Solution to PS #2.

Q1: Hecht. P: 7.23.

$$\because v_g = \frac{d\omega}{dk} \qquad \omega = \frac{c}{n}k = vk. \qquad k = \frac{2\pi}{\lambda}.$$

$$\therefore v_g = \frac{d\omega}{dk} = \frac{d(vk)}{dk} = v\frac{dk}{dk} + k\frac{dv}{dk} = v + \frac{2\pi}{\lambda}\frac{dv}{dk}.$$

$$= v + \frac{2\pi}{\lambda}\frac{dv}{d\lambda}\frac{d\lambda}{dk} = v + \frac{2\pi}{\lambda}\frac{dv}{d\lambda}\left(\frac{1}{\frac{dk}{d\lambda}}\right)$$

$$\because k = \frac{2\pi}{\lambda} \quad \therefore dk = -\frac{2\pi}{\lambda^2}d\lambda. \quad \Rightarrow \frac{dk}{d\lambda} = -\frac{2\pi}{\lambda^2}$$

$$\therefore v_g = v + \frac{2\pi}{\lambda}\frac{dv}{d\lambda}\left(\frac{1}{\frac{dk}{d\lambda}}\right) = v - \lambda\frac{dv}{d\lambda}. \qquad \left(v = \frac{c}{n}\right).$$

P. 7.24:

$$\because \omega = \frac{c}{n}k \quad \therefore k = \frac{n\omega}{c}$$

$$\therefore dk = \frac{n}{c}d\omega + \frac{\omega}{c}dn.$$

$$\therefore v_g = \frac{d\omega}{dk} = \frac{1}{\frac{dk}{d\omega}} = \frac{1}{\frac{n}{c} + \frac{\omega}{c}\frac{dn}{d\omega}} = \frac{c}{n + \omega(dn/d\omega)}$$

P 7.25. the group index of refraction is

$$n_g = c/v_g.$$

$$\because v_g = \frac{c}{n + \omega(dn/d\omega)} \quad (P. 7.24) \qquad \omega = 2\pi\nu \qquad d\omega = 2\pi d\nu.$$

frequency (ν)

$$\therefore n_g = n + \omega\, dn/d\omega = n(\nu) + 2\pi\nu\frac{dn}{d\nu}\frac{d\nu}{d\omega} = n(\nu) + \nu\frac{dn(\nu)}{d\nu}.$$

Q2: a> Hecht P. 7.29

$\because \omega^2 = \omega_p^2 + c^2k^2 \qquad \omega = \sqrt{\omega_p^2 + c^2k^2}$

$\therefore$ phase velocity $v_\phi = \frac{\omega}{k} = \frac{\sqrt{\omega_p^2+c^2k^2}}{k}$

group velocity $v_g = \frac{d\omega}{dk} = \frac{1}{2}\frac{c^2\cdot 2k}{\sqrt{\omega_p^2+c^2k^2}} = \frac{c^2k}{\sqrt{\omega_p^2+c^2k^2}}$

$\therefore \quad v_\phi v_g = c^2.$

to draw the desperosion graph, we need to know the curve is increase or decrease, curve upwards or downwards.

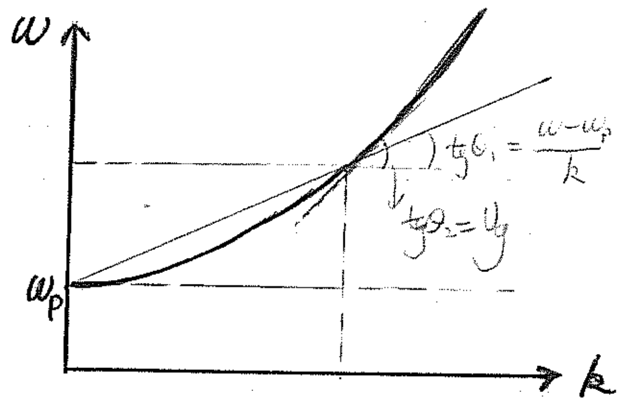
from $\omega = \sqrt{\omega_p^2+c^2k^2}$, and $v_g = \frac{c^2k}{\sqrt{\omega_p^2+c^2k^2}} = \frac{d\omega}{dk} > 0$. we know that ω is increased when k increase. There are two ways to know the curve is curved upwards or downwards, one is similar with shown on Hecht. Page ~~298~~ 298, Figure 7.20, just here we compare $\omega-\omega_p/k$ with v_g, ~~insted~~ instead of comparing $v_\phi = \frac{\omega}{k}$ with v_g. the other is to calculate the second order diveation of $\omega(k)$. that is $\frac{d\omega''}{dk''}$ or $\frac{dv_g}{dk}$. I'll show both approaches.

