

University of Toronto  
ADVANCED PHYSICS LABORATORY

**FTS**  
**Fourier Transform Spectroscopy**  
**2018F Interim Manual**

Revisions:

2017 September:	Debra Wunch added a brief introduction
2015 January:	David Bailey < <a href="mailto:dbailey@physics.utoronto.ca">dbailey@physics.utoronto.ca</a> >
2014 September:	David Bailey with contributions from students Dong Jin Shin, Hazemm Daoud, Muhammadh Ali Abdullah Salman, and Giuseppe Castiglione
2006:	Jason Harlow with suggestions from student Jerod Wagman and references update by Barbara Chu
Original:	John Pitre

Please send any corrections, comments, or suggestions to the professor currently supervising this experiment, the author of the most recent revision above, or the Advanced Physics Lab Coordinator.

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# INTRODUCTION

Fourier transform spectrometers (FTS) are scientific instruments that are used extensively to observe the composition and dynamics of Earth and planetary atmospheres, measure geophysical properties of the solid earth, and provide food quality assurance. Most famously, Michelson and Morely used an FTS to disprove the theory of the “aether” [1]. Today, FTS instruments are used on satellites, aircraft, high-altitude balloons, and on the ground. In fact, there are several state-of-the-art FTS systems in this very building! One is located at the Toronto Atmospheric Observatory, on the 16<sup>th</sup> floor of the Burton Tower. It uses the sun as a collimated light source to measure the absorption of sunlight by the atmosphere, and it is optically optimized to measure the absorption by molecules that impact Toronto’s air quality.

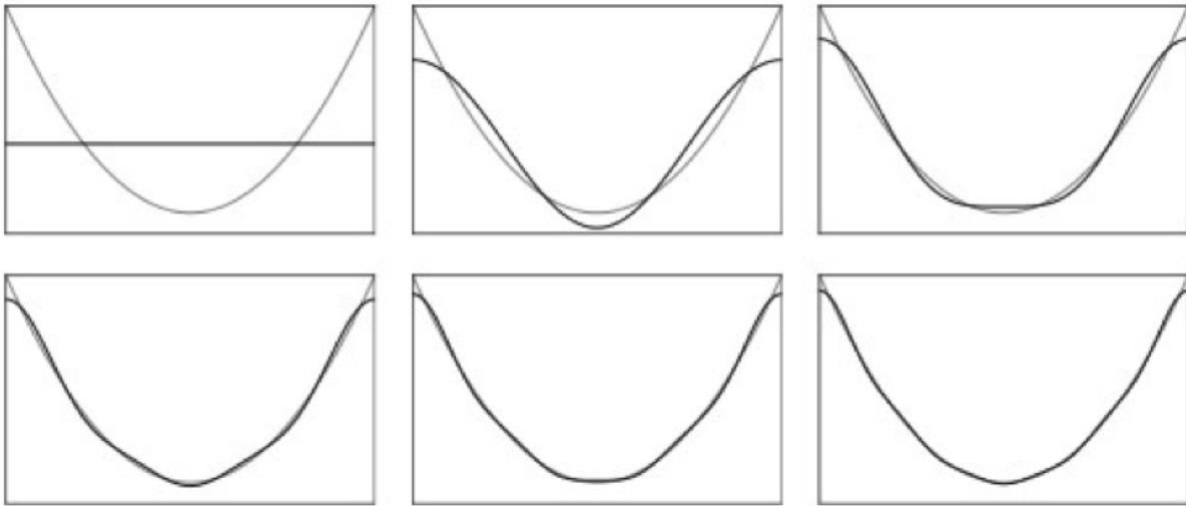
In this lab, you will learn the basic components of a Fourier transform spectrometer, create one on an optical breadboard, record interferometric data, and analyse it.

## The Fourier Transform

The idea behind the Fourier transform is to represent a function  $f(x)$  defined in an interval  $0 < x < L$  as a sum of waves:

$$f(x) = \sum_{n=0}^{\infty} a_n e^{i2\pi nx/L}$$

where the  $a_n$  are the amplitudes of the  $n^{\text{th}}$  wave. This Figure shows the how a parabola can be better and better approximated as more terms in the series are added.



