

ADVANCED UNDERGRADUATE LABORATORY

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Helicons in Metals

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Description

Helicons are low frequency electromagnetic waves that can propagate in plasma in a high magnetic field. To produce helicons in metals at reasonable magnetic fields requires reducing the conductivity by lowering the temperature. The student employs the simplest technique for achieving low temperatures, immersing the probe in liquid helium (i.e. a dip experiment), and uses a superconducting solenoid to produce the high magnetic field. The student then characterizes helicons by measuring the resonance frequency and resonance width.

Introduction

Can electromagnetic waves propagate in metals? Most people would say emphatically not, since the free electrons, which give rise to the characteristic high conductivity of a metal, screen out the electromagnetic field - hence the high reflectivity of metals to light. The electromagnetic waves are damped exponentially inside the metal, with a penetration length given by the skin depth:

$$d = \sqrt{\frac{2r}{\omega m}} \quad (SI \text{ units}),$$

Of course, at frequencies higher than the plasma frequency,

$$\omega_p = 2\pi f_p = \sqrt{\frac{n_e e^2}{m \epsilon_0}} \quad (SI \text{ units, radians/second}),$$

a metal becomes transparent because the conduction electrons cannot respond quickly enough to cancel out the electromagnetic wave. In classical terms, the displacement current dominates over the conduction current. In most metals this happens in the ultraviolet.

Helicon waves propagate with little damping as long as the electron cyclotron frequency,

$$\omega_c = 2\pi f_c = \sqrt{\frac{eB}{m}} \quad (SI \text{ units, radians/second}),$$

is much greater than the average frequency of electron scattering, i.e.

$$\omega_c \gg \frac{1}{\tau}$$

where the relaxation time, τ , is the mean time between electron collisions.

For the pure sample of indium we use in this experiment, when scattering of electrons by lattice vibrations is eliminated by cooling to liquid helium temperature, the conductivity is so high that even at a low frequency of 1 kHz the skin depth is only about 50 μm (0.005 cm). However, if a large static magnetic field is applied, the behaviour is quite different because of the alternating Hall electric field. The Hall field has the following interesting features.

- It is perpendicular to the conduction current generated by the alternating field.
- Because of (a) it is non-dissipative.
- It is much larger than the conduction (dissipative) electric field.

These features result in a propagating wave with the following properties.

- a. The wavelength is much shorter than the free space wavelength which is 3×10^7 cm at 1 kHz. Thus standing waves can be set up when the wavelength matches the dimensions of the indium sample, which in this experiment is ≈ 1 mm. This means that the velocity of the waves in the sample is only about 100 cm/s, much less than the normal speed of electromagnetic waves in vacuum ($c = 2.99792458 \times 10^8$ m/s).
- b. The waves are circularly polarised so that they can be detected by the induced electric field in a coil wrapped around the sample with its axis perpendicular to that of the coil exciting the electromagnetic waves.

These propagating waves in a conducting medium in a static magnetic field were first observed in the ionosphere and were called “whistlers” or “helicons”. The static field for whistlers is the earth's magnetic field (~ 0.5 G), which is much smaller than the field (~ 0.5 T) required in a metal. The conductivity of the ionosphere is also much less than that of a metal because the number of charge carriers is much smaller ($\sim 10^{22}/\text{cm}^3$ in metals, $\sim 10^6/\text{cm}^3$ in the ionosphere). Nevertheless, the important feature, that the alternating Hall electric field should dominate the conduction electric field, is common to both systems.

Helicons are used in the study of metals:

- a. to measure the Hall coefficient (and the magnetoresistance from the width of the standing wave resonance) when it is difficult to make electrical contacts to the sample (e.g. alkali metals).
- b. To introduce electromagnetic waves into a metal in order to study other phenomena such as nuclear magnetic resonance.

Skin Effect and skin depth

For a non magnetic, ($\mu = \mu_0$), medium of high conductivity where Ohm's law applies and we can neglect the displacement current, Maxwell's equations give

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (1)$$

and

$$\nabla \times \vec{B} = \mu_0 \vec{J} \quad (2)$$

If we take the curl of equation (1) and use equation (2) and the relation

$$\nabla \times (\nabla \times x) \equiv \nabla(\nabla \cdot x) - \nabla^2 x \quad (3)$$

plus the fact that $\nabla \cdot \vec{E} = 0$ if $\rho = 0$ then we obtain

$$\nabla^2 \vec{E} = \mu_0 \frac{\partial \vec{J}}{\partial t} \quad (4)$$

Consider a situation where we have an electric field polarized in the x direction, with a time dependence $e^{-i\omega t}$ and propagating in the z direction into the body of a good conductor as in Figure 1. For the moment we set $\vec{B}_o = 0$.