i> compare $\frac{\omega-\omega_p}{k}$ with v_g

$$\frac{\omega-\omega_p}{k} - v_g = \frac{\sqrt{\omega_p^2+c^2k^2}-\omega_p}{k} - \frac{c^2k}{\sqrt{\omega_p^2+c^2k^2}} = \frac{\omega_p^2+c^2k^2-\omega_p\sqrt{\omega_p^2+c^2k^2}-c^2k^2}{k\sqrt{\omega_p^2+c^2k^2}}$$

$$= \frac{\omega_p(\omega_p - \sqrt{\omega_p^2 + c^2k^2})}{k\sqrt{\omega_p^2+c^2k^2}} = \frac{\omega_p\left(\frac{\omega_p}{k} - \sqrt{\left(\frac{\omega_p}{k}\right)^2 + c^2}\right)}{\sqrt{\omega_p^2+c^2k^2}} < 0$$

$$\therefore \; v_g > \frac{\omega - \omega_p}{k}.$$



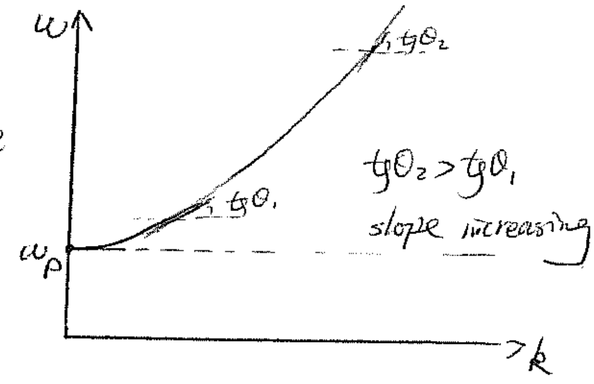
2) To calculate "$\frac{d^2\omega}{dk^2}$" to determine its sign.

$$v_g = \frac{d\omega}{dk} = \frac{c^2k}{\sqrt{\omega_p^2+c^2k^2}}$$

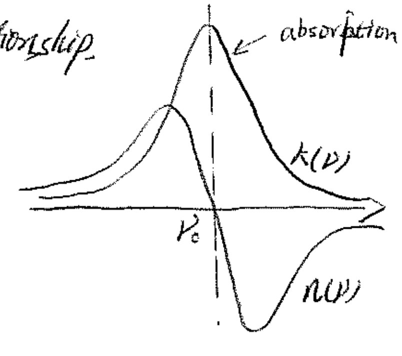
$$\therefore \frac{d^2\omega}{dk^2} = \frac{dv_g}{dk} = \frac{c^2}{\sqrt{\omega_p^2+c^2k^2}} + c^2k\cdot\left(-\frac{1}{2}\right)\frac{2c^2k}{\sqrt{\omega_p^2+c^2k^2}\cdot(\omega_p^2+c^2k^2)}$$

$$= \frac{c^4k^2+c^2\omega_p^2 - c^4k^2}{\sqrt{\omega_p^2+c^2k^2}^{\,3}} = \frac{c^2\omega_p^2}{\sqrt{\omega_p^2+c^2k^2}^{\,3}} > 0$$

$\therefore$ the curve is curved upwards. (actually, this means the slope of the curve is increased with the increase of k)



★ There is a cut-off frequency because there is no real solution for the wave-number k when $\omega < \omega_p$. here ω_p is the cut-off frequency. Remember what is discussed in PS #1 Question 3 part b), when k is imaginary, this means the light is reflected, the wave is evanescent. The Kramers-Kroning relation connects the real and imaginary part of the index of refraction. it applies not only when the light is absorbed, but also when the light is evanesent. In kramers-kronig relationship shown in the right figer. the dispersion, that is $\frac{dn(\nu)}{d\nu}$, or the slope of real part curve, becomes maximum when the absorption is maximum. that is when $\nu = \nu_0$. At $\nu = \nu_0$, $\frac{dk(\nu)}{d\nu} = 0$, the changing in the imaginary part is ~~sm~~ minimum when the ~~the~~ changing in the real part is maximum. As the light "more evanesent", the light is more dispersion. Analogue to ~~kee~~ the case in ~~mede~~ this question, when the frequency gets more closer to the cut-off frequency ω_p, the charge in the refraction ~~indd~~ index $\frac{dn}{d\omega}$ gets larger. means more dispersion. At $\omega = \omega_p$ $\frac{dn}{d\omega} \to \infty$. the largest dispersion.



