Appendix II: The Michelson Interferometer

From Fundamentals of Optics by Jenkins and White

13.7 COHERENT SOURCES

It will be noticed that the various methods of demonstrating interference so far discussed have one important feature in common: the two interfering beams are always derived from the same source of light. We find by experiment that it is impossible to obtain interference fringes from two separate sources, such as two lamp filaments set side by side. This failure is caused by the fact that the light from any one source is not an infinite train of waves. On the contrary, there are sudden changes in phase occurring in very short intervals of time (of the order of 10^{-8} s). This point has already been mentioned in Secs. 11.1 and 12.6. Thus, although interference fringes may exist on the screen for such a short interval, they will shift their position each time there is a phase change, with the result that no fringes at all will be seen. In Young's experiment and in Fresnel's mirrors and biprism, the two sources S_1 and S_2 always have a point-to-point correspondence of phase, since they are both derived from the same source. If the phase of the light from a point in S_1 suddenly shifts, that of the light from the corresponding point in S_2 will shift simultaneously. The result is that the *difference* in phase between any pair of points in the two sources always remains constant, and so the interference fringes are stationary. It is a charac-

* Good descriptions will be found in T. Preston, "Theory of Light," 5th ed., chap. 7, The Macmillan Company, New York, 1928.

teristic of any interference experiment with light that the sources must have this point-to-point phase relation, and sources that have this relation are called *coherent sources.*

While special arrangements are necessary for producing coherent sources of light, the same is not true of *microwaves*, which are radio waves of a few centimeters wavelength. These are produced by an oscillator which emits a continuous wave, the phase of which remains constant over a time long compared with the duration of an observation. Two independent microwave sources of the same frequency are therefore coherent and can be used to demonstrate interference. Because of the convenient magnitude of their wavelength, microwaves are used to illustrate many common optical interference and diffraction effects.*

If in Young's experiment the source slit S (Fig. 13C) is made too wide or the angle between the rays which leave it too large, the double slit no longer represents two coherent sources and the interference fringes disappear. This subject will be discussed in more detail in Chap. 16.

13.8 DIVISION OF AMPLITUDE. MICHELSON† INTERFEROMETER

Interference apparatus may be conveniently divided into two main classes, those based on *division of wave front* and those based on *division of amplitude*. The previous examples all belong to the former class, in which the wave front is divided laterally into segments by mirrors or diaphragms. It is also possible to divide a wave by partial reflection, the two resulting wave fronts maintaining the original width but having reduced amplitudes. The Michelson interferometer is an important example of this second class. Here the two beams obtained by amplitude division are sent in quite different directions against plane mirrors, whence they are brought together again to form interference fringes. The arrangement is shown schematically in Fig. 13N. The main optical parts consist of two highly polished plane mirrors M_1 and M_2 and two plane-parallel plates of glass G_1 and G_2. Sometimes the rear side of the plate G_1 is lightly silvered (shown by the heavy line in the figure) so that the light coming from the source S is divided into (1) a reflected and (2) a transmitted beam of equal intensity. The light reflected normally from mirror M_1 passes through G_1 a third time and reaches the eye as shown. The light reflected from the mirror M_2 passes back through G_2 for the second time, is reflected from the surface of G_1 and into the

* The technique of such experiments is discussed by G. F. Hull, Jr., *Am. J. Phys.*, 17:599 (1949).

† A. A. Michelson (1852–1931). American physicist of genius. He early became interested in the velocity of light and began experiments while an instructor in physics and chemistry at the Naval Academy, from which he graduated in 1873. It is related that the superintendent of the Academy asked young Michelson why he wasted his time on such useless experiments. Years later Michelson was awarded the Nobel prize (1907) for his work on light. Much of his work on the speed of light (Sec. 19.3) was done during 10 years spent at the Case Institute of Technology. During the latter part of his life he was professor of physics at the University of Chicago, where many of his famous experiments on the interference of light were done.

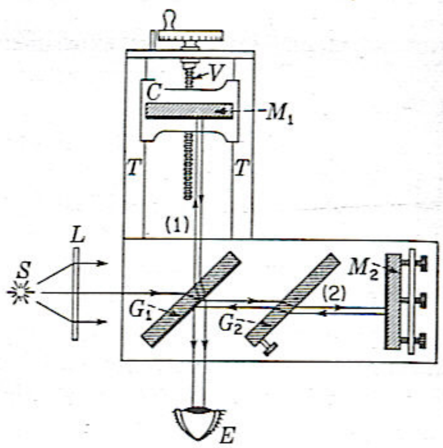


FIGURE 13N
Diagram of the Michelson interferometer.

eye. The purpose of the plate G_2, called the *compensating plate*, is to render the path *in glass* of the two rays equal. This is not essential for producing fringes in monochromatic light, but it is indispensable when white light is used (Sec. 13.11). The mirror M_1 is mounted on a carriage C and can be moved along the well-machined ways or tracks T. This slow and accurately controlled motion is accomplished by means of the screw V, which is calibrated to show the exact distance the mirror has been moved. To obtain fringes, the mirrors M_1 and M_2 are made exactly perpendicular to each other by means of screws shown on mirror M_2.

Even when the above adjustments have been made, fringes will not be seen unless two important requirements are fulfilled. First, the light must originate from an *extended* source. A point source or a slit source, as used in the methods previously described, will not produce the desired system of fringes in this case. The reason for this will appear when we consider the origin of the fringes. Second, the light must in general be *monochromatic*, or nearly so. Especially is this true if the distances of M_1 and M_2 from G_1 are appreciably different.

