UNIVERSITY OF TORONTO
Faculty of Arts and Science

DECEMBER EXAMINATIONS 2004

PHY 485H1F/1860F – Modern Optics/Foundations of Quantum Optics

Duration — 3 hours

Aids allowed: calculators, student-prepared aid sheet (one side of 8.5 x 11 inch)
(helpful formulae attached)

INSTRUCTIONS: ANSWER ALL FIVE QUESTIONS IN THE MAIN PART.
MARKS ARE SHOWN IN LEFT MARGIN. TOTAL MARKS = [110] (10 marks BONUS)

[20] 1. *Principles and terms of optical physics*
Define and explain each of the following, in about three sentences each, showing the significance and importance in the context of optics. Be brief and clear; use simple formulae and examples where appropriate.

[5] i) Fresnel and Fraunhofer diffraction. In what circumstance is each approximation appropriate?

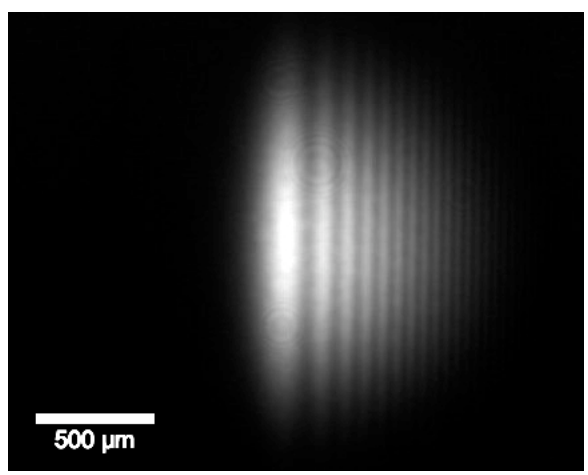
[5] ii) transform-limited pulse; for a gaussian pulse, under what circumstance is the pulse *not* transform-limited?

[5] iii) Lorentz model, and its relation to the index of refraction

[5] iv) one of: confocal parameter *or* Rayleigh range

[30] 2. *Multiple Choice Section* — In each of the following questions, select the answer A–F which best responds to the question. Please mark your answers in your exam booklet, in the order presented. Explanations are not required, but you may give a brief explanation if you think no answer is quite appropriate. Part marks may be given for certain wrong answers which have merit.

[5] i) Light from a HeNe laser (λ= 632.8 nm), collimated into a beam of diameter 1mm, was accidentally diffracted by clipping a straight edge, and produced the pattern shown at right (black is dark background, white is greater intensity). The scale of the image is shown on the figure. At approximately what distance from the detector was the straight edge?

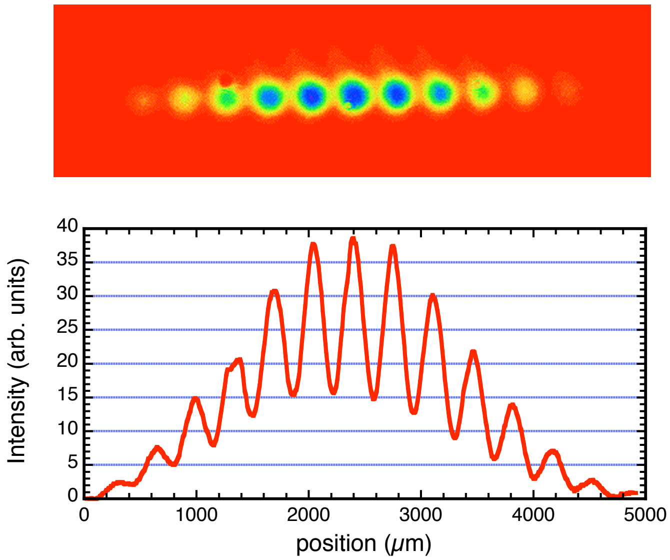


A. 1 μm
B. 100 μm
C. 5 mm
D. 5 cm
E. 50 cm
F. 5 m

[4] ii) For a 10 fs pulse (FWHM) with λ_o=780 nm (1 fs =10^{-15} s), roughly what is the minimum bandwidth (FWHM)?

A. $\Delta\lambda \cong$ 10 nm
B. $\Delta\lambda \cong$ 100 nm
C. $\Delta\lambda \cong$ 500 nm
D. $\Delta\lambda \cong$ 1500 nm
E. $\Delta\lambda \cong$ 5,000 nm
F. none of the above

[5] iii) The figure at right shows a two-slit interference pattern; the top figure is the original image, and the figure below gives the intensity profile. What is the fringe visibility?



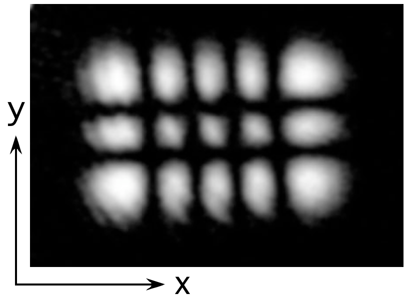
A. 0.2
B. 0.4
C. 0.5
D. 0.8
E. 1.2
F. 2.5

[4] iv) Which of these effects is associated with dispersion?

