Foundations of Quantum Optics – Modern Optics
PHY485/1860F
University of Toronto
Problem Set #2

October 7, 2008 due: Tuesday, 21 October 2008

NB: Midterm Friday 31 October 2008, 4–6pm, Room BA-2139 (Bahen building)

1. *Group speed formulae —*
   a) Hecht, Problem 7.23
   b) Hecht, Problem 7.24
   c) Hecht, Problem 7.25

2. *Dispersion for EM waves in plasma*
   a) Hecht, Problem 7.29; sketch the dispersion relation; explain why there is a cut-off frequency
   b) Hecht, Problem 7.30

3. *Malus's Law*
   Malus's Law says that when a perfect linear polarizer is placed in a beam of linearly polarized light, at an angle θ between the different orientations of polarization, the transmitted intensity is reduced by a factor $\cos^2(\theta)$.
   a) Using Jones matrices for a linear polarizer and for rotation of a matrix, prove Malus's law.
   b) Hecht, Problem 8.17, which shows that adding a polarizer can *increase* light transmission
   c) Hecht, Problem 8.18
   d) How far do the steps of (b) and (c) go? Consider a collection of N polarizers, each turned an angle $\delta = \pi/(2(N-1))$ relative to the one before (*i.e.*, two polarizers with $\pi/2$ between them, or three polarizers with $\pi/4$, etc.) Find the net transmission as $N \to \infty$.

[see nice pictures on *e.g.*, http://en.wikipedia.org/wiki/Polarizer]

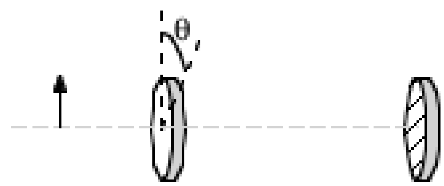
4. *Polarization and the Jones Calculus*
   In class we showed the example of an *optically active* material — a material which is capable of continuously rotating the plane of polarization of linearly polarized light. Our example was a sugar (sucrose) solution, which was *dextrorotary* (right-hand-turning). Levo-sucrose is a stereoisomer of sucrose which is *levorotary* (left-hand-turning); as for many organic materials, one stereoisomer can be digested (sucrose) while the other cannot (levo-sucrose).

   In a similar effect, the *Faraday Effect*, a strong axial magnetic field in a material can break the symmetry between right-handed circularly polarized light and left-handed circularly polarized light, giving the two different phase-speeds v_ϕ^R and v_ϕ^L. This is related to whistler waves in the magnetic field of the ionosphere.

   a) Give a description in simple physical picture of how, as a result, linearly polarized light suffers a rotation of the plane of polarization as it propagates. How can one arrange to rotate the orientation of the polarization by an angle α? What polarization states are unchanged in propagating through a Faraday-effect medium?

b) If this medium rotates the plane of polarization of linearly polarized light, can it have a Jones calculus representation which still preserves these states above as eigenstates? Show your answer using the Jones calculus, and explain your terms.

c) Using the Jones calculus, find the effect at each step on the polarization state of light as it travels through a *half-wave plate* oriented at some quite general angle θ, then retro-reflects at normal incidence from a mirror and passes back along exactly the same path through the half-wave plate.



d) Repeat the same steps above, but with a Faraday-effect element in place of the quarter-wave plate. Describe what is required to produce a general rotation of the orientation of the E-field as in question (d) just above. Compare and contrast the results of each of these polarization-rotators.