ADVANCED UNDERGRADUATE LABORATORY

EXPERIMENT 35

MUON LIFETIME

Introduction

The goal of particle physics is to understand the basic building blocks of the universe and how they interact. We currently believe that the basic constituents are quarks and leptons, and these fundamental fermions interact via forces mediated by gauge bosons corresponding to fundamental symmetries of the universe. The goal of this experiment is to measure the lifetime of one of these fundamental fermions: the muon. This lifetime determines the value of the Fermi Constant G_F, which is the fundamental parameter describing the strength of low energy weak interactions. The mass of the W gauge boson can be calculated from G_F by using the standard electroweak unified theory.

A typical particle physics experiment consists of a beam, a detector, a data acquisition system, and an offline analysis system. In this experiment, cosmic rays provide us with a free muon beam. The detector is a total absorption calorimeter consisting of a tank of liquid scintillator viewed by three photomultiplier tubes. The trigger and data acquisition system consists of a crate of standard NIM electronics readout by a multichannel analyzer. The offline analysis system is the student and a computer.

The primary cosmic rays hitting the top of the earth's atmosphere are almost all protons or heavier atomic nuclei. These primary cosmic rays interact high in the earth's atmosphere producing copious quantities of secondary particles. The most common secondary particles are pions. Charged pions decay into muons and neutrinos, and these muons typically penetrate the atmosphere. Many of the muons reaching a ground-level laboratory are very penetrating, but a few may come to rest in a solid or liquid detector.

About five muons per minute are expected to stop in this experiment. A muon coming to rest in a scintillation computer produces two pulses with a mean separation of approximately 2 micro-seconds. In this experiment we measure and analyze the distribution of this time difference so as to infer the mean lifetime of muons and deduce whatever new information can be drawn from the result. Setting up and testing the necessary data taking electronics is an important part of the experiment.

Cosmic Ray and Muon Physics

Cosmic rays, discovered in the 1930's as background radiation in laboratory experiments, provide us with a free source of muons. The flux of muons is not difficult to measure because the most penetrating ionizing particles in cosmic radiation at sea level are almost all muons. Gamma ray showers, induced by the Dopplier-shifted 70 MeV photons from $\pi°$ decay in the upper atmosphere, also occur copiously at sea level, but they can be absorbed by a simple lead shield less than 20 cm thick. The cosmic ray muon flux can thus be isolated relatively easily. A brief history of the muon has been written by Wu and Hughes (1977).

Muons are known to decay, in their own frame of reference, with a vacuum mean life of 2.19703 ± 0.00004 µs [Review of Particle Properties (1996) p. 21]. The primary decay mode

$$\mu^+ \rightarrow e^+ \nu_e + \overline{\nu}_\mu \tag{1}$$

or

$$\mu^{\bullet} \rightarrow e^{\bullet} + \overline{\nu}_e + \nu_\mu \tag{2}$$

The only other known decay mode of the muon in free space is (Hughes and Kinoshita, 1977)

$$\mu^+ \rightarrow e^+ + \nu_e + \nu_\mu + \gamma \qquad \text{or} \qquad \mu^{\bullet} \rightarrow e^{\bullet} + \overline{\nu}_e + \nu_\mu + \gamma \tag{3}$$

which has a branching ratio of about one part in 1000. The decay of muons is a weak interaction process, and, apart from coulomb interactions, the muon interacts only weakly with nucleons.

When positive muons stop in liquid scintillator or any other material, they may form muonium, a hydrogen-like atom (μ^+e^-), by the reaction

$$\mu^+ + X \rightarrow (\mu^+ e^{\bullet}) + X^+ \tag{4}$$

where X is any available molecule. A possible additional decay mode of the μ^+ bound to an electron in this way is

$$\mu^+ + e^{\bullet} \rightarrow \nu_e + \overline{\nu}_\mu \tag{5}$$

However, Pontecorvo has estimated that the branching ratio for decay mode (4) is only 1 part in 10^{10} (Hughes and Kinoshita, 1977), so that muonium formation should not affect the decay rate appreciably.