An extended source suitable for use with a Michelson interferometer may be obtained in any one of several ways. A sodium flame or a mercury arc, if large enough, may be used without the screen L shown in Fig. 13N. If the source is small, a ground-glass screen or a lens at L will extend the field of view. Looking at the mirror M_1 through the plate G_1, one then sees the whole mirror filled with light. In order to obtain the fringes, the next step is to measure the distances of M_1 and M_2 to the back surface of G_1 roughly with a millimeter scale and to move M_1 until they are the same to within a few millimeters. The mirror M_2 is now adjusted to be perpendicular to M_1 by observing the images of a common pin, or any sharp point, placed between the source and G_1. Two pairs of images will be seen, one coming from reflection at the front surface of G_1 and the other from reflection at its back surface. When the tilting screws on M_2 are turned until one pair of images falls exactly on the other, the interference fringes should appear. When they first appear, the fringes will not be clear unless the eye is focused on or near the back mirror M_1, so the observer should look constantly at this mirror while searching for the fringes.

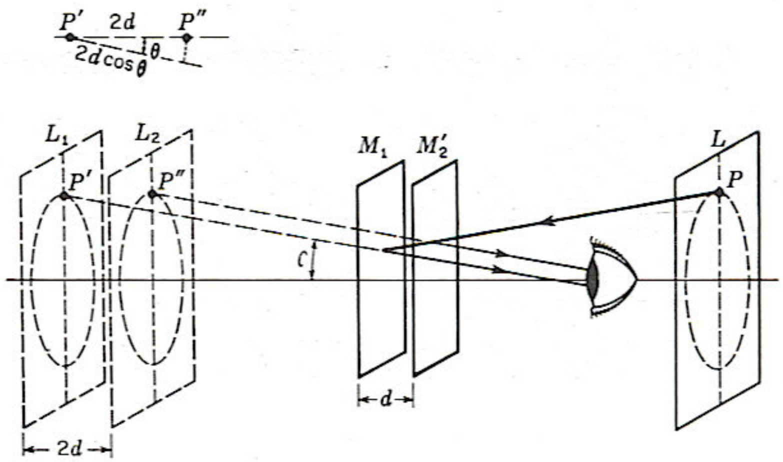


FIGURE 13O
Formation of circular fringes in the Michelson interferometer.

When they have been found, the adjusting screws should be turned in such a way as to continually increase the width of the fringes, and finally a set of concentric circular fringes will be obtained. M_2 is then exactly perpendicular to M_1 if the latter is at an angle of 45° with G_1.

13.9 CIRCULAR FRINGES

These are produced with monochromatic light when the mirrors are in exact adjustment and are the ones used in most kinds of measurement with the interferometer. Their origin can be understood by reference to the diagram of Fig. 13O. Here the real mirror M_2 has been replaced by its virtual image M_2' formed by reflection in G_1. M_2' is then parallel to M_1. Owing to the several reflections in the real interferometer, we may now think of the extended source as being at L, behind the observer, and as forming two virtual images L_1 and L_2 in M_1 and M_2'. These virtual sources are coherent in that the phases of corresponding points in the two are exactly the same at all instants. If d is the separation M_1M_2', the virtual sources will be separated by $2d$. When d is exactly an integral number of half wavelengths, i.e., the path difference $2d$ equal to an integral number of whole wavelengths, all rays of light reflected normal to the mirrors will be in phase. Rays of light reflected at an angle, however, will in general not be in phase. The path difference between the two rays coming to the eye from corresponding points P' and P'' is $2d \cos \theta$, as shown in the figure. The angle θ is necessarily the same for the two rays when M_1 is parallel to M_2' so that the rays are parallel. Hence when the eye is focused to receive parallel rays (a small telescope is more satisfactory here, especially for large values of d) the rays will reinforce each other to produce maxima for those angles θ satisfying the relation

$$2d \cos \theta = m\lambda \qquad (13g)$$

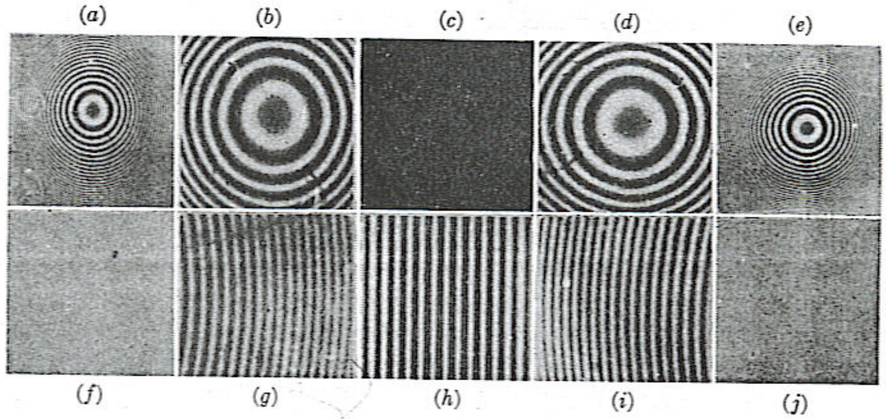


FIGURE 13P
Appearance of the various types of fringes observed in the Michelson interferometer. *Upper row*, circular fringes. *Lower row*, localized fringes. Path difference increases outward, in both directions, from the center.

Since for a given m, λ, and d the angle θ is constant, the maxima will lie in the form of circles about the foot of the perpendicular from the eye to the mirrors. By expanding the cosine, it can be shown from Eq. (13g) that the radii of the rings are proportional to the square roots of integers, as in the case of Newton's rings (Sec. 14.5). The intensity distribution across the rings follows Eq. (13b), in which the phase difference is given by

$$\delta = \frac{2\pi}{\lambda} 2d \cos \theta$$

Fringes of this kind, where parallel beams are brought to interference with a phase difference determined by the angle of inclination θ, are often referred to as *fringes of equal inclination*. In contrast to the type to be described in the next section, this type may remain visible over very large path differences. The eventual limitation on the path difference will be discussed in Sec. 13.12.