$\hat{n}(\nu) = n(\nu) + ik(\nu)$

b). the phase velocity is defined as $V_\phi = \frac{\omega}{k} = \frac{c}{n(\omega)}$ $\therefore \omega = \frac{ck}{n(\omega)}$

or $\frac{ck}{\omega} = n(\omega)$

the group velocity is defined as $V_g = \frac{d\omega}{dk} = \frac{d\frac{ck}{n(\omega)}}{dk} = \frac{c}{n + \omega\frac{dn(\omega)}{d\omega}}$ (Q1)

for dispersion equation: $n^2(\omega) = 1 + \frac{Nq_e^2}{\varepsilon_0 m_e} \sum_j \frac{f_j}{\omega_{0j}^2 - \omega^2} = 1 + \frac{Nq_e^2}{\varepsilon_0 m_e \omega^2} \sum_j \frac{f_j}{(\frac{\omega_{0j}^2}{\omega^2} - 1)}$

at very high frequency, $\omega \gg \omega_{0j} \Rightarrow \frac{\omega_{0j}}{\omega} \ll 1 \Rightarrow \frac{\omega_{0j}^2}{\omega^2} \ll 1$

$$\therefore n^2(\omega) \simeq 1 - \frac{Nq_e^2}{\varepsilon_0 m_e \omega^2} \sum_j f_j = 1 - \frac{Nq_e^2}{\varepsilon_0 m_e \omega^2} \quad (\text{with } f_j \text{ weight factor } \sum_j f_j = 1)$$

$$\therefore n(\omega) \approx \sqrt{1 - \frac{Nq_e^2}{\varepsilon_0 m_e \omega^2}} = \sqrt{1 - \frac{\omega_p^2}{\omega^2}} \quad \text{with } \omega_p^2 = \frac{Nq_e^2}{\varepsilon_0 m_e}$$

at very high frequency $\omega \gg \omega_p$. $\therefore \frac{\omega_p^2}{\omega^2} \ll 1$ & $\sqrt{1-x} \simeq 1 - \frac{x}{2}$ for $x \ll 1$

$$\therefore n(\omega) \simeq 1 + \frac{1}{2}\left(-\frac{Nq_e^2}{\varepsilon_0 m_e \omega^2}\right) = 1 - \frac{Nq_e^2}{2\varepsilon_0 m_e \omega^2} \quad \text{---} \quad (1)$$

$$\therefore \frac{dn(\omega)}{d\omega} = -\frac{Nq_e^2(-2)}{2\varepsilon_0 m_e \omega^3} = \frac{Nq_e^2}{\varepsilon_0 m_e \omega^3} \quad \text{---} \quad (2)$$

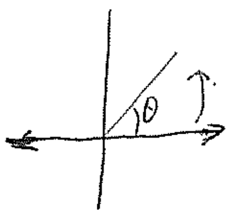
$$\therefore V_g = \frac{c}{n + \omega \frac{dn(\omega)}{d\omega}} \overset{(1)(2)}{=\!=} \frac{c}{1 - \frac{Nq_e^2}{2\varepsilon_0 m_e \omega^2} + \omega \frac{Nq_e^2}{\varepsilon_0 m_e \omega^3}}$$

$$= \frac{c}{1 + \frac{Nq_e^2}{2\varepsilon_0 m_e \omega^2}} \sim \text{group velocity}$$

the phase velocity $V_\phi = \frac{c}{n(\omega)} = \frac{c}{1 - \frac{Nq_e^2}{2\varepsilon_0 m_e \omega^2}}$

$$\therefore V_g V_\phi = \frac{c^2}{1 - \left(\frac{Nq_e^2}{2\varepsilon_0 m_e \omega^2}\right)^2} \simeq c^2 \quad \left(\frac{Nq_e^2}{2\varepsilon_0 m_e \omega^2} \ll 1 \text{ for } \omega \text{ very high}\right)$$

Q3: a) To prove Malu's Law with Jones Matrix, we need first to find out the Jone's matrix for a linear polarizer at θ.



