Physics 485/1860F
Modern Optics: Foundations of Quantum Optics
Department of Physics
University of Toronto

Term Test 28 October 2004

STUDENT-SUPPLIED AID SHEET
(hand-prepared, one side 8 1/2" x 11")
NON-PROGRAMMABLE CALCULATOR
DO ALL FIVE QUESTIONS
DURATION OF TEST: 2 HOURS

[20] 1) Define these terms, and explain as succinctly as possible (usually three sentences or less) the meaning or significance of each of the following, in an optics context:

i) Jones calculus
ii) frustrated total internal reflection
iii) lateral (transverse) coherence
iv) coefficient of finesse of a Fabry-Perot interferometer; reflecting finesse (as used by Fowles)

[20] 2) *Polarization*

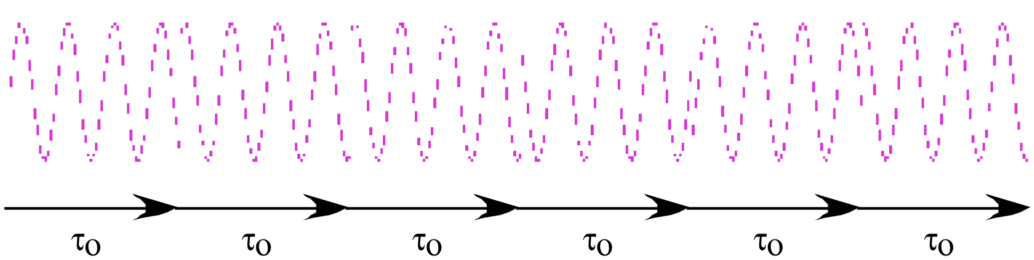
a) Light linearly polarized along $\hat{x}$ and light polarized along $\hat{y}$ together form a basis $\{\vec{\alpha}, \vec{\beta}\}$ for all polarization states. Show that is an orthogonal basis in the Jones calculus.

b) Show that light right-hand circularly polarized and light left-hand circularly polarized form an equally valid, and orthogonal, basis $\{\vec{\sigma}^+, \vec{\sigma}^-\}$ in the Jones calculus.

c) Use the Jones calculus formalism to show that two quarter-wave plates and a linear polarizer can be combined in the right order and orientation to make a circular polarizer.

[20] 3) *Coherence time and coherence length*

Consider a 'quasimonochromatic' source as follows: a source has sinusoidal variation of field in time, with a well-defined angular frequency ω. However, the phase ϕ is randomized at regular intervals τ_o. Thus the oscillation is a regular sinusoidal oscillation at angular frequency ω for a time τ_o, and then it is a similar oscillation but with a different, unrelated, phase for time τ_o, and so on.



(question 3 continues…)

Show that the degree of coherence of this source is the following:

$$|\gamma(\tau)| = \begin{cases} 1-\dfrac{\tau}{\tau_o} & \tau < \tau_o \\ 0 & \tau \geq \tau_o \end{cases}$$

[20] 4. *Coherence and Spectroscopy*

a) An idealized laser simultaneously emits three equal-amplitude monochromatic plane waves with frequencies ω_o, $\omega_o+\Omega$, and $\omega_o-\Omega$. What is the power spectrum and self-coherence function of this source? Show that although the laser output spans a frequency bandwidth of 2Ω, the coherence time τ_c is infinite.

b) Assuming $\omega_o = 10^{16}$ s^{-1} and $\Omega = 10^{10}$ s^{-1}, what must be the finesse of a 0.5m air-filled Fabry-Perot interferometer so as to resolve the peaks in the power spectrum?

c) What is the minimum free spectral range needed in order to use a Fabry-Perot interferometer to properly characterize the spectrum in (b)? What conditions does this impose on the design of the Fabry-Perot?

[20] 5. *Fabry-Perot Interferometer*

Consider two parallel partially reflecting mirrors, and plane-parallel light incident at an angle θ measured from the surface normal.

a) Sketch the path of a light ray incident at roughly 20°: its multiple reflections between mirrors and the multiple transmitted and reflected paths. Illustrate the path-length difference between successive output rays. In a separate diagram, repeat for an incidence of approximately 60°. Illustrate how this path-length difference is *less* for the higher angle of incidence.

b) Derive the simple formula for δ, the path-length difference between successive transmitted rays, and point out analytically the reduction of path-length difference for increasing angles of incidence, shown heuristically in (a).

c) Using the definition $\Delta = \delta + \delta_r$ derive the formula:

$$E_T = \frac{E_o t^2}{1 - r^2 e^{i\Delta}}$$

and subsequently the formula

$$I_T = I_o \frac{T^2}{(1-R)^2} \frac{1}{1 + F\sin^2\frac{\Delta}{2}}$$

Identify/define all terms used, including δ_r. Illustrate the significance of *F*, drawing I_T for 3 different values of *F*, with Δ ranging over at least twice the free spectral range.

[100] TOTAL