- Begin by switching on the HeNe laser. Ensure that all other light sources are switched off (it is advised you do this in a dark room).
- Place a white screen (a white piece of paper serves well) between the beam splitter and the focusing lens.
- If there is no apparent interference pattern, check the position of each output light through the interferometer by blocking each light path.
- By doing this, try to adjust the interferometer mirrors using the adjustment knobs at the back such that the position for each light output is the same.
- From here, adjust mirrors carefully using the adjustment knobs until you observe interference fringes

- Adjust the mirror screws and move until getting the widest possible circular interference patterns. (Tip. Move the screws in the direction that it gives you wider spacing between two consecutive fringes. As you do this, the fringes become more and more curved. Since the central fringe is bigger than the beam splitter cube in use, if aligned perfectly, you should be able to see rectangular uniform intensity background. Do the practice alignment exercise below to get a sense of how to do alignment for this experiment)

  1. Since seeing the circular fringe is difficult when the distances of the two arms are equal, move the actuator so that the difference in this distance is about 1cm. (The field of view of the interference pattern should increase as the distance difference between the arms increase. Why ? This means that you should be able to see more fringes per area as the distance difference increase)
  2. Now use the alignment tip mentioned above to make sure that the interference pattern is circular. As you do this, make sure that the circular interference pattern is centered with respect to the shadow of the rectangular beam splitter cube.
  3. To help you with very fine alignment, turn on the piezo controller and use the X,Y,Z knobs to center the circular interference pattern with respect to the shadow of the cube.
  4. At the end, you should be able to see many concentric circular fringes that is the characteristic of a Michelson Interferometer. (Why do you see circular fringes ?)
  5. Basically, you are trying to do the same thing when the difference in the distance of the two arms of the interferometer is zero. Except it would be harder to get the feel of alignment since the central circular fringe will take up the entire field of view.

- Set the arm distance such that the difference is zero. Use the same approach to alignment as the practice procedure above.

- Once everything is aligned, take away the sheet of paper that you used to make way for the light.

4.2 Data Acquisition

1. Use the "Signal Express" program

   - Click Add Step
     → Acquire Signal
     →DAQmx Acquire
     → Analog input
     → Voltage
   - Choose the aio and ai1 channels
   - Click on Step Setup and scroll to the Timing Settings

- Set the following:
  → Acquisition Mode: Continuous Samples
  → Samples to read: 1 K
  → Rate: 10 K
- Press on the DataView tab
- Click Run
- Expand DAQmx Acquire

2. Setting up Digital Filter in Data Acquisition:

   - Drag Dev ai0 channel into the window channel. You should now be able to observe some signal.
   - In order to isolate the noise software based filters can be applied to the signals:
     Press on Add Step
     Signal Processing
     Analog Signals
     Filters
   - Customize the filter specifications through the tool box. The following filters are recommended:
     (a) For the ai0 channel (Detector Type: DET36A): (This filter is recommended for the HeNe light source)
         i. Specify input data for the filter: ai0
         ii. Type: IIR Butterworth Bandstop Filter
         iii. Upper and Lower cutoffs based on the frequency of noise. E.g. 60Hz noise would need a bandstop of 50-70Hz
     (b) For the ai1 channel (Detector Type: PDA36A): (This filter is recommended for the Broadband light source)
         i. Specify input data for the filter: ai1
         ii. Type: IIR Butterworth Lowpass Filter
         iii. Cutoff: 200 Hz
   - Add two new displays and drag the two filtered data outputs to the displays.

3. Acquiring data: To record the data, click on the Record button next to the Add Step button. This opens up a pop-up window where the data channels to be recorded can be selected, as well as the name of the record and an optional description.

4.3 Motor Control

- Setup

  Goto: Start Menu → All Programs → Thorlabs → APT → APT User

- Set the following settings:

  Max. Velocity: 0.05 mm/s

  Step Distance: 0.002 mm

  Jogging: Single Step

- Operating:

  1. Click the position display window, a pop up will show up that asks for the position in millimeters you want the actuator to be in.
  2. Specify the position and press Enter. This should move the actuator with the velocity specified in the settings.
  3. (Warning: if you try to move to a position that exceeds the physical limits of the actuator, the software will give you warnings)

5 Analyzing data

A code is provided to the student to analyze the data using Matlab. The code is quite involved. Do not use the code as a black-box. An essential step in understanding the experiment is to go through the code and identify the analysis process.