The upper part of Fig. 13P shows how the circular fringes look under different conditions. Starting with M_1 a few centimeters beyond M_2', the fringe system will have the general appearance shown in (*a*) with the rings very closely spaced. If M_1 is now moved slowly toward M_2' so that d is decreased, Eq. (13g) shows that a given ring, characterized by a given value of the order m, must decrease its radius because the product $2d \cos \theta$ must remain constant. The rings therefore shrink and vanish at the center, a ring disappearing each time $2d$ decreases by λ, or d by $\lambda/2$. This follows from the fact that at the center $\cos \theta = 1$, so that Eq. (13g) becomes

$$2d = m\lambda \qquad (13h)$$

To change m by unity, d must change by $\lambda/2$. Now as M_1 approaches M_2' the rings become more widely spaced, as indicated in Fig. 13P(*b*), until finally we reach a critical position where the central fringe has spread out to cover the whole field

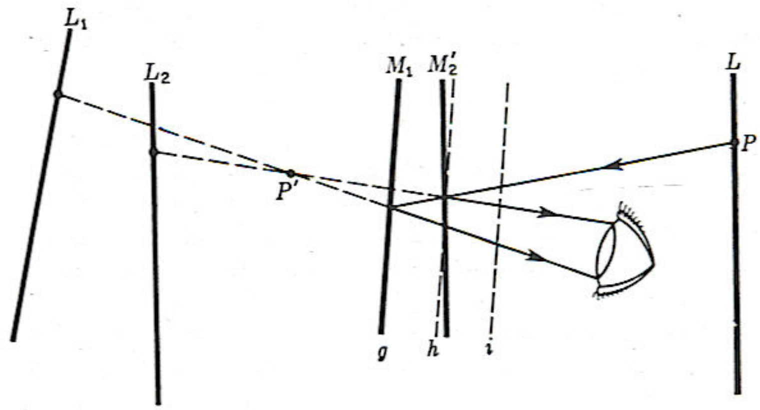


FIGURE 13Q
The formation of fringes with inclined mirrors in the Michelson interferometer.

of view, as shown in (*c*). This happens when M_1 and M_2' are exactly coincident, for it is clear that under these conditions the path difference is zero for all angles of incidence. If the mirror is moved still farther, it effectively passes through M_2', and new widely spaced fringes appear, growing out from the center. These will gradually become more closely spaced as the path difference increases, as indicated in (*d*) and (*e*) of the figure.

13.10 LOCALIZED FRINGES

If the mirrors M_2' and M_1 are not exactly parallel, fringes will still be seen with monochromatic light for path differences not exceeding a few millimeters. In this case the space between the mirrors is wedge-shaped, as indicated in Fig. 13Q. The two rays* reaching the eye from a point P on the source are now no longer parallel, but appear to diverge from a point P' near the mirrors. For various positions of P on the extended source, it can be shown† that the path difference between the two rays remains constant but that the distance of P' from the mirrors changes. If the angle between the mirrors is not too small, however, the latter distance is never great, and hence, in order to see these fringes clearly, the eye must be focused on or near the rear mirror M_1. The localized fringes are practically straight because the variation of the path difference across the field of view is now due primarily to the variation of the thickness of the "air film" between the mirrors. With a wedge-shaped film, the locus of points of equal thickness is a straight line parallel to the edge of the wedge. The

* When the term "ray" is used, here and elsewhere in discussing interference phenomena, it merely indicates the direction of the *perpendicular to a wave front* and is in no way to suggest an infinitesimally narrow pencil of light.

† R. W. Ditchburn, "Light," 2d ed., paperback, John Wiley and Sons, Inc., New York, 1963.

fringes are not exactly straight, however, if d has an appreciable value, because there is also some variation of the path difference with angle. They are in general curved and are always convex toward the thin edge of the wedge. Thus, with a certain value of d, we might observe fringes shaped like those of Fig. 13P(g). M_1 could then be in a position such as g of Fig. 13Q. If the separation of the mirrors is decreased, the fringes will move to the left across the field, a new fringe crossing the center each time d changes by $\lambda/2$. As we approach zero path difference, the fringes become straighter, until the point is reached where M_1 actually intersects M_2', when they are perfectly straight, as in (h). Beyond this point, they begin to curve in the opposite direction, as shown in (i). The blank fields (f) and (j) indicate that this type of fringe cannot be observed for large path differences. Because the principal variation of path difference results from a change of the thickness d, these fringes have been termed *fringes of equal thickness.*

13.11 WHITE-LIGHT FRINGES

If a source of white light is used, no fringes will be seen at all except for a path difference so small that it does not exceed a few wavelengths. In observing these fringes, the mirrors are tilted slightly as for localized fringes, and the position of M_1 is found where it intersects M_2'. With white light there will then be observed a central dark fringe, bordered on either side by 8 or 10 colored fringes. This position is often rather troublesome to find using white light only. It is best located approximately beforehand by finding the place where the localized fringes in monochromatic light become straight. Then a very *slow* motion of M_1 through this region, using white light, will bring these fringes into view.

The fact that only a few fringes are observed with white light is easily accounted for when we remember that such light contains all wavelengths between 400 and 750 nm. The fringes for a given color are more widely spaced the greater the wavelength. Thus the fringes in different colors will only coincide for $d = 0$, as indicated in Fig. 13R. The solid curve represents the intensity distribution in the fringes for green light, and the broken curve that for red light. Clearly, only the central fringe will be uncolored, and the fringes of different colors will begin to separate at once on either side, producing various impure colors which are not the saturated spectral colors. After 8 or 10 fringes, so many colors are present at a given point that the resultant color is essentially white. Interference is still occurring in this region, however, because a spectroscope will show a continuous spectrum with dark bands at those wavelengths for which the condition for destructive interference is fulfilled. White-light fringes are also observed in all the other methods of producing interference described above, if white light is substituted for monochromatic light. They are particularly important in the Michelson interferometer, where they may be used to locate the position of zero path difference, as we shall see in Sec. 13.13.

An excellent reproduction in color of these white-light fringes is given in one of Michelson's books.* The fringes in three different colors are also shown separately

* A. A. Michelson, "Light Waves and Their Uses," plate II, University of Chicago Press, Chicago, 1906.

