

ADVANCED UNDERGRADUATE LABORATORY

EXPERIMENT 34

HIGH ENERGY PHYSICS

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## Introduction

You are given a small roll of film showing the interactions of  $\pi^+$  mesons with the protons of a liquid hydrogen bubble chamber. Two of the frames, which show "strange" particle production, have been selected. The purpose of this experiment is to give you some opportunity to learn about high energy, or particle physics by fully identifying the secondary particles which emerge from these two reactions and by testing the validity of various conservation laws.

The section on **Theory** gives a minimum description of the things you will need to know to do this experiment. You should take some time to look at least one of the books listed in the **References**.

## Theory

### Particle Physics

Around the 1930's the only particles known were the constituents of atoms (the electron, and the nucleons - i.e. the proton and the neutron), and the quantum of light (the photon). One outstanding mystery was how nucleons manage to stick together to form the nuclei of atoms. Yukawa postulated a mechanism for the attractive interaction of two nucleons, provided by a new, hitherto unobserved particle which could be exchanged back and forth between them. This theory was confirmed in 1947 when a particle with all the correct properties, christened the pi-meson ( $\pi$ ) was observed in the high-energy interaction of cosmic rays with the nuclei of photographic emulsion. The  $\pi$  appears in three charge states,  $\pi^+$ ,  $\pi^-$  and  $\pi^0$ .

Immediately following this discovery it was realized that the high-energy collisions of nucleons produced not only  $\pi$  mesons, but a host of other particles, distinguished by different masses, spins and other properties; see Appendix I for a partial list.

These particles can interact with each other through the action of four fundamental forces of nature.

First there is the **strong** interaction which accounts for the binding of nucleons, and the production of  $\pi$  mesons in nucleon-nucleon collisions. This force is so strong that it operates in a very short time: the characteristic time for two nucleons to interact is given approximately by the time a particle, (such as the  $\pi$ ) travelling close to the speed of light ( $3 \times 10^{10}$  cm s<sup>-1</sup>), takes to cross a nucleon (of size  $\approx 10^{-13}$  cm), or about  $10^{-23}$  seconds.

Second in strength is the **electromagnetic** interaction which is responsible for the radiation of electromagnetic waves from an accelerated charge. Any interaction which involves the quantum of the electromagnetic field, the photon, is due to the electromagnetic interaction. This is also the force between charges called the Coulomb force. This interaction, being about 1000 times weaker than the strong interaction, takes longer to act, with a typical time-scale of  $10^{-16}$  seconds.

Third is the **weak** interaction, whose effects were first observed in nuclear physics. This interaction is responsible for the  $\beta$ -decay of nuclei (the emission of a positive or negative electron) and, being approximately a factor of  $10^{-11}$  weaker than the strong interaction, has a characteristic time of  $10^{-10}$  to  $10^{-8}$  seconds. The electron (positive and negative,  $e^\pm$ ), the muon,  $\mu^\pm$ , and the neutrino,  $\nu$ , and antineutrino,  $\bar{\nu}$ , belong to the class of particles referred to as leptons; neutral leptons interact only via the weak interaction while charged leptons interact via both the weak interaction and the electromagnetic interaction, the latter by virtue of their electric charge.

The fourth is the **gravitational** interaction, which is so weak (about a factor of  $10^{-39}$  less than the weak interaction) that its effects are apparent only when huge masses (by particle standards) are present.

The study of these four forces and their interrelation provide the central unsolved problems of particle physics.

To whet your appetite for further reading, it is worthwhile pointing out a few important discoveries since the middle of the 1960's.

- a. Most of the particles with which we are dealing in this experiment are not truly elementary but rather are composite particles. This is true for all hadrons (strongly interacting particles) such as the proton,  $\pi$ ,  $\Lambda^0$  and  $K^0$ . These are made of 3 quarks (proton,  $\Lambda^0$ , ...) or a quark-antiquark pair ( $\pi$ ,  $K^0$ , ...) and there are three families of quarks [2, 3, 4].
- b. As stated earlier, the electron, muon and neutrino belong to the class of particles called leptons. So far, these appear to be elementary. There are three families of leptons, each family consisting of a charged lepton and an associated neutrino. These are  $(\nu_e, e^-)$ ,  $(\nu_\mu, \mu^-)$ ,  $(\nu_\tau, \tau^-)$  [9].
- c. The electromagnetic force and the weak force are now unified under the name "Electroweak Force". The electroweak force is mediated by the combined effects of the photon,  $W^\pm$  bosons and the  $Z^0$  boson [8].

### Mass, Energy and Momentum

It is in particle physics that the theory of special relativity really comes into its own, for the particles have so little mass that they travel with speeds which approach closely that of light.

The most striking feature of the theory, that mass and energy are equivalent, is shown by the way in which, for example,  $\pi$  mesons are produced out of nucleon-nucleon collisions: some of the kinetic energy of the initial nucleons is converted to mass energy, with the consequent materialization of the  $\pi$  mesons (and perhaps other particles) in the final state.

In any interaction between particles, both vector momentum, and total energy must be conserved. We can write this symbolically as

$$\vec{P}_i = \vec{P}_f \quad (1)$$

and

$$E_i = E_f \quad (2)$$

where  $i$  and  $f$  label the initial and final states, respectively. Note that in (1), since it is a vector equation, three equations, referring to the x, y and z directions are implied, and that in (2),  $E$  means total energy (i.e. mass energy **plus** kinetic energy).

The units most commonly used for energy are multiples of the electron volt (eV) which is the energy of an electron accelerated through 1V of potential difference. 1 eV is equivalent to  $1.6 \times 10^{-19}$  joules. 1 MeV is  $10^6$  eV, and 1 GeV is  $10^9$  eV. The most recent collider experiments operate in the TeV energy range ( $10^{12}$  eV).

The mass energy of a particle of rest mass  $m_o$  (particle physicists almost always mean rest mass when they talk about mass) is given by  $E = m_o c^2$ . From this equation it is clear that if energy is given in MeV, the units of mass can be written as  $\text{MeV}/c^2$ .

If the particle is moving with speed  $V$ , and  $\beta = V/c$ , its momentum is defined by  $p = \gamma m_o V$ , or  $p/c = \gamma m_o \beta$  where  $\gamma = (1 - \beta^2)^{-1/2}$ . Thus we see from this equation that if we measure  $m_o$  in  $\text{MeV}/c^2$ , the units of momentum can be written as  $\text{MeV}/c$ .

The more general expression for the total energy of a particle of rest mass  $m_o$ , having a momentum  $p$  is

$$E^2 = p^2 c^2 + m^2 c^4 \quad (3)$$

Which is exactly equivalent to  $E = mc^2 = \gamma m_0 c^2$ , which may be more familiar to you. Now if we measure energy in MeV, mass in  $\text{MeV}/c^2$  and momentum in  $\text{MeV}/c$ , equation (3) becomes:

$$E^2 = p^2 + m^2 \quad (4)$$

**Note:** A final complication of our use of units is that some books and tables drop the  $c^2$ , or  $c$ , and quote all units as MeV or GeV. They mean the same thing!

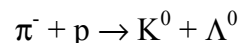
Equations (1), (2) and (4) are necessary for the calculation of the quantities you will need in this experiment.

### Strange Particles

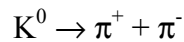
As you can see from Appendix I, most of the particles decay to other particles soon after they are formed. These then decay until we are left with the completely stable particles, the p, n, e,  $\gamma$  and  $\nu$ . This, of course, is one of the reasons that the majority of particles escaped detection for so long. If you look at the decay times for the unstable particles in the tables in Appendix I, you will see that their mean lifetime seems quite short - around  $10^{-6}$  to  $10^{-10}$  seconds. However as we have seen above, these times are long on the time-scale defined by the strong interaction ( $10^{-23}$  seconds). In fact these times are so long, that particle physicists refer to them as "Stable Particles". In fact, for example, a  $\pi^\pm$  meson, with lifetime around  $10^{-8}$  s, can still travel a long way when you consider its speed is close to  $3 \times 10^{10}$  cm s<sup>-1</sup>. That is why we can make "beams" which deliver  $\pi^\pm$  mesons to detection equipment many meters away from their point of production. (Question: In this experiment, you will see many  $\pi^+$  mesons travelling the 82 inches ( $\approx 200$  cm) of the bubble chamber without decaying. Is this consistent with a lifetime of  $10^{-8}$  s?)

