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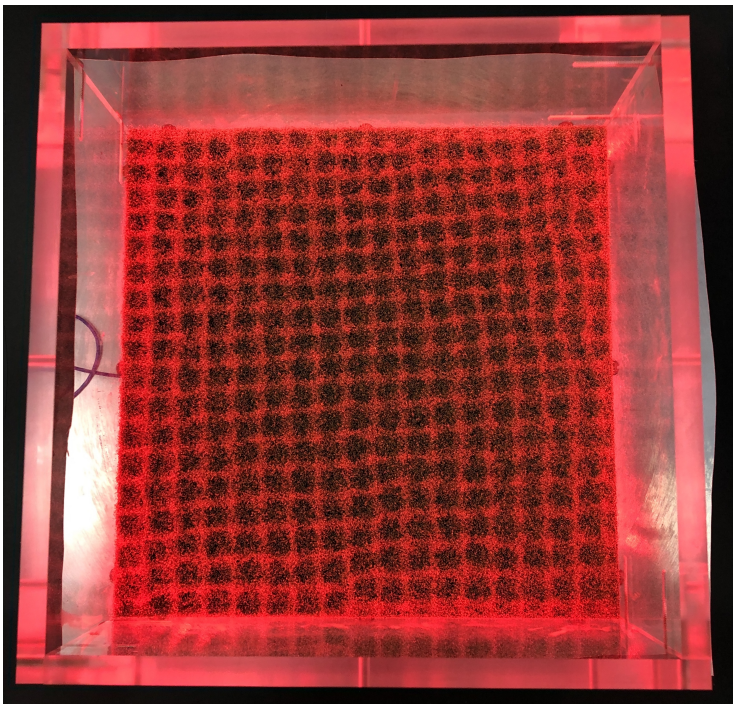
ADVANCED PHYSICS LABORATORY

GRAN

Emergent patterns in shaken granular matter



Revisions:
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Natalia Krasnopolskaia created an earlier version of this experiment, with the help of Summer Undergraduate Research Fellowship (SURF) Students Andrei Dranka (2013), Zexuan Wang and Chao Wang (2012), Jeremy McGibbon and Wenyi Zhao (2011), Yun Tao Bai and Connor Holme (2010).

Please send any corrections, comments, or suggestions to the professor currently supervising this experiment, the author of the most recent revision above, or the Advanced Physics Lab Coordinator.

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1 Introduction

Emergence is a term used to describe a process by which high-level order appears from the many-body collective behaviour of simple subunits. Familiar examples include the flocking of starlings, the shape of snowflakes and the appearance of superconductivity. In each case, the relatively simple interaction of the subunits (birds, water molecules, electrons) somehow gives rise to quite different, but organized, dynamics the level of the whole. It is often claimed of emergent processes that "the whole is more than the sum of its parts". In the present experiment, the subunits are sub-millimeter bronze spheres. These particles obey Newton's laws, with dissipative, *i.e.* frictional and inelastic, interactions during collisions [1]. To activate their motion, and to keep them in motion, requires the steady input of energy, which is constantly dissipated. Our focus is the organized state of motion of a large collection of such grains which is being driven by a shaking plate [2, 3, 4, 5, 6, 7]. The local motion of individual grains is highly random, but their large-scale collective motion is fluid-like and exhibits strikingly ordered patterns. These patterns are universal and are observed in many otherwise quite different systems [8].

2 Theory

2.1 general ideas

The behaviour of assemblies of frictionally interacting solid particles, whether in flow or locked together as a quasi-solid, is a broad area of research. Such assemblies are sometimes considered a fourth state of matter: so-called **granular matter**. The specific case of patterns in "fluidized" grains on a vertically vibrating plate can be approached theoretically and computationally in several complementary ways. The most straightforward, but computationally expensive, way is simply to simulate the motion of a large number of individual grains [6, 9]. Such simulations can be quite realistic, but fail to be explanatory, in the sense that they merely reproduce the patterns without really accounting for *why* they appear in any satisfying way. This is a very general problem with emergent phenomena in which there is always a trade-off between verisimilitude and explanatory simplicity. A different, but still computational intensive, approach is to reduce the motion of the grains to that of a smooth continuum by deriving the appropriate fluid equations [10]. It turns out that the patterns exist in a difficult limit of the continuum equations in which extreme compressibility effects such as shock waves are found [11]. The fluid of the continuum model has no surface tension, so the instability is quite different than the one driving Faraday waves (see **the FAR experiment**). Finally, it is possible to employ very general, symmetry-based, phenomenological model equations [1]. This type of model emphasizes the universal nature of the patterns and makes connections with similar patterns in other systems [8], at the expense of realism. A complete discussion of these various approaches may be found in Ref. [1], chapter 5.