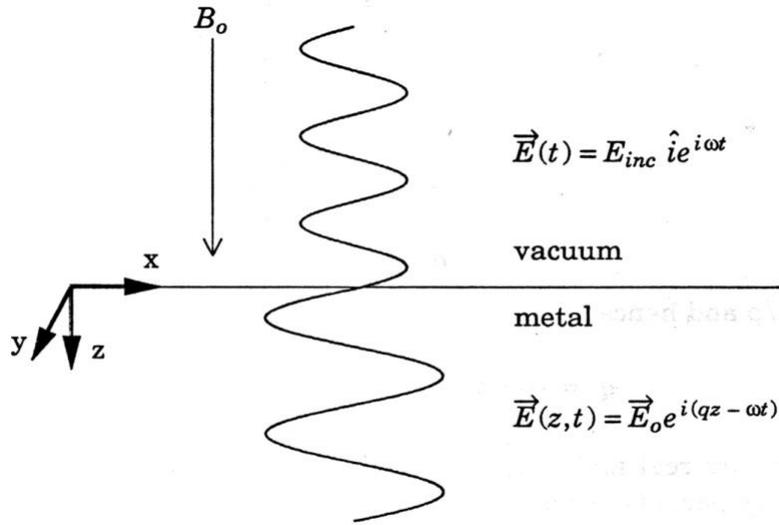


Figure 1: A Transverse Linearly Polarized Wave Incident Normally on a Surface.

Inside the metal, the electric field \vec{E} and the current density \vec{J} have the same time dependence $e^{-i\omega t}$ as the incident wave so

$$\frac{\partial \vec{J}}{\partial t} = -i\omega \vec{J} \quad (5)$$

Since the electric field is travelling in the z direction, using equation (5), equation (4) becomes

$$\frac{d^2 \vec{E}}{dz^2} = -i\omega m_0 \vec{J} \quad (6)$$

Taking $q = 2\pi/\lambda$ to be the wave vector, a solution of (6) is

$$\vec{E}(z,t) = \vec{E}_0 e^{i(qz - \omega t)} \quad (7)$$

if

$$\frac{d^2 \vec{E}}{dz^2} = i^2 q^2 \vec{E} = -i\omega m_0 \vec{J} \quad (8)$$

or

$$q^2 \vec{E} = i\omega m_0 \vec{J} \quad (9)$$

But since Ohm's law applies then

$$\vec{E} = r \vec{J} \quad (10)$$

Equations (9) and (10) can be true simultaneously only if

$$r = \frac{i\omega m_0}{q^2} \quad (11)$$

or

$$q^2 = i\omega m_0 S \quad (12)$$

where $\sigma = 1/\rho$ and hence

$$q = (i\omega\mu_o\sigma)^{1/2} = \left(\frac{\omega\mu_o\sigma}{2}\right)^{1/2} (1+i) \quad (13)$$

so that there are real and imaginary parts for q and they have equal magnitudes. The imaginary part of q corresponds to an attenuation, and we call this δ so that

$$\delta = \frac{1}{q_i} = \left(\frac{2}{\omega\mu_o\sigma}\right)^{1/2} \quad (14)$$

is the distance for $E(z)$ to drop to E_0/e in the conductor. We call δ the skin depth of the conductor.

Helicons

We now allow \vec{B}_0 in the Figure 1 to be non-zero and follow the approach of Maxfield.