This code uses data provided by the user for the intensity of the light being examined and data for the HeNe laser taken at the same time by the two detectors. Due to the non-constant speed of the stepper-motor, and the need for the data to be taken at constant phase intervals, one cannot just Fourier transform the obtained data without modification.

With this in mind, the code does two things: a) It accounts for the non-constant velocity by comparing the data of the examined light with the HeNe data and then creates a vector of new data points corresponding to points at constant phase intervals. b) It performs a Fast Fourier Transform

As the data is taken in time domain and the motor moves at a non-constant speed the data for the HeNe laser deviates from a sinusoidal by being narrower or wider between different zero crossings. By assuming a phase of pi between consecutive zero crossings one can assign phases to the data obtained and in this way one can then take data points corresponding to constant phase intervals.

One then performs a discrete Fourier-transform on the data to obtain the power spectra of the examined light.

A more detailed description of the code can be found in appendix III. To run your analysis, simply specify the file path to your data in the provided code.

6 Questions

1. Before beginning the experiment, trace out the path the light will follow on the bench.

2. Explain why changing the path difference does not change the brightness of the fringes for the HeNe laser, but it does for non-coherent sources (e.g. white/filtered white light).

3. The fringes in the Michelson interferometer are called fringes of equal inclination. What does the term equal inclination refer to?

4. Prove the **Convolution theorem**. Namely, given a function $h(\xi) = \int f(x)g(\xi - x)dx$ (the convolution of f and g), its Fourier transform is given by $\tilde{h}(k) = \tilde{f}(k)\tilde{g}(k)$. Use this to show that the Fourier Transform of $\Gamma_{11}(\tau)$ does indeed yield the power spectrum.

5. In the case of a perfectly monochromatic beam with $E(t) = E_0 \mathrm{e}^{-i\omega_0 t}$, find the form of $\Gamma_{11}(\tau)$, and $P(\omega)$. Is this what you would expect? Why?

6. A more realistic coherence function is given by $\Gamma_{11}(t) = |E_0|^2 \mathrm{e}^{-i\omega_0 \tau} \mathrm{e}^{-\frac{l_0 \tau}{c}}$, where l_0 is the coherence length of the source (i.e. a length characterizing the decay of the coherence function). Show that the resulting Power Spectrum is a Lorentzian centered on $\omega = \omega_0$, with FWHM $\frac{l_0}{c}$.

7. Use the previous result to compute the coherence length of the red light.

8. Show that for a filter who transmission curve is either a rectangle or triangle with a FWHM of Λ, the interferogram will have its first node for a path difference:

$$\delta = \frac{\lambda_0^2}{\Delta\lambda}$$

   Where λ_0 is the central wavelength of the transmission curve.

   From one of your interferograms, find δ at the first node in terms of λ_0 (that is, the number of rapid oscillations) and hence determine $\Delta\lambda$. How does this value compare with your FT data?

References

[1] Eugene Hecht, *Optics*. Addison-Wesley 4th Edition 2001

[2] Henry van Driel, *Modern Optics and Laser Physics*. Rev. 1.7B 1996

[3] Steven W. Smith, *The Scientists and Engineer's Guide to Digital Signal Processing*. www.dspguide.com

[4] J. F. James, *A Student's Guide to Fourier Transforms: With Applications to Physics and Engineering*. Cambridge University Press 2nd edition 2002

[5] P. Hariharan *Optical Inteferometry* Academic Press 2nd edition 2003

[6] For device specifications, go to www.thorlabs.com, or ask the Lab Technicians for the device manuals.

