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Please send any corrections, comments, or suggestions to the professor currently supervising this experiment, the author of the most recent revision above, or the Advanced Physics Lab Coordinator.
INTRODUCTION

This challenging experiment gives you an opportunity to join a cohort of experimentalists involved into creation of scientific theory governing the behavior of vertically vibrating layers of granulated materials. Principles of non-linear dynamics and theory of chaos form theoretical background of this experiment.

The objects of study are standing waves in a thin layer of fine grains that will be sometimes called “sand” for short. Almost perfect spherical grains however are produced of brass and have nothing in common with real sand on a beach! Placing a container with a layer of grains on a vibrating shaker, you will observe a variety of patterns; stripes, squares, hexagons and their combination. Your goal is to find the relationship between a set of parameters of the pattern (wavelength, frequency and amplitude), properties of the substance (diameter of grains and thickness of the layer) and conditions of the pattern formation (frequency and amplitude of the driving force, surface area and the shape of the container).

Besides aesthetic pleasure while creating and modifying a variety of granular patterns, you can find a lot of amazing features of the patterns in the sand: 2D crystal lattice behavior; dislocations in the pattern structure, multi-domain structure, spinning of the pattern, slowly growing pattern of one type that is steadily transforming in the other type of the pattern without any visible change in the condition of shaking the container, and many other.

You are encouraged to make your own search on the Internet for new discoveries or observations in the field of this physical phenomenon, which is under current study by a number of laboratories world wide. Some important articles and books for reading are in the list if references.

Theory of parametric oscillator

A complete review of all latest discoveries related to experiments with standing waves in granulated substances is given in the book [1]. The book is available at the department library. Access to electronic version at http://simplelink.library.utoronto.ca/url.cfm/150354 is possible from the St.George campus. On-line version represents some videos of the vibrating granular layers.

The basic equation governing the behavior of a layer of sand in a vertically vibrating system is same as for a fluid and describes a parametric oscillator. The parametric oscillator equation for an infinitely small fraction of the surface is given by:

\[ z'' + 2\mu \dot{z} + \omega_0^2 [1 + \alpha(t)] z = 0, \]  

(1)

where \( z \) is the vertical position of the infinitesimal fraction of the surface; \( \mu \) is the damping rate [damping rate = damping ratio multiplied by \( \omega_0 \)], similar to the viscosity of a fluid; \( \omega_0 \) is the frequency of oscillation of the fraction of the surface; and \( \alpha(t) \) is the non-dimensionalized oscillating parameter function. This equation is to some extent similar to that of a damped harmonic oscillator. The oscillating parameter is the gravitational field, which in the frame of the shaker can be expressed as

\[ g(t) = g + A\omega^2 \cos(\omega t) = g [1 + \Gamma \cos(\omega t)] \]  

(2)
if the driving force is a harmonic function; \( \omega = 2\pi f \); \( f \) is the frequency of the driving force; and \( A \) is the amplitude of vibrations of the shaker. The ratio \( \Gamma \) (gamma) of the maximum linear acceleration of a shaker and the free fall acceleration is widely used under the term “amplitude”. Eq. (1) cannot be solved analytically in general, i.e. for any possible combinations of \( \mu, A, \omega_0, \omega \) and function \( \alpha(t) \). Substitution of Eq. 2 into Eq. 1 gives

\[
\ddot{z} + 2\mu \dot{z} + \omega_0^2 \left[ 1 + \Gamma \cos(\omega t) \right] z = 0 \tag{3}
\]

Certain combinations of \( \Gamma \) and \( \omega \) form the resonance regions (‘resonance tongues’), where \( z \) grows exponentially with time. It means that equilibrium state with \( z = 0 \) is unstable in these regions, and even a negligible deviation from this state can take the system into parametric resonance with an exponential growth of \( z \).