this can be achieved by acting the rotation matrix on the one for horizontal linear polarizer. The Rotation matrix is

$$R(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

thus the Jone's matrix for a linear polarizer along θ is.

$$P(\theta) = R(\theta)P(0)R(-\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$$

For a horizontal polarized light, the light passing through such a polarizer is:

$$\tilde{E}_t = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \sin\theta\cos\theta, & \sin^2\theta \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos^2\theta \\ \cos\theta\sin\theta \end{bmatrix} = \cos\theta \underbrace{\begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}}, \text{ along } \theta \text{ direction}$$

$\therefore$ The intensity $\propto |\tilde{E}_t|^2 = \cos^2\theta$, so ~~there is~~ a the intensity reduces as a factor of $\cos^2\theta$, where θ is the angle between the linearly polarized light and linear polarizer.

a) easier way: Instead of calculating $P(\theta)$, you can also assum the polarizer is horizontally polarized, while the ~~a~~ incident light is polarized at angle θ. The Jone's vector for such polarized light is

$$\tilde{E}_\theta = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}.$$

passing through horizontal linear polarizer, the ~~EM~~ passed light is

$$\tilde{E}_t = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} = \begin{bmatrix} \cos\theta \\ 0 \end{bmatrix} = \cos\theta \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

polarized along horizontal.

but the intensity is ~~as~~ $\propto |\tilde{E}_t|^2$, thus a $\cos^2\theta$ decrease.

b7. Hecht P8.17. ~~A~~ The Jone's vector for a linear polarized light at θ is $E_1\begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix}$. from Table 8.6 P_{378}, ~~the~~ This light passing through horizontal linear polarizer and vertical linear polarizer the ~~transmit~~ transmitted light is.

$$\tilde{E}_t = \begin{bmatrix}0&0\\0&1\end{bmatrix}\begin{bmatrix}1&0\\0&0\end{bmatrix}E_1\begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix} = E_1\begin{bmatrix}0\\0\end{bmatrix} \quad \therefore I_2 = 0$$

if insert a linear polarizer ~~at~~ whose ~~the~~ transmission direction is $+45°$

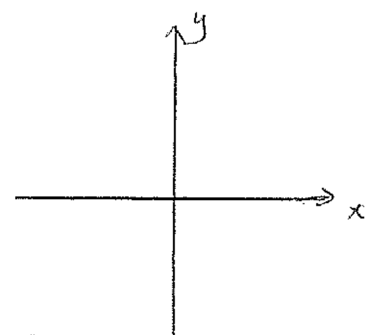
thus: $$\tilde{E}_t = \begin{bmatrix}0&0\\0&1\end{bmatrix}\frac{1}{2}\begin{bmatrix}1&1\\1&1\end{bmatrix}\begin{bmatrix}1&0\\0&0\end{bmatrix}E_1\begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix}$$

$$= \frac{E_1}{2}\begin{bmatrix}0&0\\0&1\end{bmatrix}\begin{bmatrix}1&1\\1&1\end{bmatrix}\begin{bmatrix}\cos\theta\\0\end{bmatrix}$$

$$= \frac{E_1}{2}\begin{bmatrix}0&0\\0&1\end{bmatrix}\begin{bmatrix}\cos\theta\\ \cos\theta\end{bmatrix} = \frac{E_1}{2}\begin{bmatrix}0\\ \cos\theta\end{bmatrix} = \frac{E_1\cos\theta}{2}\begin{bmatrix}0\\1\end{bmatrix}$$

$$\therefore I_2 \propto \left|\frac{\cos\theta}{2}E_1\right|^2 = \frac{\cos^2\theta}{4}I_1$$

c7. Hecht P8.18.



For natural light is unpolarized light, it's x component and y component of E-field has the same amplitude. but no phase relationship. Thus the intensity $I_{natural} = I_x + I_y = 2I_x = 2I_y$

So the intensity transmitted from the first linear polarizer (at 0°) is $I_{0°} = \frac{1}{2} I_{natural} = 500\ W/dm^2$, the Jone's vector for this polarized light is $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Then $I_{0°}$ passing through a polarizer alligned at 50°. from the previouse problem (a), we knew that. ~~P(θ)~~ the Jone's matrix for ~~the~~ such a polarizer is:

$$P(\theta) = \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$$

thus the light transmitted is:

$$\hat{E}_{50°} = P(50°)\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos^2 50° & \sin 50°\cos 50° \\ \sin 50°\cos 50° & \sin^2 50° \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \cos 50° \begin{bmatrix} \cos 50° \\ \sin 50° \end{bmatrix}$$