In the presence of a magnetic field, in addition to the electric field, the charge experiences an additional field due to the Lorentz force. Ohm's law ($\vec{E} = r\vec{J}$), has an additional term and is now written

$$\vec{E} + \vec{v} \times \vec{B} = \vec{J} \quad (15)$$

where \vec{v} is the drift velocity. Since

$$\vec{J} = ne\vec{v} \quad (16)$$

where n is the carrier density and R_H the Hall coefficient is defined by

$$R_H = \frac{1}{ne} \quad (17)$$

then the modified Ohm's law of equation (10) becomes

$$\vec{E} + R_H \vec{J} \times \vec{B} = r\vec{J} \quad (18)$$

Just as before when we had to solve equations (9) [the relationship between \vec{E} and \vec{J} if \vec{E} has the form assumed in equation (7)] and (10) [Ohm's law] simultaneously, we now have to solve the simultaneous equations (9) and (18) [Ohm's law modified]. A standard means of solving such a set of equations is make the substitution

$$E^\pm = E_x \pm iE_y \quad (19a)$$

$$J^\pm = J_x \pm iJ_y \quad (19b)$$

Note that these substitutions correspond to representing a linearly polarized wave as the sum of right and left circularly polarized waves as illustrated in Figure 2.

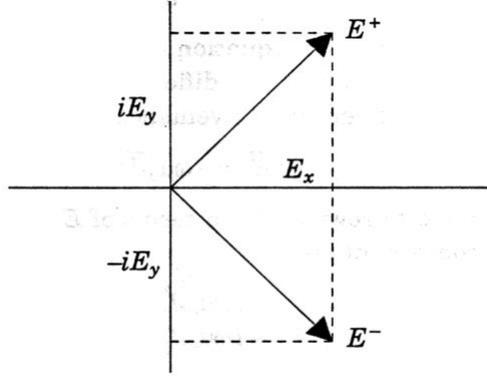


Figure 2: Visualization of E^\pm .

If we write equation (18) in terms of its components it becomes

$$E_x = \rho J_x - R_H J_y B_0 \quad (20a)$$

$$E_y = \rho J_y + R_H J_x B_0 \quad (20b)$$

$$E_z = \rho J_z \quad (20c)$$

We define, (see Appendix II)

$$u = \frac{R_H B_0}{r} \quad (21)$$

where u is called the tangent of the Hall angle (the angle between \vec{E} and \vec{J}) i.e.

$$u = \frac{\text{Hall electric field}}{\text{conduction electric field}} \quad (22)$$

Using equations (21), [the definition of u], and (20) in equation (19) we obtain E^\pm .

$$\begin{aligned} E^+ &= \rho(J_x - uJ_y) + i\rho(J_y + uJ_x) \\ &= \rho[J_x(1 + iu) + iJ_y(1 + iu)] \\ &= \rho(1 + iu)(J_x + iJ_y) \\ E^+ &= \rho(1 + iu)J^+ \end{aligned}$$

We obtain E^- in a similar way so that

$$E^\pm = \rho(1 \pm iu)J^\pm \quad (23)$$

It should be remembered that (23) is really a set of two equations

$$E_x + iE_y = \rho(1 + iu)(J_x + iJ_y) \quad (24a)$$

$$E_z - iE_y = \rho(1 - iu)(J_x - J_y) \quad (24b)$$

Equation (23) is a modified form of equation (18) which is itself a modified form of Ohm's law. It is Ohm's law (or its modified form) that we have to solve with equation (9). We rewrite (9) in terms of E^\pm and J^\pm , so equation (9) which in terms of its components is

$$q^2 E_x - i\omega\mu_0 J_x = 0 \quad (25a)$$

$$q^2 E_y - i\omega\mu_0 J_y = 0 \quad (25b)$$

Becomes, if we multiply (25b) by I and add it to (25a)

$$q^2(E_x + iE_y) - i\omega\mu_0(J_x + iJ_y) = 0 \quad (26)$$

or

$$q^2E^+ - i\omega\mu_0J^+ = 0 \quad (27)$$

We obtain E^- in a similar way (by subtracting) so that

$$q_{\pm}^2E^{\pm} = i\omega m_0 J^{\pm} \quad (28)$$

where q_+ is the q obtained when E^+ and J^+ are used. Equation (23) [essentially Ohm's law] and equation (28) [essentially the relationship between \vec{E} and \vec{J} if \vec{E} has the form given in equation (7)] can be true simultaneously only if

$$q_{\pm}^2 = \frac{i\omega m_0}{r(1 \pm iu)} \quad (29)$$

To have a propagating wave we must have $|\text{Re}(q)| \gg |\text{Im}(q)|$ which means that q^2 must be positive and largely real. For q^2 to be largely real, u must be large compared to 1. Recalling equation (21), the definition of u , this means that the magnetic field \vec{B}_0 must be large and the resistivity ρ must be small. If $u \gg 1$ then equation (29) becomes

$$q_{\pm}^2 = \pm \frac{\omega m_0}{ru} = \pm \frac{\omega m_0}{R_H B_0} \quad (30)$$