Appendix I: Sample spectra

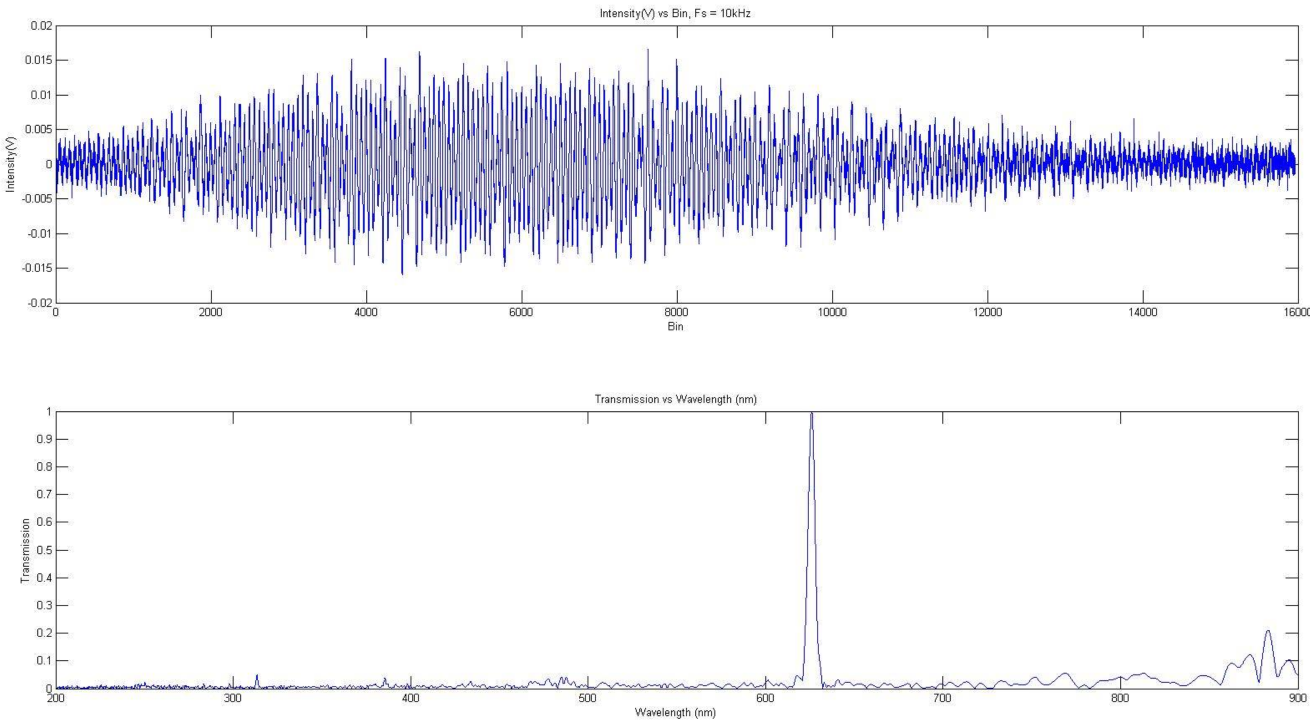


Figure 2: Spectrum obtained for white light passed through a red filter

Appendix II: Additional Background on the Fourier Transform

The idea behind the Fourier transform is to represent a function as a sum of waves. To picture this physically, one recalls the principle of superposition. Here, when the two waves are made to interact, their waveforms are summed, resulting in constructive interference where they have the same sign, and destructive interference when they have opposite sign. In such a way, for a function defined in an interval $0 < x < L$, we can represent it as a sum of basic waveforms (a Fourier series):

$$f(x) = \sum_{n=0}^{\infty} a_n \mathrm{e}^{\frac{i2\pi nx}{L}} \tag{4}$$

where the a_n are the amplitudes of the n^{th} wave.

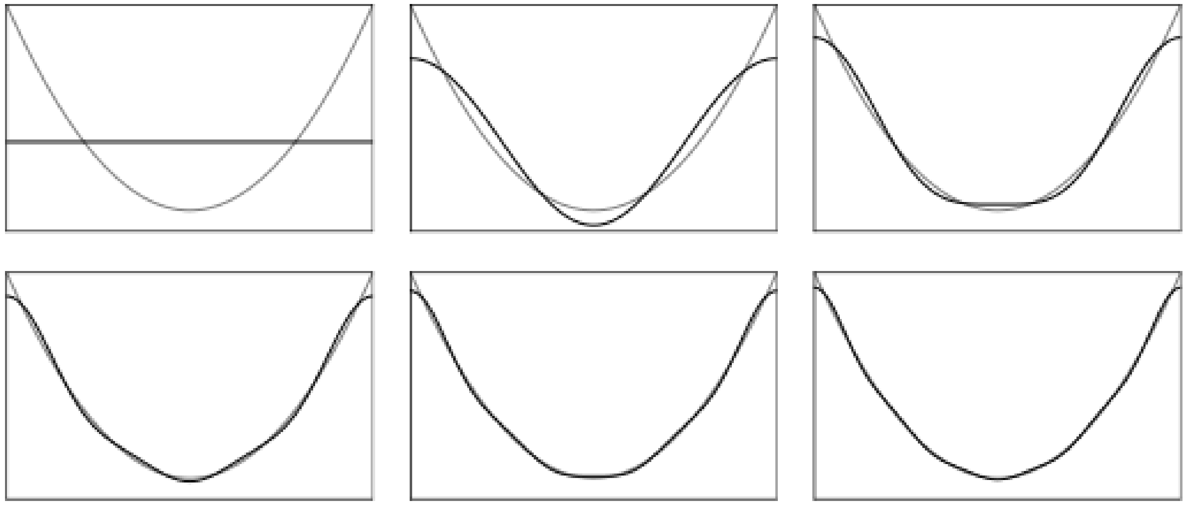


Figure 3: Partial fourier sums for $f(x) = x^2$

The trouble with using a discrete sum is that we are limited to functions which are periodic, or are defined within a finite region. To accomodate the full range of functions[3], we allow the wavelength to take on continuous values. Defining the wavenumber $k = \frac{2\pi}{\lambda}$, the sum becomes an integration over k:

