Problem Set 3 2008
Problem 1

Hecht 8.48

Q1 a) The rotary power for 10 cm of 1 g/cm^3 solution is +66.45°

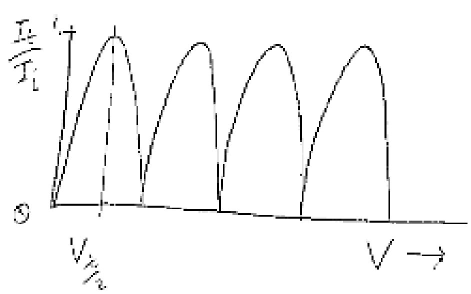
The rotary power for 1 m of 10g/1000 cm^3 solution is thus

$$+66.45 \times \left(\frac{1\,m}{0.10\,m}\right) \times \left(\frac{10}{1000}\right) = +6.645^\circ \text{ to vertical.}$$

b) Hecht 8.65 $\quad I_t = I_i \sin^2\left(\frac{\Delta\varphi}{2}\right)$

and $\quad \Delta\varphi = 2\pi n_0^3 \frac{r_{63}}{\lambda_0} V$

So



The voltage which provides maximum transmission is the half-wave voltage, which causes the Pockels cell to rotate the incident polarisation through 90°.

For $I_t = 0$, $V \neq 0$, we need $\frac{\Delta\varphi}{2} = \pi$

$$\Rightarrow \Delta\varphi = 2\pi$$

$$\cancel{2\pi} = \frac{\cancel{2\pi}\, n_o^3\, r_{63}}{\lambda_o} V$$

$$\cancel{=2\pi} \quad V = \frac{\lambda_o}{n_o^3 r_{63}}$$

$$= \frac{5.461\times10^{-9}}{(1.52)^3\, 8.5\times10^{-12}}$$

$$= 182.94\ V$$

To provide max $\frac{I_t}{I_i}$ at zero voltage, the input + output polarisers at the cell should be oriented in the same direction.

Then at $V_{\frac{\lambda}{2}}$ the output would be ~~zero~~.

Problem 2

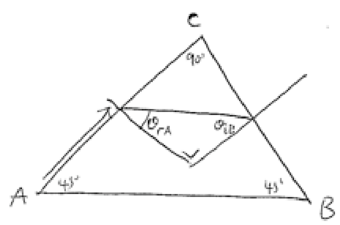
a) Hecht Q 4.55

$n = \sin(90)/\sin(48) = 1.35$

Q2 b)

We don't know the angle of incidence on AC – but we know we want the minimum possible index of refraction, meaning we need the maximum angle of incidence at BC, meaning we need the maximum angle of incidence at AC (looking at the diagram) – so we are incident at 90 degrees, and

Q2 b)



A beam traversing AC will be bent into the prism at angle $\theta_{rA} = \sin^{-1}\left(\frac{1}{n_p}\right)$

The angle od incidence at BC must be at least the critical angle to give TIR

$$\theta_{iB} = \theta_c = \sin^{-1}\left(\frac{1}{n_p}\right) = \theta_{rA}$$

and, from the diagram

$$\theta_{iB} = 90 - \theta_{rA}$$

Hence, θ_{rA} must be at ~~least~~ most 45° so that θ_{iB} can be greater than it

$$45 = \sin^{-1}\frac{1}{n_p}$$

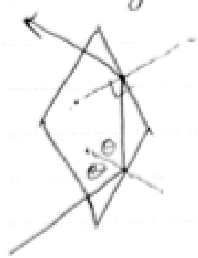
$$n_p = \frac{1}{\sin 45} = 1.41$$

c)

Hecht Q 4.61
Solution from Hecht page 663:
The beam scatters off the wet paper and is mostly transmitted until the critical angle is attained, at which point the light is reflected back toward the source.
$\tan\theta_C = (R/2)/d$, and so $n_{ti} = 1/n_t = \sin\left[\tan^{-1}(R/2d)\right]$.