**Figure 1: Partial Fourier sums for  $f(x) = x^2$**

In the continuous limit, a Fourier series becomes a Fourier Transform. Two functions defined by

$$f(x) = \int_{-\infty}^{\infty} g(s) e^{i2\pi sx} ds \quad (1)$$

and

$$g(s) = \int_{-\infty}^{\infty} f(x) e^{-i2\pi sx} dx \quad (2)$$

are said to constitute a Fourier transform pair.  $f(x)$  and  $g(s)$  are called Fourier transforms of each other.

If  $f(x)$  and  $g(s)$  are even functions then the imaginary parts of (1) and (2) disappear and thus

$$f(x) = \int_{-\infty}^{\infty} g(s) \cos(2\rho sx) ds \quad (3)$$

and

$$g(s) = \int_{-\infty}^{\infty} f(x) \cos(2\rho sx) dx \quad (4)$$

Equations (3) and (4) are called cosine Fourier transforms. Cosine Fourier transforms are more commonly introduced in the following way.

If  $f(x)$  is defined only for  $0 < x < \infty$  then  $f(x)$  can be represented by

$$f(x) = 2 \int_0^{\infty} g(s) \cos(2\rho sx) ds \quad (5)$$

and

$$g(s) = 2 \int_0^{\infty} f(x) \cos(2\rho sx) dx \quad (6)$$

Equations 5 and 6 constitute a cosine Fourier transform pair. An analogous set of equations define the sine Fourier transform.

## The Fourier Transform and the Michelson Interferometer

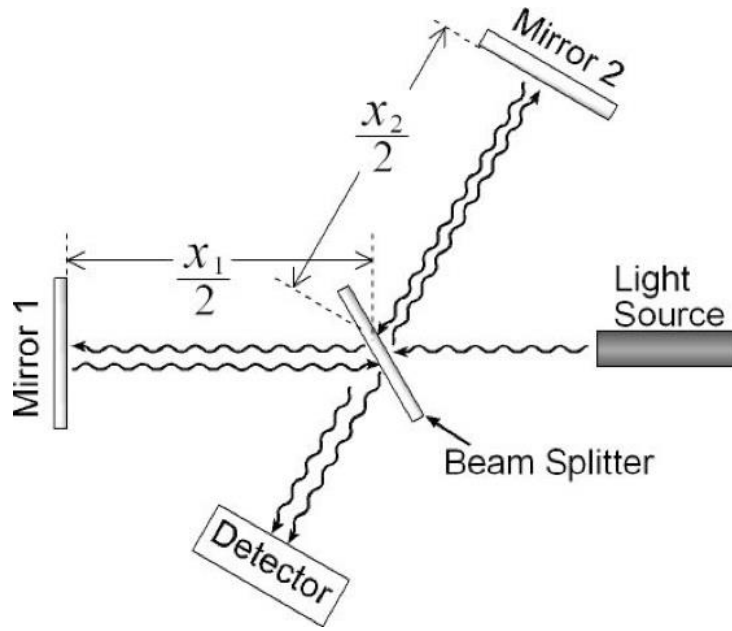
Consider a beam of light incident on the beam splitter with an electric field given by

$$E(x, k) = E_0(k) e^{i(kx - \omega t)} \quad (7)$$

where  $k = 2\pi/\lambda$  is the wave number. The flux density  $B(k)$  is proportional to  $E_0^2(k)$ . In this derivation we will ignore all proportionality constants since we will eventually only be interested in the variation in a signal and not in its absolute magnitude.

In the Michelson interferometer (Figure 1), after amplitude splitting there are two beams which have traveled distances  $x_1$  and  $x_2$ . They recombine to form a resultant electric field  $E_R(x_1, x_2, \sigma)$ . Each beam has undergone one reflection from, and one transmission through the beam splitter so both should have the same amplitude. The resultant electric field, aside from a proportionality factor to account for losses is

$$E(x_1, x_2, k) = E_0(k) [e^{i(kx_1 - \omega t)} + e^{i(kx_2 - \omega t)}] \quad (8)$$



**Figure 2: Schematic of a Michelson Interferometer**

The intensity (e.g. in W/m<sup>2</sup>) for a particular value of  $k$  is given (within a multiplying factor) by the square of the electric field:

$$I_R = E_R^* E_R \quad (9)$$

which, written explicitly, becomes

$$I_R(x_1, x_2, k) = E_0^2(k) [2 + e^{ik(x_1 - x_2)} + e^{-ik(x_1 - x_2)}] \quad (10)$$

or

$$I_R(x_1, x_2, k) = 2E_0^2(k) [1 + \cos(k\delta)] \quad (11)$$

where  $\delta = x_1 - x_2$ . The total flux for any path difference  $\delta$  is obtained by integrating over  $k$ :

$$I_R(\delta) = \int_0^\infty I(\delta, k) dk \quad (12)$$

$$I_R(\delta) = 2 \int_0^\infty E_0^2(k) dk + 2 \int_0^\infty E_0^2(k) \cos(k\delta) dk \quad (13)$$