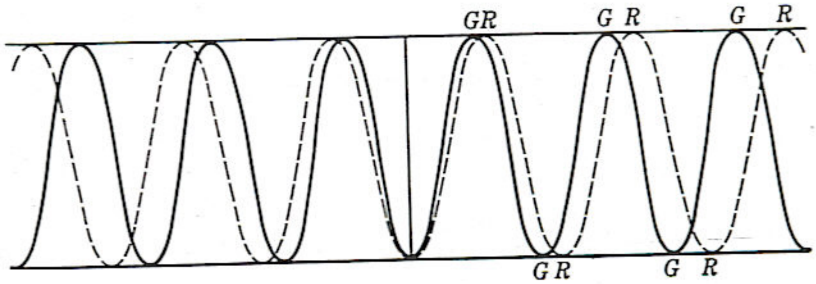


FIGURE 13R
The formation of white-light fringes with a dark fringe at the center.

and a study of these in connection with the white-light fringes is instructive as showing the origin of the various impure colors in the latter.

It was stated above that the central fringe in the white-light system, i.e., that corresponding to zero path difference, is black when observed in the Michelson interferometer. One would ordinarily expect this fringe to be white, since the two beams should be in phase with each other for any wavelength at this point, and in fact this is the case in the fringes formed with the other arrangements, such as the biprism. In the present case, however, it will be seen by referring to Fig. 13N that while ray 1 undergoes an internal reflection in the plate G_1, ray 2 undergoes an external reflection, with a consequent change of phase [see Eq. (14d)]. Hence the central fringe is black if the back surface of G_1 is unsilvered. If it is silvered, the conditions are different and the central fringe may be white.

13.12 VISIBILITY OF THE FRINGES

There are three principal types of measurement that can be made with the interferometer: (1) width and fine structure of spectrum lines, (2) lengths or displacements in terms of wavelengths of light, and (3) refractive indices. As explained in the preceding section, when a certain spread of wavelengths is present in the light source, the fringes become indistinct and eventually disappear as the path difference is increased. With white light they become invisible when d is only a few wavelengths, whereas the circular fringes obtained with the light of a single spectrum line can still be seen after the mirror has been moved several centimeters. Since no line is perfectly sharp, however, the different component wavelengths produce fringes of slightly different spacing, and hence there is a limit to the usable path difference even in this case. For the measurements of length to be described below, Michelson tested the lines from various sources and concluded that a certain red line in the spectrum of cadmium was the most satisfactory. He measured the *visibility*, defined as

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \qquad (13i)$$

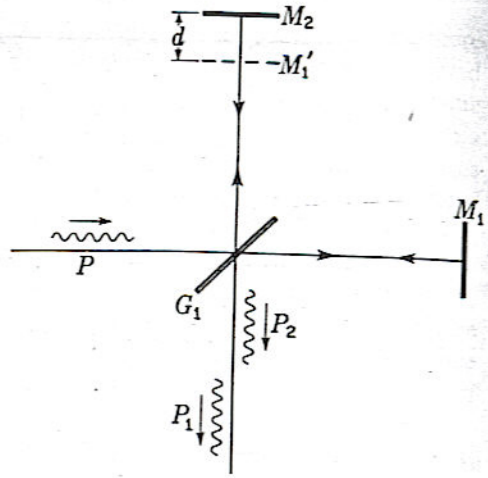


FIGURE 13S
Limiting path difference as determined by the length of wave packets.

where I_{max} and I_{min} are the intensities at the maxima and minima of the fringe pattern. The more slowly V decreases with increasing path difference, the sharper the line. With the red cadmium line, it dropped to 0.5 at a path difference of some 10 cm, or at $d = 5$ cm.

With certain lines, the visibility does not decrease uniformly but fluctuates with more or less regularity. This behavior indicates that the line has a fine structure, consisting of two or more lines very close together. Thus it is found that with sodium light the fringes become alternately sharp and diffuse, as the fringes from the two D lines get in and out of step. The number of fringes between two successive positions of maximum visibility is about 1000, indicating that the wavelengths of the components differ by approximately 1 part in 1000. In more complicated cases, the separation and intensities of the components could be determined by a Fourier analysis of the visibility curves.* Since this method of inferring the structure of lines has now been superseded by more direct methods, to be described in the following chapter, it will not be discussed in any detail here.

An alternative way of interpreting the eventual vanishing of interference at large path differences is instructive to consider at this point. In Sec. 12.6 it was indicated that a finite spread of wavelengths corresponds to wave packets of limited length, this length decreasing as the spread becomes greater. Thus, when the two beams in the interferometer traverse distances that differ by more than the length of the individual packets, these can no longer overlap and no interference is possible. The situation upon complete disappearance of the fringes is shown schematically in Fig. 13S. The original wave packet P has its amplitude divided at G_1 so that two similar packets are produced, P_1 traveling to M_1 and P_2 to M_2. When the beams are reunited, P_2 lags a distance $2d$ behind P_1. Evidently a measurement of this limiting path difference gives a direct determination of the length of the wave packets. This

* A. A. Michelson, "Studies in Optics," chap. 4, University of Chicago Press, Chicago, 1927.