2.2 dimensionless description

To understand any mechanical system in a general way, it helps to reduce the dynamics to dimensionless form. In this way, its fundamental scaling behaviour may be revealed. This is always possible, according to the **Buckingham Pi Theorem**.

The plate is driven by the shaker with a sinusoidal force at a frequency f. In the rest frame of the plate, the grains experience both gravity and an oscillating fictitious force so that they have a total acceleration

$$\vec{\mathbf{a}} = \vec{\mathbf{g}} + \vec{\mathbf{a}}_{\max} \cos \omega t \qquad \text{with} \qquad a_{\max} = \omega^2 A = 4\pi^2 f^2 A, \tag{1}$$

where $g = 9.8$ m/s^2, $\omega = 2\pi f$, and A is the amplitude of the motion. The peak acceleration $a_{\max}$ can be measured directly using a calibrated accelerometer. There is no need to measure A.

The plate acceleration can be made dimensionless using

$$\Gamma = \frac{a_{\max}}{g} = \frac{4\pi^2 f^2 A}{g} \ . \tag{2}$$

Γ is one of the key dimensionless control parameters of the experiment. The grains leave the plate when $\Gamma > 1$ and patterns are observed when Γ exceeds a critical value $\Gamma_c \gtrsim 2.5$. The patterns are typically *subharmonic*, *i.e.*, they oscillate at a frequency $f/2$. Some patterns are even *subsubharmonic* and oscillate at $f/4$. Other periods and chaotic patterns may also be observed [5]. Otherwise identical $f/2$ patterns on different parts of the plate may be one cycle of f out of phase with one another, leading to phase boundaries, which are one kind of defect in the pattern. Small, localized structures called *oscillons* may be observed [4].

Important experimental parameters other than Γ are the frequency f, the size d of the grains and the depth h of the granular layer when at rest. In fact, our grains have a mixture of sizes with 425 μm $\leq d \leq$ 600 μm. We will use the average grain size $\bar{d} = 513$ μm. The grain depth h can be measured by introducing a known volume V of packed grains onto the plate, which has an area $\mathcal{A} = 645.2$ cm^2. Then $h = V/\mathcal{A}$. The depth h can be made dimensionless by

$$H = h/\bar{d} = V/(\mathcal{A}\bar{d}\ ) \ . \tag{3}$$

In the experiment, we will vary H, but keep $\bar{d}$ constant. Putting in the numbers, we find that $H = V/(33.1$ mL). The volume V can be directly measured with a graduated cylinder.

The material of the grains (here bronze, which is chosen because it is not magnetic) also matters. Softer materials, like lead, have more dissipative collisions. The inelasticity of collisions can be modeled [6] with a velocity dependent **coefficient of restitution** $e \sim 0.7$. Since we will always use the same grains, this will also not be varied in the experiment, but is an important parameter in simulations [6].

The frequency f can be made dimensionless by multiplying by the time required for a grain to fall a distance h,

$$F = f\sqrt{h/g} \ . \tag{4}$$

Using the dimensionless variables Γ, F and H, the phase diagram of the ordered states of motion of the layer can be mapped out in the F-Γ plane a general way. Examples of such diagrams are given in Refs. [2, 3, 4, 5, 6, 7].