Problem 3

Mooney's rhomb



2 reflections identical angle, at max symmetry (vertical, of drawing) then on each

$$\frac{\pi}{4} = \Delta = \delta_p - \delta_s \quad \text{(want } \tfrac{\pi}{2} \text{ net)}$$

Thus

$$0.4142 = \tan\frac{\pi}{8} = \tan\frac{\Delta}{2} = \frac{\cos\theta\sqrt{\sin^2\theta - n^2}}{\sin^2\theta}$$

where $n = \frac{1}{1.65}$

$$\tan^2\frac{\pi}{8} = \frac{\cos^2\theta(\sin^2\theta - n^2)}{\sin^4\theta}$$

$$= \frac{(1-\sin^2\theta)(\sin^2\theta - n^2)}{\sin^4\theta}$$

$$\tan^2\frac{\pi}{8}\cdot\sin^4\theta = \sin^2\theta - n^2 - \sin^4\theta + n^2\sin^2\theta$$

$$s \equiv \sin^2\theta$$

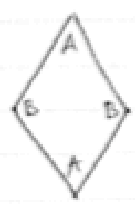
$$s^2\left(1+\tan^2\frac{\pi}{8}\right) - (1+n^2)s + n^2 = 0$$

$$s = \frac{(1+n^2) \pm \sqrt{(1+n^2)^2 - 4\left(1+\tan^2\frac{\pi}{8}\right)n^2}}{2} \qquad n = \frac{1}{1.65}$$

$\sin\theta = 0.4192,\ 0.7478$

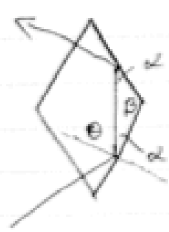
$\theta = 0.7043$ or 1.045 radians

$= 40.35°$ or $59.86°$



$2A + 2B = 2\pi$

$\Rightarrow A + B = \pi$



$A = \pi - 2\theta$

$\Rightarrow A = 99.3°$ or $\underline{\underline{60.3°}}$

$\theta + \alpha = \pi/2$

$2\alpha + B = \pi$

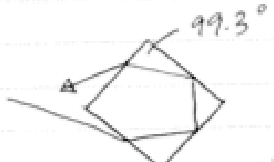
$B = \pi - 2\alpha$

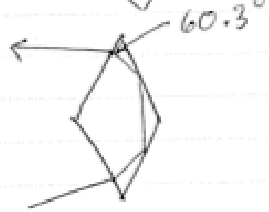
$= \pi - 2(\pi/2 - \theta)$

$B = 2\theta$

$A = \pi - B$

$= \pi - 2\theta$





Problem 4

X-ray mirrors $\quad N^2 = \varepsilon = 1 - \dfrac{\omega_p^2}{\omega^2 + i\omega\nu}$

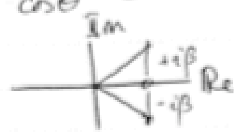
$$r_s = \frac{\cos\theta - \sqrt{n^2 - \sin^2\theta}}{\cos\theta + \sqrt{n^2 - \sin^2\theta}}$$

Note first how it works with $n \in \mathbb{R}$:

when $n^2 - \sin^2\theta = 0$, then $r_s = \dfrac{\cos\theta}{\cos\theta} = 1$

when $n^2 - \sin^2\theta < 0$, then

$$r_s = \frac{\cos\theta - i\beta}{\cos\theta + i\beta}$$



so ~~as~~ long as the radical is imaginary, numerator & denominator have the same magnitude, thus $|r_s|^2 = 1$ [illegible]

