Modern Optics
PHY485F/1485F
University of Toronto
Problem Set #4

21 November 2008 due: 4 December 2008

1. *Transverse coherence — Van Cittert-Zernike theorem*

The Van Cittert-Zernike theorem gives transverse coherence of light coming from a source that is not two separate points, but an extended source. For a circular aperture of radius R, the theorem reduces to a statement that the transverse coherence length at a plane distance L away is:

$$L_C = 0.26\frac{\lambda L}{R}$$

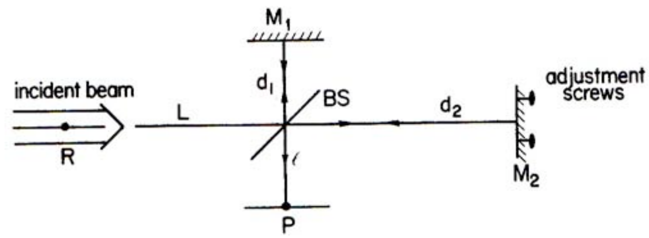
a) Hecht 12.9

b) Hecht 12.15

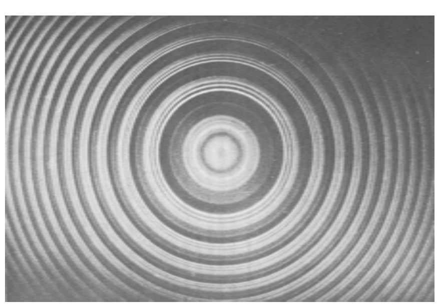
b) Hecht 12.17

2. *Interferometry—*

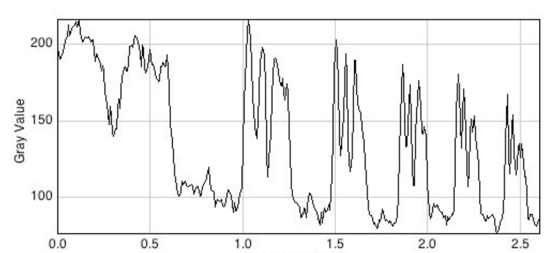
a) In an actual Michelson interferometer using an extended source of light (see figure below), we observe circular interference fringes when viewing M1 through BS if M1 and M2 are perpendicular (one of the mirrors of the interferometer has tilting screws so that the mirror orientation is adjustable). Explain the appearance of circular fringes. Do you expect to see circular fringes if the mirrors are not perpendicular? (*Lasers*, Milonni & Eberly)



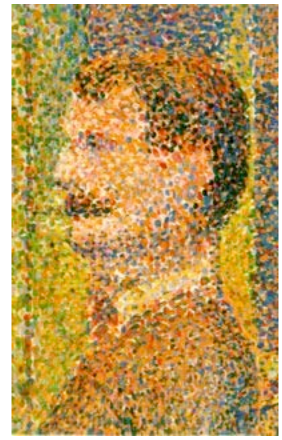
b) Verify that the reflectance of a glass or other dielectric substrate can be *increased* by coating it with a $\lambda/4$, high-index layer, that is, $n_1 > n_S$. Show that the reflected waves interfere constructively. The quarter-wave stack $g(HL)^m Ha$ (glass, high, low, air) can be thought of as a series of such structures.



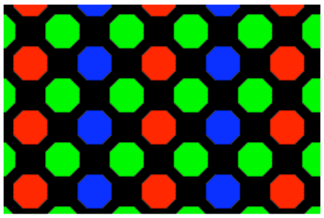
c) The well-known yellow D-lines from sodium are at wavelengths 588.9950 nm & 589.5924 nm. What must be the configuration of a Fabry-Perot spectrograph, illuminated by a point source, so that these lines can be resolved while still affording a free spectral range equal to the whole visible spectrum. Is it possible? (Figure at left shows an output for a different spectrum, exhibiting many orders; at right is an intensity line-out from the centre outwards showing the spectrum repeated in different orders)



3. *Quick diffraction questions—*



a) Then neoimpressionist painter Georges Seurat was a member of the *pointillist school*. Rather than creating his colours by mixing pigments on his pallette, before application on the canvas in solid areas of paint, his paintings consist of an enormous number of closed spaced small dots (1–2 mm) of single-colour pigment (see right). The observed colours are

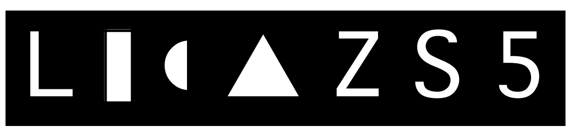


therefore created by mixing *light*, as the tiny points of colour light are mixed in the eye by blurring together through diffraction. This scheme anticipates colour television (see left), which is similarly based on *additive* primary colours, rather than *subtractive* (absorption-based) primaries.