When a periodic pattern appears, it can be minimally described as having a wavelength λ. Hexagonal and square patterns can be thought of as superpositions of sets of straight standing wave patterns at 60° and 90°, respectively, with λ being the basic periodicity of the pattern. We can make λ dimensionless by dividing by the depth,

$$L = \lambda/h = \lambda\,(\mathcal{A}/V) \ . \tag{5}$$

The relationship between the wavelength and the frequency of the standing wave pattern is called the **dispersion relation**. Since our waves are nonlinear, this relation might also depend on Γ. A simple thing to measure is the dimensionless dispersion relation $L(F;\Gamma_c)$, near the onset of the pattern at Γ_c. In particular, Umbanhowar and Swinney [7] found a scaling law

$$L = a + bF^{\alpha}, \quad \text{for} \quad F < 0.4, \tag{6}$$

with $a = 1.0$, $b = 1.1$ and $\alpha = -1.32 \pm 0.03$, independent of H. For larger F, the data collapse fails and the dispersion curve begins to depend also on H. The fractional value of α is interesting; for standing waves on normal fluids, one expects a F^{-2} dependence. For much more discussion, see Ref. [12].

2.3 air effects

A more subtle experimental parameter is the atmospheric pressure P. Most of the published experiments on these patterns [3, 4, 5, 6, 7] used evacuated cells to avoid air effects. Our cell is open. The grains dilate as they are thrown upward by the plate and tend to compact as they fall down and collide with the plate again [13]. This compaction traps air under the layer and modifies its dynamics. Entrained air can cause heaping effects in which conical piles of grains form on the plate (an effect first identified by Michael Faraday in 1831). Heaping dominates for small grains, like fine powders. A detailed study [13] of air effects suggests that they decrease as $1/\bar{d}^4$. Air effects are also increased in deeper layers, *i.e.* for larger values of $h/\bar{d}$. For these reasons, we have chosen relatively large grains. Nevertheless, some air effects are to be expected.

3 Experiment

3.1 important safety notices

- **This experiment has rapidly moving parts.** The shaker is powerful and you should avoid putting your fingers under it, or in any position where pinching might occur. It **is** safe to put your fingers into the layer of grains from above to stir them around while shaking, if you wish.
- **It is possible to overdrive the shaker and break the stinger.** Using too large an acceleration, especially at high frequency, can cause the thin rod connecting the shaker to the plate to buckle and break. If this happens, the rod must be replaced, which requires technical help. See below for the recommended procedure for avoiding this.
- **Frequency and acceleration limits.** The range of frequencies is limited to roughly 10 - 50 Hz. The maximum peak acceleration at any frequency should not exceed $8g$. The amplifier has a limited current range: do not exceed the full scale of the LED current indicator on the front panel of the amplifier. The shaker has a limited amplitude in its motion. At the lowest frequencies, when the amplitude of the plate motion is maximum, it should not make contact with the leveling plate.

3.2 overview

The objective of this experiment is to observe and measure the patterns that appear as a function of dimensionless amplitude Γ, frequency F and depth H. For periodic patterns, we want to measure the dimensionless wavelength L as a function of Γ, F and H, as well as observing various kinds of defects.

3.3 suggested procedure

Familiarize yourself with the theory of operation of the shaker, the stinger and the air bearing by carefully reading Appendix A.

3.3.1 compressed air

The square air bearing uses compressed air to precisely locate the axis of the shaking. Before doing anything else, turn on the compressed air using the valve on the wall. The pressure gauge should read 80 psi. The amplifier is interlocked so that it will not operate if the air bearing pressure is too low. The compressed air can be left on for the duration of the experiment, but should be turned off when the experiment will not be used for a while.

3.3.2 the signal generator and amplifier

Always begin with the amplifier gain knob turned to zero ("reset") before presenting any signal to the amplifier. The signal generator can then be turned on and set to a frequency in the range 10 - 50 Hz and an amplitude of about 1V, peak to peak. Always use a sinusoidal signal. Then the main gain knob of the amplifier can be slowly increased to the desired level. The main gain is only useful as a coarse control. With the amplifier gain at about half way, the signal generator amplitude functions as a fine control of the acceleration amplitude.

The air blower cooling system for the shaker will operate whenever the amplifier is turned on, even when it is set to "reset". It is a good idea to operate the blower for a few minutes to cool the shaker any time it has been running for a while, even when not shaking.

The front panel of the amplifier has an LED bar indicator of the drive current. This should never exceed the full scale (but you can safely use most of the scale). The amplifier will "trip" (stop operating) if its maximum current is exceeded. A trip condition is indicated by a flashing LED on the front panel of the amplifier. The amplifier will also trip if the air bearing pressure is too low. If the amplifier is tripped, turn the main gain knob to "reset" to start over.