For the case of the  $\pi$  meson, its relatively long lifetime has a natural explanation: the  $\pi$  meson mass (energy) is so low that it cannot decay into any other particles except the  $\mu$ , the e and the neutrino. These particles participate only in the weak interaction, so this decay must occur through this interaction, and is thus expected to take place over the time-scale characteristic of the weak interaction. However, for example, the K meson, which is produced via the strong interaction, has enough mass ( $494 \text{ MeV}/c^2$ ) to decay to two  $\pi$  mesons, each of mass  $140 \text{ MeV}/c^2$ . The  $\pi$  meson participates in the strong interaction and in fact it was first postulated to explain this interaction. Thus we might expect the K to decay to two pions in a time characteristic of the strong interaction ( $\approx 10^{-23}$  seconds). Instead, as is shown in appendix I, the K meson lifetime is close to what we would expect from a weak interaction. The K meson, and the  $\Lambda$ ,  $\Sigma$ ,  $\Xi$ , and  $\Omega$  particles, all of which exhibit similar puzzling behavior, were dubbed "strange" for this reason.

To provide a mechanism for explaining the properties of these strange particles, a new quantum number, S, "strangeness", was invented. The value assigned to each particle is shown in appendix I and is appropriate only for particles that can interact strongly. The anti-particles have S values which are the negative of that of the particles. The rule for strong interactions is that the value of S may not change from one side of the interaction to the other (i.e. S is conserved). For weak interactions, S may change by one unit. Thus



proceeds via the strong interaction strong (total S on both sides is zero) and



proceeds via the weak interaction (S is +1 on the left, but 0 the right).

(Question: How about the decay  $\Lambda^0 \rightarrow p + \pi^-$ ?)

This classification does not explain the underlying physics, but it is a useful empirical rule which has been well tested.

### Conservation Laws

In writing down a particle reaction there are a number of quantities which must always be conserved (i.e. have the same value in the initial and the final state). Some of these are discussed below:

- a. **Energy and momentum:** equations (1) and (2) always apply.
- b. **Charge:** the total charge must be the same before and after the interaction.
- c. **Baryon Number:** it is observed (from the fact that we exist!) that the proton (or the bound neutron) cannot decay<sup>1</sup> into two pions, or to an electron and a pion, even though it has enough mass energy to do so. The proton, the neutron, and all particles which contain either of these in their decay products are called **baryons**, and are assigned a baryon number, B. The baryon number of particles is indicated in Appendix I and as with S, the anti-particles have B values which are the negative of the values for the particles. The sum of the B numbers for all the particles on one side of an interaction must be the same as the sum for all particles on the other. This is a complicated way of saying that the number of baryons is conserved in a particle interaction. No such conservation law holds for the photon or for the mesons. Many of them can be produced or absorbed in an interaction as long as overall energy is conserved.
- d. **Strangeness.** As discussed above, S must be conserved in all strong interactions. This means that if we produce one strange particle in a strong interaction, we must produce at least one other with it (more specifically, if a particle containing a strange quark appears in the final state (of a strong interaction process) there must be another containing and anti-strange quark. If the strangeness number does change by one unit however, we know the interaction is weak.

## Apparatus

### Bubble Chamber Theory

The bubble chamber was for many years one of the major pieces of equipment for detecting particles. In modern experiments it has been replaced by electronic detectors, but bubble chamber results still provide an excellent environment in which to visualize particle interactions. A bubble chamber basically consists of a tub of liquid hydrogen, kept superheated under pressure. A beam of particles, produced by a particle accelerator, is fired into the chamber. The particles, if they are charged, ionize (i.e. knock out electrons from) the hydrogen atoms which they encounter along their path. The electrons are too light and too loosely bound to affect the particle's momentum in any significant way. The electric (Coulomb) field which the charged particles carry along with them effectively strips the electron from the atom. Now and again one of the beam particles will interact through the strong interaction with one of the protons in the hydrogen, and new particles may be produced. These, if they

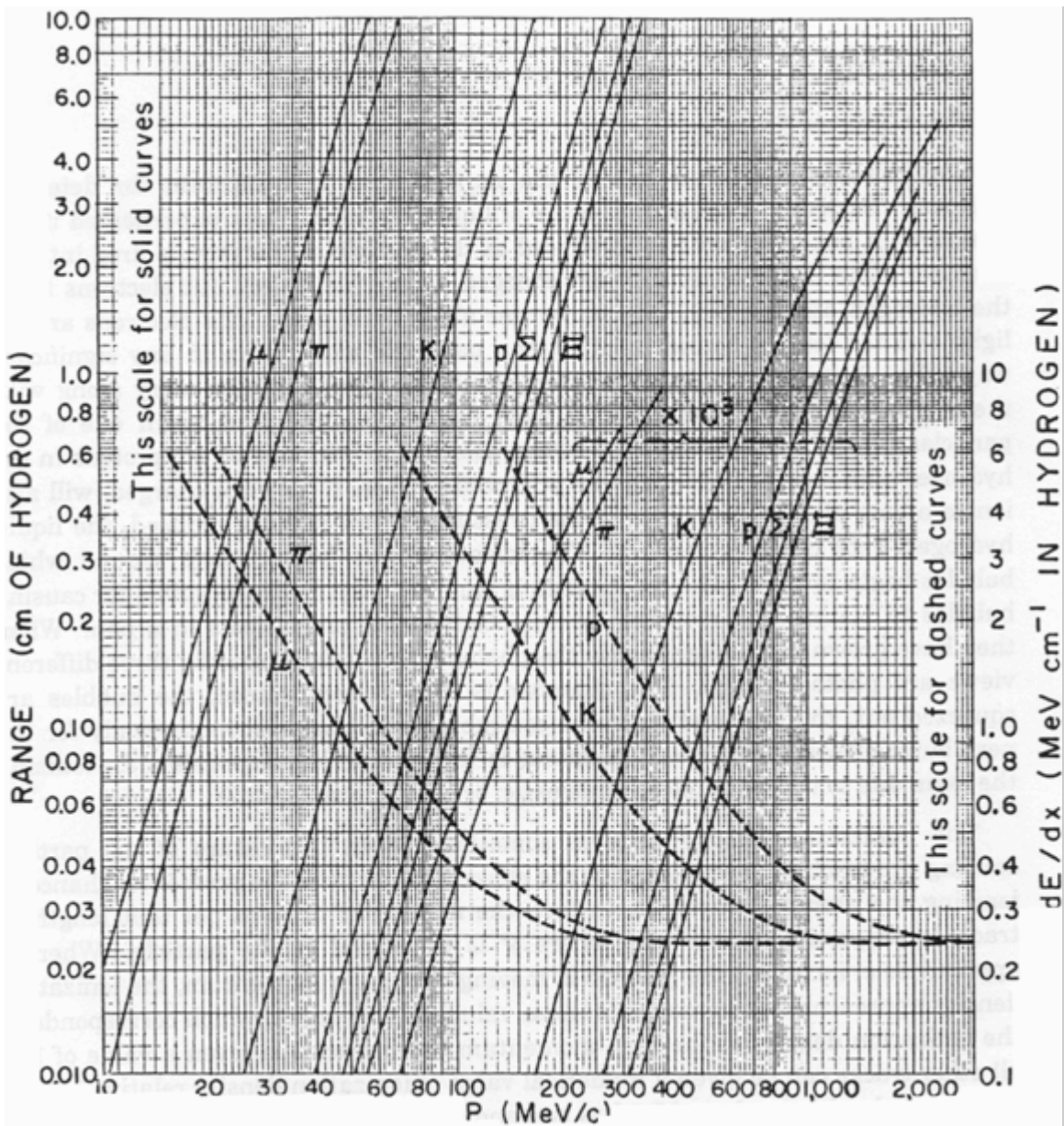
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<sup>1</sup> As of 2011 the lower limit on the proton lifetime is about  $10^{35}$  years. However, proton decay is predicted by many theories of physics beyond the Standard Model, and experimental searches for proton decay continue.

