The TA is still preparing the solutions for PS#4 and they should be ready on Sunday or early Monday. Meanwhile here are some materials and comments from me.   -RSM

Q2 (Michelson) - solution here

some notes/comments
a) an extended source means different angles are available through the Michelson -- so the orders like a Fabry-Perot are explored by changing through angles where different wavelengths are resonant. Tilting one mirror should just tilt the whole angle-set of one arm, and shift the centre. Note that this is available in the 2nd year labs.

b) adding a layer coating makes TWO Fresnel reflections; getting the index right (the geometric mean of the air and the glass) makes the two reflections of equal strength (equal steps in ratio of index, at each surface). Getting the thickness right makes the phasing correct for destructive interference going forward. Note that the thickness is not a quarter-wavelength of the VACUUM wavelength, it's the wavelength in the material.

c) the visible spectrum is roughly 400nm; any value within 25% of this is fine. Set the distance between mirrors to meet the FSR, the finesse will determine the resolution; we sometimes refer specifically to the "Rayleigh criterion" for the resolution, but any approximate form with the right principles is acceptable here.

Q3 (Diffraction, pointillism) - solution here

Q4 some notes/comments only
a) Babinet's principle is about complementary apertures -- that means OPENINGS. People, even texts, make silly errors adding the screens (blocking parts) and say, for instance, that the diffraction pattern for a slit and a wire are the same. They're not, of course, they look like E(u,v)^2 and (1-E(u,v))^2. On the Cornu spiral, which is a particular "vibration curve", the vibration-curve for straight-edge apertures, light for a given aperture looks like a line between two points on the Cornu spiral (each edge of the aperture, relative to the observation point, determines the parameterization point along the curve -- you should quote the formula). Two apertures then means two straight lines on two portions of the spiral.

For Babinet's principle on the slit, the TA will accept any version of complementary apertures which are relevant -- the most 'natural' is perhaps:
    (slit from A to B) U (half-plane from B to infinity) = (half-plane from A to infinity)
where U is "union" in set-theory sense; the first two are 'complementary' in the last; this requires the intersection of the first two be null/empty.

Graphically this then looks like this for a specific observation point: parameterize A, parameterize B, draw a straight line between these two on the Cornu spiral; draw also a straight line from the parameter for B to the centre of the spiral, the horn, which corresponds to INFINITY on the open side. Babinet's principle is that the VECTOR SUM of these two lines is the line from parameterization for A direct to the INF-point.

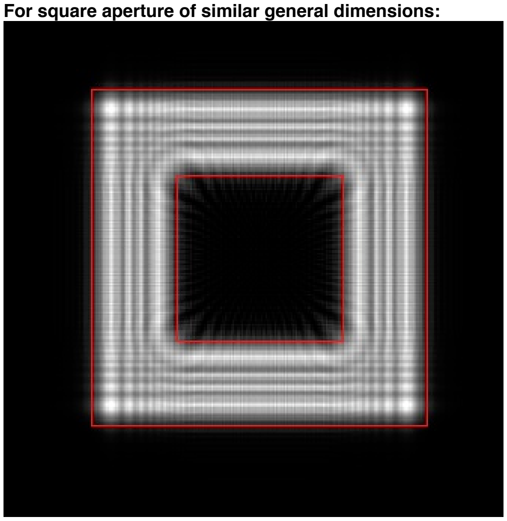
b) The diagram contains two rectangles RectLarge and RectSmall. Babinet's principle says:
    E-RectLarge(u,v) = E-Diagram(u,v) + E-RectSmall(u,v)
so that
    E-Diagram(u,v) =E-RectLarge(u,v) - E-RectSmall(u,v)
and
    lE-Diagram(u,v)l^2 =lE-RectLarge(u,v) - E-RectSmall(u,v)l^2

Each of E-RectLarge(u,v) and E-RectSmall(u,v) give a slit-pattern; you are looking for a product of separate patterns for x and y. Each breaks down into a simpler slit-pattern in the open areas, and two-slit interference where the line crosses the central bit.

There are many fringes, and any sketch showing many fringes is OK -- the TA will give full marks for any effort to draw it, if the principles are right. He'll accept numerical solutions too, if the student notes first that it's quite complicated to do manually.