I. the rainbow produced from a prism
II. Faraday rotation of polarization, for glass in a strong magnetic field
III. absorption of red light in blue glass
IV. partial reflection at an interface
V. Joule heating in a conductor

A. all of I–V
B. none of I–V
C. I only
D. I, II, III
E. IV, IV
F. none of choices A–E is a correct option

[4] v) The figure at right shows a laser output intensity pattern. Which label below correctly describes this pattern?



A. normal mode $NM_{4,2}$
B. Hermite-Gaussian $TEM_{5,3}$
C. Laguerre mode $TEM_{3,5}$
D. Laguerre mode $TEM_{4,2}$
E. Hermite-Gaussian mode $TEM_{2,4}$
F. none of the above

[Question #2 continues ...]

[4] vi) A 100 fs optical pulse of frequency ω_o travels through a piece of glass (n=1.6) which has $\frac{\omega_o^2 k''(\omega_o)}{k(\omega_o)} \cong 0.1$. Approximately how much glass must it travel through before it broadens to 1 ps?

A. 1 mm
B. 1 cm
C. 10 cm
D. 1 m
E. 10 m
F. none of the above

[4] vii) Which properties are to be expected, for a high finesse Fabry-Perot etalon?

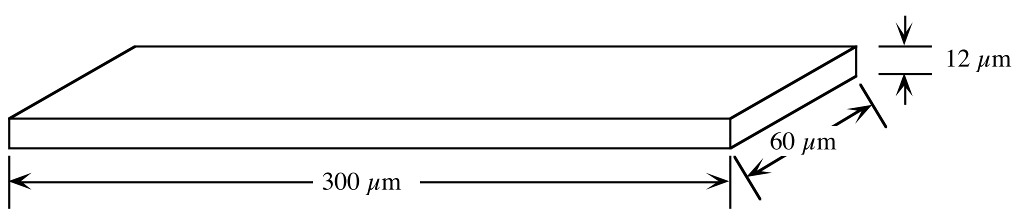
I. high-reflectivity mirrors
II. a rapid vibration curve
III. strong spectral discrimination
IV. long optical-field decay-time, inside
V. operation at ultraviolet wavelengths

A. all of I–V
B. none of I–V
C. I only
D. I, II, III
E. I, III, IV
F. none of choices A–E is a correct option

[20] 3. *Diffraction*

[5] i) Estimate how far away a typical automobile can be at night and still permit you to barely resolve the two headlights by eye. You will need to estimate certain dimensions, for the headlights and in the eye—a wide range of reasonable values will be accepted.

The gain volume of a diode laser is 12 μm $\times$ 60 μm $\times$ 300μm long. The shortest dimension is oriented vertically, and the operating wavelength is 820 nm.



[5] ii) Determine the number of optical modes in the gain volume, for a 10nm gain bandwidth.

[5] iii) The finite output aperture size, determined by the 12μm $\times$ 60 μm output face, will lead to diffraction of the output beam. If this output aperture were illuminated by a perfect plane wave at normal incidence, what would be the diffracted beam pattern on a wall, located 2.5m from the laser output?

[5] iv) The actual, observed, beam has a minimum width of 10 cm on the wall at 2.5 m. What must be the transverse coherence length of the laser?

[20] 4. *Gaussian beams*

In class we developed the paraxial wave equation:

$$\nabla_T^2 \, \mathcal{E}_0 + 2ik \frac{\partial \mathcal{E}_0}{\partial z} = 0$$

[15] i) Starting with the form:

$$\mathcal{E}_0(x,y,z) = A\, e^{ik(x^2+y^2)/2q(z)} e^{ip(z)}$$

show how this leads to the Gaussian beam formula:

$$\mathcal{E}_0(x,y,z) = \frac{A\, e^{i\phi(z)}}{\sqrt{1+z^2/z_o^2}} \; e^{ik(x^2+y^2)/2R(z)} \; e^{-(x^2+y^2)/w^2(z)}$$

Be sure to identify all terms.

[5] i) Describe how this also describes a beam with flat wavefronts and a gaussian transverse intensity profile. (The gaussian beam in general is therefore the solution of the diffraction equation for a plane wave apodized with a gaussian transverse intensity profile.)

[20] 5. *Dispersion and the Lorentz model*

From the Lorentz model, the complex polarizability is related to the index of refraction by:

$$\left(1 + \frac{N\alpha(\omega)}{\epsilon_o}\right) = n^2(\omega)$$

and thus

$$n^2(\omega) = \frac{Ne^2 / m \epsilon_o}{\omega_o^2 - \omega^2 - 2i\beta\omega}$$

[10] i) Derive the real and imaginary parts of the index of refraction near a resonance

[10] ii) Find a formula, at line centre, for the group velocity dispersion:

$$k'' \equiv \left.\frac{d^2 k}{d\omega^2}\right|_{\omega=\omega_o} = \left.\frac{d}{d\omega}\left(\frac{1}{v_g(\omega)}\right)\right|_{\omega=\omega_o}$$

TOTAL MARKS = [110] (10 marks BONUS)

APPENDIX of formulae and equations follows on next three pages