The parametric resonance occurs at frequency ratios \( \omega/\omega_0 \approx 2/n \), for \( n = 1, 2, 3, \ldots \) The odd-\( n \) modes are called subharmonic, while the even-\( n \) modes are called harmonic. The stable pattern is observed under the resonance condition regardless the frequency ratio.

### Safety Reminders

- **Grains on the floor can create a dangerous trip hazard.** Using the brush and pan provided, immediately clean up any grains that escape the shaker or otherwise end up on the table or floor; these can. If the brush and pan are missing, contact the lab technologist.
- Do not touch the moving parts of the shaker when the shaker is operating.
- Turn off the amplifier before touching the power leads to the shaker or otherwise working on the shaker.

**NOTE:** This is not a complete list of every hazard you may encounter. We cannot warn against all possible creative stupidities, e.g. juggling cryostats. Experimenters must use common sense to assess and avoid risks, e.g. never open plugged-in electrical equipment, watch for sharp edges, don’t lift too-heavy objects, … If you are unsure whether something is safe, ask the supervising professor, the lab technologist, or the lab coordinator. When in doubt, ask! If an accident or incident happens, you must let us know. More safety information is available at [http://www.ehs.utoronto.ca/resources.htm](http://www.ehs.utoronto.ca/resources.htm).

### Experimental setup and equipment (See Fig. 1)

a. **Camera**
   Two different kinds of cameras are used in this experiment. The Logitech Quickcam Pro 9000, produces 30 frames per second; however it easily takes quality pictures for measuring the wavelength of the pattern. (The User Guide is available at [http://www.logitech.com/repository/1403/pdf/25618.1.0.pdf](http://www.logitech.com/repository/1403/pdf/25618.1.0.pdf)).
   The Casio EX-FS10 is used to take videos for frequency analysis. It is suggested to be set at 210 frames per second to obtain the best quality video. Its specifications and user instructions are available at [http://www.physics.utoronto.ca/~phy326/far/camera.pdf](http://www.physics.utoronto.ca/~phy326/far/camera.pdf).

b. **Container**
   Two containers are used for this experiment: a circular container with an inner diameter of 17.1 ± 0.1 cm and a square container with length 17.8 ± 0.1 cm on each side. Both of the containers have an inner height of 11.0 ± 0.1 cm.
c. **Sample**
There is a variety of samples with different diameters of bronze grains available for this experiment. Some cans contain calibrated grains, while diameters of the other must be studied. The recommended grain size for this experiment varies between 0.1 mm and 0.5 mm.

d. **Light Source**
Two light sources may be used to illuminate the surface of the grains. They are placed around the containers, as shown in Fig.2. The first is a rectangular white light source with a side length of 25 ± 1 cm, and the other one is a circular red light source with a diameter of 25± 1 cm. Both light sources have approximately one LED per centimetre of the perimeter.

e. **Light Power Supply (not shown)**
The power supply used is a HP 6216A power supply. (Datasheet available at http://www.teknetelectronics.com/DataSheet/HP_AGILENT/HP_6216A.pdf).

f. **Accelerometer**
Accelerometer MMA1220 is attached to the side of the container with a screw. The accelerometer is used to measure the vertical linear amplitude of the container. (Datasheet available at http://pdf1.alldatasheet.com/datasheet-pdf/view/188021/FREESCALE/MMA1220.html). The screw can be attached and removed using a 7/64” Allen key.

g. **Sticky Blanket**
The sticky blanket is placed under the base for the loudspeaker to prevent it from walking around.

h. **Retort Stand**
Retort stand is used to hold the light sources in place. Height can be adjusted.
i. **Wave Function Generator**
   DDS Signal Generator UD-B is very simple in use.

j. **Oscilloscope**
   The oscilloscope Tektronix TDS2001C is used to measure the linear vertical acceleration of the container. It can be reattached to the lateral surface to measure the horizontal vibrations.

k. **Loudspeaker**
   A loudspeaker is used as a powerful shaker for granular materials.

l. **Amplifier**
   A Speaker Crown XLS1000 is used to magnify the signal produced by the function generator.

m. **Base**
   The base is used to level the loudspeaker. There are three screws used to adjust the levelling.

