Modern Optics PHY485/1485F
University of Toronto
Problem Set #1

September 18, 2008 due: 6 October 2008

1. *Light numbers*

a) The threshold of sensitivity of the human eye is about 100 photons per second. The eye is most sensitive at a wavelength of around 550nm. For this wavelength, determine the threshold in watts of power. Estimate the size of the pupil of the eye in a dark room, and find the irradiance.

b) What is the energy, in electron volts, of light photons at the ends of the visible spectrum, that is, at the wavelengths of 380 nm and 770 nm?

c) Determine the wavelength and momentum of a photon whose energy equals the rest-mass energy of an electron.

d) A 100 W light bulb is in the geometrical centre of a cubical room with sides of 3 m. Find the time-averaged total energy within the room from the light bulb, assuming that no light is absorbed or reflected by the walls (*i.e.*, the walls are perfectly transparent).

e) Determine the magnitude of the magnetic field B in an electromagnetic wave in vacuum that carries a power density of 100 mW/cm^2. How much current would be required in a long straight wire so that a magnetic field of this same magnitude would be generated in vacuum at a distance of 1 mm from the wire?

2. *Electron motion in relativistic EM fields*

We saw in class that the ratio of forces on a charged particle, due to magnetic and electric fields in light, has the relation: $F_B/F_E \leq v/c$. That means for most EM fields we encounter, we can ignore the dynamics that are due to the B field.

When the EM field is strong, however, the E-field of the light may drive an oscillating electron to a speed approaching the speed of light.

a) Show that the condition for this is not just the field strength E_0 — find a relationship for the field strength, particle charge, particle mass, etc. to pass a rough threshold for onset of relativistic effects

b) Show, qualitatively, that for linearly polarized light, of relativistic-strength fields, an electron driven *transversely* at the light frequency ω_0 will also oscillate *longitudinally* at $2\omega_0$.

c) Starting with the Lorentz equation of motion:

$$\frac{d\vec{p}}{dt} = -e\left(\vec{E} + \frac{1}{c}\vec{v} \times \vec{B}\right) \quad \text{where momentum } \vec{p} \equiv m\vec{v} = \gamma m_0 \vec{v}$$

and the relativistic energy equation:

$$\frac{d\varepsilon}{dt} = -e \cdot \left(\vec{v} \cdot \vec{E}\right) \quad \text{where energy } \varepsilon \equiv mc^2 = \gamma m_0 c^2$$

find separate equations of motion for $p_\perp$ and $p_\parallel$, the momenta perpendicular to, and parallel to, the direction of propagation of the light.

d) For an electron initially at rest, integrate to find the electron coordinates at times after the light is turned on. Show that the strong EM field leads to an average drift speed v_D in the direction of the propagating light:

$$\frac{v_D}{c} = \frac{a_0^2}{4 + a_0^2} \quad \text{where } a_0 = \frac{eE_0}{m\omega c}$$

plus a figure-eight motion superposed on top of that drift, with an equation in the drift frame:

$$16x^2 = y^2\left(4q^2 - y^2\right) \quad \text{where } q \equiv \frac{a_0}{2\gamma_0}$$

3. *Solid-state plasmas*

a) The DC conductivity of silver is $\sigma = 6.8 \times 10^7$ MKS units and the free electron density is $N_e = 1.5 \times 10^{28}$ m^{-3}. Considering silver to be a free-electron metal what is its plasma frequency? What is the electron damping time τ?

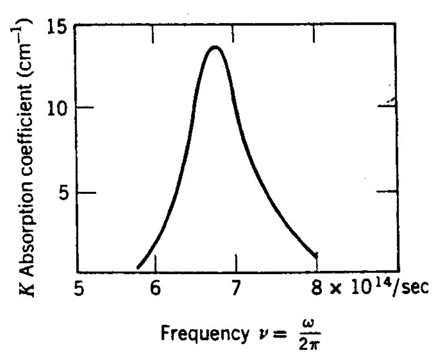
b) For the model of a metal discussed in class, the plasma frequency $\omega_p = 6 \times 10^{15}$ rad s^{-1} corresponds to a wavelength $\lambda_p = 314$ nm. How important are collisions in determining the attenuation of the field into the metal? Calculate numerical values for the penetration depth, with and without an electron collision rate of $\gamma = 10^{13}$ s^{-1}, for light at normal incidence on a slab of metal at the following wavelengths: 349 nm, 314 nm, and 286 nm.

4. *Group and phase velocity*

Consider a fictitious material with a single resonance at $\lambda = 300$ nm and with a damping coefficient of 10^{13} s^{-1}. Take a density $N = 1$ mole/m^3. Determine the percentage difference between the group and phase velocity at $\lambda = 320$ nm and $\lambda = 500$ nm. What is the maximum absorption coefficient of this material?

5. *Colour centres*

There is a class of electronic impurity centres in crystals called *colour centres*. The absorption coefficient produced by one such centre, the '*F* centre' in a sodium chloride crystal, is shown the figure at right. Treat this centre as a harmonic oscillator with the mass of an electron, imbedded in the crystal (of background refractive index $n = 1.55$). Assume the experimental lineshape to be Lorentzian, and from it estimate the lifetime of the oscillator and the oscillator strength. Calculate a value for the maximum change in the real part of the index of refraction caused by the presence of the *F* centres in this sample. Estimate the percentage difference between the group and phase velocity at $\nu = 7 \times 10^{14}$ Hz and $\nu = 9 \times 10^{14}$ Hz.



[see, *e.g.*, http://en.wikipedia.org/wiki/F-center Though water-clear colour is one of the "four-C's" of valuation, diamonds are sometimes deliberately coloured pale blue by treating them in an electron accelerator to create crystal vacancy-defects, thus colour centres.]