Equation 13, which gives  $I_R(\delta)$  will have a constant part and a varying part. The constant part could be determined by measuring  $I_R(0)$  when the path difference is zero or by measuring  $I_R(\infty)$  which is the constant level to which the signal tends when the path difference becomes very large (see Bell p.41 for further information on this point). In actual fact, if the beam splitter in the interferometer does not split the incoming beam into two equal components then there will be a constant flux,  $I$ , which is independent of the path difference and this will add another constant term. If we agree to interpret  $I_R(\delta)$  as the fluctuation of the interferogram from its final constant level and not from zero then equation 13 becomes

$$I_R(\delta) = 2 \int_0^\infty E_0^2(k) \cos(k\delta) dk \quad (14)$$

Again, ignoring proportionality constant we may replace  $2E_0^2(k)$  by  $B(k)$ :

$$I_R(\delta) = \int_0^\infty B(k) \cos(k\delta) dk \quad (15)$$

But this is just one of a cosine Fourier transform pair and so we may also write:

$$B(k) = \int_0^{\infty} I_R(d) \cos(kd) dd \quad (16)$$

So within a multiplicative constant the incident power spectrum can be computed by a cosine Fourier transform of the interferogram.

### Safety Reminders

- Do not look directly into the laser beam. This laser is not intense enough to require eye protection, but forcing oneself to stare into any laser beam could cause eye damage.
- DO NOT TOUCH ANY OPTICAL SURFACES! This is not dangerous to you, but fingerprints on the surface of mirrors, lenses, beam-splitters, ... will degrade your signals and damage the components.

*NOTE: This is not a complete list of every hazard you may encounter. We cannot warn against all possible creative stupidities, e.g. juggling cryostats. Experimenters must use common sense to assess and avoid risks, e.g. never open plugged-in electrical equipment, watch for sharp edges, don't lift too-heavy objects, .... If you are unsure whether something is safe, ask the supervising professor, the lab technologist, or the lab coordinator. When in doubt, ask! If an accident or incident happens, you must let us know. More safety information is available at <http://www.ehs.utoronto.ca/resources.htm>.*

