UNIVERSITY OF TORONTO
Faculty of Arts and Science

DECEMBER EXAMINATIONS 2003

PHY 485F/1860F – Modern Optics

Duration — 3 hours

Aids allowed: calculators, student-prepared aid sheet (one side of 8.5 x 11 inch)
(helpful formulae attached)

INSTRUCTIONS: ANSWER ALL FIVE QUESTIONS IN THE MAIN PART.
MARKS ARE SHOWN IN LEFT MARGIN. TOTAL MARKS = [100]

[15] 1. *Principles and terms of optical physics*

Explain each of the following, in about three sentences each, showing the significance and importance in the context of optics. Be brief and clear; use simple formulae where appropriate.

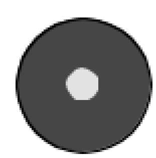
[3] i) Paraxial approximation, and its use in diffraction
[3] ii) Coherence function
[3] iii) Rayleigh range
[3] iv) Hermite-Gaussian modes
[3] v) ABCD matrices, and their use both for ray-tracing and for Gaussian beams.

[20] 2. *Multiple Choice Section* — In each of the following questions, select the answer A–E which best responds to the question. Please mark your answers in your exam booklet, in the order presented. Explanations are not required, however, you may give a brief explanation if you think no answer is quite appropriate. Part marks may be given for certain wrong answers which have merit.

[4] a) For a 100 fs pulse with λ_o=780 nm (1 fs =10^{-15} s), roughly what is the minimum bandwidth?

A. $\Delta\lambda \cong$ 10 nm
B. $\Delta\lambda \cong$ 128 nm
C. $\Delta\lambda \cong$ 6,000 nm
D. the entire visible spectrum
E. none of the above

[4] b) Light (λ=1 μm) from a point source travels 50 cm to be intercepted by a small opaque disk. 50 cm beyond, the diffracted light falls on a screen and produces the intensity pattern sketched at right, where the shaded portions represent shadow (not to scale).



[Question #2(b) continues ...]

What can be said about the diffraction?

A. Fresnel diffraction should describe the pattern to a good approximation
B. it requires refraction and cannot be produced using an opaque obstacle
C. the radius of the disk is nearly exactly 0.4 mm
D. the pattern can be simply explained with reference to the Cornu spiral (see aid sheet)
E. none of the above

[4] c) Which of these effects is associated with dispersion?

I. the rainbow produced from a prism
II. Faraday rotation of polarization, for glass in a strong magnetic field
III. absorption of red light in blue glass
IV. partial reflection at an interface
V. Joule heating in a conductor

A. all of I–V
B. none of I–V
C. I only
D. I, II, III
E. none of choices A–D is a correct option

[4] d) Inside a typical helium-neon laser we might find:

power (circulating)	$P = 50$ mW
wavelength	$\lambda_o = 632.8$ nm
bandwidth	$\Delta\omega = 4 \times 10^9\ \mathrm{s}^{-1}$
beam diameter	0.5 mm

Roughly what temperature would a *blackbody* need to be to produce the same spectral density $\rho_{BB}(\omega_o)$ (*i.e.*, energy per unit volume per unit frequency range) at the wavelength of the laser:

A. < 10 K
B. $10 - 10^3$ K
C. $10^3 - 10^5$ K
D. $10^5 - 10^7$ K
E. $> 10^7$ K

[4] e) Which properties are expected with a high-finesse Fabry-Perot etalon?

I. high-reflectivity mirrors
II. compact size
III. strong spectral discrimination
IV. long lifetime for light inside the cavity
V. operation at ultraviolet wavelengths

A. all of I–V
B. none of I–V
C. I, II, IV
D. I, III, IV
E. II, III, V

[25] 3. *Optical resonators and Hermite-Gaussian Modes*

In constructing a solution to the paraxial wave equation, which gave us Gaussian beams, we started with a plane wave, and described transverse variation by way of the envelope function u(x,y,z) which we assumed had only adiabatic variation along z

$$E(x,y,z) = u(x,y,z)\, e^{ikz}$$

[5] i) Show *simply* (*i.e.*, don't work hard) that for the lowest-order spatial mode of a Gaussian beam one of the adiabatic effects of this variation is to introduce an additional phase change of π radians as the wavefront travels through the beam waist along z. Thus in being focussed a beam picks up an extra 180° phase shift compared to a plane wave. What is the analogous phase shift associated with a given higher-order mode TEM_{mn}? This is called the *Guoy shift*.

Consider a laser cavity which consists of a planar mirror at one end and a spherically curved mirror with R_2 = 1.5 m at the other end. The mirrors are separated by 1 m.