∴ The transmitted intensity is:

$$I_{50°} = \cos^2 50°\ I_{0°} \approx 0.41\, I_0 = 206.6\ W/dm^2$$

If there is a third polarized placed between them, along 25°. then the ~~the~~ transmitted field turns out to be.

$$\hat{E}_t = P(50°)P(25°)\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos^2 50° & \sin 50°\cos 50° \\ \sin 50°\cos 50° & \sin^2 50° \end{bmatrix}\begin{bmatrix} \cos^2 25° & \sin 25°\cos 25° \\ \sin 25°\cos 25° & \sin^2 25° \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \cos 25° \begin{bmatrix} \cos^2 50° & \sin 50°\cos 50° \\ \sin 50°\cos 50° & \sin^2 50° \end{bmatrix}\begin{bmatrix} \cos 25° \\ \sin 25° \end{bmatrix}$$

$$= \cos 25° \begin{bmatrix} \cos^2 50°\cos 25° + \sin 50°\cos 50°\sin 25° \\ \sin 50°\cos 50°\cos 25° + \sin^2 50°\sin 25° \end{bmatrix} = \cos^2 25° \begin{bmatrix} \cos 50° \\ \sin 50° \end{bmatrix}$$

∴ The transmitted field has an intensity

$$I_t = (\cos^2 25°)^2 I_0 = 337.3\,W/m^2$$

Problem 3 (b) and (c) shows us, we can always increase the transmitted intensity by rotating the polarization step by step. Part d) will discuss how far this approach can go.

d) There are N polarizer placed in sequence with the angle between the 1st and the N th polarizer is $\frac{\pi}{2}$. and the angle between the two neighbour polarizer is $\frac{\pi}{2(N-1)}$. According Malus's Law, the light passing through two linear polarizer with angle θ, the intensity will be reduced by a factor $\cos^2\theta$. Thus, when light passing through # N polarizer angled $\frac{\pi}{2(N-1)}$, the intensity will be $\left[\cos^2\left(\frac{\pi}{2(N-1)}\right)\right]^N$. If $N=2$, that is the case that the light passing through two perpendicular polarizer,

$$\underset{(N=2)}{I_{out}} = \left[\cos^2\left(\frac{\pi}{2}\right)\right]^2 I_{in} = 0$$

when $N \to \infty$, $\lim_{N\to\infty} I_{out} = \lim_{N\to\infty} \left[\cos^2\left(\frac{\pi}{2(N-1)}\right)\right]^N I_{in} = I_{in}$.

$N \to \infty \quad \frac{\pi}{2(N-1)} \to 0 \quad \cos^2(0°) \to 1. \quad \underset{N\to\infty}{(1)^N} = 1.$

Q4: a) birefringent material, 2 polarizations have E fields in different directions; interaction of E fields with the different direction in the material $\Rightarrow$ EM wave sees different indices of refraction, n_o and n_e (ordinary and extraordinary) say here that $n_e < n_o$ and n_e along $\hat{x}$, n_o along $\hat{y}$.

Light input with $\vec{E}$ polarized at θ, so theres is a decomponation into $\vec{E} = E_x\hat{x} + E_y\hat{y} = E\cos\theta\,\hat{x} + E\sin\theta\,\hat{y}$.

$v_\phi = \frac{c}{n}$, so v_ϕ (ordinary) $< v_\phi$ (extraordinary)

for length of material L, the optical path-length is nL.

so for a fixed length, the ordinary component sees a greater optical path length than does the extraordinary component. ~~These~~

Thus the ordinary component emerges with a phase retardation

$$\phi = kz - \omega t = \phi(z,t) \qquad k = \frac{2\pi}{\lambda} \qquad \lambda = \frac{\lambda_0}{n_0}$$

So phase difference is.

$$\Delta\phi = k_o L - k_e L = \frac{2\pi L}{\lambda_0}(n_o - n_e)$$

When this phase difference is π, we call it a half-wave plate. ie.