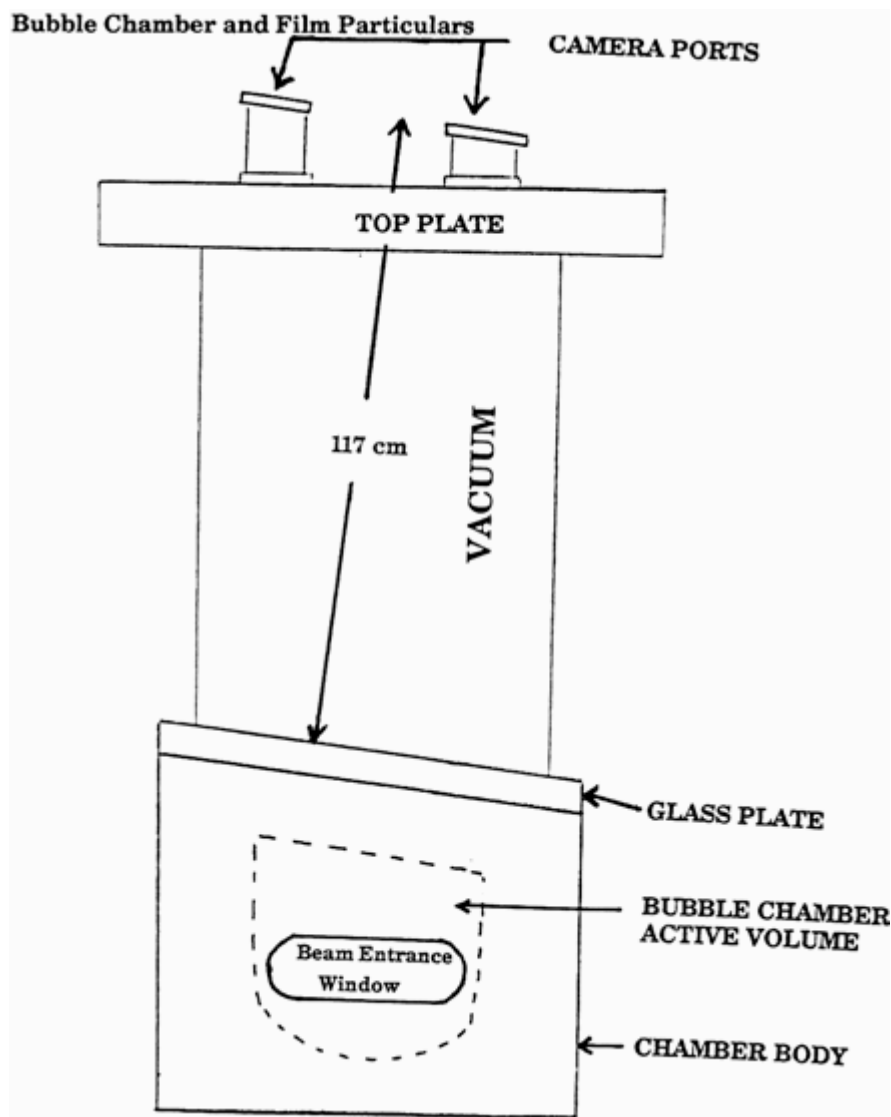
are charged, will also ionize atoms along their path. If the pressure is now slightly reduced, the liquid hydrogen gets closer to boiling. The ionized atoms form centres around which bubbles begin to grow. The mechanism is similar to the one responsible for causing bubbles in boiling water to form preferentially from rough spots in a pan. When they have grown big enough to see, the photograph is taken (usually three different views are photographed). The chamber is then re-compressed, the bubbles are squeezed out, the ions and electrons recombine and the chamber is ready for the next burst of beam particles. (Question: is the interaction responsible for ionizing the atom due to the weak, electromagnetic, or strong interaction?)

The "ionization density" tells us something about the nature of the particle causing the track. It turns out that the slower the particle, the greater its chance of ionizing the atoms it traverses. In fact, the number of bubbles per unit length of track is proportional to  $c^2/V^2$  where  $V$  is the speed of the particle. When  $V$  approaches  $c$ , i.e. when the particle is moving with high momentum, the ionization density approaches a constant minimum value. Such particles are referred to as "minimum-ionizing". If this value, which corresponds to the ionization density of the high momentum beam tracks, is given a value of 1.0, all other tracks can be given a numerical value of ionization density relative to this. Thus a track with a value of 2.5 has two and a half times as many bubbles per unit length as a minimum ionization beam track. In order to calibrate your eye, note that when two beam tracks happen to overlap in the chamber, then the ionization density is 2.0.

Figure 1 shows the range and energy loss of  $\pi$  and K mesons and also of protons as a function of momentum. Relative ionization density values are obtained from the energy loss curves by the method described in the caption of Figure 1.



**Figure 1.** Range and Energy Loss ( $dE/dx$ ) in liquid Hydrogen. Dashed curves give  $dE/dx$ , its scale is given on the right side. To obtain relative ionization density, one divides the  $dE/dx$  value by the  $dE/dx$  value of the pion at 10 GeV/c, which is seen to be approaching 0.24 MeV/cm. For example, at 700 MeV/c, the proton has  $dE/dx$  of 0.48 MeV/cm which corresponds to 2.0 in relative ionization density.



**Figure 2.** View of the 82 inch Chamber (viewed along the beam path).

Make sure you understand the gross features of Figure 1. Why, for instance does the curve for the proton lie above that for the K, and that above the curve for the  $\pi$ ? Where would you expect the curve for the electron to lie? Note that the abscissa of this graph is momentum rather than velocity, since momentum is the quantity which is experimentally available. Thus for a known momentum, the ionization density sometimes allows the particle to be identified.

The film is from the 82 inch long hydrogen bubble chamber shown schematically in Figure 2 and located at the two mile long Stanford Linear Accelerator Centre (SLAC). The beam consists of  $\pi^+$  mesons of momentum  $10.3 \text{ GeV}/c$ . The entire chamber sits in an almost uniform magnetic field of  $15.5 \text{ kG}$ , which explains the curvature of the tracks. (Question: Which tracks curve the most - those of high or low momentum particles? Is the field into or out of the plane?)

Figure 3, shows how the chamber looks from the cameras; the following things are worth noting:

1. There are three cameras whose locations are indicated in figures 2 and 3. Each camera has its own roll of film, and each roll of film is labeled by view number 1, 2, or 3 (view 1 is specified if the first two digits are 10 in the picture number at the bottom of the film frame). In the present experiment, two pieces of film from two different views (1 and 2) have been spliced together.



2. A set of numerals are used to produce the picture number. In addition to the view number, the roll number, (a typical experiment will take 500 rolls of film per cameras, each with 1000 exposures), and the frame number or the number of exposure within that roll. The way these numbers appear is shown in figure 3.
3. The "rake" numbers, which run along the side of each frame, give a rough way of locating an event of interest within a frame.
4. In order to allow the full 3-dimensional reconstruction of an event from the measurement of 2 or 3 one-dimensional views, it is convenient to have some points which appear on all views, and whose absolute location in space can be very well measured before or after the experimental run. These are called "fiducial marks" and appear as crosses on the film. Since this experiment has been arranged so that a 3-dimensional reconstruction is not necessary, these are of little interest to you.

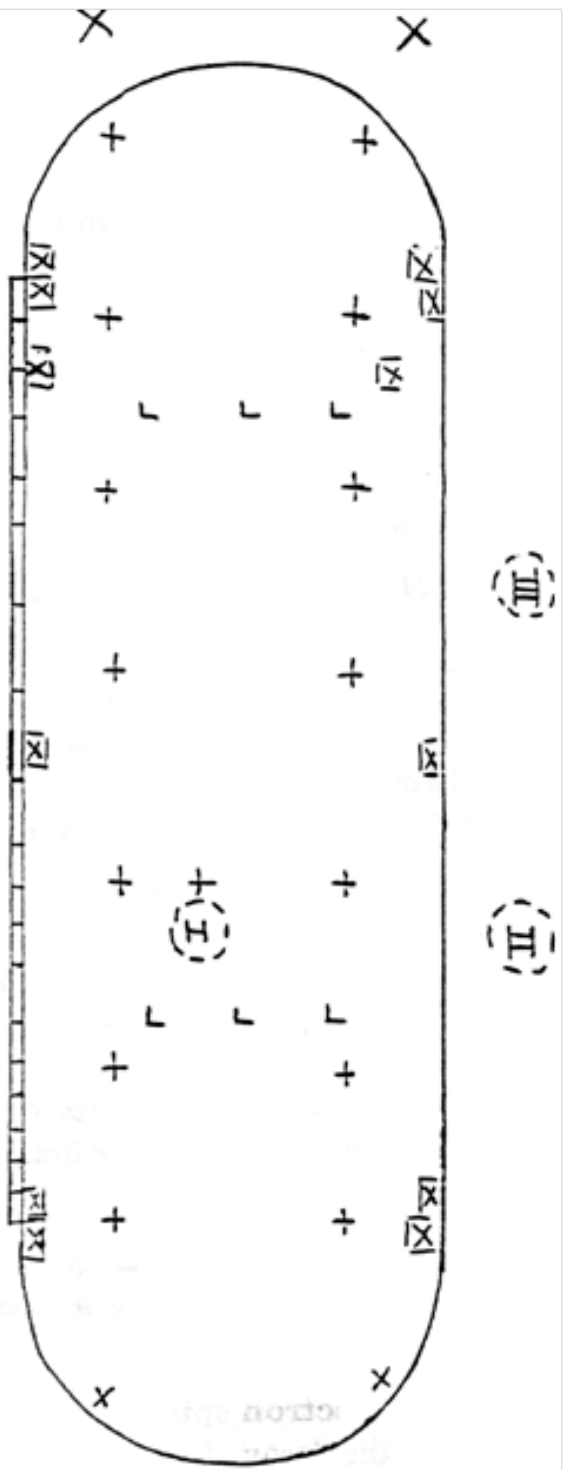
Figure 3

View of the 82 Inch Chamber From the Top  
(As Seen on the Scan Table)