For negative muons the alternative decays in the presence of matter are quite different. A negative muon stopped in a piece of matter forms a "muonic atom". (Hüfner, et al., 1977).

$$\mu^- + X \rightarrow (\mu^- X^+) + e^- \tag{6}$$

The μ^- fall rapidly into hydrogen-like orbits around the nuclei of atoms. Since μ^- and e^- are not identical particles, the fact that an electronic orbit having particular quantum numbers is occupied does not prevent a μ^- from occupying an orbit having the same quantum numbers. Furthermore, the μ^- orbits have much smaller radii, $-\dfrac{1}{207}$ of the corresponding electron orbits. Thus the Bohr radius of a C^{5-} ion is 8.8×10^{-10} cm whereas the orbit radius of μ^- in the lowest Bohr orbit of the muonic carbon atom is 4.3×10^{-12} cm. Thus, direct interaction is to be expected between μ^- and atomic nuclei, and in particular the decay mode within such nuclei

$$\mu^- + p \rightarrow n + \nu_\mu \tag{7}$$

is proportional to Z^4 and so becomes significant for sufficiently high Z atoms[1]. An early

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1 The rate is proportional to the number of protons in the nucleus and inversely proportional to the volume of the

measurement of the μ⁻ lifetime in carbon was made at Chalk River by Bell and Hincks (1952).

The Experiment

The principle of the experiment is to obtain a time distribution of consecutive pulse pairs coming from a large liquid (or solid) detector. Such a time distribution will contain a random component, indicating that some pulse pairs were unrelated in time, and an exponentially decaying component corresponding to muon decays. Only when the first pulse of a pair corresponds to a muon entering the detector, and the second to the decay of a muon which stopped in the detector will we get an event contributing to the exponential decay.

The source of the pulse pairs is a large liquid scintillator, of about 9 litres volume and 12 cm vertical thickness[2]. When a charged particle passes through matter it loses energy by ionizing or exciting atoms or molecules. A scintillator is a material in which some of the lost energy is promptly emitted as visible light by the de-exciting atoms or molecules. The ionization energy loss for high energy particles varies from about 2 to 1 MeV per gm/cm^2 going from low to high atomic number materials [Review of Particle Properties (1996) p. 72]. In this experiment the scintillator is viewed by three XP1040 photomultiplier tubes. The latter have been coupled so as to sum their output pulses and trigger a fast discriminator circuit (Fig. 1). Ionization energy loss, scintillators, photomultipliers, discriminators, and almost all other technical aspects of this experiment are very well explained in Leo (1987).
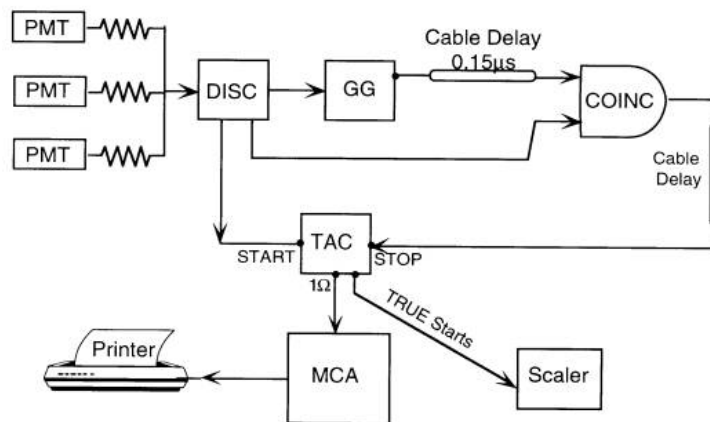
Figure 1   Outline of electronics layout. PM = photomultiplier, DISC = discriminator, GG = gate generator, COINC = coincidence unit, DG = delay generator, TAC = time-to-amplitude converter, MCA = multichannel analyzer.

The discriminator provides logic pulses of approximately -0.7V amplitude and 30 ns duration, which are suitable for triggering the electronic equipment that follows.
The following logical requirements must be met:

a)   the first pulse of a pulse pair must start a timing circuit, currently a time-to-

muon orbitals.
2 It is not necessary to use a scintillator; a tank of water is a cheap and flexible alternative [Ward et al. (1985)].

amplitude converter (TAC);

b)   the second pulse of the pulse pair must go to the STOP input of the timing circuit

c)   to avoid confusing the timing circuit with starts and stops coming at the same time, all STOP pulses must be delayed by a fixed amount.

d)   if a STOP fails to occur within the time range of the timing circuit, the timing and logic circuits must be reset so as to be ready to accept a new START pulse.