The other equipment includes: stands and holders for cameras and lights; vessels; beakers; meshes to sieve grains; a stroboscope; PASCO motion sensor and interface, and a microscope to find a distribution of sizes of grains and to verify their shape.

![Motion sensor, container with sand, strobe, white light source, shaker](image)

**FIG. 2:** Motion sensor, container with sand, strobe, white light source, shaker

**Calibration of accelerometer**

The shaker is expensive and delicate. Students must take care to set the amplifier gains to minimum before turning on the system, gradually turning up the gain knob during use. The input frequency should never exceed 50 Hz. The range of interest is $10 \text{ Hz} \leq f \leq 30 \text{ Hz}$, but due to limited power of the shaker and heavy load it is driving the upper limit can be smaller than 20 Hz. When under specific conditions the beats are generated in the system, the shaker can be broken. If you see and hear the typical features of beats instead of a harmonic mode of vibration, immediately set the amplitude to zero with following adjustment of the driven frequency, just to shift the mode of vibration away from the dangerous state.

Before measurements and observations can be made, the calibration of the accelerometer must be studied. It is recommended to always have a supply voltage $V_{DD} = 5\text{V}$ which however can be
smaller. Use a multimeter to measure $V_{DD}$ from time to time to correctly choose the accelerometer sensitivity. The full accelerometer calibration is attached to the manual.

The output signal of accelerometer is measured with oscilloscope in millivolts. For a sinusoidal signal, the amplitude of acceleration $a_{\text{max}}$ of the plate is $a_{\text{max}} = (2\pi f)^2 A$, where $A$ is the linear amplitude of vibration of the plate of a shaker. The function $\Gamma = (2\pi f)^2 A / g$ is used instead of the amplitude $A$ as a parameter of vibration. $\Gamma$ is a convenient measure because it can be easily obtained by dividing the amplitude $a_{\text{max}}$ measured in millivolts by the value of sensitivity.

**Measurements and calibrating sizes of the grains**

![FIG.: 3 – Apparatus for measuring sand grains.](image)

1. Magnifying glass – Finescale magnifying comparator (must be borrowed in MP 250)
2. Light source – AmerTec LED Soft Glo Continuous On Nite Lite 120V – 60 Hz, 0.3W max
   (Must be borrowed in MP 251 A)
3. Glass slide

Set up apparatus as shown in Figure 2.

1. Make sure all surfaces are clean.
2. Measure each grain by taking a few grains and dropping them onto the glass slide creating a monolayer of grains.
3. Bring magnifying glass against the grains on the glass slide. Screw the eyepiece clockwise or counter clockwise to adjust it to and measure the grains. Do this by scanning the area, measuring the grains one by one.
4. Take as many measurements as you can ($\geq 50$ measurements) to draw a histogram. If the distribution of diameters is not bell-shaped, use meshes to create a set of grains with a
narrow bell-shaped distribution function. The mean value of the diameter will be used in all your calculations.

**Measuring the thickness of the layer of grains in the container**

1. Recommended grain depths should be between 1.1 mm and 4.0 mm to form a layer of 8 – 11 diameters of the grains. Multiplying this value in centimetres by $316.84 \pm 1 \text{ cm}^2$ if it is in the square container and $229.66 \pm 1 \text{ cm}^2$ if it is in the circular container, you obtain the desired amount of sand in cm$^3 = \text{mL}$.
2. Make sure all equipment is clean and there is no sand leftover in the container, graduated cylinder or funnel.
3. Pour the grains into the graduated cylinder using the funnel marked “for grains”.
4. Try to empty the contents of the graduated cylinder as much as possible (some will be sticking to the bottom). It has been noted that flipping over the graduated cylinder and tapping it down against the bottom of the container very gently helps in trying to remove the contents.
5. Once the desired amount is obtained, try to make the layer of grains as flat as possible.
6. Measure the height of the grains in the container as best as possible. Use a ruler and place it outside of the container vertically and measure the height at equal intervals. It is suggested to measure the height at each of the screws (therefore 12 measurements for the square container and 8 measurements for the circular container).
7. Take an average of the grain height. Compare it to the calculated height.