$$f(x) = \int_{-\infty}^{\infty} \mathrm{e}^{ikx} \tilde{f}(k) dk \tag{5}$$

It also possible to perform the reverse operation. That is, given a function $f(x)$, we can find the amplitudes $\tilde{f}(k)$ as:

$$\tilde{f}(k) = \int_{-\infty}^{\infty} \mathrm{e}^{-ikx} f(x) dt \tag{6}$$

In such a way, the function $\tilde{f}(k)$ contains the same amount of information as $f(x)$ - they are complimentary descriptions, existing in different domains. Whereas $f(x)$ describes the intensity of a signal at a point in the "space domain", the fourier transform $\tilde{f}(k)$ describes the amplitude from a particular point in the "wavelength domain."

In the experiment, the characteristic frequencies, that is, the peaks in the spectra, will appear as peaks in $\tilde{f}(k)$.

[3]There is a good deal of mathematical subtlety we are ignoring here, but for the purpose of this lab all signals can be fourier transformed

Appendix III:

- Importing intensity data for the light under examination and the HeNe laser. Of course, the student needs to change the paths to the files containing obtained data.

```
W_intensity = importdata('path\to\sample_data.txt');
L_intensity = importdata('path\to\hene_data.txt');
```

- Subtract the average of the intensities to make the oscillations around 0

```
W_avg = mean(W_intensity);
W_intensity = W_intensity - mean(W_intensity);
L_avg = mean(L_intensity);
L_intensity = L_intensity - mean(L_intensity);
```

-Searching for the zero point crossings

- First look for exact zeros (Laser Light)

```
ind0_L = find( L_intensity == 0 );
```

-> Now `ind_0_L` contains the indices of the HeNe points that have intensity value 0

- Then look for zero crossings between data points (Laser Light), the idea is that we are multiplying every point with the next point. Since we made the curves oscillate around zero the resulting value would be negative if the zero point has been crossed.

```
L1 = L_intensity(1:end-1) .* L_intensity(2:end);
ind1_L = find( L1 < 0 );
```

-> Now `ind_1_L` contains the indices of negative values that are followed by positive ones or vice versa

- Bring exact zeros and "in-between" zeros together and order them in one vector (Laser Light)

```
ind_L = sort([ind0_L ind1_L]);
```

-> Now ind_L contains the indices of points which have intensity value of exactly zero and for each zero crossing the index for the point right below or above the crossing

- Get "fractional" numbers of sample points at which the zeroes are estimated to occur through linear interpolation. There are quite a few subtleties in this function because many cases are being handled.

- If the point index in ind_L corresponds to an exact zero no change is made

```
for (i=1:length(ind_L))
        if (L_intensity(ind_L(i))==0)
                ind_L(i)=ind_L(i);
```

- But if the point in ind_L lies behind the zero crossing then we linear interpolate to find the index for the zero crossing which would a fractional number. There are two cases to be handled. The point lying behind the zero crossing could be negative or positve.

```
elseif (L_intensity(ind_L(i))<0 && L_intensity(ind_L(i)+1)>0)
            ind_L(i)=ind_L(i)+(abs(L_intensity(ind_L(i))))/
(abs(L_intensity(ind_L(i)))+abs(L_intensity(ind_L(i)+1)));
 elseif (L_intensity(ind_L(i))>0 && L_intensity(ind_L(i)+1)<0)
ind_L(i)=ind_L(i)+(abs(L_intensity(ind_L(i))))/
(abs(L_intensity(ind_L(i)))+abs(L_intensity(ind_L(i)+1)));
```