## EXPERIMENT

### Michelson Interferometer

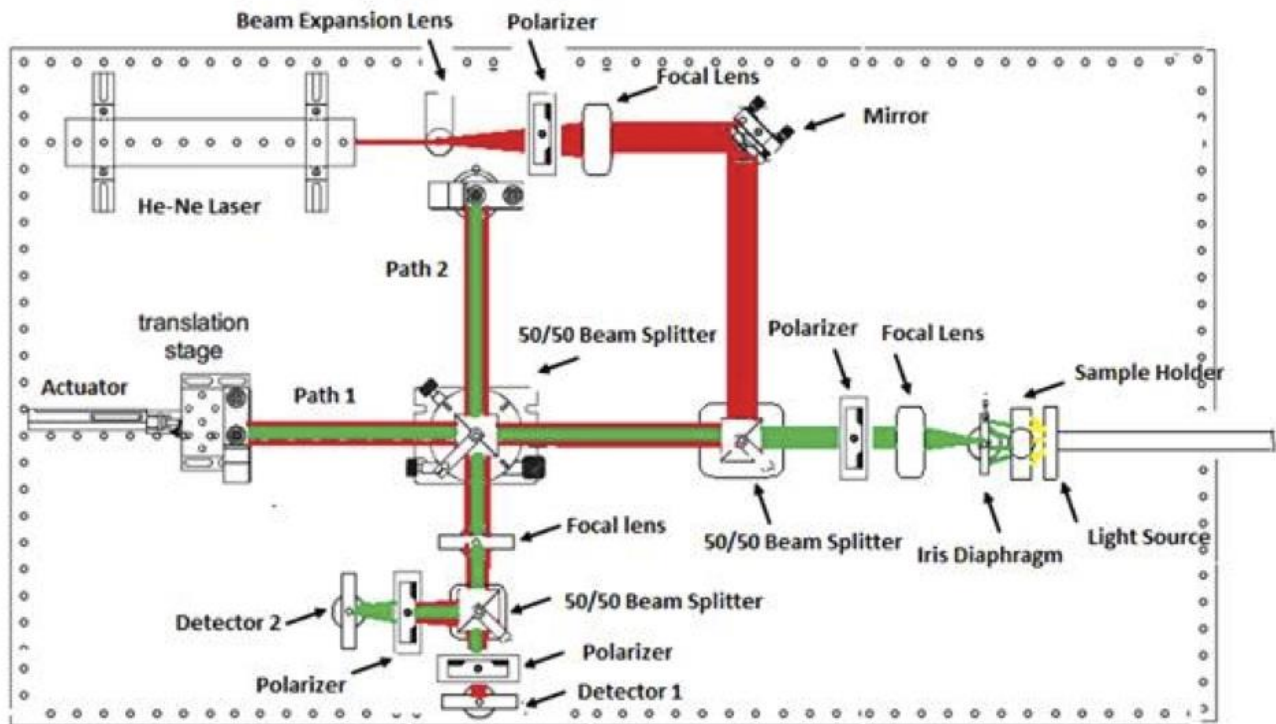


Figure 2: A possible schematic of the experimental apparatus.

## Components

- For device specifications, search for the model numbers at [www.thorlabs.com](http://www.thorlabs.com), or ask the Lab Technologist for the device manuals.

### HeNe Laser



- Source of red laser light, with a well defined peak at 630 nm
- Used to calibrate data and correct noise from non-constant stepper velocity

### Piezo Controller



- Allows extremely fine control of orientation and position of fixed mirror

### OSL1 White Light Source



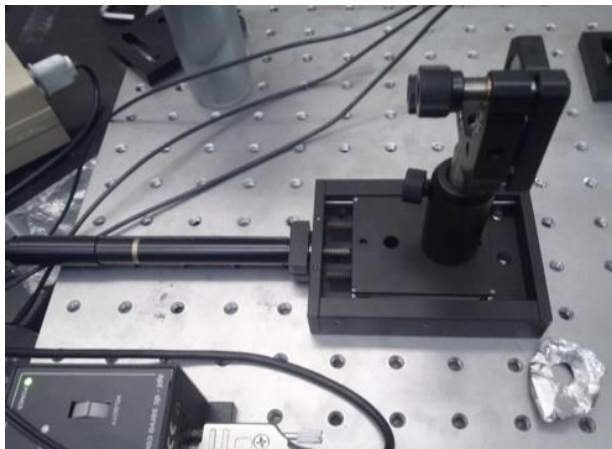
- White light source includes all wavelengths in visible spectrum
- Connected to setup with fibre optic cable
- Focused through samples to perform optical spectroscopy

### Silicon Detector



- Measures intensity of light incident on detector.
- This experiment contains two: one to acquire interferograms (with white light); the second to detect the laser light to correct for deviations from non-constant stepper motor.

### Translation Stage + Actuator



- Responsible for holding and controlling motion of movable mirror
- Motion can be controlled with provided software, or manually with onboard controller
- Stepper motor may have non-constant velocity, which can complicate time-domain measurements. Data Analysis software employs algorithms to correct this deviation based on HeNe calibration data.

### Translation Stage + Actuator



- Used to manually adjust position of movable mirror



### Relay lenses, beam splitters, and mirrors



- Steer beams to determine optical path
- It is essential that the laser light be aligned properly, otherwise fringes cannot be produced.