3.3.3 the shaker

Tune up the shaker with no grains first. The shaker must be precisely levelled and adjusted to minimize lateral accelerations.

With no shaking signal applied, but with the air bearing pressurized, check that the stinger is not bent and that it is precisely vertical. This can be achieved by adjusting the lateral position of the levelling plate. Next, use the vertical screws to adjust the levelling plate so that the cell is level

axis	Sensitivity (mV/g)
x	96.0
y	95.8
z	101.3

Table 1: Calibration information for the three axis accelerometer. The z axis is vertical. The x and y axes are horizontal, but (unfortunately) skewed with respect to the major axes of the square cell.

according to the large bubble level. In fact the grains themselves are a more sensitive test of level than any bubble level. You can sprinkle a few gains in the cell and observe their motion when the acceleration is just enough to make them mobile. The grains will migrate to the lowest point of the cell.

Finally, you can monitor one of the lateral acceleration components (x and y) with the accelerometer and adjust the levelling plate to minimize those acceleration components.

When the cell is full of grains, imperfect levelling may be indicated by a tendency of the grains to persistently heap on one side of the cell. This can also happen due to air effects, even if the cell is perfectly level, in which case the heap forms in a random location. You can distinguish these cases by intentionally heaping the grains on different sides of the cell, to see if a heap persists on a particular side. The position of the levelling plate may be adjusted even while shaking, but only small adjustments should be made to avoid bending the stinger too much.

3.3.4 the accelerometer system

A three-axis accelerometer is attached to bottom side of the main square cell. The accelerometer converts acceleration (in units of g) into a voltage according to the calibration shown in Table 1. The output of each axis of the accelerometer can be monitored and measured by the oscilloscope. Use the sync output of the signal generator to externally trigger the oscilloscope. The x and y components should be minimal, while the z component shows the acceleration of the main square cell, and hence Γ (in voltage units).

When the cell is full of bouncing grains, the impact of the grains produces a noisy-looking signal in the acceleration. It is convenient to filter the accelerometer output with a 1 kHz low-pass filter to suppress the part of the signal due to individual grain impacts. It is also possible to use signal averaging on the oscilloscope to further improve the acceleration measurement. The "measure" feature of the oscilloscope can be used to determine Γ.

The peak value of the acceleration should not exceed about $8g$, to avoid overdriving the stinger. Except at the highest frequencies, this should be well above the threshold for patterns to appear.

3.3.5 handling the grains

The dimensionless depth H of the grains in the cell is determined by the volume V of grains introduced to the cell according to Eqn. 3 above, such that $H = V/(33.1\text{ mL})$. Measure the volume V of grains using a dry graduated cylinder, making sure to tap the cylinder a few times to settle the grains into a close packed configuration.

To remove the grains from the cell, use the small shop vac provided. Check that the inside of the shop vac is empty, clean and dry before using it to suck up the grains. Any obvious bits of dust or schmutz should be removed. Used grains can be returned to the supply container with a metal funnel.

For small values of H and high frequencies, you might not observe any pattern at all, even for a large Γ.

3.3.6 operating the lights and the camera

The LED lighting system should be operated at 12v DC.

The camera is controlled by the `FlyCapture2` program. You can choose to shoot single frames or image sequences at a set frame rate. First, set the aperture, focus and zoom level of the camera lens and then take a length calibration image with the white ruler in the cell. This must be redone whenever the lens settings are changed.

To take image sequences, you can use all the default settings of the program except the frame rate. In the `camera control` dialog window, uncheck the `auto` box on the frame rate. The frame rate can be set to a precise number by typing in the box. The maximum frame rate is 162 fps.

When you press record, you will get the `record settings` window. Here you should set the directory and file name for the image sequence. It is a good idea to use a name that contains information not already recorded by the camera, such as the frame rate, the layer depth and the shaker amplitude and frequency. Something like `100fps_H8.3_f22Hz_a3.6g`. The program will add a date and time stamp and a frame number to the name. Use a new directory to hold the images for each image sequence. `Saving options` should be set to capture a fixed number frames; a few seconds worth is usually plenty. `Image format` should be set to TIFF, with `compression method` set to LZW. Click the `Start Recording` button to take the sequence of images.