Now with $n \in \mathbb{C}$

$$r_s = \frac{\cos\theta - \sqrt{\left(1 - \frac{\omega_p^2}{\omega^2 + i\omega\nu}\right) - \sin^2\theta}}{\cos\theta + \sqrt{\left(1 - \frac{\omega_p^2}{\omega^2 + i\omega\nu}\right) - \sin^2\theta}}$$

$$= \frac{\cos\theta - \sqrt{\cos^2\theta - \frac{\omega_p^2}{\omega^2(1 + i\nu/\omega)}}}{\cos\theta + \sqrt{\cos^2\theta - \frac{\omega_p^2}{\omega^2(1 + i\nu/\omega)}}}$$

in our case $\dfrac{\nu}{\omega} \ll 1$ so

$$r_s \simeq \frac{\cos\theta - \sqrt{\cos^2\theta - \frac{\omega_p^2}{\omega^2}(1 - i\nu/\omega)}}{\cos\theta + \sqrt{\cos^2\theta - \frac{\omega_p^2}{\omega^2}(1 - i\nu/\omega)}}$$

when $\cos^2\theta = \dfrac{\omega_p^2}{\omega^2}$ this corresponds to the onset of TIR in the case of a conductor/plasma without collisions ($\nu = 0$). Beyond this point the radicals are imaginary and $|r_s|^2 = 1$ even though this is not a dielectric.

b) x-rays 10 keV : $\hbar\omega = 10\text{keV}$ $\quad \hbar = 6.57\times10^{-16}\,\text{eV-s}$

$\omega = 1.5\times10^{19}\,s^{-1}$

$\omega_p = 10^{15}\,s^{-1}$

$\nu = 10^{13}\,s^{-1}$

$\sqrt{\frac{\nu}{2\omega}} = 5.8\times10^{-4}$

Then $|r_s|^2 = 0.998$ at $\cos\theta = \frac{\omega_p}{\omega} = 3.3\times10^{-5}$

θ is near $\pi/2$; write $\cos\theta = \cos(\pi/2 - \delta) = \sin\delta$

Grazing angle $\delta = \sin^{-1}(3.3\times10^{-5})$

$= 6.7\times10^{-5}$ radians

$= 0.003$ degrees

$= 1.38$ arc-seconds

Compare for 1 keV

$= 0.03$ degrees

$= 13.8$ arc-seconds

Both quite small grazing angles - difficult to get high efficiencies.

c) $\hbar\omega = 50\text{eV}$

$\omega = 7.5\times10^{16}\,s^{-1}$

$\delta = 13$ mRad

$= 0.8$ degrees.

Any longer wavelength will also reflect, at this angle, so this makes a low-pass filter. K-edge absorption filters make a high-pass filter, so in the right combinations x-ray bandpass filters can be constructed.

Problem 5

A thick lens can be broken down into 2 curved interfaces and one free propagation region. This is modeled using ray matrices as follows:

$$\begin{bmatrix} 1 & 0 \\ (n-1)/R_2 & n \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \left(\frac{1}{n}-1\right)\Big/R_1 & 1/n \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ (n-1)/R_2 & n \end{bmatrix} \begin{bmatrix} 1+d\left(\frac{1}{n}-1\right)\Big/R_1 & d/n \\ \left(\frac{1}{n}-1\right)\Big/R_1 & 1/n \end{bmatrix}$$

$$= \begin{bmatrix} 1+d\left(\frac{1}{n}-1\right)\Big/R_1 & d/n \\ (n-1)/R_2 + d(n-1)\left(\frac{1}{n}-1\right)\Big/(R_1R_2) + n\left(\frac{1}{n}-1\right)\Big/R_1 & d(n-1)/(nR_2)+1 \end{bmatrix}$$

$$C = (n-1)/R_2 + d(n-1)\left(\frac{1}{n}-1\right)\Big/(R_1R_2) + n\left(\frac{1}{n}-1\right)\Big/R_1$$

$$= (n-1)/R_2 + d(n-1)\left(\frac{1}{n}-1\right)\Big/(R_1R_2) - (n-1)\Big/R_1$$

$$= (n-1)\left[\frac{1}{R_2} - \frac{1}{R_1} - \frac{d(n-1)}{nR_1R_2}\right]$$

$\frac{1}{f} = -C = (n-1)\left[-\frac{1}{R_2} + \frac{1}{R_1} + \frac{d(n-1)}{nR_1R_2}\right]$ is the focal length formula.

Also, $A \neq 1$, $B \neq 0$, and $D \neq 1$ (unless d=0 which in that case is the thin lens approximation).