### Interference filters

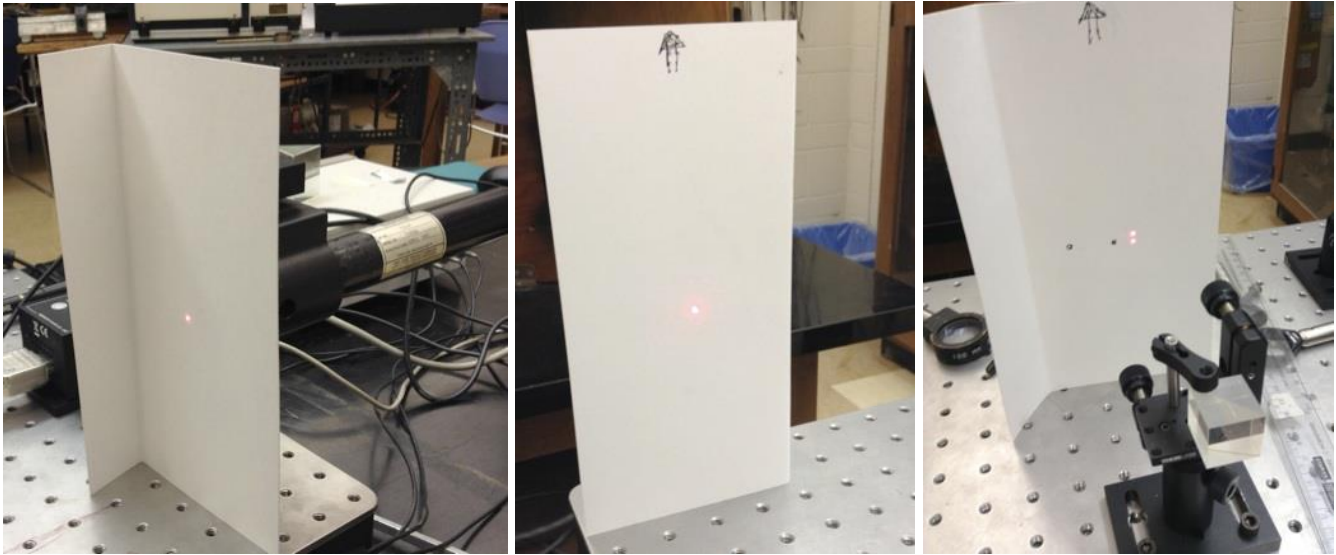


- Placed in sample holder to filter white light source in initial experimental investigations.
- Note that the transmission characteristics of interference filters change if the incident light is not parallel. The wavelength of maximum transmission shifts to lower wavelengths and the transmission curve broadens. You will be measuring the transmission characteristics of filters and comparing them to measurements made using a monochromator, so it is important that the light be parallel.
- Green filter is recommended for finding the zero point, since the light passed is the brightest among the filters.
- Replaced by other samples, e.g. ink in water, for optional investigations.

### Alignment

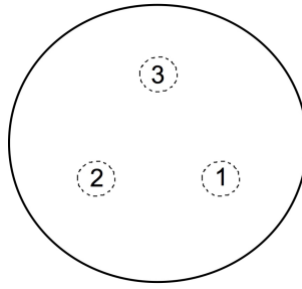
- Find a rectangular piece of stiff white piece of paper and a plastic ruler.
- Switch on the HeNe laser and turn off unnecessary lighting.
- Place a rectangular piece of stiff white paper flush with the front of the laser and mark the beam spot. The paper can now be used to check that the beam is horizontal throughout the set-up. the middle picture below is good (the mark is invisible under the laser spot). The third picture shows a problem since there are two spots, neither at the right height; a mirror needs adjustment.





- A transparent ruler can be used to check alignment of reflected beams; the picture to the right shows a reflected beam that is not aligned to the initial beam.
  - Place a white screen (e.g. a white piece of paper or ground glass) at the location of the detector.
  - If there is no apparent interference pattern, check the position of each output light through the interferometer by blocking each light path.
  - Adjust the interferometer mirrors using the knobs on their backs such that the position for each light output is the same.
  - From here, adjust mirrors carefully using the adjustment knobs until you observe interference fringes
  - Adjust the mirror screws and move until getting the widest possible circular interference patterns. (Tip. Move the screws in the direction that it gives you wider spacing between two consecutive fringes. As you do this, the fringes become more and more curved. Since the central fringe is bigger than the beam splitter cube in use, if aligned perfectly, you should be able to see rectangular uniform intensity background.)
1. Since seeing the circular fringes is difficult when the distances of the two arms are equal, move the actuator so that the difference in this distance is about 1 cm. (The field of view of the interference pattern should increase as the distance diff between the arms increase. Why? This means that you should be able to see more fringes per area as the distance difference increase)
  2. Now use the alignment tip mentioned above to make sure that the interference pattern is circular. As you do this, make sure that the circular interference pattern is centered with respect to the shadow of the rectangular beam splitter cube.
  3. To help you with very fine alignment, turn on the piezo controller and use the X,Y,Z knobs to center the circular interference pattern with respect to the shadow of the cube.