Note: It has been observed that usually the measured height is slightly smaller than the calculated height because of effect of the packing parameter, which is different in the graduated cylinder and the container. The container appears to have a larger packing factor.

**Turning-on procedure**

a) Before turning on the power button of a signal generator, set amplitude to minimum
b) Level the shaker’s plate by adjusting the screws at the shaker’s platform
c) Firmly stick the empty container to the shaker with screws
d) Turn on the power supply for signal generator, amplifier and accelerometer
e) Set frequency to 10 Hz and choose the sine wave generation
f) Turn on an oscilloscope and set it to measure the amplitude of a signal in millivolts in the range of frequencies between 10 Hz and 30 Hz.
g) Adjust illumination of the sample: quick camera demand higher luminosity and perhaps, additional lights.

**Onset of patterns in cylindrical and rectangular containers**

Make preliminary experiments with different setting to obtain different patterns and find the approximate range of frequencies and amplitudes where the majority of patterns can be observed. You are expected to report squares, stripes and hexagons in two containers. If you managed to see a clear pattern, adjust the camera position and illumination to improve the image on the screen and make a snapshot. Also sketch the wave profile from the side view. Begin your measurements with a square pattern in both containers.
Measurement of wavelength and frequency of the standing wave.

Recording video using Logitech Quickcam Pro 9000

1. On the desktop there will be an icon called Video Capture. Double click it.
2. When window pops up, bring down the capture menu and make sure Capture Audio is checked off. If it is checked on, check it off.
3. From the capture drop down menu click Set Frame Rate… A window will appear. Enter 30 f/sec. This is the maximum number of frames per second on the Logitech Quickcam.
4. Go to the File dropdown menu. Click Set Capture File… Type in the file name. Make sure to add .avi to the end of the name, otherwise it will not save it in avi format.
5. Set File Size window will open. It will allocate space for the video. Typically a 5 second video will take about 80 MB of data. If one goes over the capture file size, it will adjust and allocate more space.
6. Click on the Capture drop down menu and click Start Capture.
7. Ready to Capture window will pop up. Click Ok when you want to start recording. To stop recording, click the drop down menu Capture and click Stop Capture.

Taking snapshots for Wavelength Analysis with Logitech Quickcam Pro 9000

1. Open up the video using VLC Media Player. Pause the video where one wants to start looking through frame by frame.
2. Find the button that has a reel and down arrow. Every time this button is pressed, it will move to the next frame. If this button is not visible, right click under the video screen, hit view and click Advanced Controls.
3. Once the desired frame is in view go to the Video drop down and click Take Snapshot. Check to make sure the snapshot looks like it’s suppose to. Oftentimes the colours are inverted with the reason unknown. If this happens repeatedly, take second approach:
   - On keyboard press Print Screen button. Open Paint. Paste the image into Paint. Crop out the desired picture from the image.

Wavelength calculating software

1. Adjust frequency and amplitude to obtain a stable standing pattern. Make sure it is illuminated by either the red or white strobe.
2. Position the camera at an optimal height and take a video of the standing wave pattern for approximately 10 seconds with at 30 fps.
3. Using VLC Media Player (or the media player of your choice) view the video frame by frame and choose the best images where the peaks are clearly shown.
4. Open up the best images in MS Paint and measure the diameter (or side length) of the container in terms of pixels.
5. Divide the real-life side length of the container (in mm) by the measured side length (in pixels) to obtain a unit to pixel conversion ratio. Error of the conversion ratio can be obtained by measuring many pictures with this camera position and then finding the standard deviation.
6. Crop the picture such that only the pure pattern is left and save all the cropped pictures in one folder.
7. Open up enhanced_image_fft.py and enter the conversion ratio with error. Click on “Select Directory” to select the folder that contains all the cropped pictures.