For a painting such as described above, how far back should one stand to achieve the effect of mixing? (Look in a mirror to estimate any eye-sized parameters needed) http://en.wikipedia.org/wiki/Pointillism

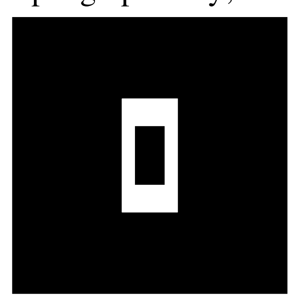
b) The Hubble Space Telescope has a primary mirror which is 2.4 m in diameter. Determine its angular resolution limit at a wavelength of 550 nm (green light, middle of visible spectrum) in radians, and in degrees, minutes and seconds of arc. How far apart must be two objects on the surface of the Moon, at the minimum, if they are to be resolved by Hubble? Take the Earth-Moon distance to be 3.844×10^{8} m. On the other hand, how far apart must two objects be on the Moon, at the minimum, in order to be resolved by eye?

c) For the apertures shown below, sketch the Fraunhofer diffraction patterns for each. Note that disks are associated with Airy-function diffraction patterns, and include rings.



4. *Fresnel diffraction*

a) Give a precise statement of Babinet's principle. Illustrate Babinet's principle graphically, on a Cornu spiral, in the context of a one-dimensional slit and for two different detection points lying behind it which have different displacements relative to the slit.



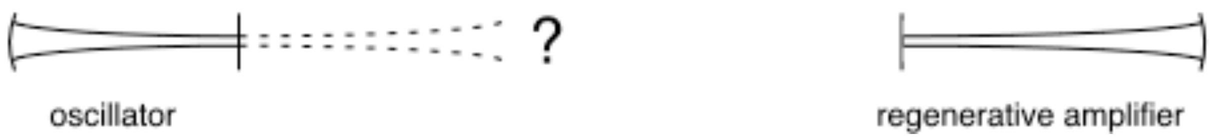
b) The aperture in the figure at right has outer dimensions of 100 μm x 50 μm and inner dimensions of 50 μm x 25 μm. For illumination at normal incidence by a plane wave with $\lambda = 500$ nm, at approximately what distance does Fraunhofer diffraction become valid? Use Babinet's principle and a Cornu spiral to sketch the diffraction pattern in an observation plane which is 2 mm beyond the aperture.

5. *Mode-matching gaussian beams in two resonators*

a) Consider a laser oscillator cavity 1m long, with one flat end-mirror and the other with $R = 3$ m. Show that this configuration is stable, and find the far-field divergence angle of the TEM_{00} beam output through the flat mirror.

The output pulse from this oscillator is to be amplified in another laser-cavity (a *regenerative* amplifier). If the pulse is injected in an arbitrary way, the gaussian beam will not be a TEM_{00} mode in the second cavity. In that case it will be equivalent to some linear combination of different normal modes of the second cavity.

b) The regen cavity is 1.25m long, with one flat mirror and one mirror of curvature R = 5 m. The laser pulse is injected through the flat mirror (an over-simplification, actually). The distance between the oscillator and regen is 4 m. What lens or lenses are required to take the output of the cavity and introduce it into the second cavity as a matched mode? Specify values and locations.



c) Amir tries this but finds he has a small error, and find that in the regen he has both TEM_{00} and some TEM_{10} modes operating. A portion of the light output from the regen is collected onto a photodiode, and he records the signal on a fast oscilloscope. Describe the signal that Amir will observe on his oscilloscope. If he cannot get the mode-matching perfect, what else can he do to minimize this problem?