**Figure .** Location of the piezoelectric crystals on the fixed mirror.

4. At the end, you should be able to see many concentric circular fringes that is the characteristic of a Michelson Interferometer. (Why do you see circular fringes ?)
  5. Basically, you are trying to do the same thing when the difference in the distance of the two arms of the interferometer is zero. Except it would be harder to get the feel of alignment since the central circular fringe will take up the entire field of view.
- Set the arm distance such that the difference is zero. Use the same approach to alignment as above.
  - Once everything is aligned, take away the sheet of paper that you used to make way for the light.

### Data Acquisition

Use the "Signal Express" program

1. Click Add Step → Acquire Signal → DAQmx Acquire → Analog input → Voltage
2. Choose the aio and ai1 channels
3. Set Step Setup:Timing Settings:Acquisition Mode to Continuous Samples
4. Choose your sample Rate and number of Samples to read.
5. Press on the DataView tab
6. Click Run
7. Expand DAQmx Acquire
8. Setting up Digital Filter in Data Acquisition:
9. Drag Dev ai0 channel into the window channel. You should now be able to observe some signal.
10. In order to isolate the noise software based filters can be applied to the signals: Press on Add Step Signal Processing Analog Signals Filters
11. Customize the filter specifications through the tool box. The following filters are recommended:
  - (a) For the ai0 channel (Detector Type: DET36A): (This filter is recommended for the HeNe light source) i. Specify input data for the filter: ai0 ii. Type: IIR Butterworth Bandstop Filter iii. Upper and Lower cutoffs based on the frequency of noise. E.g. 60Hz noise would need a bandstop of 50-70Hz
  - (b) For the ai1 channel (Detector Type: PDA36A): (This filter is recommended for the Broadband light source) i. Specify input data for the filter: ai1 ii. Type: IIR Butterworth Lowpass Filter iii. Cutoff: 200 Hz
12. Add two new displays and drag the two filtered data outputs to the displays.
13. Acquiring data: To record the data, click on the Record button next to the Add Step button. This opens up a pop-up window where the data channels to be recorded can be selected, as well as the name of the record and an optional description.

## Moving Mirror Motor Control

Double click on APT User<sup>1</sup> program on computer Desktop. If it is not there, go to: Start Menu → All Programs → Thorlabs → APT → APT User.

- Set the Jogging to Single Step, and choose the Max. Velocity and Step Distance; 0.05 mm/s and 0.001 mm are reasonable initial values.
- Operating
  1. Click the position display window, a pop up will show up that asks for the position in millimeters you want the actuator to be in.
  2. Specify the position and press Enter. This should move the actuator with the velocity specified in the settings.
  3. (Warning: if you try to move to a position that exceeds the physical limits of the actuator, the software will give you warnings.)

## Experimental Procedure

### HeNe Laser

1. Align the setup for the HeNe light. Make sure the arm distance difference is zero.
2. Use the Data Acquisition procedures to set up data acquisition for one of the detectors. For the HeNe part of the experiment, you only need one detector.
3. Make sure the output of the interferometer hits one of the detectors. An iris or home-made aluminum foil cover with a pin-hole may be used later reduce noise or improve resolution.
4. Before recording the actual data, set up the motor control using the Motor Control Procedures.
5. Set the actuator to move about 1mm. (If your alignment is not excellent, moving the actuator too far may move your system out of alignment.) Do not move the detector just yet.
6. Record the data and while it is recording, quickly go back to the actuator control and move the actuator by pressing Enter.
7. After the stop recording the data when the actuator is stopped.
8. The recorded data can be found in the Logs section in the Signal Express software. Right click the individual data streams (usually named filtered data, filtered data1) to open their file location in Windows. The file can be opened only with Excel, so remember to save them as .xlsx or any compatible format if planning on analyzing the data later. Isolate the points corresponding to the interference pattern and create two files with the data points from both detectors. Note: It is important that there is a one-to-one correspondence between the HeNe data and the data for the light under examination in order for the phase correction software to work properly.
9. Make a FFT of the data to assess its quality.

### White Light

Finding the zero point

- To observe interference fringes for broadband light such as the white light, you need to set the two arm length distance to be exactly equal within few micron of tolerance. (Why?)