8. The output file will display the wavelength with its corresponding angles in the window. Two files will be created, output.txt which contains file names, wavelengths and uncertainties and outputWithAngle.txt which contains file names, wavelengths, angles and their uncertainties.

Troubleshooting

- To test out the software at first, users should use graph paper or lined paper to make sure the software accurately measures the wavelength. Try to estimate the pixel length of the wavelengths themselves, and then compare it to what the software shows.
- The software works better with red light compared to white light, since the intense white light produces many shadows thus producing inconsistent results.
- Angle should always be checked to determine if software has done its job correctly.
- Results that do not make any sense should be discarded.
- Larger wavelengths produce greater uncertainty. This is because in the 2-D FFT, the wavelengths are converted to frequencies. The larger the wavelength, the smaller the frequency. The software calculates the frequencies by measuring the distance to the brightest peak and then converting it back into a wavelength. The uncertainty used in the software is ±1 pixel in the 2-D FFT. Since the frequency is small, the ±1 pixel creates a huge uncertainty. Smaller wavelengths produce larger frequency and therefore the ±1 pixel does present as much uncertainty.

Wavelength Analysis Program

Steps taken:
1. The image is taken in as data.
2. A 2D FFT is performed on the image pixels, giving all the coordinates a certain weight.
3. Only points above a certain threshold are chosen (default threshold = 3)
4. Find the n most prominent peaks in the 2D FFT (default n = 4)
5. Take each peak and its surrounding 8 pixels and find a weighted center and an average weight.
6. Determine the x distance, y distance and angle between the peaks. The distance represents the frequency.
7. Use \( T = \frac{1}{\sqrt{f_x^2 + f_y^2}} \) to find the period.
8. Convert period to wavelength using conversion ratio provided by the user.

For more information on 2D FFT, see http://www.qsimaging.com/ccd_noise_interpret_ffts.html

Calculation of uncertainty

Uncertainty is calculated using the pixels in the 2D Fast Fourier Transform. Uncertainty is given by ±1 pixel, and the error propagates through to the final result. This means that images that have larger wavelengths have larger errors.
Instructions on how to use the frequency calculating software

1. Double click on the m-file VideoSpectrum.m and the MATLAB interface will appear.
2. Enter “VideoSpectrum(‘’);” where ‘’ is the absolute path to the video that is analyzed.
3. A window will appear showing a picture of the video and one should click on a good part of the image around which the analysis will take place.
4. The frequency and estimate of the uncertainty will then be displayed in the MATLAB interface.

Troubleshooting

- Use videos that only have a length between 4-5 seconds to prevent the program from running out of memory.
- Try to use the highest frame rate possible to get best results for the frequency. Software is programmed for 210 Hz, but can be changed within the code.

FFT of signal on the oscilloscope Tektronix 2001C

An FFT of the signal is created because we must make sure that the driving frequency for the shaker only has one mode. If other frequencies exist, then parametric resonance for stable patterns will occur at different frequencies as well.

Make sure entire waveform appears on screen of the Tektronix TDS2001C oscilloscope. It is recommended to have the SEC/DIV set at 2.50 s to get the proper scope in the FFT. Click Math and make sure the FFT option operation is selected and along with the correct source.

Sand grains sticking to the walls of container

When the container is shaken, the bronze grains stick to the sides of the container. This is a problem because not all of the mass participates in the patterns. One possibility as to why this occurs could be because electrostatic forces are created by the grains rubbing against the walls of the container. To try and mitigate this effect, the grains should be rubbed off the sides of the container, just to make sure that all the mass participates in the experiment.