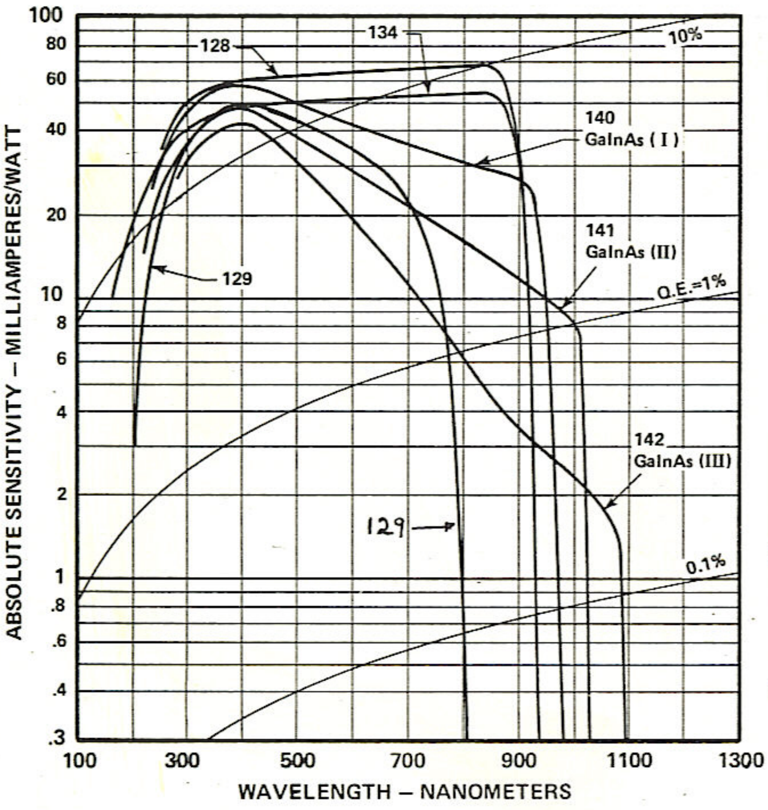
interpretation of the cessation of interference seems at first sight to conflict with the one given above. A consideration of the principle of Fourier analysis shows, however, that mathematically the two are entirely equivalent and are merely alternative ways of representing the same phenomenon.

Appendix III: Photomultiplier Characteristics

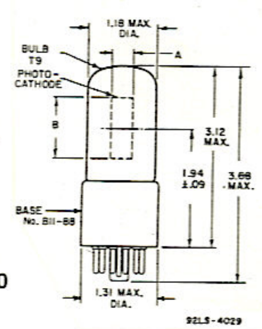
1-1/8"-Diameter QUANTACON Side-On Types
Ga-As, Ga-As-P, and Ga-In-As Photocathodes
Electrostatic-Focus, Circular-Cage Dynode Structure

Spectral Response	RCA Type No.	Mechanical: No. of Stages and Cage Structure[a]	Mechanical: Dynode Secondary Emitting Surface Material	Maximum Ratings: Supply Voltage V	Maximum Ratings: Average[b] Anode Current mA	Operating Supply Volts and Distribution[c]	Sensitivity Radiant[d]: Anode A/W	Sensitivity Radiant[d]: Cathode mA/W	Sensitivity Luminous[e]: Anode A/lm	Sensitivity Luminous[e]: Cathode µA/lm	Gain (Approx.) × 10^6	Anode Dark Current nA@ Anode Luminous Sensitivity A/lm	Anode Pulse Rise Time[f] ns	Remarks[g]
128	C31025C	9 C	Be-O	1500	0.01	1250 C	2700	61	20	450	0.045	0.3 @ 10	≤ 1.5	Ga-As photocathode, UV-transmitting-glass window type. Has essentially "flat" sensitivity throughout its spectral range of 200 to 930 nanometers.
129	C31025B	9 C	Be-O	1800	0.01	1250 C	2000	48	7	170	0.041	0.4 @ 10	≤ 1.5	Ga-As-P photocathode, UV-transmitting-glass window type. Has high sensitivity throughout its spectral range of 200 to 800 nanometers.

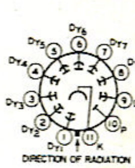
Electrical columns: Typical Characteristics at specified operating supply voltage, voltage distribution, and 22°C