For q to be real and thus have a wave propagating without attenuation, q^2 must be positive. Recalling the definition of R_H in equation (17) we see that the sign of u has the same as the charge carriers. Thus waves of right helicity[†] propagate for positive carriers with

$$q_+ = \left(+ \frac{\omega\mu_0}{\rho u} \right)^{1/2} \quad (31)$$

and waves of left helicity propagate for negative charge carriers with

$$q_- = \left(- \frac{\omega\mu_0}{\rho u} \right)^{1/2} \quad (32)$$

Remember that the q_+ and q_- of (31) and (32) are just wave vectors ($q = 2\pi/\lambda$) and the subscripts merely serve to identify the helicity of the wave with which they are associated.

Both waves will be excited if the incident wave is linearly polarized. For indium, where the carriers are holes, the wave of right helicity (+) propagates whereas the wave of left helicity (−) is strongly attenuated. Thus the helicon is a circularly polarized electromagnetic wave, where electric and magnetic vectors rotate about the magnetic field (the direction of propagation) in the same direction as the cyclotron resonance rotation.

Before taking any data, reading the paper by Merrill et al. in Appendix II is recommended.

[†] It is a matter of convention whether waves of right helicity are considered right or left circularly polarized. Engineers, quantum physicists and astronomers tend to use the convention that right helicity corresponds to right circular polarization, but many optics textbooks use the opposite convention.

We shall concentrate on the resonant modes corresponding to standing waves that are set up when the thickness d of the metal sample is an integral number N of half wavelengths, i.e.

$$d = \frac{N\lambda}{2} = \frac{N\pi}{q} = N\pi \left(\frac{\rho u}{\mu_o \omega} \right)^{1/2} \quad (33)$$

or

$$f_{res} = \frac{\omega}{2\pi} = \frac{N^2 \rho}{2d^2} \frac{ru}{m_o} = \frac{N^2 \rho}{2m_o d^2} R_H B_o \quad (34)$$

Equation 34 is in S.I. Units, and corresponds to Eq. 2a from Merrill et al. when $\rho \ll R_H B_o$. Each resonance produces a peak in the response of the sample to the exciting alternating field, but the pickup coil will give a peak response only for the odd harmonics due to flux cancellation for even harmonics. The successive harmonics can be observed by keeping the magnetic field constant and varying the frequency, giving a curve like Figure 3 of Merrill et al..

Note that equation 2c in Merrill et al. is incorrect; when correctly derived from equations 2a & 2b, it may be written in S.I. units as:

$$\rho = \frac{\mu_o}{N^2 \pi} d^2 \Delta f \quad (35)$$

Safety Reminders

- Eye protection, gloves, and proper footwear, e.g. no sandals, must be worn when working with cryogenic liquids or dewars.
- When pouring liquid nitrogen (LiN) into a small dewar, it is best to place the small dewar on the floor so that any spills or splatters will be far away from your eyes and from clothes that might absorb and hold spilled LiN against your skin causing more severe burns.
- Sealed cryogenic containers build up pressure from the evaporating gas, so eye protection must always be worn when opening the valves on the liquid helium dewar. You must ask for instruction from the supervising professor, the Demo, or the Lab Technologist, when first using a liquid helium dewar. Never leave the dewar with all valves closed for long periods; the safety valve should prevent an explosion, but such a blow-out is not desirable.

NOTE: This is not a complete list of all possible hazards; we cannot warn against all possible creative stupidities, e.g. juggling cryostats. Experimenters must use common sense to assess and avoid risks, e.g. never open plugged-in electrical equipment, watch for sharp edges, If you are unsure whether something is safe, ask the supervising professor, the lab technologist, or the lab coordinator. If an accident or incident happens, you must let us know. More safety information is available at <http://www.ehs.utoronto.ca/resources.htm>.