- X are "Body" Fiducials
- + are "Glass" Fiducials
- L are "Coathanger" (Bottom) Fiducials

The Dotted Circles Indicate Approximately  
The Positions Of The 3 Cameras Vertically  
Above The Front Of The Chamber

"Rake Numbers" →



View(10) - Roll(164) - Frame(0872) ←→ 1 0 1 6 4 0 8 7 2

↑  
B  
E  
A  
M

You are also provided with a **momentum template**. Knowing the camera distance and focal length, the focal length of the projection lens on the scan table and the magnetic field in the chamber, we can calculate how much a particle with a given momentum will curve. A series of these curves, labeled with the appropriate momentum value, are provided to enable you to measure the momentum of the particles whose tracks appear on the scan table.

### Preliminary Film Investigations

You should now familiarize yourself with the operation of the scan table and with the film. Make sure you know how to read the view, roll and frame numbers. Scan some of the film to look at some typical interactions, some of which are described below. Sketch those events relevant for the preliminary investigations. Answer all specific questions given below.

- a. **Strong interactions.** Find a few examples of  $(\pi^+ + p)$  interactions. Check for charge conservation. Can you identify a proton track emerging from one such interaction? Does there seem to be a limit to the number of particles produced in such interactions? In bubble chamber terminology such interactions are often labeled as " $N$ -prong events" where  $N$  counts the number of outgoing tracks. What values of  $N$  are possible for small values of  $N$ ? What factor would set an upper limit on  $N$ ?
- b. **Delta-rays.** Occasionally a charged particle will ionize an atom of the hydrogen in such a way that the emitted electron is energetic enough to emerge and make a track in the bubble chamber. See if you can find a few examples.
- c. **Electron spirals.** Occasionally a low energy electron produced as in (b) or by the decay of a particle, will have enough energy to spiral several times, before it loses all its energy or exits the chamber. Find examples of this. Is the ionization density what you would expect? Devise a method and measure the  $dE/dx$  of a spiraling electron by measuring the momentum change over a measured distance. The following scale factors are useful for this purpose. For a flat track, 1 cm on the projection table corresponds to 1.13 cm in the bubble chamber. The radius of curvature and the momentum are related by  $r$  (in cm) =  $0.189 \times p$  (in MeV/c). Does your measured  $dE/dx$  make sense when compared to the plots in figure 1?
- d.  **$\gamma$  conversions.** A photon ( $\gamma$  ray) can "convert" to an electron-positron ( $e^+ - e^-$ ) pair in the presence of nuclear matter, as long as the  $\gamma$  energy is sufficiently high. Why does "sufficiently high" mean at least 1.022 MeV? In this film, high-energy photons are produced when  $\pi^0$  mesons, produced in the strong interactions  $(\pi^+ + p)$ , decay via  $\pi^0 \rightarrow \gamma + \gamma$ . These  $\gamma$  rays can then convert some distance from the decay point. Give the view-roll-frame and rake numbers for at least one of these events. Could a  $\gamma$  conversion take place in vacuum? Justify your answer.
- e. Neutral strange particle decays ( $\Lambda^0 \rightarrow p + \pi^-$ ;  $K^0 \rightarrow \pi^+ + \pi^-$ ). These will have features that are similar to a  $\gamma$ -conversion, since in both cases a neutral particle which leaves no track, decays into two charged particles. The  $\gamma$  conversions however, always have an opening angle between the two charged tracks of exactly  $0^\circ$  so the "V" always has a sharp point (for kinematic reasons). In  $\gamma$  conversions also, both charged tracks must be minimum ionizing (why?), and, most probably, the available energy will be shared in a closely symmetric way between the positron and the electron. Neutral strange particle decays are featured in the events you will study in this experiment.
- f. **Charged particle decays.** Occasionally charged particles produced in the strong interactions will decay before leaving the chamber. If the particle decays "in flight" such behavior may be difficult to distinguish from small angle scatters (see (g) below). However,

if the particle comes to rest in the chamber before decaying, the identification is much easier. One characteristic decay, of which there are at least 3 examples in the film you have, is that of a  $\pi^+$  meson. The  $\pi^+$  slows down and comes to rest in the chamber. In the examples in this film, some of the  $\pi^+$  mesons are not produced in beam interactions, so don't limit your search. The  $\pi^+$  then decays via  $\pi^+ \rightarrow \mu^+ + \nu$ . The  $\mu^+$  has a unique energy, and travels a distance of about 1 cm before coming to rest in the chamber. It then decays via  $\mu^+ \rightarrow e^+ + \nu + \bar{\nu}$ , and the low-energy electron typically spirals before leaving the chamber. Give the view-roll-frame and rake numbers for at least one of these events. Why might we expect not to see this sort of characteristic decay for a  $\pi^-$  meson:  $\pi^- \rightarrow \mu^- + \bar{\nu}$ , then  $\mu^- \rightarrow e^- + \nu + \bar{\nu}$ ?

Measure the range of the  $\mu$  from a  $\pi \rightarrow \mu$  decay and deduce the momentum of the muon using the momentum curve of figure 1. Is this momentum consistent with the hypothesis of a two-body decay of the  $\pi$ , via  $\pi^+ \rightarrow \mu^+ + \nu$ ?

- g. **Secondary interactions.** Occasionally a particle produced in a strong interaction will interact strongly with another proton in the hydrogen before it leaves the chamber. In some cases, a multi-prong secondary interaction is thus produced. Often the produced particle scatters "elastically" (i.e. bounces off the target proton, producing no new particle). In these cases, a 2-prong event will be observed with at least one outgoing track having the characteristics of a proton. In rare cases a track may be observed to suddenly deviate from its smooth path at some point. This is often a "small angle scatter", in which the target proton does not receive enough energy from the scattered particle to leave an observable track in the chamber. Such small angle scatters are often hard to distinguish from the decay of a charged particle.