e)   since START and STOP pulses are **a priori** indistinguishable, and STOP pulses are to be delayed, a special mechanism is required to prevent START pulses themselves appearing as STOPS.
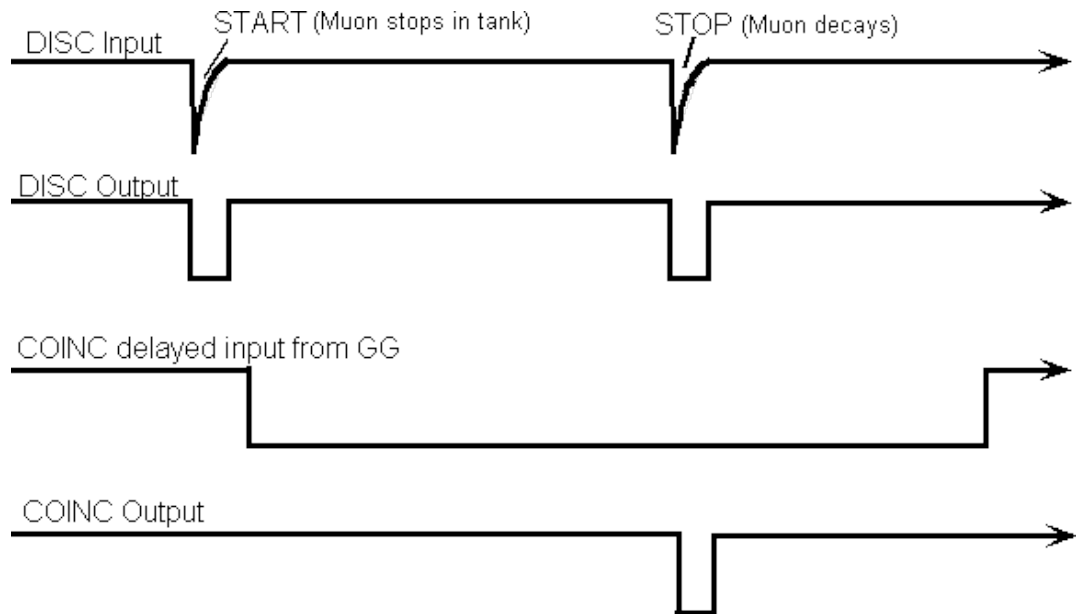

Figure 2: Trigger logic circuit pulse timing

A trigger logic circuit must be set up with the set of modular units provided; namely, a discriminator, a gate/delay generator, and a coincidence circuit. The output of the timing circuit is accumulated in a multichannel analyser to produce the spectrum of time separations between the START and STOP pulses.

Several additional measurements are necessary to complete this experiment. Generally, it will not be necessary to carry out all of these tasks, as there is continuity and experience accumulating in the use of the equipment.

a) **Equalizing the photomultiplier gains**

If the gains of the three photomultipliers are equalized, then sensitivity to muons and their decay electrons is almost
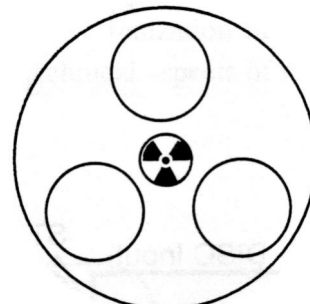
independent of where they decay in the scintillator. The
photomulitpliers view the scintillator from above as shown
in the diagram. Gain equalization can be achieved by
connecting the photomultiplier outputs one by one direct to
the discriminator input without going through the pulse
adding network. If the discriminator is set to its minimum,
about 30 mV, then the detector system should be sensitive to nuclear gamma rays (^{60}Co or ^{208}Tl)
when the high voltage across the photomultipliers is as low as 1750 V. The gain of each
multiplier can be changed slightly by adjusting a potentiometer in series with the high voltage.
If the gains are equalized, each multiplier will give the same count rate when a radioactive
source is placed beneath the centre of the bottom of the tank, as in the figure.

If your photomultiplier high voltage is too low, some relevant pulses may not pass the
discriminator even at its lowest setting. If the voltage is too high, some of your pulses may
saturate your discriminator and you may also fry the PMT. The ideal choice of voltage and
discriminator are those values which produce the spectrum which gives you the smallest errors
on the muon lifetime.

b) Equalizing the photomultiplier delays

The current pulses at the anodes of the photomultipliers
are extremely fast rising and are not integrated on any
input capacitance in this experiment. On a voltage-time
scale, seen by passing this current signal through 50
ohms, a pulse might look like the sketch shown here,
where the time marks are roughly at 10 ns intervals.
When adding several pulses of this nature together, the
leading edge of each needs, for optimum effect, to be
aligned with the others. This can be achieved by adjustin;
the relative cable length between each photomultiplier
and the pulse adding circuit. When the cable delays are optimized, the average signal voltage
peaks will be maximized (in the negative voltage direction).
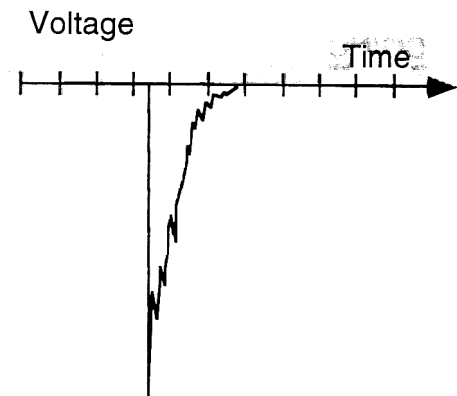
c) Choosing the discriminator level