$$\pi = \frac{2\pi L}{\lambda_0}(n_o - n_e)$$

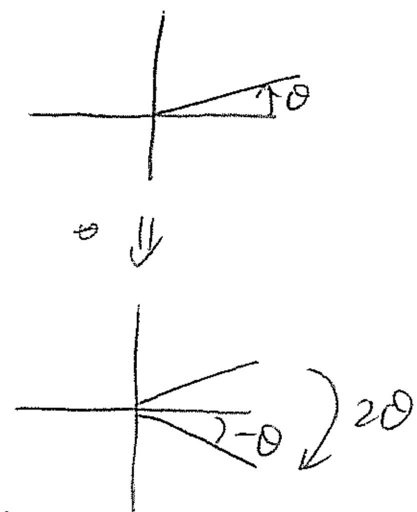
$$L = \frac{\lambda_0}{2}\frac{1}{n_o - n_e}$$

Here, after propagation. $\vec{E} = E\cos\theta\,\hat{x} + E\sin\theta\,\hat{y}$ becomes

$$\vec{E}' = E\cos\theta\,\hat{x} - E\sin\theta\,\hat{y} = E\cos(-\theta)\,\hat{x} + E\sin(-\theta)\,\hat{y}.$$

comes from the π phase shift.

as ~~if~~ if the polarization along $-\theta$.



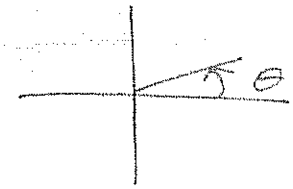
Thus a linear polarized light incident to a ~~θ~~ half wave plate with the angle between ~~the~~ the polarization and the optical axies θ, it will be rotated by 2θ after this half wave plate. Thus, if one want to rotate the angle α of a linear polarized light, ~~be~~ what to do is to put ~~the~~ a half wave plate infront of the light with its optical axies angled $\frac{\alpha}{2}$ to the polarization of the light.

Faraday Effect: In Faraday medium, the light can be regarded as componented by right-handed circularly polarized light and left-handed circularly polarized light. $\vec{E} = E_{RHC}\,\hat{e} + E_{LHC}\,e^{i\phi}\,\hat{j}$

If the input light is RHC or LHC polarized, only one component exist, so the polarization state is unchanged. This is similarly to ~~the~~ a linear polarized light passing through a linear polarizer with its transmission directio parallel to the polarization direction of incident light.

Then, after propagation

$$\vec{E} = E\cos\theta\,\hat{x} + E\sin\theta\,\hat{y}$$

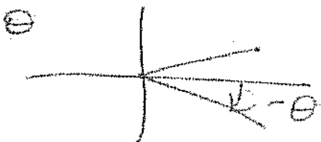


becomes, relatively

$$\vec{E} = E\cos\theta\,\hat{x} - E\sin\theta\,\hat{y}$$

$$= E\cos(-\theta)\,\hat{x} + E\sin(-\theta)\,\hat{y}$$

as if the polarization were at $-\theta$



Thus the polarization has rotated through a total angle of $\underline{2\theta}$.

~~4a)~~ $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ fast axis is horizontal <u>or</u> vertical

$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ eigenvector with eigenvalue $+1$

$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ eigenvector with eigenvalue -1

4b) The light interacts with electrons which are affected by the magnetic field. RHC light and LHC light interact differently because RHC can resonantly drive electrons in cyclotron orbits while LHC cannot.

Thus there is different dispersion for the two ~~the~~ polarizations, and so it is <u>dispersion</u> that leads to different indices of refraction, n_L and n_R. (It is basically the same for linear polarizations since the two directions in the crystal have different electron "spring constant

b) we have these eigenstates

"BREAK SYMMETRY" $\Rightarrow$ NO LONGER TREAT THE SAME

LHC: $\begin{bmatrix} 1 \\ i \end{bmatrix}$ $\quad \sigma^-$

RHC: $\begin{bmatrix} 1 \\ -i \end{bmatrix}$ $\quad \sigma^+$

Want to find ABCD such that

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} 1 \\ i \end{bmatrix} = e^{i\theta} \begin{bmatrix} 1 \\ i \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} 1 \\ -i \end{bmatrix} = e^{-i\theta} \begin{bmatrix} 1 \\ -i \end{bmatrix}$$

NB we could make the constants 1 and $e^{-2i\theta}$ - only the difference in ϕ matters ~~crzp~~.