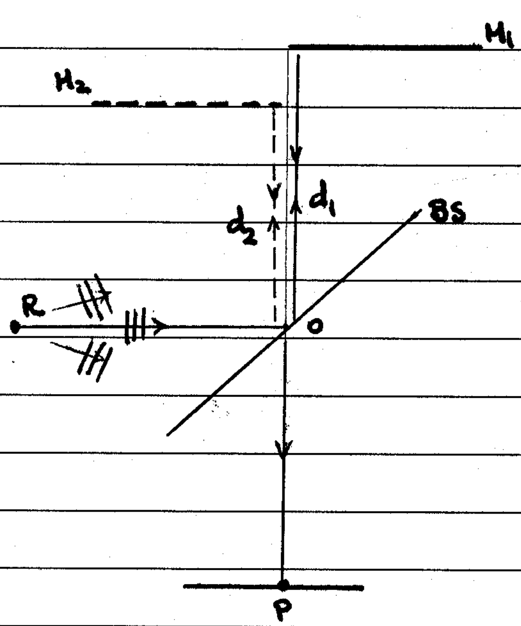
Explore with Java applet for rectangles (you have to trick it: multiply all lengths by 100x): http://www.falstad.com/diffraction/
Also see Demo posted on website (lower part of site, look for Diffraction Demo).

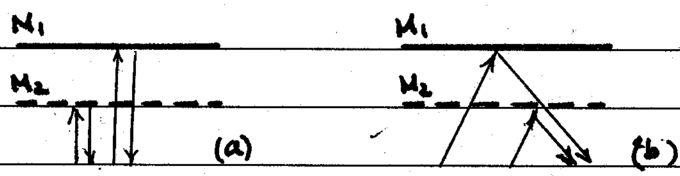
For square aperture of similar general dimensions:



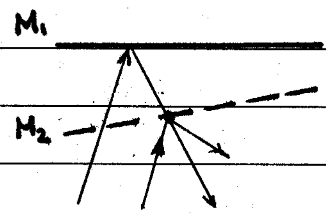
Q5 (Gaussian mode-matching) - solution here

Q1 a) Let us consider first the case when M_1 and M_2 mirrors are perpendicular to each other. To clarify the situation we redraw the Michelson's interferometer scheme as below. The redrawn scheme shows that a situation similar to reflection from a dielectric slab is occuring. To explain the appearance of the ringlike interferences patterns we have to consider that rays from the source fall on BS at angles different from 45° as well.



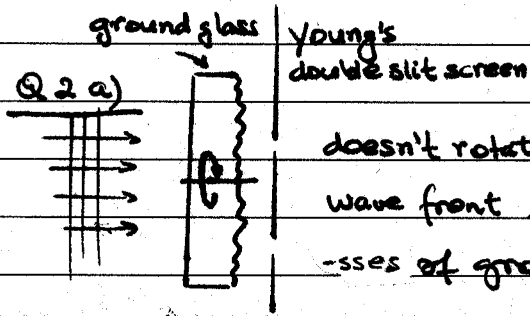


To observe the interference of the rays falling on the BS at angles ≠ 45°, the eye of the observer has to focus at infinity, or likewise the source (extended) should be put at the focus of a lens, and the interferometer at the other side of the lens. This situation (of perpendicular M_1 and M_2) has cylindrical symmetry about the RO axis of the picture. Bundles of parallel rays, falling on BS at different angles, fulfill different interference conditions as can be seen from (a) and (b) pictures. Bundles of parallel rays coming toward P are then focused on the retina by the eyeball and should result in circular interference patterns. If M_1 is not perpendicular to M_2 then we have a wedge like situation



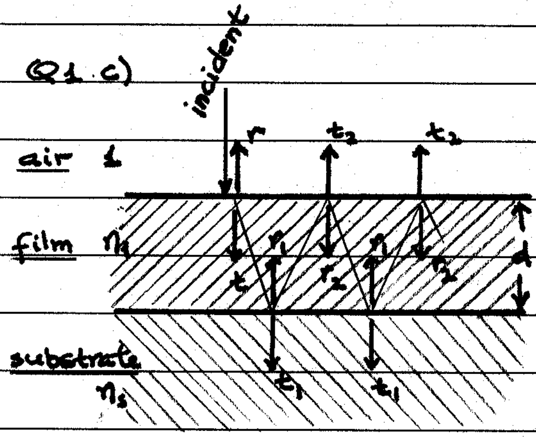
The interference pattern can be observed if the eye is focused on the surface of M_2 where reflected rays meet. In this case the interference pattern is made up of stripes lined in the direction out of the page.