Suggested Exercises

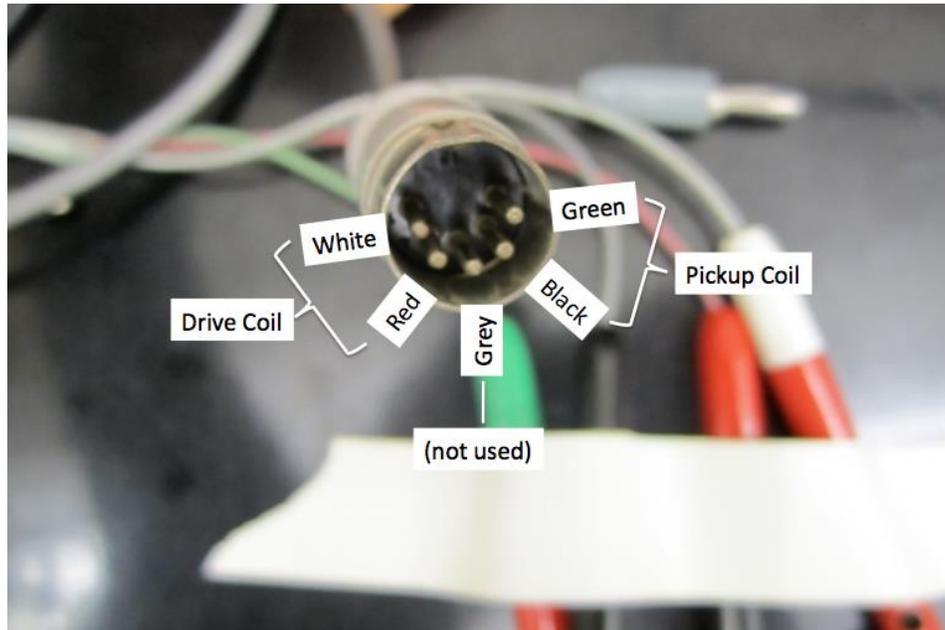


Figure 3. The 5-pin wire which connects to the coils in the probe.

1. The indium foil is surrounded by the pickup coil, and the whole assembly is held in the drive coil. An example from a disassembled old third probe is available in a little transparent plastic box. At room temperature, check the continuity of the drive and pickup coils with an ohmmeter. These coils should have resistances between 100 and 200 Ω at room temperature. Also, observe low amplitude pickup due to the coils not being exactly perpendicular.

WARNINGS:

- To avoid burning out the drive coil, **never turn on the frequency generator unless** the drive coil is immersed in liquid nitrogen or liquid helium.
- **Do not turn on the superconducting coil's power supply unless** the coil is immersed in liquid helium; it won't work, and might damage the coil.

2. Check the continuity of the superconducting coil with an ohmmeter. The experiment formerly used a magnetoresistor to measure the magnetic field, but as of Feb. 15, 2011, the magnetoresistors in both probes are broken, so students must measure the current through the superconducting magnet, and then refer to the graph in Appendix I to infer magnetic field.
3. Precool the lower portion of the helicon probe in a dewar of liquid nitrogen (until major boiling stops) before lowering it carefully into the helium storage dewar. A laboratory supervisor should assist you in this operation.
4. Repeat the checks on electrical continuity after the probe has been lowered into the helium storage dewar.
5. Measure the amplitude of the signal from the pickup coil as a function of magnetic field for several frequencies, such as: 100, 150, 200, 400, 1000, 2000, ... Hz.

6. Measure the amplitude of the signal from the pickup coil as a function of frequency for several values of the magnetic field. Start with higher magnetic fields since they give better results. Initially focus on the range from 50 to 300 Hz to determine the $n = 1$ resonance. Verify the existence of $n = 3$ resonance as in Figure 3 of Merrill et al. in Appendix II. You may find higher order resonances as well. Once you think you have found a peak, it is best to reduce your frequency interval and go over the peak again, so that you characterize the maximum.
7. For each value of the magnetic field, determine f_{res} and $Q = f_{\text{res}}/\Delta f$ for the resonances by the method demonstrated in Figure 2 of Merrill et al. in Appendix II. The rationale behind this method is given in Appendix IV.
8. Using the results of the previous exercise, find the resistivity, ρ , and the Hall coefficient, R , for indium at 4.2 K. Place limits of error on your measurements of ρ and R .
9. The indium foil has a purity of 99.9999% has dimensions $8.5 \text{ mm} \times 9 \text{ mm} \times \text{thickness}$ where thickness may be calculated from the following table of measured values.