ADD the two equations

$$\Rightarrow \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} e^{i\theta} + e^{-i\theta} \\ i(-e^{i\theta} + e^{-i\theta}) \end{bmatrix}$$

thus

$$2 \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 2 \begin{bmatrix} \cos\theta \\ -\sin\theta \end{bmatrix}$$

$$\begin{bmatrix} A \\ C \end{bmatrix} = \begin{bmatrix} \cos\theta \\ -\sin\theta \end{bmatrix} \qquad A = \cos\theta \quad C = -\sin\theta$$

SUBTRACT the two eq$^{\underline{ns}}$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} 0 \\ 2i \end{bmatrix} = \begin{bmatrix} e^{i\theta} - e^{-i\theta} \\ i(e^{i\theta} + e^{-i\theta}) \end{bmatrix}$$

$$2i \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 2i \begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix}$$

) cnt'd

$$\begin{bmatrix} B \\ D \end{bmatrix} = \begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix} \qquad \begin{matrix} B = \sin\theta \\ D = \cos\theta \end{matrix}$$

Thus our matrix is

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \equiv R(\theta)$$

But this is the rotation matrix! Does this make sense?

Sure when two circular polarizations propagate in free space, they make linearly polarized light of a fixed orientation. BUT if we rotate our axes this discriminates between the two chiralities (handed-ness) LEFT and RIGHT — the rotation advances the phase of one circular pol$^{\underline{n}}$ and retards the phase of the other.

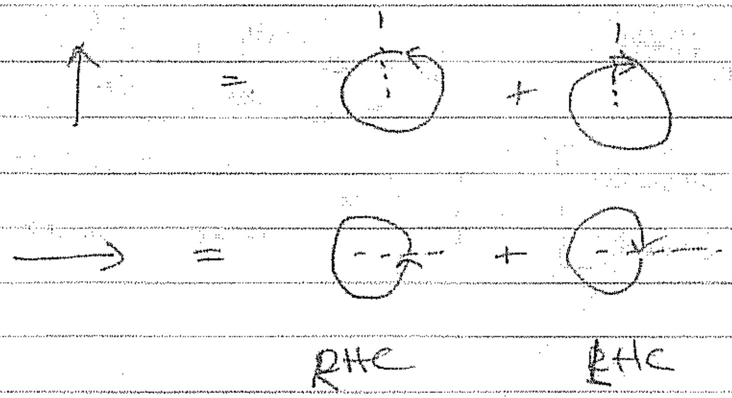
So a rotation of θ adds phase θ to one, subtracts phase θ from the other; the relative phase difference is 2θ, and thats exactly whats needed to rotate the linear polarization by θ.

Our magnetic field does this, in the ionosphere — the phase speeds are diffrent for RHC and LHC, so the linearly polarized light stays linear but steadily rotates orientation and their dispersion relations differ, to cause n_e and n_o!)

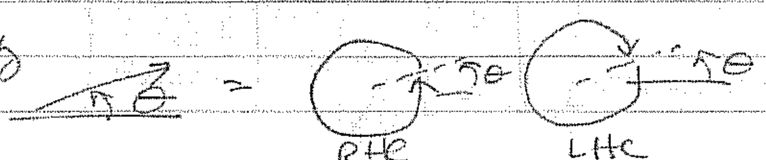
Then with any linearly polarized light decomposed into RHC and LHC with some phase difference:

$$\vec{E} = E(\sigma^+ e^{i\delta} + \sigma^-)$$

If



generally



$$= \sigma_+ e^{i\theta} + \sigma_- e^{-i\theta}$$

If we cause an additional phase delay δ of σ^- relative to σ^+

$$\sigma_+ e^{i\theta} + \sigma_- e^{-i\theta} e^{i\delta}$$

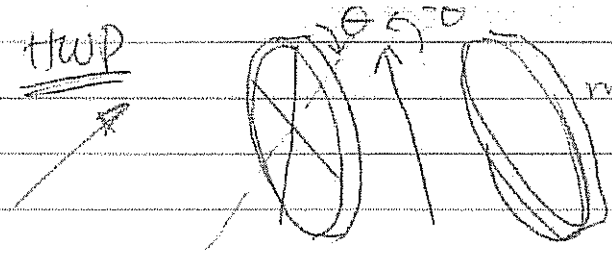
$$= \left(\sigma_+ e^{i(\theta - \delta/2)} + \sigma_- e^{-i(\theta - \delta/2)}\right) e^{i\delta/2}$$

Thus $\theta \to \theta - \delta/2$: we cause a rotation of $-\delta/2$ in the linear polarization state