Q 2 a)



Suppose for a moment that the ground glass doesn't rotate. The wave front deforms away from a plane wave front since parallel rays go through different thicknesses of ground glass. Nevertheless the spatial coherence will be preserved and an interference pattern observed on the projection screen. This interference pattern is shifted from the one observed if the glass isn't there at all. Now if we start rotating the ground glass, interference patterns, shifted from each other will follow one another on the projection screen, giving an, in average, uniformly illuminated screen.

Q1. c)



$\underline{n_1 > n_s > 1}$

have: $r = \dfrac{1-n_1}{1+n_1}$ (since $n_1 > 1$ we have a phase change of $180°$ as the wave goes from less dense to dense medium)

$r_1 = \dfrac{n_1 - n_s}{n_1 + n_s}$ (no phase change)

$r_2 = \dfrac{n_1 - 1}{n_1 + 1} = -r$

$t = \dfrac{2}{n_1+1}$; $t_1 =$ (do not need this) ; $t_2 = \dfrac{2n_1}{n_1+1} = n_1 t$

For each pass through the film, the normally incident ray changes the phase by e^{ikd} where d is the film thickness. Now $kd = n_1 \frac{2\pi}{\lambda_0} d = \frac{\pi}{2} \dfrac{d}{\left(\frac{\lambda_0}{4n_1}\right)}$
If $d = \dfrac{\lambda_0}{4n_1}$ we have a quarter-wave film $\Rightarrow e^{ikd} = e^{i\pi/2} = i$.

The reflected ray has an amplitude factor (where subindex c is for coating)

$r_c = r + t\, i r_1 i\, t_2 + t\, i r_1 i r_2 i r_1 i\, t_2 + t\, i r_1 i r_2 i r_1 i r_2 i r_1 i\, t_2 + \cdots$

$= r + t t_2\, i^2 r_1 + t t_2\, i^4 r_1^2 r_2 + t t_2\, i^6 r_1^3 r_2^2 + \cdots$

$= r + \dfrac{t t_2}{r_2}(i^2 r_1 r_2) + \dfrac{t t_2}{r_2}(i^4 r_1^2 r_2^2) + \dfrac{t t_2}{r_2}(i^6 r_1^3 r_2^3) + \cdots$

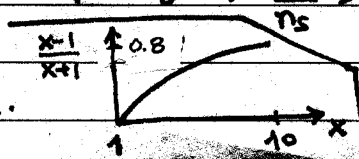
$= r + \dfrac{t t_2}{r_2}(i^2 r_1 r_2)^1 + \dfrac{t t_2}{r_2}(i^2 r_1 r_2)^2 + \dfrac{t t_2}{r_2}(i^2 r_1 r_2)^3 + \cdots$

$= r + \dfrac{t t_2}{\cancel{r_2}}\, i^2 r_1 \cancel{r_2} \left[1 + (i^2 r_1 r_2) + (i^2 r_1 r_2)^2 + \cdots\right]$; since $i^2 r_1 r_2 = -r_1(-r) = r_1 r$

$= r - r_1 t t_2 \left[1 + (r_1 r) + (r_1 r)^2 + \cdots\right] = r - r_1 n_1 t^2 \dfrac{1}{1 - r_1 r} = \dfrac{r - r_1 r^2 - r_1 n_1 t^2}{1 - r_1 r}$

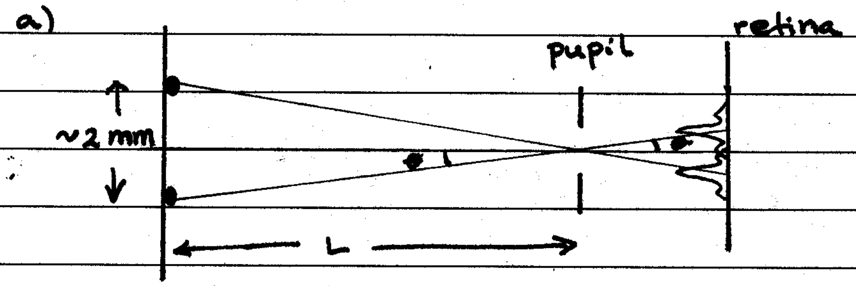
$= \dfrac{r - r_1 \overbrace{\left[r^2 + n_1 t^2\right]}^{=1}}{1 - r_1 r} = \dfrac{r - r_1}{1 - r_1 r} = \dfrac{n_s - n_1^2}{n_s + n_1^2} = \dfrac{1 - \frac{n_1^2}{n_s}}{1 + \frac{n_1^2}{n_s}}$

This amplitude factor has to be compared with the amplitude factor of the bare substrate $r_b = \dfrac{1 - n_s}{1 + n_s}$; Since $n_1 > n_s \Rightarrow n_1^2 > n_s^2 \Rightarrow \dfrac{n_1^2}{n_s} > n_s$



we get $|r_c| > |r_b|$. The reflectance is increased.