Measurement	Thickness (inches)
1	0.0315
2	0.0315
3	0.0320
4	0.0318
5	0.03175
6	0.0318
7	0.0315
8	0.0315
9	0.0319

10. Compare your experimental value of R_H with published values (see Kittel). Also compare it with the value obtained assuming one free electron per atom for indium. Indium has a tetragonal crystal structure with 4 atoms per unit cell and lattice parameters: $c = 0.495 \text{ nm}$, and $a = 0.459 \text{ nm}$.
11. From what you can find out about the behaviour of indium as an electrical conductor at 4.2 K, give a reasonable argument for the sign of R_H .
12. Following Ashcroft and Mermin, $\omega_c \tau$ can be estimated from relations quoted in Chapter I. Assuming that Figure 1.4 of Chapter I holds for indium, show where the conditions for the helicon experiment would be represented on a plot of $-1/(R_H n e)$ versus $\omega_c \tau$. (Note: In the non-S.I. electrostatic units used by Ashcroft & Mermin, $R_H = 1/(nec)$., with e in statcoulombs.)
13. Compare your experimental value of $\rho_{4.2}$ for indium with values given in both Swenson and also White and Woods.

Questions

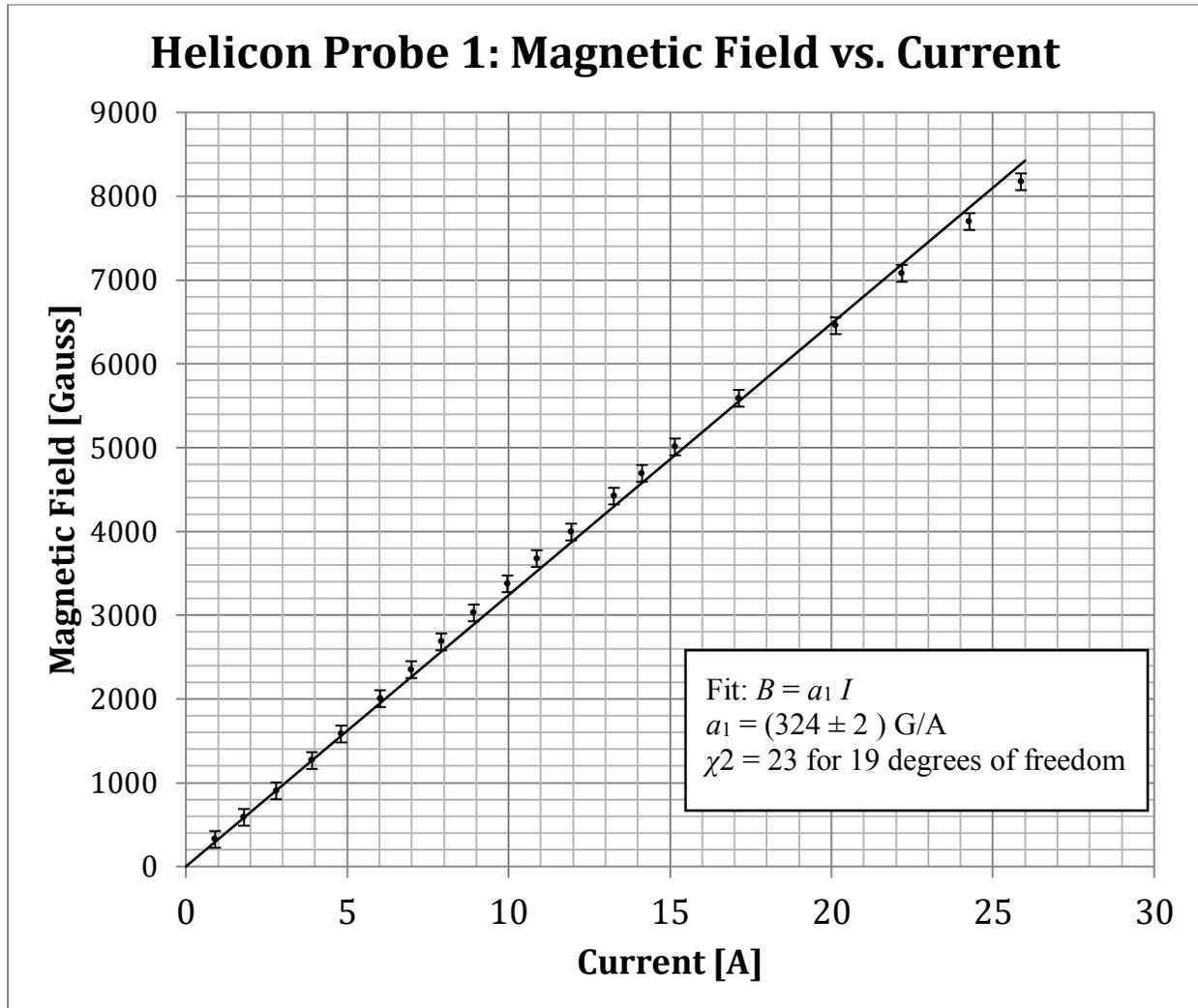
1. Show that the imaginary part of q in equation (13) results in attenuation.