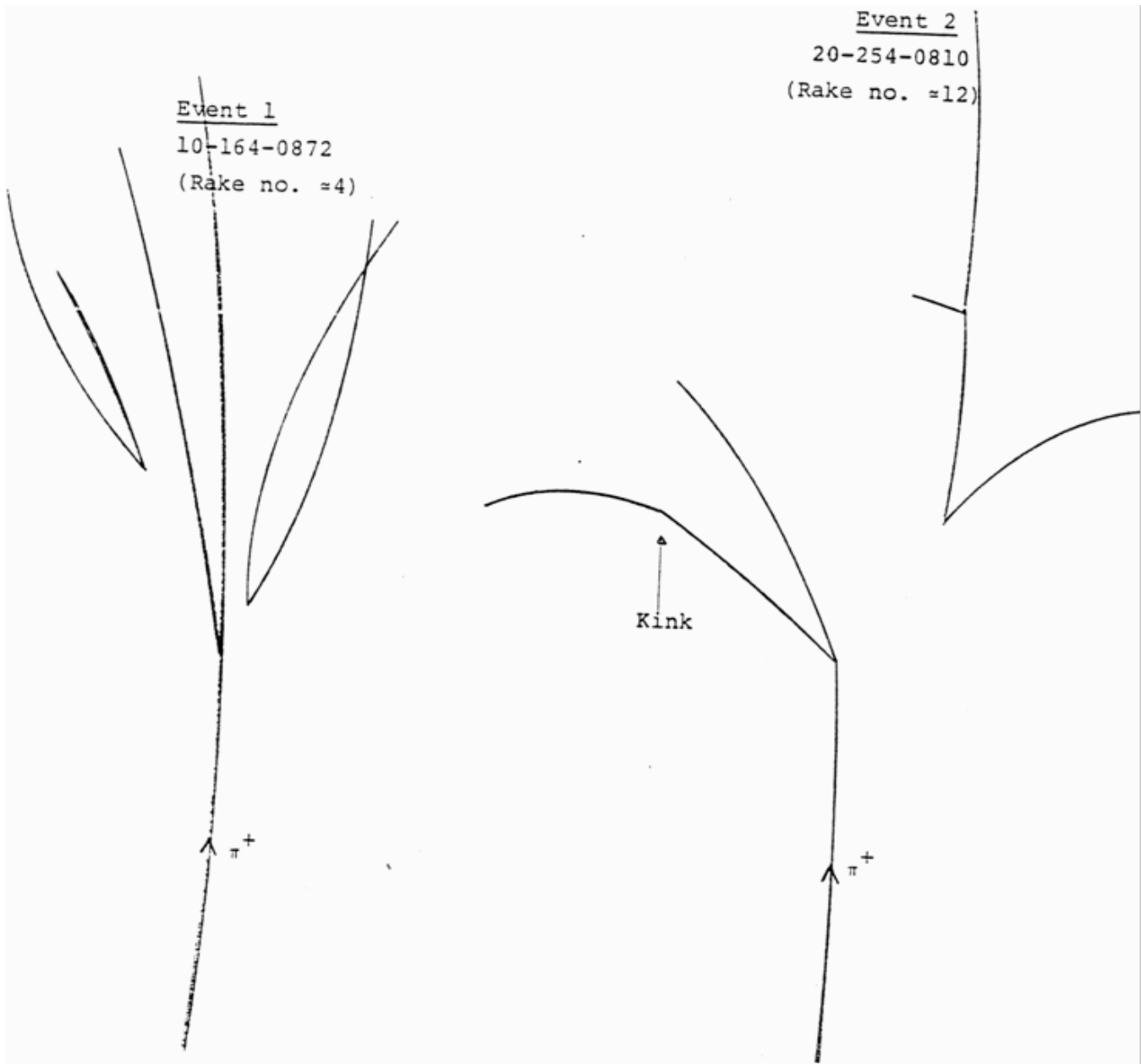
## The Experiment

You are now ready to attempt the experiment. Two events have been chosen in which, to a good approximation, all outgoing tracks lie in the plane of the viewing camera. This plane is close to perpendicular to the magnetic field. The view-roll-frame number, the rake number and a sketch of these events is provided in figure 4. The purpose of the experiment is to reconstruct the interactions and identify all the final state particles (i.e. all the particles emerging from the collision). Normally this is a complicated process, requiring information from at least 2, and usually 3, views, and a computer program for processing this information. Due to the fact that, in the two selected events, all tracks are within  $10^\circ$  of the plane of the photograph, the view provided is sufficient, and the required measurements can be made using the momentum template and an ordinary protractor.

The ultimate aims of the detailed study of these events are to:

- Identify all visible charged and neutral final-state particles (remember, only the charged particles will produce visible tracks). Identification could be positive or by process of elimination of other possible interpretations.
- Measure the vector momenta and energy of the particles. This will allow you to verify conservation laws (both in collisions and decays) of charge, baryon number, lepton number, strangeness and energy-momentum. Once you have learned how to measure and calculate things it would be a good idea to make a table of all relevant variables for quick reference.

Keeping these issues in mind, the following is the outline of the experimental procedures.



**Figure 4.** Sketch of the two events to be analyzed in this experiment.

1. Look at an event. Identify which tracks are caused by positive and which by negative particles.
2. Look at the density of the tracks and use the  $dE/dx$  graph (figure 1) to obtain some preliminary information on the identity of the charged particles (where possible). Of course the graph is really not too useful until you know the momenta of the particles.
3. Using momentum template provided, determine the momenta of the particles including an estimate of possible errors in these measurements. Tabulate your measurements and calculated results for quick reference.
4. Using the  $dE/dx$  information from point 1 (above), the measured charged particle momenta from 2, and the particle properties listed in Appendix I, identify the strange particles

produced in the events you are analyzing. During all of this you should be considering only reactions which take into account all you know and conserve the necessary quantum numbers discussed in the THEORY section. You may assume that the neutral particles which decay are either  $K^0$  or  $\Lambda^0$  particles.

Remember, that in these strong interactions,  $\pi$  mesons are by far the most copiously produced particles. So if you have to guess at the nature of a particle, the best guess is that it is a pion.

In the event number 254-810, the track of the charged particle from the primary interaction which has a small "kink" in it, is a  $\pi^+$ .

5. In resolving the momentum component along a certain direction you may find it helpful to tape a sheet of paper to the table and draw the initial directions of the particles.

If you have difficulty determining the exact spot where the primary interaction took place then err in the upstream direction rather than the downstream direction. Think about how the uncertainty on your ability to locate this primary vertex affects your results.

Finding the initial direction of the particles by estimating the tangent to the particle's track at the point where it appears introduces another source of error. An alternative method, which can help to reduce this error, is suggested in Appendix II. Keep a record of the uncertainties in your measurements and don't be discouraged by the large values of some of them.

6. First, resolve the momenta of the decay products of the neutral particles both parallel and perpendicular to the direction of the neutral particle. Is momentum conserved in the direction perpendicular to the neutral particle's path? How would an error in determining the position of the primary interaction affect your results? Determine the momenta of the neutral particles. The effective mass (more properly referred to as the invariant mass) of a system of particles is defined as  $m^2 = (\sum E_i)^2 - (\sum \vec{p}_i)^2$ . Calculate the effective mass and its uncertainty for the daughter particles of each "V" and check this against the masses of the  $K^0$  and  $\Lambda^0$ .
7. For decaying particles, it is suggested that you compare the observed decay distance with the expected mean decay distance which is given by  $c * \tau * p/m$ , where  $c$  is the speed of light,  $\tau$  is the mean lifetime,  $p$  is the momentum in MeV/c and  $m$  is the mass in MeV/c<sup>2</sup>.
8. Now resolve the momenta of the particles emerging from the primary interaction vertex, including the strange particles, both parallel and perpendicular to the beam direction. Is momentum conserved? Use a value of 10.3 GeV/c for the beam momentum. Does the uncertainty in the momentum of any one particle influence the balance in the momentum equation more than the others?
9. If momentum is not conserved then you may postulate a missing neutral particle to conserve momentum. You may assume that there is, at most, one missing neutral particle. If there is a missing particle, do the uncertainties in your measurements allow you to determine its mass?

10. Postulate a reaction and check that energy is conserved as well as all the necessary quantum numbers. Be able to explain how the uncertainty in your measurements may affect your postulate for the reaction.
11. If there is a missing particle, calculate the direction in which it would be moving and **look at the film to see if there is anything on the film that might be interpreted as decay products of the neutral particle.**

## References

1. Chew G.F., Gell-Mann M. and Rosenfeld A.H., Strongly Interacting Particles, Scientific American, February 1964.
2. Glashow, S.L., Quarks with Color and Flavor, Scientific American, October 1975.
3. Isgur N., and Gabriel K., Hadron Spectroscopy and Quarks, Physics Today, November 1983, p. 36.
4. Schwitters A.F., Fundamental Particles with Charm, Scientific American, October 1977.
5. Hughes I.S., Elementary Particles, Penguin, 1972. (QC 793.2 H83) ON RESERVE.
6. Longo M.J., Fundamentals of Elementary Particles, McGraw Hill, 1973. (QC 721 L8).
7. Perkins D.H., Introduction to High Energy Physics, 3<sup>rd</sup> ed., Addison Wesley, 1987, (QC 721 P329) ON RESERVE.
8. Cline D.B., Rubbia C. and Van der Meer S., The Search for Intermediate Vector Bosons, Scientific American, March 1982.
9. Perl M., Heavy Leptons, Scientific American, March 1978.

The film investigated in this experiment was selected from film actually used in particle physics research. The following two papers which are available in the physics library are examples of publications which used this film.

- a) Goddard, M.C., Gilbert D., Key A.W., Gordon H.A. and Lai K.W., Search for  $\pi^+ p \rightarrow \bar{D}^0 C^{++}_1$  Near Threshold, Phys. Rev. D, 16, 2730, 1977.
- b) Kennedy C.N., Zeman P.D., and Key A.W., Study of the Reaction  $\pi^+ p \rightarrow \pi^+ p \bar{p}$  at 10.3 GeV/c Phys. Rev. D 16, 2083, 1977.