Q2 a)



painting

Take the pupil size to be $D_p \sim 5\,\text{mm}$. The angular radius of the Airy disk is $\sin\theta_a = 1.22\dfrac{\lambda}{D_p}$. From the picture, if $\theta < \theta_a$ then Airy disks of different colors overlap on the retina and color mixing occurs. $\tan\theta \simeq \dfrac{1\,\text{mm}}{L}$; also $\sin\theta_a \simeq 1.22\,\dfrac{480\,\text{nm}}{5\,\text{mm}}$ for blue lights

$\Rightarrow \sin\theta_a \simeq 1.2\times10^{-4} \Rightarrow \theta_a \simeq 1.2\times10^{-4}\,\text{rad}$. since $\theta < \theta_a \Rightarrow \tan\theta \simeq \sin\theta$

and $\sin\theta \simeq \dfrac{1\,\text{mm}}{L} < \sin\theta_a \simeq 1.2\times10^{-4} \Rightarrow L \simeq 8.5\,\text{m}$

Q2 b) The angular resolution of the Hubble telescope is

$$\sin\theta_a = 1.22\,\frac{550\,\text{nm}}{2.4\,\text{m}} \simeq 2.8\times10^{-7} \Rightarrow \theta \simeq 2.8\times10^{-7}\,\text{rad}$$

$$\theta_a = 2.8\times10^{-7}\,\frac{180^\circ}{\pi} \simeq 1.6\times10^{-5}\ \text{degrees of arc} \simeq 1.6\times10^{-5}\times3600\ \text{sec of arc}$$
$$= 0.058\ \text{sec of arc.}$$

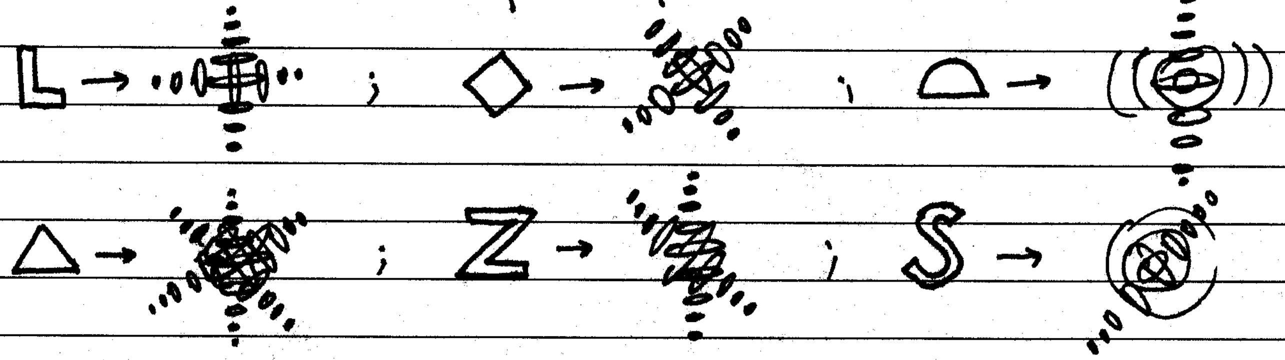
The condition of resolving two objects on the Moon's surface is

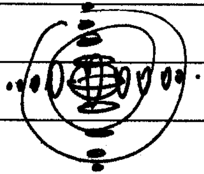
$$\frac{d/2}{L} = \tan\theta \simeq \sin\theta > \sin\theta_a \simeq 2.8\times10^{-7} \Rightarrow \underline{d > 215\,\text{m}}\ \text{for}$$

an Earth-Moon distance of $L \simeq 3.844\times10^{8}\,\text{m}$, and d is the objects' separation.

On the other hand, to resolve two objects on the Moon's surface by bare eye, they should be $d > 2L\sin\theta_a = \underline{92\,\text{km}}$ appart.