Thus to rotate by α, must delay by 2α

σ_+ and σ_- are unchanged except for a phase —

c d) HWP



Half waveplate

① Incoming polarizatio at angle θ Leaves HWP at $-\theta$

② hits mirror, goes back to waveplate again hits at angle $+\theta$ ~~(turn around direction of k)~~

input at angle θ

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} = \begin{bmatrix} \cos\theta \\ -\sin\theta \end{bmatrix}$$

③ exits at θ, but this is same orientati it went in at

HWP

$$\cancel{\begin{bmatrix} \cos\theta \\ -\sin\theta \end{bmatrix} \quad \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}}$$

$$\begin{bmatrix} \cos\theta \\ -\sin\theta \end{bmatrix} \rightarrow \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$ reverse z off reflector

goes back

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \cos\theta \\ +\sin\theta \end{bmatrix} = \begin{bmatrix} \cos\theta \\ -\sin\theta \end{bmatrix}$$

same state but with z reversed (see comment at bottom

So double-passing HWP leaves the polarization unchanged.

d) Faraday

$$\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\alpha + \sin\theta\sin\alpha \\ -\cos\theta\sin\alpha + \sin\theta\cos\alpha \end{bmatrix}$$

~~mirror reverses z, so y flips also~~

$$\begin{bmatrix} \cos\theta\cos\alpha + \sin\theta\sin\alpha \\ \cos\theta\sin\alpha - \sin\theta\cos\alpha \end{bmatrix}$$

but now the angle α of the Faraday element is $-\alpha$, when z changes direction (RHC→LHC & LHC→RH d (cont'd)

Thus, going backwards

$$\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\theta\cos\alpha + \sin\theta\sin\alpha \\ \cos\theta\sin\alpha - \sin\theta\cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha(\cos\theta\cos\alpha + \sin\theta\sin\alpha) - \sin\alpha(\cos\theta\sin\alpha - \sin\theta\cos\alpha) \\ \sin\alpha(\cos\theta\cos\alpha + \sin\theta\sin\alpha) + \cos\alpha(\cos\theta\sin\alpha - \sin\theta\cos\alpha) \end{bmatrix}$$

$$= \begin{bmatrix} (\cos^2\alpha - \sin^2\alpha)\cos\theta + 2\cos\alpha\sin\alpha\sin\theta \\ (\sin^2\alpha - \cos^2\alpha)\sin\theta + 2\sin\alpha\cos\alpha\cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos(2\alpha)\cos\theta + \sin(2\alpha)\sin\theta \\ +\sin(2\alpha)\cos\theta - \cos(2\alpha)\sin\theta \end{bmatrix}$$

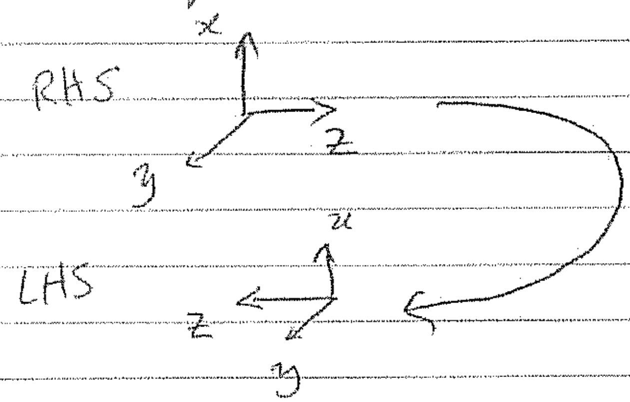
which we can recognize as the reflection $(z \to -z)$ of:

$$= \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix} \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

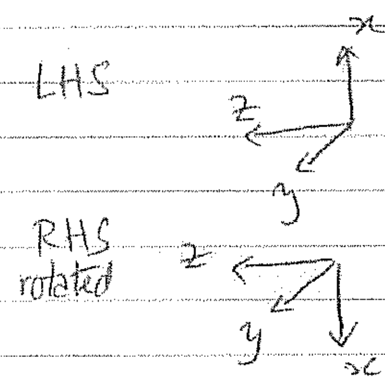
ie the original polarization rotated by 2α & then reversed.

Comment on reversing on reflection

On reflection, a right-hand coordinate system becomes a mirror image — a left-hand coordinate system



We can rotate our LH coordinate system around $\hat{y}$ so that $\hat{z}$ agrees, we see:



So a reflection this way means our x-component changes sign in the RH coordinate system

(we could have changed $\hat{y}$ instead)

For this reason also, right-hand circular polarized light becomes left-hand circular polarized light.