# APPENDIX I

## TABLES OF PARTICLE PROPERTIES

April 1976

N. Barash-Schmidt, A. Barbaro-Galtieri, C. Bricman, V. Chaloupka,  
 R. J. Hemingway, R. L. Kelly, H. J. Losty, A. Rittenberg,  
 M. Roos, A. H. Rosenfeld, T. G. Trippe, G. P. Vost

(Closing date for data: Jan. 1, 1976)

### Stable Particle Table

For additional parameters, see Addendum to this table.  
 Quantities in *italics* have changed by more than one (old) standard deviation since April 1974.

Particle	$I^G(J^P)C_n$	Mass (MeV) Mass <sup>2</sup> (GeV) <sup>2</sup>	Mean Life (sec) $c\tau$ (cm)	Partial decay mode			S	B	
				Mode	Fraction <sup>a</sup>	p or p <sub>max</sub> <sup>b</sup> (MeV/c)			
$\gamma$	$0,1(1^-)$	$0(<7 \times 10^{-22})$		stable			-		
$\nu$	$J=\frac{1}{2}$	$\nu_e: 0(<0.00006)$ $\nu_\mu: 0(<0.65)$	stable	stable			-	○	
e	$J=\frac{1}{2}$	$0.5110034$ $\pm 0.000014$	stable ( $>5 \times 10^{21}$ y)	stable			-	○	
$\mu$	$J=\frac{1}{2}$	$105.65948$ $\pm 0.00035$ $m^2=0.01116$ $m_\mu - m_e = -33.909$ $\pm 0.006$	$2.197134 \times 10^{-6}$ $\pm 0.00077$ $c\tau = 6.5868 \times 10^4$	$e\nu^+$ $e\nu^-$ $3e$ $e\nu_\mu^-$ $e^+\nu_\mu^-$	100 ( $<4$ ) ( $<6$ ) ( $<2.2$ ) ( $<25$ ) (%)	% ( $1.267 \pm 0.023$ ) $\times 10^{-4}$ ( $1.24 \pm 0.25$ ) $\times 10^{-4}$ ( $1.02 \pm 0.07$ ) $\times 10^{-8}$ ( $3.0 \pm 0.3$ ) $\times 10^{-8}$ (%)	53 53 53 53	-	○
$\pi^\pm$	$1^-(0^-)$	$139.5688$ $\pm 0.0064$ $m^2=0.0195$	$2.6030 \times 10^{-8}$ $\pm 0.0023$ $c\tau = 780.4$ ( $\tau^+ - \tau^-$ )/ $\tau =$ ( $0.05 \pm 0.07$ )% (test of CPT)	$\mu\nu$ $e\nu$ $\mu\nu\gamma$ $\pi^0 e\nu$ $e\nu\gamma$ $e\nu^+e^-$	100 ( $1.267 \pm 0.023$ ) $\times 10^{-4}$ ( $1.24 \pm 0.25$ ) $\times 10^{-4}$ ( $1.02 \pm 0.07$ ) $\times 10^{-8}$ ( $3.0 \pm 0.3$ ) $\times 10^{-8}$ (%)	% ( $1.15 \pm 0.05$ )% ( $1.15 \pm 0.05$ )% ( $1.15 \pm 0.05$ )% ( $1.15 \pm 0.05$ )% ( $1.15 \pm 0.05$ )% ( $1.15 \pm 0.05$ )%	30 70 30 5 70 70	○	○
$\pi^0$	$1^-(0^-)$	$134.9645$ $\pm 0.0074$ $m^2=0.182$ $m_\pi - m_\mu = 4.6043$ $\pm 0.0037$	$0.828 \times 10^{-16}$ $\pm 0.057$ S=1.8 <sup>b</sup> $c\tau = 2.5 \times 10^{-6}$	$\gamma\gamma$ $\gamma e^+e^-$ $\gamma\gamma\gamma$ $e^+e^-e^+e^-$ $\gamma\gamma\gamma\gamma$ $e^+e^-$	( $98.85 \pm 0.05$ )% ( $1.15 \pm 0.05$ )% ( $<5$ ) ( $3.32$ ) ( $<5$ ) ( $<2$ )	% ( $1.15 \pm 0.05$ )% ( $1.15 \pm 0.05$ )% ( $1.15 \pm 0.05$ )% ( $1.15 \pm 0.05$ )% ( $1.15 \pm 0.05$ )%	67 67 67 67 67 67	○	○

- a. Quoted upper limits correspond to a 90% confidence level.
- b. For two body decays; p is the momentum of each of the decay particles in the rest frame of the decaying particle. For decays to more than two particles, p<sub>max</sub> is the maximum momentum that any of the decay particles can have, in the frame of the decaying particle.