C31025C
C31025B
C31025K
C31025M
C31025N

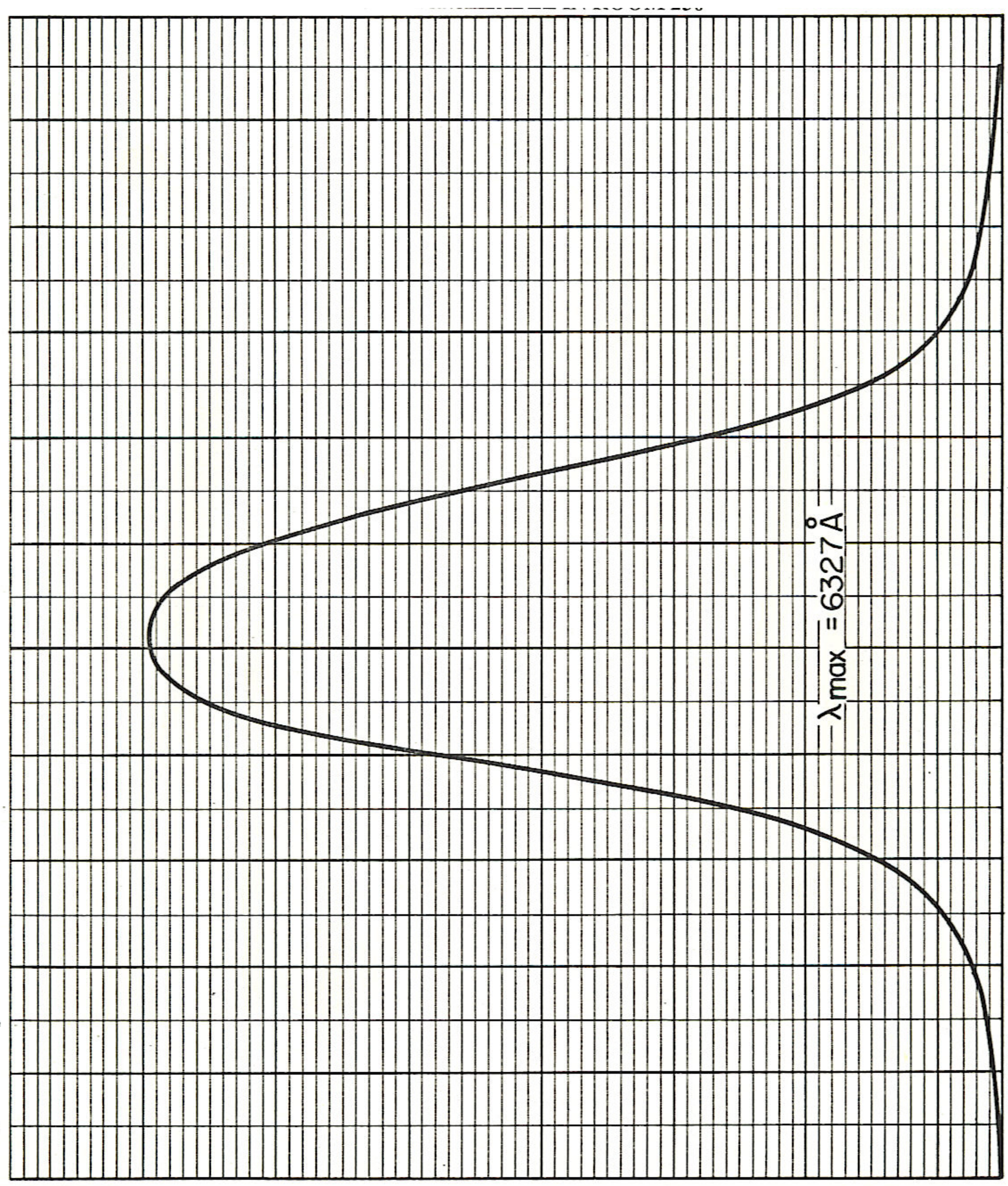


Basing (Bottom View)