2. The high purity indium used in this experiment has an electrical conductivity at 4.2 K of about $1.25 \times 10^{12} (\Omega\text{m})^{-1}$. Using this information, calculate the skin depth in indium at a frequency of 100 Hz for a temperature of 4.2 K.

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APPENDIX I: Magnetic Field vs Current



These data were taken in March 1998 by Thomas Szkopek, using a magnetoresistor [not operational as of Feb.2011], operating at 4.2 K. The fit was performed by Jason Harlow in Feb.2011. We expect a linear relation between B and I, but the residuals of this fit are not random: the measured values are systematically above the fit by up to 150 G in the region $I = 5$ to 17 A, and systematically below the fit by up to 200 G for $I = 17$ to 26 A. Therefore, any magnetic field which is inferred from the current is uncertain by at least ± 100 Gauss.

Appendix II: "[A Helicon Solid-State Plasma Experiment for the Advanced Laboratory](http://dx.doi.org/myaccess.library.utoronto.ca/10.1119/1.1976357)" by J.R. Merrill, D. Pierce and D. Giovanielli, 1970 American Journal of Physics Vol. 38 Pg. 417 - 5 pages; <http://dx.doi.org/myaccess.library.utoronto.ca/10.1119/1.1976357>.

Appendix III: "[Helicon Waves in Solids](http://dx.doi.org/myaccess.library.utoronto.ca/10.1119/1.1975500)" by B.W. Maxfield, 1969 American Journal of Physics Vol. 37 Pg. 241 - 29 pages; <http://dx.doi.org/myaccess.library.utoronto.ca/10.1119/1.1975500>.

APPENDIX IV

Rationale Behind Analysis of Experimental Data

When circularly polarized standing waves are set up in the indium plate a voltage V_p is induced in the pickup coil by the time dependent field. μ_T , which is called the standing wave permeability, represents the peak value of the time dependent flux through the pickup coil. The subscript T indicates that the pickup coil axis is perpendicular (transverse) to the external magnetic field, V_p is proportional to ω the angular frequency of the time dependent field (due to the time derivative in Faraday's law) and also to μ_T which is itself a function of frequency. μ_T , of course, is maximum for frequencies for which standing waves are set up. V_p is given by

$$V_p(\omega) = iG\omega\mu_T(\omega)I_d \quad (\text{IV-1})$$

Where G is a geometrical factor depending on the number of turns in the drive and pickup coils and the sample volume, I_d is the current in the drive coil and $i = (-1)^{1/2}$. Simplifying (IV-1) we write

$$V_p(f) = Kf\mu_T(f) \quad (\text{IV-2})$$

Where f is the frequency and K is a constant.

Although we measure V_p we wish to find the resonant frequency for μ_T . The maximum in μ_T occurs at a frequency for which a straight line through the origin is tangent to V_p . The proof is as follows.

Call f_r the frequency for which μ_T is a maximum. Taking the derivative of equation (IV-2) with respect to frequency we obtain

$$\frac{dV_p(f)}{df} = K[f\mu_T'(f) + \mu_T(f)] \quad (\text{IV-3})$$

and hence

$$\left. \frac{dV_p(f)}{df} \right|_{f_r} = K[f_r\mu_T'(f_r) + \mu_T(f_r)] \quad (\text{IV-4})$$

But by definition $\mu_T'(f_r) = 0$. Therefore

$$\left. \frac{dV_p(f)}{df} \right|_{f_r} = K\mu_T(f_r) \quad (\text{IV-5})$$

Consider the straight line

$$V_p(f) = K\mu_T(f_r)f \quad (\text{IV-6})$$

This is a straight line through the origin with slope $K\mu_T(f_r)$. But according to equation (IV-5) it is also a tangent to $V_p(f)$ at f_r where f_r is the frequency for which $\mu_T(f)$ is a maximum. This completes the proof.