You can use the camera in two different ways. One way is to shoot image sequences at a frame rate much higher than the shake frequency. Then the image sequence is effectively a high-speed view of the grain motion during a few oscillations. You can select the best image(s) from such a series for later pattern analysis. You can also animate a fast sequence of images to create a slow-motion video of the grain motion, as described in section 3.4 below.

Another way to use the camera is to set the frame rate to be close, or equal to, the shake frequency, or a whole number fraction of it. Then an image sequence is a stroboscopic view of the motion over many pattern oscillations. Since we do not control the relative phase relationship between the camera and shake frequencies, it's useful to make their frequencies slightly different so that the sequence slowly explores the full range of phases. Again, individual frames can be analyzed, or longer sequences can be animated.

3.4 data analysis

In the experiment the data are mostly in the form of *images*. An image is a 2D position measurement. **Every pixel is a measured data point.** The camera captures 8 bit black and white images.

All images you intend to analyze must be in a format that preserves pixel-level information with lossless compression. Tagged Image Files (.tif) format with LZW compression achieves this. Do not make .jpg images except to be used qualitatively as frames in movies.

To analyze images, we will use a combination of python codes and an open source application called **Image J**. To determine the length of a line in Image J, you simply draw a straight line on the image and use the **menu command** *Analyze* → *Measure* to record its length. You can use any image application that allows the length of a line to be measured (in pixels).

First, using your calibration image of the white ruler, measure the number of pixels in a long line whose length is a known number of mm. The number of pixels per mm can be used to convert pixel measurements to physical length, or *via* Eqn. 5, directly to dimensionless wavelengths L.

To measure the wavelength of a pattern in pixels, measure a long line between peaks in the pattern and divide by the number of pattern units along that line. You may need to do this a number of times with different lines to get good statistics. A bit of judgement is needed to locate pattern peaks. A more sophisticated and objective method is to use the Image J **menu command** *Process* → *FFT* to Fourier analyze a portion of the image which contains a good sample of the pattern. Peaks in the resulting transformed image give the wavenumbers and harmonics of the pattern. See **this Image J example page** for how to use this to measure the wavelength. The FFT method is more precise in principle, because it uses more of the image. This analysis could also be done in python, using the `numpy` command `fft2`, after converting the subimage to an array.

To make movies from image sequences, you can use the python code `crop_movie_GRAN.py` provided on the experiment web page. See the comments in that code for how to use it. You can use movies to diagnose the temporal structure of the motion.

4 questions

The following are some rough ideas for things to study.

- Measure the critical acceleration Γ_c at the onset of the pattern formation, as a function of f, for various values of the depth H. Then try to collapse all the data onto one plot by non-dimensionalizing f to get F. Do the various values of H collapse onto one curve?
- Examine the patterns you see at and above the critical value of the acceleration Γ. Plot these as a phase diagram in the F-Γ plane, with one diagram for each value of the depth H. Locate the boundaries between the different types of pattern (straight waves, squares, hexagons, disorder *etc.*). Do these agree for different values of H? What features are universal, and which depend on H?
- Looking at your phase diagram, determine which phases oscillate at what frequency, relative to f. Which are at $f/2$? Which at $f/4$? Or other frequencies?
- Identify the various kinds of defects that appear in otherwise spatially periodic patterns. Which defects are purely *topological* in the sense that they separate regions of the patterns oscillating one period of f out of phase with each other? Which are merely analogous to grain boundaries?

  You can produce and destroy defects by muddling up the pattern with your fingers while shaking. See if you can observe transient defects, like dislocations of opposite signs in a square pattern, that move and annihilate each other. Try also to produce "perfect" defect free patterns. These may take some time to "anneal".

- Is the onset of pattern formation hysteretic? Can you nucleate a patch of pattern in an otherwise unpatterned layer by disturbing the layer locally while shaking? Are such patches stable?

  An extreme example of a localized pattern is an *oscillon* [4], which consists of a single oscillating unit of a square pattern, surrounded by a flat layer. Oscillons have only been observed in a narrow region of the phase diagram and are often only transient features. Can you make and study the interactions of oscillons?