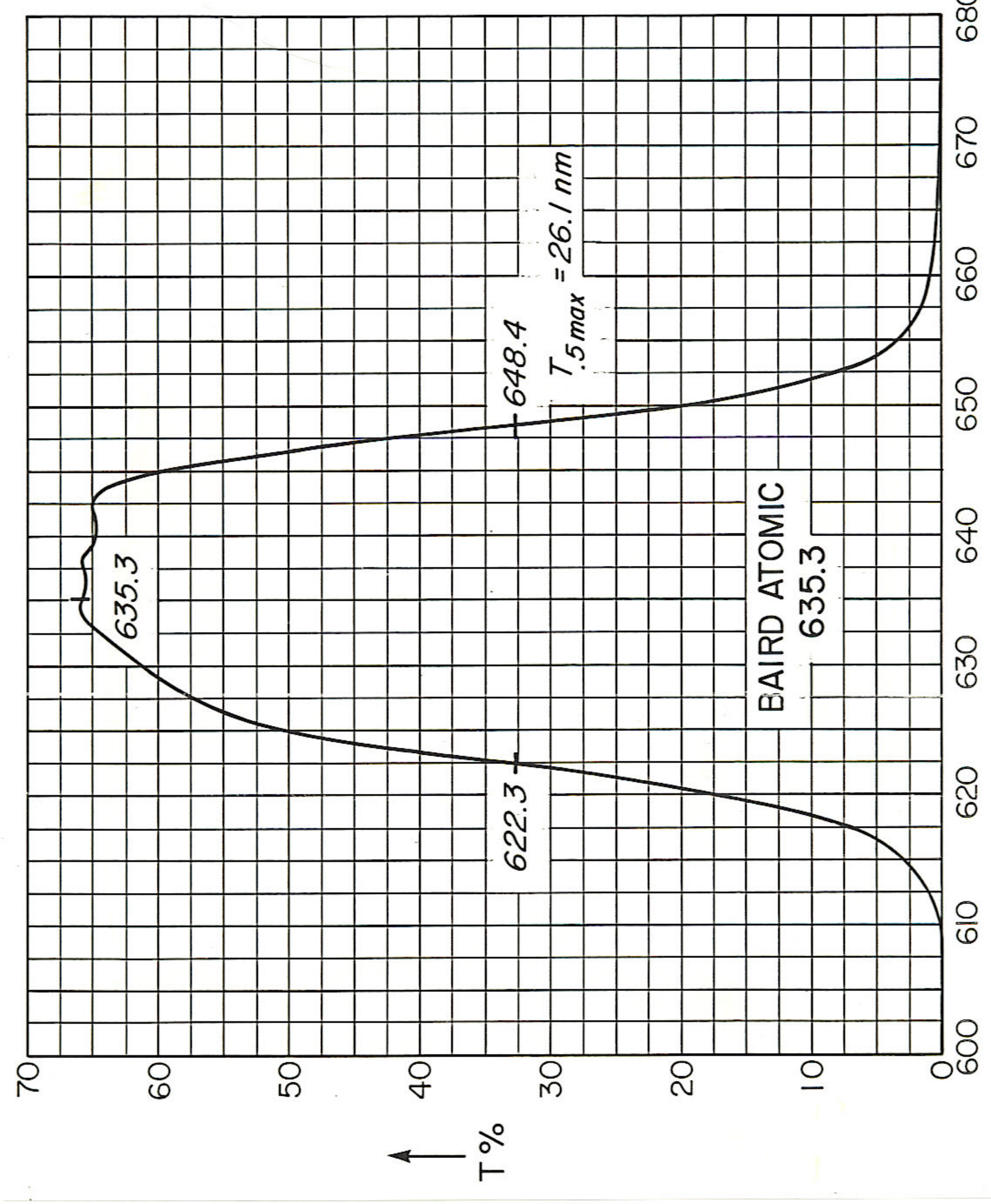


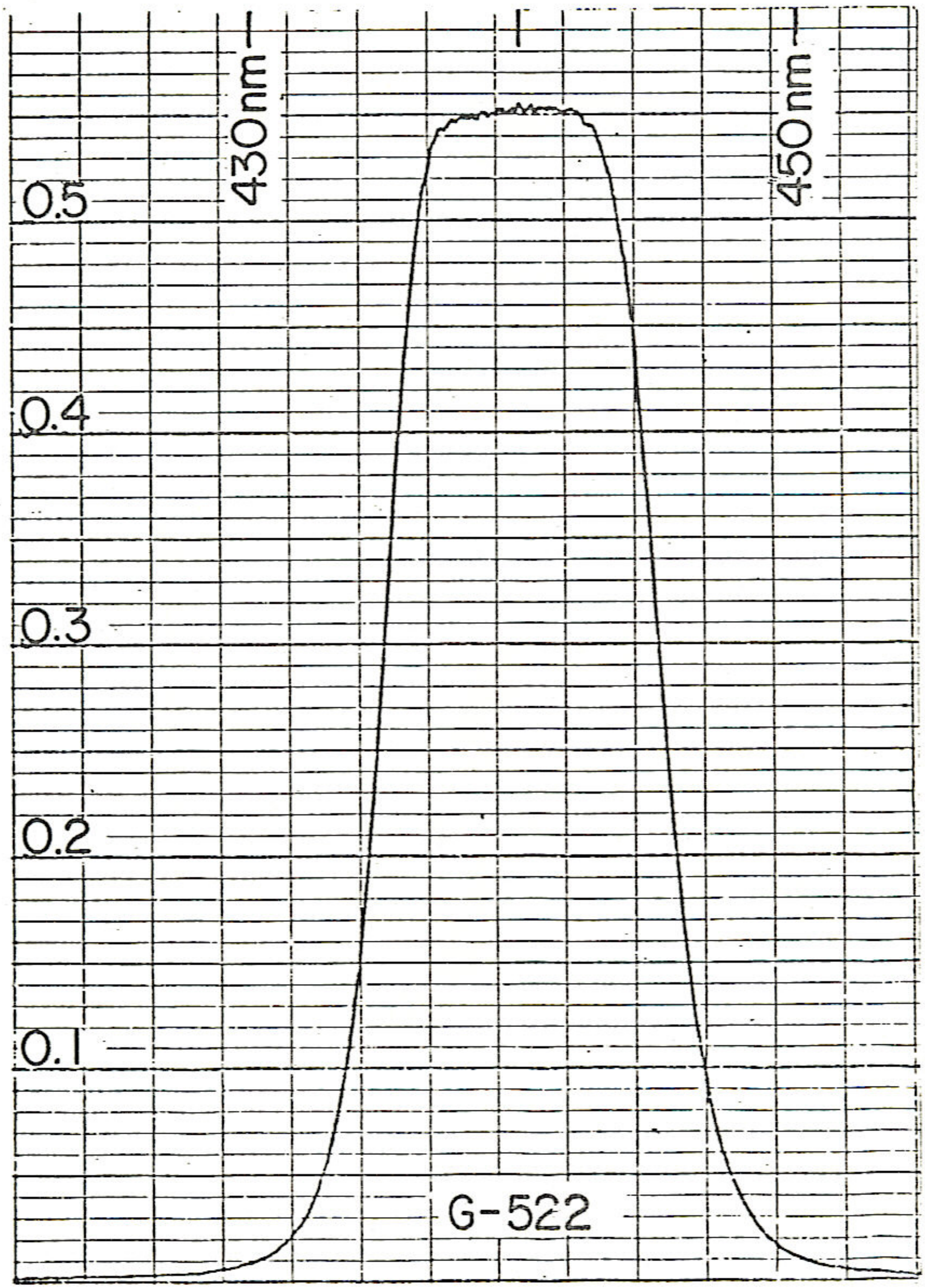
	C31025B	C31025C C31025M C31025K C31025N
A	.31 Min.	.2 Min.
B	.94 Min.	.5 Min.

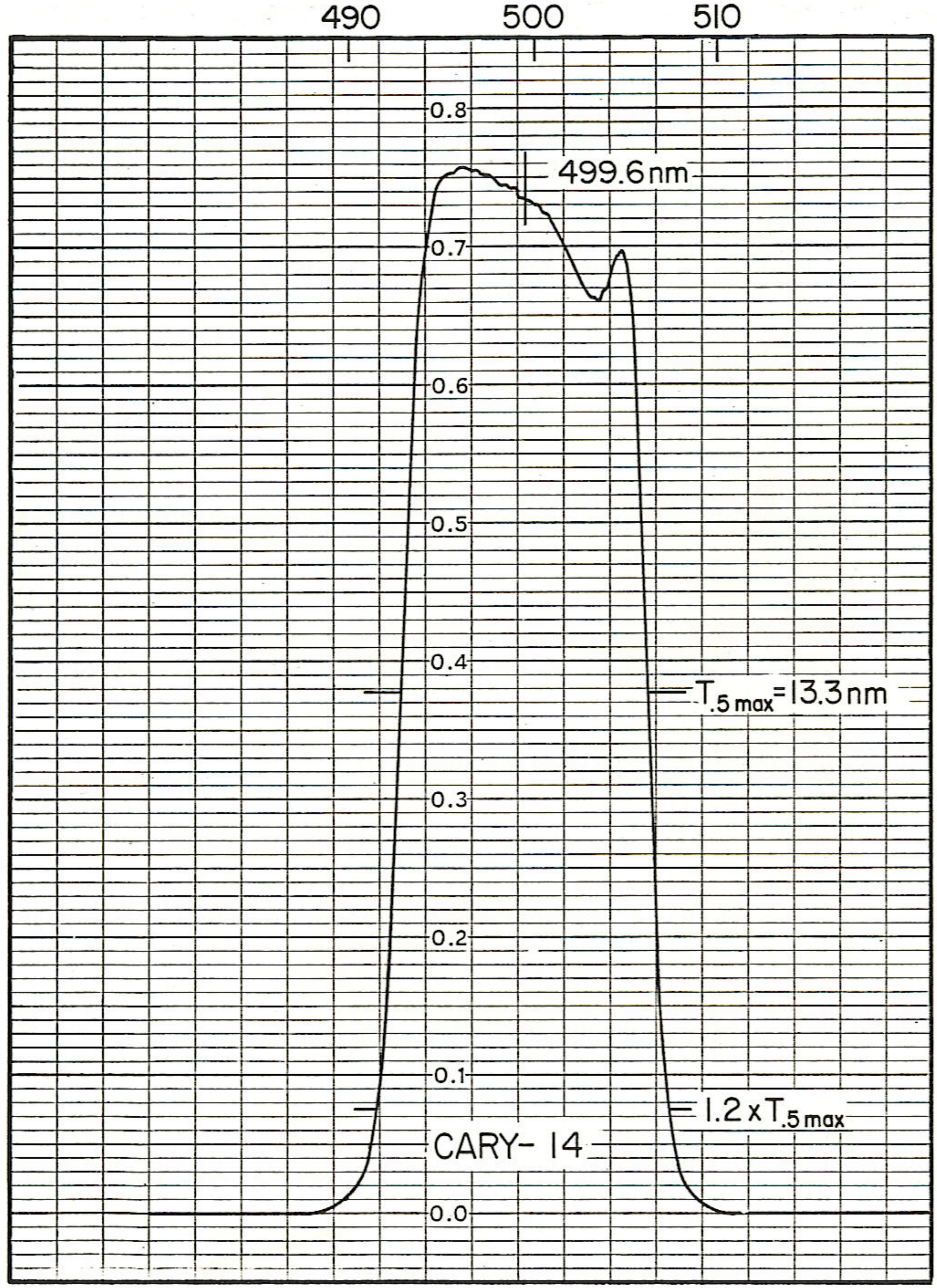
Typical Photocathode Spectral Response Characteristics■