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<sup>1</sup> [http://www.thorlabs.com/software\\_pages/ViewSoftwarePage.cfm?Code=APT](http://www.thorlabs.com/software_pages/ViewSoftwarePage.cfm?Code=APT)

- To find the position of the actuator where the broadband interference fringes can be observed, first carefully measure the distance of each arm length using a plastic ruler.
- After recording the positions, move the actuator so that the arm distances are roughly the same.
- Using the HeNe light only, adjust the setup at this position using the Alignment procedures. The alignment for the HeNe light should also align the setup for the broadband light source.
- Turn off the HeNe light and turn on the Broadband light source. To see the interference fringes for this light source, use the J-Jog feature of the actuator controller, or the controller software and move the actuator around the expected zero difference position.
- As you do this, you should be able to see interference fringes for the broadband light source if you put a piece of white paper like you did in the alignment procedure section.
- Now, use the Alignment Procedures again to align for the broadband light this time. Use the procedure with the broadband light only.
- Make sure the alignment is good across all positions where the interference can be observed. (Note, the position where the interference fringes are the brightest is where the difference in the two arm of the interferometer is exactly zero. However this is not always true, why ?).

#### Detector setup

- Turn on the HeNe laser light and the Broadband light.
- The output of the interferometer should split into two beams of light at the last beamsplitter cube after the focal lens.
- For the HeNe light detection, put the detector on the right side of the beamsplitter cube such that the ONLY HeNe light dot hits the detector.
- For the Broadband light, put the detector on the bottom of the beamsplitter cube such that only the broadband light dot hits the detector.
- This ensures that each detector reads light from individual light source independently.
- (Note: If the interference signal is weak when you take data, try narrowing the light dots by moving the detectors backwards and forwards. This should focus more light into the detector for improved intensity)

#### Running and measuring data

- About 0.5mm away from your measured position where the arm length diff is zero.
- Go to the actuator controller software, set the actuator to move about 1mm. This should scan all range of the interferogram. However, do not move the actuator just yet.
- Use the procedures for the data acquisition to acquire data for both detectors (Note, record the data that is filtered using the recommended digital filters.)
- Press record and afterwards quickly move the actuator to your specified position.
- Once the actuator stops moving, stop recording the data.

#### Pre-analysis

- In the acquired data, you should crop the regions where there is actual interference for both detector signals (Why? Hint: Alignment). (When you crop the data, make sure you crop both HeNe and broadband light data at the same places in time, and make sure that they have the same number of data points)

- Analyze the interferograms and obtain the spectrum of the light under examination.

## ANALYSIS

You may use a variety of programs to analyze the data. Sample Python analysis code is provided (but as of 11 September 2014 it has not been debugged). The code is quite involved. Do not use the code as a black-box. An essential step in understanding the experiment is to go through the code and identify the analysis process.

This code uses data provided by the user for the intensity of the light being examined and data for the HeNe laser taken at the same time by the two detectors. Due to the non-constant speed of the stepper-motor, and the need for the data to be taken at constant phase intervals, one cannot just Fourier transform the obtained data without modification.

With this in mind, the code does two things: a) It accounts for the non-constant velocity by comparing the data of the examined light with the HeNe data and then creates a vector of new data points corresponding to points at constant phase intervals. b) It performs a Fast Fourier Transform

As the data is taken in time domain and the motor moves at a non-constant speed the data for the HeNe laser deviates from a sinusoidal by being narrower or wider between different zero crossings. By assuming a phase of  $\pi$  between consecutive zero crossings one can assign phases to the data obtained and in this way one can then take data points corresponding to constant phase intervals.

One then performs a discrete Fourier-transform on the data to obtain the power spectra of the examined light.

A more detailed description of the code can be found in appendix III. To run your analysis, simply specify the file path to your data in the provided code.