- Measure the wavelength λ of the patterns near onset as a function of f for various depths H. Try to collapse the data onto one curve by making a non-dimensional log-log plot of wavelength L against F. Do the data for different H collapse onto one curve? Over what range of F? Fit your data to a power law of the form of Eqn. 6. What value of the exponent α do you observe?

- For fixed F and H, how does the dimensionless wavelength L depend on Γ for $\Gamma > \Gamma_c$? Do all patterns eventually become disordered? By what mechanism(s)?

- You can find suggestions for more advanced experiments with the shaker in Appendix B.

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(Electronic versions, if available, are accessible by clicking on the title.)

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Part (top to bottom)	Mass (kg)
main square experimental cell	3.31
connecting plate below square cell	0.520
moving part of air bearing	1.877
top fixture holding stinger	0.229
bottom fixture holding stinger	0.149
shaker armature	0.454

Table 2: Masses of the various parts of the shaker and experimental cell. See Fig. 1. These are used by the code `stinger.py` to calculate the possible failure modes of the stinger.

A The design of the shaker

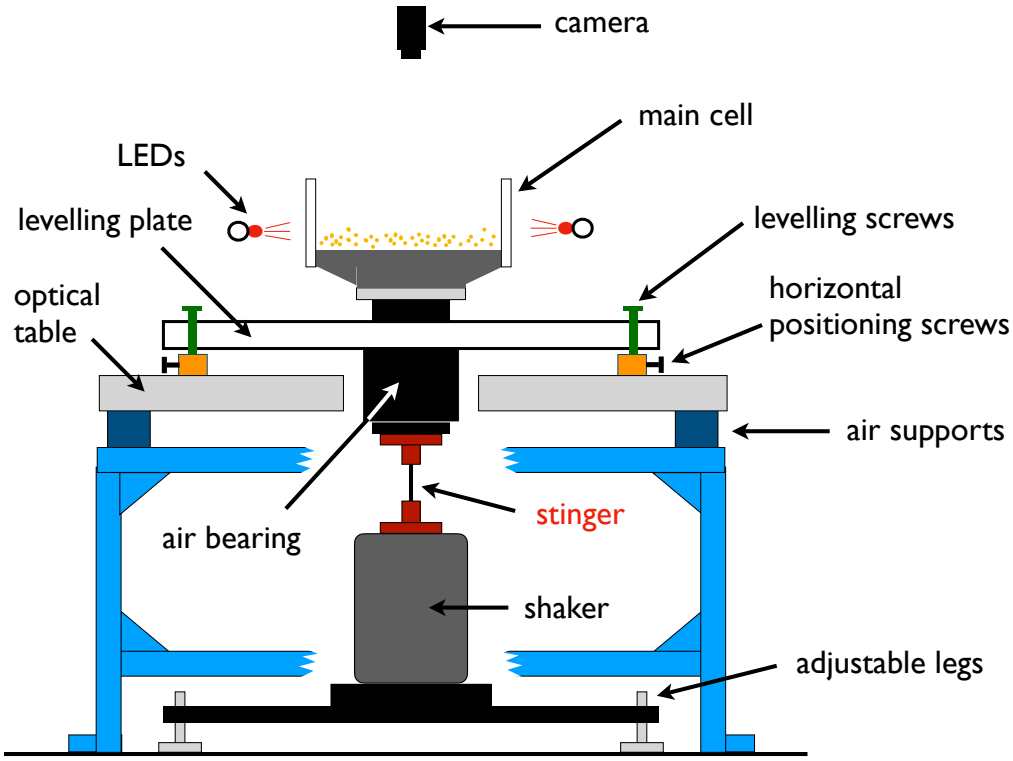

Figure 1: The parts of the shaker system. The masses of the moving parts are listed in Table 2.