- But if the point in ind_L lies after the zero crossing then we linear interpolate to find the index for the zero crossing which would a fractional number. There are two cases to be handled. The point lying after the zero crossing could be negative or positve.

```
elseif (L_intensity(ind_L(i))<0 && L_intensity(ind_L(i)-1)>0)
ind_L(i)=ind_L(i)-(abs(L_intensity(ind_L(i))))/
(abs(L_intensity(ind_L(i)))+abs(L_intensity(ind_L(i)-1)));
elseif (L_intensity(ind_L(i))>0 && L_intensity(ind_L(i)-1)<0)
ind_L(i)=ind_L(i)-(abs(L_intensity(ind_L(i))))/
(abs(L_intensity(ind_L(i)))+abs(L_intensity(ind_L(i)-1)));
     end
end
```

-> Now ind_L is expected to contain "mainly" non-integer indices of the points corresponding to zero point crossings

- Now we assign phases to all points lying between the first zero crossing and the last zero crossing. The ceil function rounds up any non-integer values.

```
first=ceil(ind_L(1));
last=ceil(ind_L(end))-1;
```

- Initiate a vector of associated phases and set them all to zero. We assign a phase of zero to the first zero crossing and then assume there's a pi phase difference between consecutive zero crossing and assign the phases to all the obtained data accordingly. This function contains quite a few subtleties as it handles many cases.

```
L_phase = zeros([1,last-first+1]);
```

-The index i runs over the points obtained and the index j keeps track of the index values of the zero point crossings.

-A phase value is assigned to the point if it lies between two zero crossings. A pi- phase is always assumed between consecutive zero crossings.

```
for (j=2:length(ind_L))
   for (i=first:last)
   if (i>ind_L(j))
    j=j+1;
```

```
   break
 end

             if (i>=ind_L(j-1) && ind_L(j)>=i)

       if (ind_L(j-1)>i-1 && i ~= first)

             L_phase(i-first+1)= L_phase(i-first)+
((i-ind_L(j-1))/(ind_L(j)-ind_L(j-1)))*pi+((ind_L(j-1)-(i-1))/
(ind_L(j-1)-ind_L(j-2)))*pi;

       elseif (ind_L(j-1)>i-1 && i == first)

             L_phase(i-first+1)= L_phase(i-first+1)+
((i-ind_L(j-1))/(ind_L(j)-ind_L(j-1)))*pi;

                       elseif (ind_L(j-1)<=i-1)

             L_phase(i-first+1)= L_phase(i-first)+(1/
(ind_L(j)-ind_L(j-1)))*pi;

       end

     end

    end

   end
```

-> Now L_phase should contain the phases of all data points

- Define segments of equal phase
-Phase vector is the same length as the actual intensity vector (between the first and last zero crossing) to prevent FFT problems

```
phases=linspace(L_phase(1), L_phase(end),length( W_intensity
(first:last)));
```

-Fit points using cubic spline interpolation (White Light)

New_W_intensity = W_intensity (first:last);
W_proper=spline(L_phase,New_W_intensity,phases);
-> Now W_proper contains data points corresponding to equal phase intervals. It has the same length as the original intensity vector between the first and last zero crossing.

Fourier Transform Part

-We set the values to be Fourier transformed equal to the variable I and make it oscillate around 0

```
I = W_proper;

I_avg = mean(I);

I = I - mean(I);
```

-Set fs to ½ the sample rate in units of Hz
-Set V equal to the motor speed in mm/s
-The nyquist frequency is ½ the sampling rate. It is the highest frequency that can be coded at a given sampling rate in order to be able to fully reconstruct the signal. It is defined here in THz.

-Frequencies between 0 THz and 100 THz are windowed out

```
fs = 5000;

T = 1/fs;

V = 0.0499942e-3;

D = T*V;

Tao = D/3e8;

fs_tao = 1/Tao;

nyquist = fs_tao/(2*1e12);
```

- m is equal to the window length
- n is equal to the length of the vector containing the transformed points
- y contains the points that are Fourier transformed
- nm is the wavelength range in nm
- f is the frequency in THz
- power is the the normalized power spectrum to be plotted

```
m = length(I);

n = pow2(1.5*nextpow2(m))
```

```
y = fft(I,n);

f = (-n/2:n/2-1)*(fs_tao/n)/1e12;

nm = (1./(f*1e12))*3e8*1e9;

power = abs(y)/n;

power = power/max(power);
```

-Plotting the data

```
subplot(2,1,1);

plot(I);

xlabel('Bin');

ylabel('Intensity(V)');

title('Intensity(V) vs Bin: White light filter, Fs = 10kHz');

subplot(2,1,2);

plot(nm,fftshift(power));

axis([200 900 0 1]);

xlabel('Wavelength (nm)');

ylabel('Transmission');

title('Transmission vs Wavelength (nm)');
```