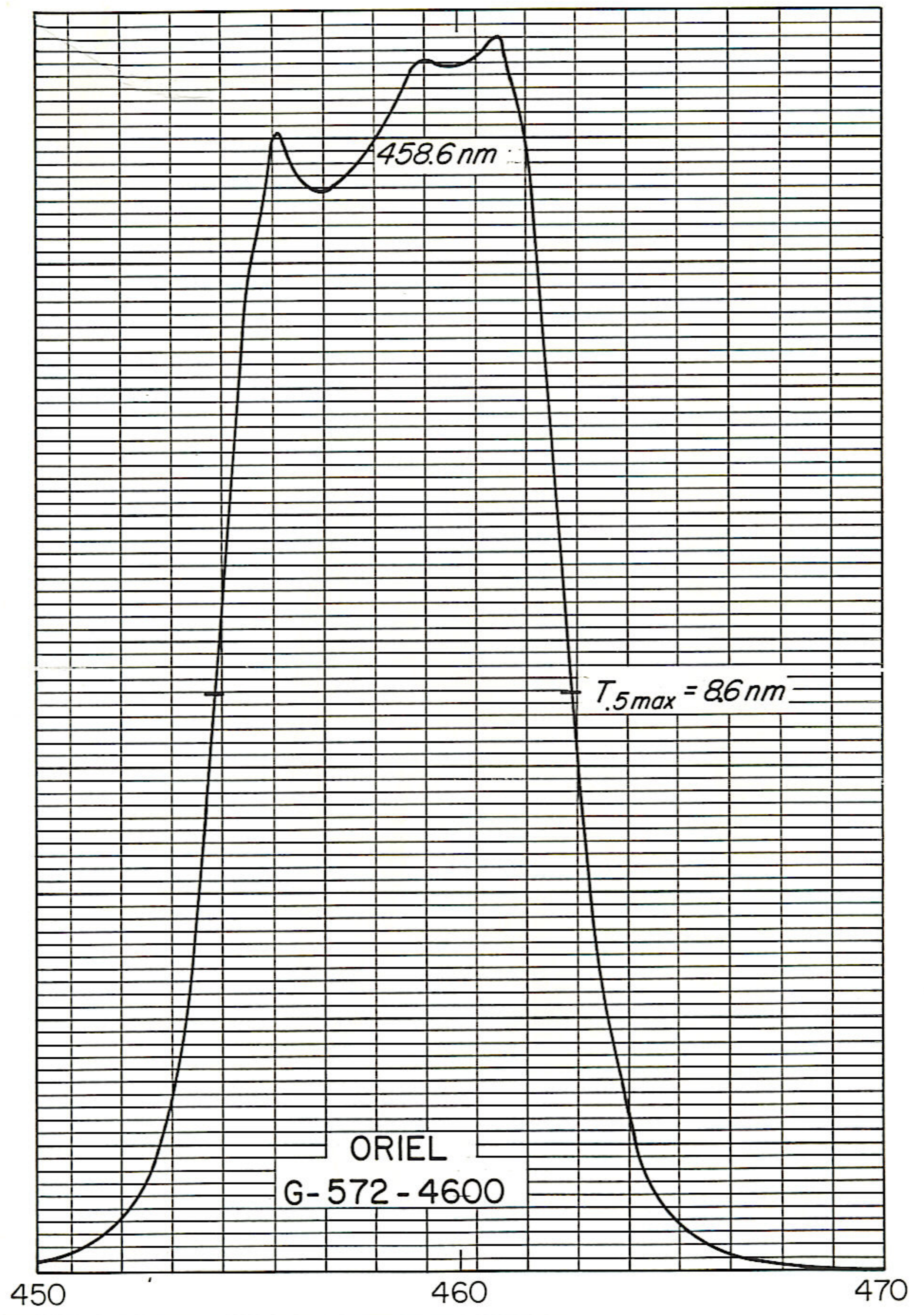
6277
6289
6301
6313
6325
6337
6349
6361
6373
6385
λmax = 6327 Å


635.3
622.3
648.4
$T_{.5max}$ = 26.1 nm
BAIRD ATOMIC
635.3
T %
0
10
20
30
40
50
60
70
600
610
620
630
640
650
660
670
680


430nm
450nm
0.5
0.4
0.3
0.2
0.1
G-522



490
500
510
0.8
499.6 nm
0.7
0.6
0.5
0.4
$T_{.5\,max}$=13.3 nm
0.3
0.2
0.1
1.2 x $T_{.5\,max}$
CARY- 14
0.0



458.6 nm
T.5max = 8.6 nm
ORIEL
G-572-4600
450
460
470