[5] ii) Show that stable modes *can* exist for this cavity.

[5] iii) From the condition that the radius of curvature of the beam must match that of each mirror, find the fundamental Gaussian beam which is a normal mode of this cavity for λ = 700 nm. Repeat for λ = 1 μm.

[5] iv) Write the ABCD matrix for a round-trip in this cavity, starting from the curved mirror. The normal mode of the cavity must be reproduced after one round trip, so the Gaussian solution must be an *eigenmode* of the round-trip operation *T*, *i.e.*, *T(q)* = *q* where *q* is the Gaussian beam parameter and T is the round-trip transformation. Write the proper operator expression and solve it for *q*. Show that this *q* is the same solution as found in a) above.

[5] v) Find the far-field divergence θ of the beam, from either (b) or (c) above.

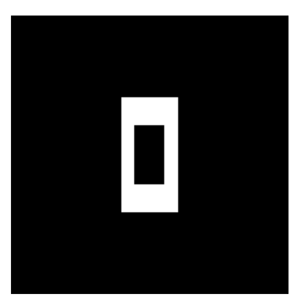
[20] 4. *Diffraction*

On the first page of the Appendix to this exam, please find an unlabelled Cornu Spiral, which should be detached from your exam questions. Mark your answers on this page, where required, and include it in your answer booklet as part of your answer to this question.

[5] i) Describe what the Cornu spiral represents in diffraction theory. Give simple instructions of how to use this graph in practical solutions, and when to use it. For the attached copy of the Cornu spiral, please add the coordinate axes and other information needed to use the spiral in diffraction. Answers should be qualitatively correct, but may be somewhat rough.

[5] ii) Give a precise statement of Babinet's principle. On the Cornu spiral supplied or on a sketch of your own, illustrate Babinet's principle graphically in the context of a one-dimensional slit and for two different detection points lying behind it which have different displacements relative to the slit.

[10] iii) For the aperture in the figure at right, illuminated at normal incidence by a plane wave with λ = 500 nm, calculate the diffraction pattern in an observation plane which is 5 m beyond the aperture. The aperture has outer dimensions of 100 μm x 50 μm and inner dimensions of 50 μm x 25 μm. Outside this aperture, the sheet is very large and blocks all other light. Sketch the pattern of intensity that would be viewed at the observation plane.



[20] 5. *Density Matrix and Rate Equations*

In class, we developed these equations for the elements of the density matrix:

$$\dot{\rho}_{11} = -\Gamma_1 \rho_{11} + A_{21} \rho_{22} - \frac{i}{2}\left(\chi \rho_{12} - \chi^* \rho_{21}\right)$$

$$\dot{\rho}_{22} = -(\Gamma_2 + A_{21})\rho_{22} + \frac{i}{2}\left(\chi \rho_{12} - \chi^* \rho_{21}\right)$$

$$\dot{\rho}_{12} = -(\beta - i\Delta)\rho_{12} + i\frac{\chi^*}{2}(\rho_{22} - \rho_{11})$$

$$\dot{\rho}_{21} = -(\beta + i\Delta)\rho_{21} - i\frac{\chi}{2}(\rho_{22} - \rho_{11})$$

$$\textit{where} \quad \beta = \frac{1}{\tau} + \frac{1}{2}(\Gamma_1 + \Gamma_2 + A_{21})$$

[5] i) Give the definitions for each element ρ_{ij}. What does each represent, or how should each be viewed, in the context of optical physics? What is the significance of Δ, χ and β, and how should each term τ, Γ_1, Γ_2 and A_{21} be understood?

[10] ii) In the context of these equations, what is meant by *'adiabatic following'*? In the approximation that:

$$\beta \approx \frac{1}{\tau} >> \frac{1}{2}(\Gamma_1 + \Gamma_2 + A_{21})$$

show that the equations above can be reduced to the *population rate equations*:

$$\dot{\rho}_{11} = -\Gamma_1 \rho_{11} + A_{21} \rho_{22} + \frac{|\chi|^2 \beta / 2}{\Delta^2 + \beta^2}(\rho_{22} - \rho_{11})$$

$$\dot{\rho}_{22} = -(\Gamma_2 + A_{21})\rho_{22} - \frac{|\chi|^2 \beta / 2}{\Delta^2 + \beta^2}(\rho_{22} - \rho_{11})$$

[5] iii) Describe in words the physical meaning of this mathematical step. Give rate equations for the sum $(\rho_{11} + \rho_{22})$ and difference $(\rho_{22} - \rho_{11})$ of level populations, and identify their physical meaning.

TOTAL MARKS = [100]

APPENDIX of formulae and equations follows on next three pages