## QUESTIONS

1. Before beginning the experiment, trace out the path the light will follow on the bench.
2. Explain why changing the path difference does not change the brightness of the fringes for the HeNe laser, but it does for non-coherent sources (e.g. white/filtered white light).
3. The fringes in the Michelson interferometer are called fringes of equal inclination. What does the term “equal inclination” refer to?
4. Consider the situation where the mirrors are exactly parallel. If the lens and detector were removed from the output arm of the interferometer and replaced by a white screen, would interference fringes be seen on the screen? Why or why not? Does your answer depend on whether or not the separation between the mirrors is zero?
5. As the distance between the mirrors becomes smaller what happens to the ring pattern? Why?
6. The Convolution theorem states that given a function  $h(\tau) = \langle f(t)g(t-\tau) \rangle$  (the convolution of  $f$  and  $g$ ), then its Fourier transform is given by  $\tilde{h}(\omega) = \tilde{f}(\omega)\tilde{g}(\omega)$  (where  $\tilde{h}$ ,  $\tilde{f}$ ,  $\tilde{g}$  are the Fourier transforms of  $h$ ,  $f$ ,  $g$ ). This can be used to show that the Fourier Transform of  $\Gamma_{11}(\tau)$  is the power spectrum. The correlation function for two electric fields,  $E_i$  and  $E_j$  is defined as  $\Gamma_{ij}(\tau) = \langle E_i(t)E_j^*(t+\tau) \rangle$ , where the electric fields are averaged over  $t$ . In the case of a perfectly monochromatic beam with  $E_1(t) = E_0 e^{-i\omega_0 t}$ , the autocorrelation function is  $\Gamma_{11}(\tau) = \langle E_1(t)E_1^*(t-\tau) \rangle = |E_0|^2 e^{i\omega_0 \tau}$ . What is the power spectrum  $P(\omega)$ ? Is this what you would expect? Why?
7. A more realistic coherence function is given by  $\Gamma_{11}(\tau) = |E_0|^2 e^{-i\omega_0 \tau} e^{-l_0 \tau/c}$ , where  $l_0$  is the coherence length of the source. The resulting Power Spectrum is a Lorentzian centered on  $\omega = \omega_0$ , with FWHM  $l_0/c$ . Use this to compute the coherence length of the red light.

8. [Answer this question if you have used the red (632.8 nm) filter with the narrow (3.6 nm) bandwidth.] The transmission curve for this filter is approximately Gaussian. For a Gaussian,  $I(\delta)$  is given by

$$I(\delta) = ke^{-\left(\frac{\rho}{2}\right)^2 \left(\frac{\Delta\lambda}{\lambda_0}\right)^2 \left(\frac{\delta}{\lambda_0}\right)^2}$$

where  $k$  is a constant,  $\Delta\lambda$  is the full width at  $1/e$  points in the transmission curve, and  $\lambda_0$  is the central wavelength of the transmission curve. When  $I(\delta)$  falls to  $1/e$  of its maximum at  $\delta = \delta_e$  then from the equation above,

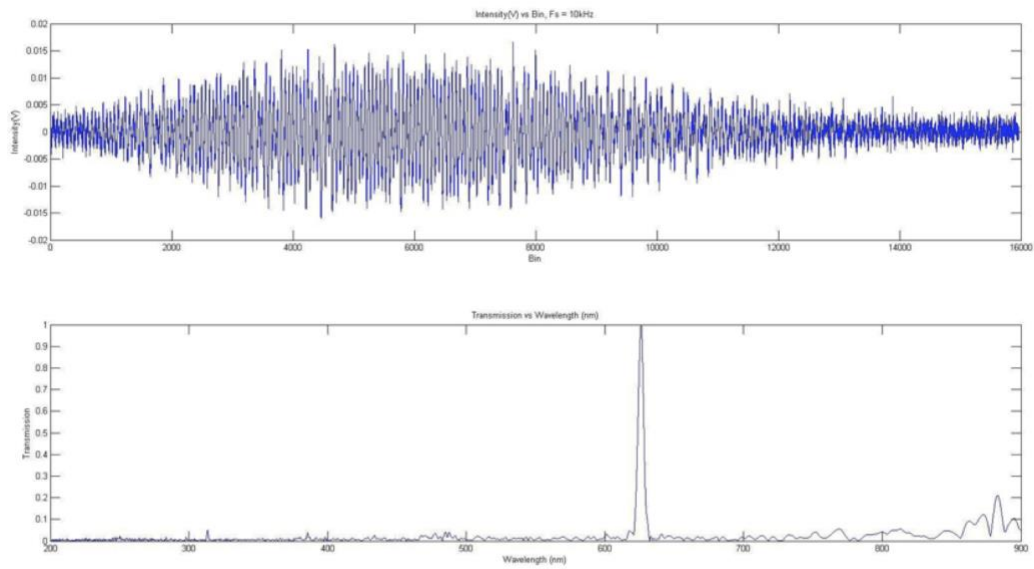
$$\frac{\Delta\lambda}{\lambda_0} = \frac{2}{\pi(\delta_e / \lambda)}$$

Find  $\delta_e$  in units of  $\lambda_0$  from your interferogram and calculate  $\Delta\lambda$  by assuming a value for  $\lambda_0$ . Does this value agree with your  $FT$  data?

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## APPENDIX I : SAMPLE SPECTRA



**Figure I.1:** Spectrum obtained for white light passed through a red filter