The shaker system, shown in Fig. 1, follows a design published by Harris *et. al.* [14]. It uses a powerful electromagnetic shaker made for vibration testing. Such shakers are not precise enough in their motion to be used for our experiment. The Harris design uses a flexible, compliant rod, known as a "stinger", and a precise air bearing to achieve better vibration control. The stinger transmits the force between the shaker and the experimental cell and the air bearing allows the cell to be independently and precisely levelled. The air bearing also suppresses unwanted lateral accelerations by tightly constraining the motion to be along one axis. The task of the levelling plate is to make $\vec{\mathbf{a}}_{\max}$ exactly parallel to $\vec{\mathbf{g}}$ in Eqn. 1. Without the decoupling effect of the stinger, off-axis motion of the grains in the cell tends to create lateral accelerations in the shaker, resulting in a feedback loop

that makes the whole system unstable to heaping. The stinger and air bearing prevent this "live load" instability. In addition, the shaker is attached to a heavy steel plate, which rests on the floor independent of the blue stand holding up the experimental cell. Air supports in the stand prevent any stray lateral vibration from getting back into the cell.

The most important consideration in this design is the stability of the stinger. It must be as long and thin as possible to be maximally compliant, but without failing by bending or breaking. Harris *et. al.* [14] have carefully considered all the failure modes of the stinger, which allows us to choose a safe operating point (a length and diameter for the stinger). See Ref. [14] for details. The most dangerous failure mode turns out to be buckling, in which the stinger bends to one side due to axial compression. The results of the full Harris analysis, which are reproduced in the python code `stinger.py`, are plotted in Fig. 2. We have chosen an operating point such that the stinger should withstand a maximum acceleration of $8g$, at a frequency 50 Hz. The calculations show that a stinger diameter of 1.65 mm, and length of 45 mm is safe.