The energy loss of the initial stopping muon and the final decay electron pulse can be estimated
from the kinematics of muon decays and the ionization energy loss in liquid scintillator of about
2 MeV/cm for muons or low energy electrons. Natural radioactivity will produce background
pulses in the tank, with the highest energy common gamma rays being 2.6 MeV gamma rays
from ^{208}Tl. A ^{60}Co source yields gamma rays of 1.17 and 1.33 MeV; when both are detected
simultaneously, their Compton continua sum to a maximum absorbed energy of 2.5 MeV. The
initial discriminator setting should be based on the size of the observed ^{60}Co pulses and the
energies of the background and signal pulses. An improved value can be achieved by observing
actual decay spectra taken with different discriminator thresholds.

d) Setting the delays, gates, and time range

The STOP cable delay and gate generator width are often already chosen at suitable values. The TAC time range should be chosen to mathc the gate generated by the electronic gate generator, typically 20 microseconds. The student is advised to spread this time range over no more than 256 analyzer channels, otherwise the counts per channel will be too few for the application of Gaussian statistics. The Ortec 437A TAC will not produce an output signal and will automatically reset itself if it does not receive a stop pulse within the set time range.

e) Calibrating the average time/channel of the combined TAC-MCA system

START pulses from the Ortec time Calibrator may be fed into the TAC START input, and STOP pulses into the TAC STOP input via the sTOP delay cable and delay box provided. The Calibrator puts out pulses with START/STOP time spacings which can be set to multiple of some constant time. In a linear system this gives rise to coincidence signals equally spaced throughout the available analyzer channels. When only 256 channels of the analyzer are used, counts corresponding to any given delay normally are found to be in a single channel, so that there is an uncertainty of ± half a channel in the location. If necessary, this uncertainty (typically ±20 to 40 ns in this experiment) can be greatly reduced by using the adjustable delay box an an additional vernier. By adding or subtracting delays of the delay box, which is adjusted using switches, the Calibrator pulses can be made to fall on the borderline between adjacent channels, yielding typically a 40-fold increase in precision.[3]

f) Examining the nonlinearities of the combined TAC-multichannel analyzer

The method of calibrating just described permits sections of the spectrum to be determined to ± 0.025 of a channel. Thus each such interval can be calibrated with an accuracy of about one part per thousand, sufficient to investigate the integral nonlinearity of the system. Of greater importance, these data can be used to correct the spectrum obtained for nonlinearities, prior to final analysis of the data.

Analysis and interpretation of the data

The background component of the observed time spectrum can include both random and non-random components. A random background event can occur, for example, when one cosmic ray starts the TAC and a second cosmic ray stops it. Uncorrelated starts and stops from cosmic rays, gamma rays and electronic noise can all contribute to this random background. Non-random background can occur from electronic pickup and glitches. For example, many photomultipliers, especially if old and somewhat gassy, give unwanted additional after pulses, typically about half a microsecond after the normal pulse arising from an external event.

If there are no non-random backgrounds, the expected time spectrum is of the form

$$\frac{dn}{dt} = n_o e^{-\lambda t} + b \tag{8}$$

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3 This precision could, of course, be limited by an inaccuracies in the Calibrator.

where dn/dt is the count rate of pulse pairs corresponding to time separation t, n_o is a constant, λ is the muon decay rate[4], and b is the rate due to the random background component. Any part of the observed spectrum which has no non-random backgrounds can be fitted to equation (8) so as to obtain τ, the laboratory lifetime of the muon. An alternative to the use of equation (8) is the simple linear regression analysis

$$\ln\left(\frac{dn}{dt} - b\right) = \ln n_o - \lambda t \qquad (9)$$

Use of (9) requires determination of b separately. The easiest way is to make the time window for your data long enough that all muons have decayed before the end of the time window, so b is simply measured from the final flat part of the time distribution.[5]

Although equation (9) correctly describes the vacuum decay of muons at rest, whether μ^+ or μ^-, in the liquid scintillator capture of negative muons by the carbon adds to the decay rate of negative muons:

$$\lambda_- = \lambda_o + \lambda_c \qquad (10)$$

where λ_- is the decay rate of negative muons in the scintillator, λ_o the vacuum decay rate and λ_c the capture rate.