It has been shown with the high speed camera that some of the grains on wall move but do not detach themselves from the wall. Since there is movement in the horizontal direction as well as the vertical direction, there must be some forces in the horizontal direction as well.

Mapping onset conditions for circular and rectangular containers

The pattern onset condition of a particular container and sample is best represented using a phase diagram with $\Gamma$ vs. driving frequency system of coordinates (see [2]). You can choose a sample of a particular size of grains and thickness of the layer on your own. We can recommend an 8-layer bronze sample with a diameter of 0.33mm as one of the best suitable to observe all possible patterns for both containers. Fig. 23 is a copy of the phase diagram using the cylindrical container with the above condition. Moreover, it should be noted that phase diagrams as such should be produced by taking discrete values for frequency and slowly decreasing the dimensionless acceleration $\Gamma$ to record data points where a new pattern starts to form. Phase diagram is created this way as changing the frequency also varies the dimensionless acceleration $\Gamma$. 
**Dispersion relation**

The dispersion relation gives the wavelength of the pattern as a function of driving frequency. It is important to note that while doing these experiments, the dimensionless acceleration $\Gamma$ must be fixed. Different sources reported different functions for the dispersion relation with $\Gamma$. You can try to verify the following two of them published for discussion in [3]:

1. \[ \frac{\lambda}{d} \propto \left( f \sqrt{\frac{d}{g}} \right)^{\alpha} \] where $\alpha \approx -2$ for lower frequencies but $\alpha \approx -2/3$ for higher frequencies;

and

2. \[ \frac{\lambda}{h} \propto \text{const} + \left( f \sqrt{\frac{h}{g}} \right)^{\alpha} \] where $\alpha \approx -1.32 \pm 0.03$.

Use the depth of the sand by about an order of magnitude larger than the diameter of a grain. Report your results in graphs of normalized wavelength ($\lambda/h$) or ($\lambda/d$) vs normalized frequency $f \sqrt{\frac{h}{g}}$.

Discuss the results.

**Oscillons (challenging)**

Unlike a well developed theory for Faraday waves in a fluid, standing waves in a vibrating layer of grains are not yet described satisfactory with one particular theory. There are three main approaches [4]: Molecular Dynamics (MD) model; Statistical Mechanics model; and Continuum Mechanical Models. We will not study theoretical details in this experiment.

In some respect, the granular patterns are very similar to Faraday waves in a fluid. However, there is one phenomenon that cannot be observed in a fluid, but has been recently discovered in a vibrating layer of grains and observed in the Advanced Labs. This phenomenon is called oscillons. The oscillon is a localized subharmonic excitation that can be referred to either a peak or a crater. It was experimentally found that under some conditions the oscillons of different kinds interact like oppositely charged particles. The origin of onset of oscillons is still unclear. In this experiment, you get an opportunity to find the onset conditions for mysterious oscillons.

The motion of particles inside the oscillon is very exotic. Take video and several photos for a report that can clarify the character of this motion and describe it in your report. Measure the lifetime of an oscillon.

The recommended sample to begin is a layer of bronze spheres of about 0.15 – 0.20 mm in diameter. In general, the recommended diameter of the grains is between 0.1 mm and 0.5 mm. The recommended frequency range is between 10 Hz and 25 Hz. The frequency of the driving force should be tuned very slowly with a step of not more than 0.2 Hz. The recommended thickness of the layer is between 8 particles to about 18 particles. In this experiment $\Gamma > 1$. Oscillons were observed in our lab for non-magnetic stainless steel spheres and $10.5 \text{ Hz} < f < 12 \text{ Hz}$; $3.0 < \Gamma < 3.4$. They were reported under different conditions in the research [2].
REFERENCES

APPENDIX
Granular patterns and 2D power spectrum obtained by Zexuan Wang and Chao Wang in SURF 2012