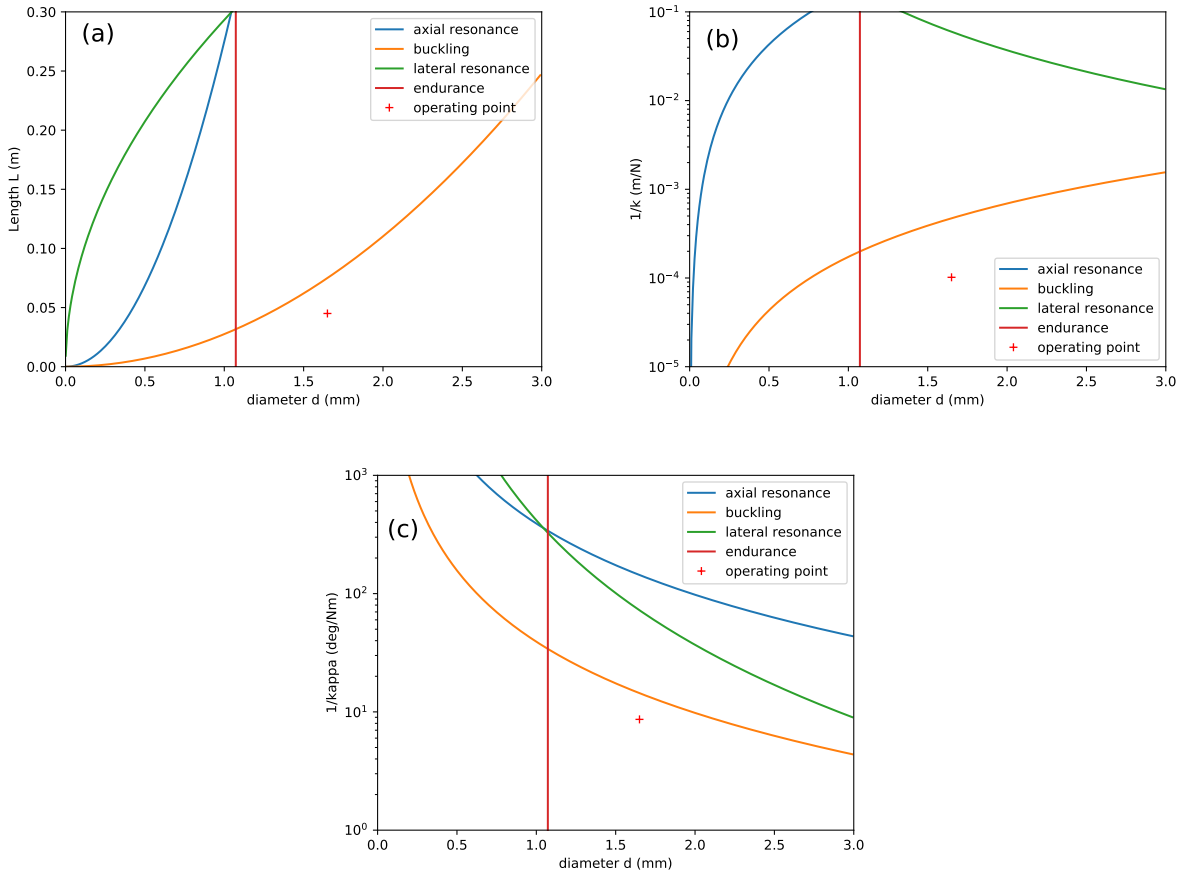

Figure 2: A recalculation of Fig. 7 from Harris *et. al* [14]. The red cross shows the operating point. The stinger will not fail if the operating point is below and to the right of the thresholds given by the curves for the various failure modes. This code is available as `stinger.py`.

B Other shaking experiments

This Appendix lists some other experiments that could be done using the shaker system. These might be pursued as special projects by some students. Some require the construction of significant new apparatus.

- **Shaking grains, while varying the grain sizes, or using mixtures of sizes.**
  An obvious omission of our experiment is that grain size was not varied. In order to do this systematically, a **set of standard sieves** could be used to sort grains into a series of samples of different mean sizes. Standard sieves should be shaken by a dedicated bench-top sieve shaker. Then a wide range of sizes, or selected mixtures of sizes, could be investigated. One new effect that may appear is that air effects will dominate for the smaller sizes.

- **Shaking grains in an evacuated cell.**
  To eliminate air effects, a sealed cell would have to be constructed and connected to a good vacuum pump. This would allow smaller sized grains and a much better comparison with the published literature. Gas effects could be investigated by running the cell at different pressures of air, or even by using different gases, such as very dense SF_6.

- **Shaking grains with multiple frequencies.**
  Then patterns produced my shaking with multiple frequencies that have a fixed ratio and phase relationship has been investigated [15]. A rich variety of new "superlattice" patterns are observed.

- **Shaking grains of different shapes.**
  Grains in the form of rods or nails (rods with heads) may be shaken in a near monolayer or in thick layers. For a near monolayer, the rods tend to align for low Γ, forming increasingly complex **liquid crystal** -like phases [16]. When the nails have heads, a head-to-tail configuration appears. For thick layers, rods align more vertically and spectacular swarming effects can be observed [17]. It would also be interesting to try small disk shaped particles, like washers. These may form "discotic" liquid crystal -like phases.

- **Shaking chains that unmix.**
  The shaker could be used to do more precise versions of experiments with ball chains, in the manner of the KNOTS experiment. If two identical ball chains are spread in a non-crossing monolayer on the plate, and shaken so they become mobile but not hard enough to cross, a curious segregation effect occurs. The two chains completely separate so they occupy different areas of the cell [18]. Tight spirals can also form. You can see a video **here**.

- **Shaking chains on a sinusoidally corrugated plate.**
  It is interesting to vary the forcing of ball chains by corrugating the plate on which they are shaken [19]. If the plate has long sinusoidal undulations, chains become confined in the minima for gentle shaking. With harder shaking, they can sometimes hop over the barrier to occupy an adjacent minima. This forms a topological object known as a "kink". A long chain, formed in a circle on a circularly corrugated cell forms a 1D gas of such kinks, which have topological signs and are created and destroyed in pairs. A plate with the right corrugations has been made and can be attached to the existing shaker in place of the main cell.

- **Shaking oil with bouncing droplets.**
  Probably the most interesting experiment that could be done with the shaker is a precision variant of the FAR experiment which creates a fluid mechanical analog of de Broglie pilot wave quantum mechanics [20, 21]. When a horizontal layer of light silicon oil is shaken, Faraday waves appear at a threshold acceleration. Shaking just below, but very close to, this threshold, and introducing a tiny droplet of the oil, results in a bouncing droplet that does not coalesce with the layer. Instead, the bouncing droplet interacts with decaying Faraday surface waves in a manner that is very much like the interaction of a quantum particle with a pilot wave in de Broglie's original interpretation of quantum mechanics. Many quantum analog experiments are possible. For a visual exposition, see this MIT video.

  To do this experiment, a new experimental cell would have to be built.