Physics 485/1485F
Modern Optics
Department of Physics
University of Toronto

Midterm Test, 31 October 2008

STUDENT-SUPPLIED AID SHEET
(hand-prepared, one side 8 1/2" x 11")
NON-PROGRAMMABLE CALCULATOR
DO ALL FOUR QUESTIONS
DURATION OF TEST: 2 HOURS

[20] 1) Define these terms, and explain as succinctly as possible (usually three sentences or less) the meaning *and* significance of each of the following, in an optics context:

- i) dielectric function
- ii) Kramers-Kronig relation
- iii) optical activity and Faraday effect
- iv) Stokes parameters

[30] 2) *Lorentz Model*

a) What is the Lorentz model, and why is it useful in optics? What phenomena was it introduced in order to explain?

b) When the bound electron in hydrogen is displaced from its equilibrium position a small distance, a restoring force with this coefficient (force per unit distance) results:

$$k = \frac{e^2}{4\pi\varepsilon_o r^2}$$

where the constant $r = 0.05$ nm may be taken as the radius of the atom. Show that this leads to a resonance and find the frequency.

c) An oscillating dipole radiates power, as an antenna does. The rate at which a dipole $p(t) = -e\,x(t)$ radiates power is given by:

$$P_{rad} = \frac{1}{4\pi\varepsilon_o}\frac{2}{3c^3}\left(\frac{d^2p}{dt^2}\right)^2$$

Find the linewidth from this natural damping, and give the maximum index-of-refraction change which results. You may assume a density for hydrogen atoms of $6\times10^{19}$ cm$^{-3}$, if you need it.

[30] 3) *Polarization*

a) Light linearly polarized along $\hat{x}$ and light polarized along $\hat{y}$ together form a basis $\{\vec{\alpha}\ ,\ \vec{\beta}\}$ for all polarization states. Show that is an orthogonal basis in the Jones calculus.

b) Show that light right-hand circularly polarized and light left-hand circularly polarized form an equally valid, and orthogonal, basis $\{\vec{\sigma}^+\ ,\ \vec{\sigma}^-\}$ in the Jones calculus.

c) Use the Jones calculus formalism to show that two quarter-wave plates and a linear polarizer can be combined in the right order and orientation to make an ideal polarizer for left-hand circular polarized light.

*(Question 4 follows on next page…)*

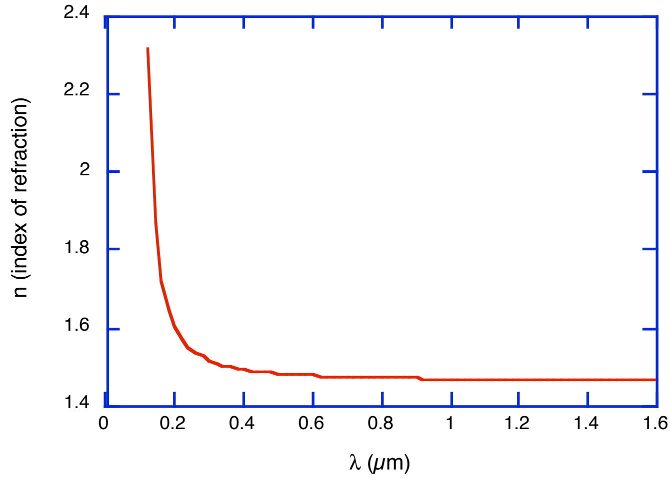
[30] 4) *Optical dispersion*

A standard way of representing the index of refraction for an optical glass, as a function of free-space wavelength λ, is to give its *Sellmeier coefficients*. The Sellmeier equation is:

$$n^2(\lambda) = 1 + \frac{B_1\lambda^2}{\lambda^2 - C_1} + \frac{B_2\lambda^2}{\lambda^2 - C_2} + \frac{B_3\lambda^2}{\lambda^2 - C_3}$$

and Sellmeier coefficients for the borosilicate crown glass known as BK7 are (λ in $\mu$m):

| Coefficient | Value |
|---|---|
| *B1* | 1.15150 |
| *B2* | $1.18584 \times 10^{-1}$ |
| *B3* | 1.26301 |
| *C1* | $1.05984 \times 10^{-2}$ |
| *C2* | $-1.18225 \times 10^{-2}$ |
| *C3* | $1.29618 \times 10^{2}$ |




For this question, it is sufficiently accurate to keep only the first two terms: take *B2=B3=0*.

$$n^2(\lambda) = 1 + \frac{B_1}{1 - \left(C_1 / \lambda^2\right)}$$

a) From the figure above right, of index *vs*. wavelength, *roughly sketch* the corresponding ω–*k* curve. Illustrate schematically on this curve the phase speed and group speed, *i.e*., how each is found from the ω–*k* curve. (You'll probably need to exaggerate the small differences, in drawing your graph)

b) What is the phase speed for a laser pulse of 0.01 nm bandwidth at $\lambda_o$=500 nm? What is the group speed for that same pulse? (Bear in mind the usefulness of series expansions, in approximating the formulae needed.)

c) A 25 fs transform-limited (*i.e*., minimum bandwidth) gaussian pulse, centred on wavelength 400 nm, passes through 5 cm of this glass. *Estimate* roughly the duration of the transmitted pulse. (Your answer need be accurate only within a factor of two or three, but show your work.)

[110] TOTAL