## Stable Particle Table (cont'd)

Particle	$I^G(J^P)C_{\eta}$	Mass (MeV) Max <sup>2</sup> (GeV) <sup>2</sup>	Mean life (sec) $\tau$ (cm)	Partial decay mode		p or Pmax <sup>b</sup> (MeV/c)	S	B
				Mode	Fraction <sup>a</sup>			
$K^{\pm}$	$\frac{1}{2}(0^-)$	493.707 $\pm 0.037$ $m^2 = 0.244$	$1.2371 \times 10^{-8}$ $\pm 0.0026$ $S=1.9^*$ $\tau = 370.9$ $(\tau^+ - \tau^-) / \tau =$ $(.11 \pm .09)\%$ (test of CPT) $S=1.2^*$	$\mu\nu$	( 53.61 $\pm$ 0.16)%	235	+1 for $K^+$  -1 for $K^-$	0
				$\pi\nu^0$	( 21.05 $\pm$ 0.14)%	205		
				$\pi\pi^+\pi^+$	( 5.59 $\pm$ 0.03)% $S=1.1^*$	125		
				$\pi\pi^0\pi^0$	( 1.73 $\pm$ 0.05)% $S=1.4^*$	133		
				$\mu\pi^0\nu$	( 3.20 $\pm$ 0.09)% $S=1.7^*$	215		
				$e\pi^0\nu$	( 4.82 $\pm$ 0.05)% $S=1.1^*$	228		
				$\mu\nu\gamma$	$( 5.8 \pm 1.5 ) \times 10^{-3}$	236		
				$e\pi^0\pi^0\nu$	( 1.8 $\pm$ 0.4 ) $\times 10^{-5}$	207		
				$\pi\pi^+\pi^+\nu$	( 3.7 $\pm$ 0.2 ) $\times 10^{-5}$	203		
				$\pi\pi^+\pi^+\nu$	( < 5 ) $\times 10^{-7}$	203		
				$\pi\pi^+\mu^+\nu$	( 0.9 $\pm$ 0.4 ) $\times 10^{-5}$	151		
				$\pi\pi^+\mu^+\nu$	( < 3.0 ) $\times 10^{-6}$	151		
				$\nu\nu$	( 1.54 $\pm$ 0.09 ) $\times 10^{-5}$	247		
				$e\nu\gamma$	$( 1.62 \pm 0.47 ) \times 10^{-5}$	247		
				$\pi\pi^0\gamma$	$( 2.71 \pm 0.19 ) \times 10^{-4}$	205		
				$\pi\pi^+\pi^-\gamma$	$( 1.0 \pm 0.4 ) \times 10^{-4}$	125		
				$\mu\pi^0\nu\gamma$	( < 6 ) $\times 10^{-5}$	215		
				$e\pi^0\nu\gamma$	$( 3.7 \pm 1.4 ) \times 10^{-4}$	228		
				$\pi e^+\pi^+$	( 2.6 $\pm$ 0.5 ) $\times 10^{-7}$	227		
				$\pi^+\pi^+\pi^+$	( < 1.5 ) $\times 10^{-5}$	227		
				$\pi\mu^+\mu^-$	( < 2.4 ) $\times 10^{-6}$	172		
				$\pi\gamma\gamma$	( < 3.5 ) $\times 10^{-5}$	227		
				$\pi\gamma\gamma$	( < 3.0 ) $\times 10^{-4}$	227		
				$\pi\nu\nu$	( < 0.6 ) $\times 10^{-6}$	227		
				$\pi\gamma$	( < 4 ) $\times 10^{-6}$	227		
				$e\pi^+\mu^+$	( < 2.8 ) $\times 10^{-8}$	214		
				$e\pi^+\mu^+$	( < 1.4 ) $\times 10^{-8}$	214		
				$\mu\nu\nu$	( < 6 ) $\times 10^{-6}$	235		
$K^0$	$\frac{1}{2}(0^-)$	497.70 $\pm 0.13$ $S=1.1^*$ $m^2 = 0.248$	50% $K_{\text{Short}}$ ; 50% $K_{\text{Long}}$				+1	0
$K_S^0$	$\frac{1}{2}(0^-)$		$0.8930 \times 10^{-10}$ (t) $\pm 0.0023$ $\tau = 2.68$	$\pi^+\pi^-$ ( 68.67 $\pm$ 0.25 )% $S=1.1^*$ $\pi^0\pi^0$ ( 31.33 )% $\mu^+\mu^-$ ( < 3.2 ) $\times 10^{-7}$ $e^+e^-$ ( < 3.4 ) $\times 10^{-4}$ $\pi^+\pi^-\gamma$ ( 2.0 $\pm$ 0.4 ) $\times 10^{-3}$ $\gamma\gamma$ ( < 0.4 ) $\times 10^{-3}$	206 209 225 249 206 249			
$K_L^0$	$\frac{1}{2}(0^-)$		$5.181 \times 10^{-8}$ $\pm 0.040$ $\tau = 1553$	$\pi^0\pi^0\pi^0$ ( 21.4 $\pm$ 0.7 )% $S=1.2^*$ $\pi^+\pi^-\pi^0$ ( 12.25 $\pm$ 0.18 )% $S=1.1^*$ $\pi\nu\nu$ ( 27.1 $\pm$ 0.5 )% $\pi\nu\nu$ ( 39.0 $\pm$ 0.5 )% $S=1.1^*$ $\pi\nu\nu\gamma$ ( 1.3 $\pm$ 0.8 )% $\pi^+\pi^-$ ( 0.201 $\pm$ 0.006 )% $\pi^0\pi^0$ ( 0.094 $\pm$ 0.019 )% $S=1.5^*$ $\pi^+\pi^-\gamma$ ( 6.0 $\pm$ 2.0 ) $\times 10^{-5}$ $\pi^0\gamma\gamma$ ( < 2.4 ) $\times 10^{-4}$ $\gamma\gamma$ ( 4.9 $\pm$ 0.5 ) $\times 10^{-4}$ $e^+\mu^-$ ( < 2.0 ) $\times 10^{-9}$ $\mu^+\mu^-$ ( 1.0 $\pm$ 0.3 ) $\times 10^{-8}$ $\mu^+\mu^-\gamma$ ( < 7.8 ) $\times 10^{-6}$ $\mu^+\mu^-\pi^0$ ( < 5.7 ) $\times 10^{-5}$ $e^+e^-$ ( < 2.0 ) $\times 10^{-9}$ $e^+e^-\gamma$ ( < 2.8 ) $\times 10^{-5}$ $\pi^+\pi^-\pi^0$ ( < 7.2 ) $\times 10^{-6}$ $\pi^0\pi^+\pi^-\nu$ ( < 2.2 ) $\times 10^{-1}$	139 133 216 229 229 206 209 206 231 249 238 225 225 177 249 249 206 207			
$\eta$	$0^+(0^-)^+$	548.8 $\pm 0.6$ $S=1.4^*$ $m^2 = 0.301$	$\Gamma = (0.85 \pm 0.12) \text{keV} (1)$ Neutral decays ( 71.0 $\pm$ 0.7 )% $S=1.1^*$  $\tau < 10^{-20}$  Charged decays ( 29.0 $\pm$ 0.7 )% $S=1.1^*$	$\gamma\gamma$ ( 38.0 $\pm$ 1.0 )% $S=1.2^*$ $\pi^0\gamma\gamma$ ( 3.1 $\pm$ 1.1 )% $S=1.2^*$ $3\pi^0$ ( 29.9 $\pm$ 1.1 )% $S=1.1^*$ $\pi^+\pi^-\pi^0$ ( 23.6 $\pm$ 0.6 )% $S=1.1^*$ $\pi^+\pi^-\gamma$ ( 4.89 $\pm$ 0.13 )% $S=1.1^*$ $e^+e^-\gamma$ ( 0.50 $\pm$ 0.12 )% $\pi^0e^+e^-$ ( < 0.04 )% $\pi^+\pi^-$ ( < 0.15 )% $\pi^+\pi^-\pi^0$ ( 0.1 $\pm$ 0.1 )% $\pi^+\pi^-\pi^0\gamma$ ( < 6 ) $\times 10^{-4}$ $\pi^+\pi^-\gamma\gamma$ ( < 0.2 )% $\mu^+\mu^-$ ( 2.2 $\pm$ 0.8 ) $\times 10^{-5}$ $\mu^+\mu^-\pi^0$ ( < 5 ) $\times 10^{-4}$	274 258 180 175 236 274 258 236 236 175 236 253 211	0	0	

## Stable Particle Table (cont'd)

Particle	$I^G(J^P)C_n$	Mass (MeV) Mass <sup>2</sup> (GeV) <sup>2</sup>	Mean Life (sec) $c\tau$ (cm)	Partial decay mode			S	B
				Mode	Fraction <sup>a</sup>	p or P <sub>max</sub> <sup>b</sup> (MeV/c)		
$p^+$	$\frac{1}{2}(\frac{1}{2}^+)$	938.2796 $\pm 0.0027$ $m^2 = 0.8804$	stable ( $> 2 \times 10^{30}$ y)				0	+1
$n$	$\frac{1}{2}(\frac{1}{2}^+)$	939.5731 $\pm 0.0027$ $m^2 = 0.8828$ $m_p - m_n = -1.29343$ $\pm 0.00004$	$918 \pm 14$ $c\tau = 2.75 \times 10^{13}$	$p e^- \nu$	100 %		0	+1
$\Lambda$	$0(\frac{1}{2}^+)$	1115.60 $\pm 0.05$ $S = 1.2^*$ $m^2 = 1.245$	$2.578 \times 10^{-10}$ $\pm 0.021$ $S = 1.6^*$ $c\tau = 7.73$	$p \pi^-$ $n \pi^0$ $p e^- \nu$ $p \mu^- \nu$ $p \pi^- \gamma$	( 64.2 $\pm$ 0.5 ) % ( 35.8 $\pm$ 0.5 ) % ( 8.13 $\pm$ 0.29 ) $\times 10^{-3}$ ( 1.57 $\pm$ 0.35 ) $\times 10^{-4}$ ( 0.85 $\pm$ 0.14 ) $\times 10^{-3}$	100 104 163 131 100	-1	+1
$\Sigma^+$	$1(\frac{1}{2}^+)$	1189.37 $\pm 0.06$ $S = 1.8^*$ $m^2 = 1.415$	$0.800 \times 10^{-10}$ $\pm 0.006$ $c\tau = 2.40$	$p \pi^0$ $n \pi^+$ $p \gamma$ $n \pi^+ \gamma$ $\Lambda e^+ \nu$ $\Sigma^+ \rightarrow \pi^+ p \nu$ $\Sigma^+ \rightarrow \pi^+ n \nu$ $p e^+ \nu$	( 51.6 $\pm$ 0.7 ) % ( 48.4 $\pm$ 0.7 ) % ( 1.24 $\pm$ 0.18 ) $\times 10^{-3}$ ( 0.93 $\pm$ 0.10 ) $\times 10^{-3}$ ( 2.02 $\pm$ 0.47 ) $\times 10^{-5}$ ( < 3.0 ) $\times 10^{-5}$ ( < 0.5 ) $\times 10^{-5}$ ( < 7 ) $\times 10^{-6}$	189 185 225 185 71 202 224 225	-1	+1
$\Sigma^0$	$1(\frac{1}{2}^+)$	1192.47 $\pm 0.08$ $m^2 = 1.422$	$< 1.0 \times 10^{-14}$ $c\tau < 3 \times 10^{-4}$	$\Lambda \gamma$ $\Lambda e^+ e^-$ $\Lambda \gamma \gamma$	100 % ( 5.45 ) $\times 10^{-3}$ ( < 3 ) %	74 74 74	-1	+1
$\Sigma^-$	$1(\frac{1}{2}^+)$	1197.35 $\pm 0.06$ $m^2 = 1.434$	$1.482 \times 10^{-10}$ $\pm 0.017$ $S = 1.5^*$ $c\tau = 4.44$	$n \pi^-$ $n e^- \nu$ $n \mu^- \nu$ $\Lambda e^- \nu$ $n \pi^- \gamma$	100 % ( 1.08 $\pm$ 0.04 ) $\times 10^{-3}$ ( 0.45 $\pm$ 0.04 ) $\times 10^{-3}$ ( 0.60 $\pm$ 0.06 ) $\times 10^{-4}$ ( 4.6 $\pm$ 0.6 ) $\times 10^{-4}$	193 230 210 79 193	-1	+1
$\Xi^0$	$\frac{1}{2}(\frac{1}{2}^+)(1)$	1314.9 $\pm 0.6$ $m^2 = 1.729$	$2.96 \times 10^{-10}$ $\pm 0.12$ $c\tau = 8.87$	$\Lambda \pi^0$ $\Lambda \gamma$ $\Sigma^0 \gamma$ $p \pi^-$ $p e^- \nu$ $\Sigma^0 e^- \nu$ $\Sigma^0 \mu^- \nu$ $\Sigma^0 \pi^- \nu$ $\Sigma^0 \mu^- \nu$ $\Sigma^0 \mu^- \nu$ $p \mu^- \nu$	100 % ( 0.5 $\pm$ 0.5 ) % ( < 7 ) % ( < 3.6 ) $\times 10^{-5}$ ( < 1.3 ) $\times 10^{-3}$ ( < 1.1 ) $\times 10^{-3}$ ( < 0.9 ) $\times 10^{-3}$ ( < 1.1 ) $\times 10^{-3}$ ( < 0.9 ) $\times 10^{-3}$ ( < 1.3 ) $\times 10^{-3}$	135 184 117 299 323 120 112 64 49 309	-2	+1
$\Xi^-$	$\frac{1}{2}(\frac{1}{2}^+)(1)$	1321.29 $\pm 0.14$ $m^2 = 1.746$	$1.652 \times 10^{-10}$ $\pm 0.023$ $S = 1.1^*$ $c\tau = 4.95$	$\Lambda \pi^-$ $\Lambda e^- \nu$ $\Sigma^0 e^- \nu$ $\Lambda \mu^- \nu$ $\Sigma^0 \mu^- \nu$ $\Sigma^0 \mu^- \nu$ $n \pi^-$ $n e^- \nu$ $n \mu^- \nu$ $\Sigma^- \gamma$ $p \pi^- \pi^-$ $p \pi^- e^- \nu$ $p \pi^- \mu^- \nu$ $\Sigma^0 e^- \nu$	100 % ( 0.69 $\pm$ 0.18 ) $\times 10^{-3}$ ( < 0.5 ) $\times 10^{-3}$ ( 3.5 $\pm$ 3.5 ) $\times 10^{-4}$ ( < 0.8 ) $\times 10^{-3}$ ( < 1.1 ) $\times 10^{-3}$ ( < 3.2 ) $\times 10^{-3}$ ( < 1.5 ) % ( < 1.2 ) $\times 10^{-3}$ ( < 4 ) $\times 10^{-4}$ ( < 4 ) $\times 10^{-4}$ ( < 4 ) $\times 10^{-4}$ ( < 2.3 ) $\times 10^{-4}$	139 190 123 163 70 303 327 313 118 223 304 250 6	-2	+1
$\Omega^-$	$0(\frac{3}{2}^+)(1)$	1672.2 $\pm 0.9$ $m^2 = 2.796$	$1.3 \pm 0.3 \times 10^{-10}$ $\pm 0.2$ $c\tau = 4.0$	$\Xi^0 \pi^-$ $\Xi^0 \pi^-$ $\Lambda K^-$	Total of 43 events seen	293 290 211	-3	+1