In theory this means that a mixed μ^- and μ^+ beam would decay in the scintillator according to the expression

$$n = n_+ e^{-\lambda_o t} + n_- e^{-\lambda_- t} + b \qquad (11)$$

where n_+ is the number of μ_+ decaying per second at time t = 0 and n_- is the observable number of μ_- decaying per second at t = 0.

In practice we cannot accumulate sufficient data in a reasonable time (for example, one week) to permit the analysis of those data into two exponential components and a constant background, because the two rates λ_o and λ_- are too close. At best we can determine an average λ (using (8) or (9)) and assume it to be a weighted average of λ_o and λ_-.

To determine the weighting, we make use of two facts: the approximately known ratio of μ^+ to μ^- in cosmic rays at sea level, and the fact that muons captured by nuclei in the scintillator will escape direct observation — even though these decays directly affect λ_- through equation (10).

Let the fractions of μ^+ and μ^- stopping in the scintillator be f_+ and f_-, and let the fraction of μ_- that escape observation be f_c. Then the average λ corresponding to the weighted mean of λ_o (for μ^+)

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4 The decay rate is simply the inverse of the mean lifetime, i.e. $\lambda = 1/\tau$

5 It is also recommended to use a scaler to count the total number of STARTS, including those without stops. If n is the START rate per second, and τ is the channel width, then for a random, low rate background we expect $b \cong n^2\tau$

and λ_- must be

$$\lambda = \frac{f_+ \lambda_o + f_- (1 - f_c) \lambda_-}{f_+ + f_- (1 - fSUBc)} \qquad (12)$$

Note that, while $f_+ + f_- = 1$, the denominator of (12) is not in general unity.

The value of f_c follows directly from the decay channel widths for μ^-; hence

$$f_c = \frac{\lambda_c}{\lambda_o + \lambda_c} \quad \text{and} \quad 1 - f_c = \frac{\lambda_o}{\lambda_o + \lambda_c} \qquad (13)$$

Eliminating f_c and λ_- from (12), using (13) and (10)

$$\lambda = \lambda_o \left\{ 1 + \frac{\lambda_c f_-}{(\lambda_o + \lambda_c) f_+ + \lambda_o f_-} \right\} \qquad (14)$$

Solving for λ_c

$$\lambda_c = \frac{\lambda - \lambda_o}{1 - f_+ \lambda / \lambda_o} \qquad (15)$$

In (15) we shall assume that λ is the single value given by fitting the summed exponentials in the experiment, and that λ_o is the well known vacuum value already cited. The lowest energy muon component in cosmic rays contains somewhat more μ^+ than μ^-. According to Thompson's article in Wolfendale (1973), the ratio is about 6:5. Assume this value is accurate to calculate the λ_c.

Note that equation (14) assumes we are able to view the entire muon decay spectrum, whereas the circuit logic of the present experiment forces us to blank out the first few channels of the muon spectrum. The omission of these few channels weights slightly the positive muon component which has the longer half life, thus introducing a small systematic error in the use of equation (14).
Conclusions

By the end of the experiment you should have answered the following questions:

1) What is the average lifetime of the muons decaying in the liquid scintillator?
2) Are the data consistent with a single lifetime and a random background? If not, what are the other features?
3) What is the lifetime of negative muons captured by nuclei in the scintillator?

From your average muon lifetime and the known muon mass, m_μ, you can calculate the lowest order value for the Fermi constant [Perkins (1987)]

$$G_F = \sqrt{\frac{192\pi^3}{\tau_\mu m_\mu^5}}$$

The lowest order calculation of the mass of the W boson is

$$M_W = \sqrt{\frac{\pi\alpha}{\sqrt{2}G_F \sin^2\theta_w}}$$

where α is the fine structure constant, and $\sin^2\theta_w = 0.2315 \pm 0.0004$ [Review of Particle Properties (1996)] is the weak mixing angle.

Historical Note

The scintillation tank used in this experiment was originally purchased as a fast neutron detector for the 50-metre neutron time-of-flight vacuum beam in the McLennan sub-basement laboratory. Fast neutron detection in such a medium is though proton recoil. The equipment was first used in an experiment directed by Prof. K.G. McNeill and D.J.S. Hewitt -- late professor of Nuclear Engineering. John Hewitt died prematurely in April 1995, while still in his fifties. Kenneth McNeill retired in 1992.

References

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