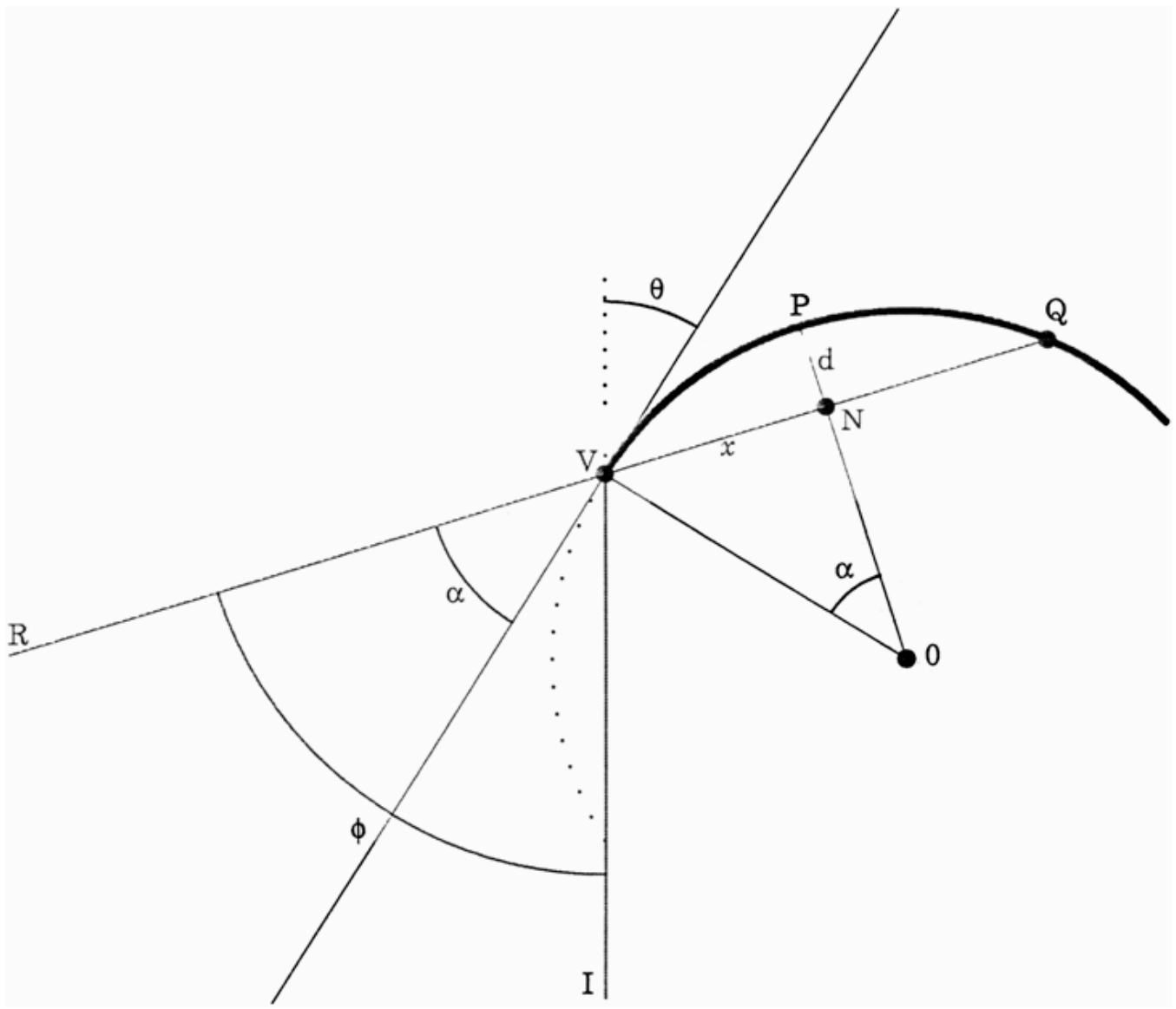
## APPENDIX II

### Determination of Particle Direction

When using the template to find the momenta of particles one should write down, for each measurement, the uncertainty in the momentum. In order to check momentum conservation, the angles between the direction of the initial particle and the daughter particles must be found. If the initial directions of the daughter particles are found by drawing tangents to the tracks at the vertex, the results will have large errors. Also, angles found by this method have, in general, been found to be too large. The following procedure is suggested.

Take a large clean white sheet of paper and tape it onto the scanning table and trace out or draw the relevant geometrical construct as follows. Consult figure 5 before you proceed. Let V be the vertex of the interaction.

1. First draw a tangent to the incident beam direction such that IV represents the incident direction with respect to which we wish to measure the angle of the secondary tracks.
2. Pick a chord QV, long enough so that you can measure its length with a few percent accuracy, but short enough so that the curvature does not decrease noticeably from the point V to the point Q. Extend the chord QV backward to draw a line QVR.
3. Measure the angle IVR =  $\varphi$ . Measure the chord length VQ. Then the desired angle is  $\theta = \varphi - \alpha = \arcsin(x/r)$ , where  $r$  is the radius of curvature of the track and  $x = VQ/2$ . The value of  $r$  can be determined from the momentum measurement using the formula  $r(\text{in cm}) = 189 p(\text{in GeV}/c)$ . The value of  $r$  can also be obtained by measuring the sagitta  $d$  and using  $r = (x^2 + d^2)/(2d)$ .



**Figure 5.** Geometrical construction to determine a particle's direction.