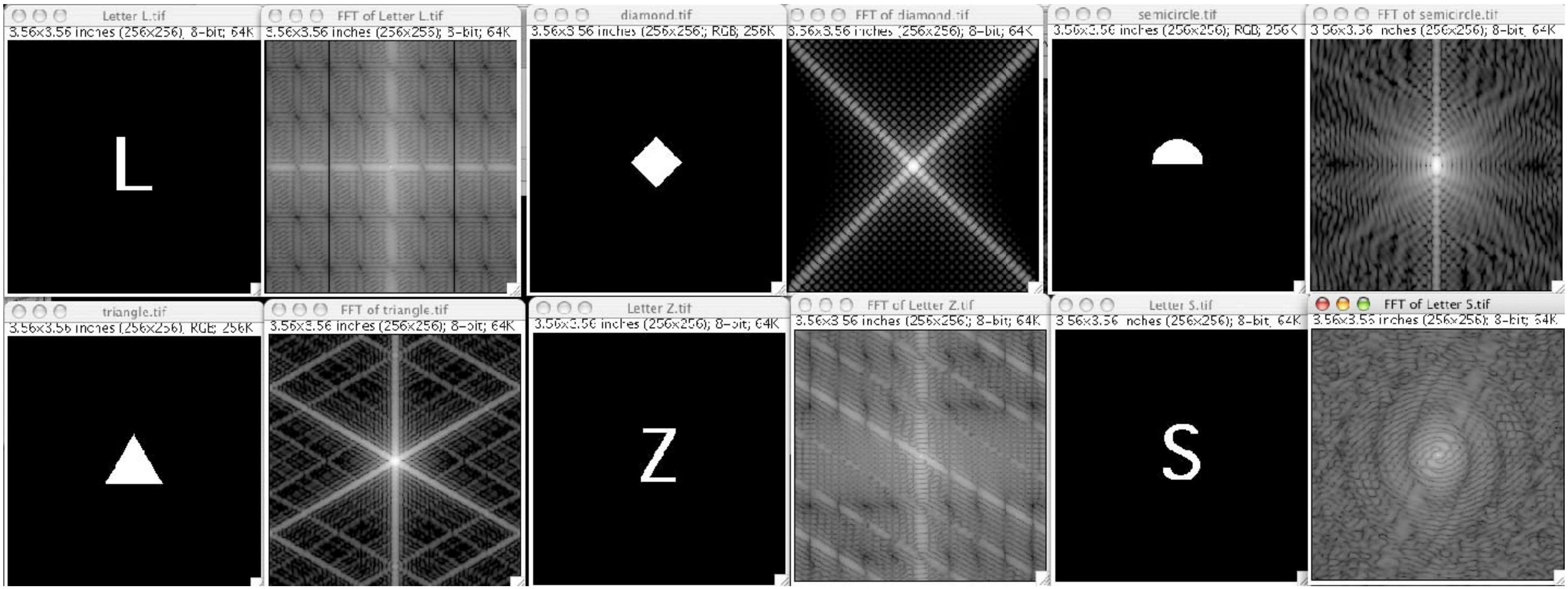
Q2 c) Whenever straight edges are present, diffraction lines perpendicular to them will show on the diffraction pattern:



5 → 

<u>Note</u>: Exact diffraction patterns corresponding to the above shapes can be drawn using the computer. You can use a free software: ImageJ (a Java image processor) available for Mac, PCs, Linux, etc. Please visit: http://rsb.info.nih.gov/ij/

These figures are Fast Fourier Transforms (FFTs; an optimized algorithm that approximates Fourier Transforms), so effectively what they do is fold back (or reflect) the images at the walls of the box. So for instance, the hand-drawn figure for the diamond is better and clearer. (This note is from Prof. R. Marjoribanks)



Letter L.tif
3.56x3.56 inches (256x256); 8-bit; 64K
FFT of Letter L.tif
3.56x3.56 inches (256x256); 8-bit; 64K
diamond.tif
3.56x3.56 inches (256x256); RGB; 256K
FFT of diamond.tif
3.56x3.56 inches (256x256); 8-bit; 64K
semicircle.tif
3.56x3.56 inches (256x256); RGB; 256K
FFT of semicircle.tif
3.56x3.56 inches (256x256); 8-bit; 64K
triangle.tif
3.56x3.56 inches (256x256); RGB; 256K
FFT of triangle.tif
3.56x3.56 inches (256x256); 8-bit; 64K
Letter Z.tif
3.56x3.56 inches (256x256); 8-bit; 64K
FFT of Letter Z.tif
3.56x3.56 inches (256x256); 8-bit; 64K
Letter S.tif
3.56x3.56 inches (256x256); 8-bit; 64K
FFT of Letter S.tif
3.56x3.56 inches (256x256); 8-bit; 64K

Q5

a) For stability, find parameters g_1, g_2:

$$g_1 = 1 - \frac{L}{R_1} = 1 - \frac{1}{\infty} = 1$$

$$g_2 = 1 - \frac{L}{R_2} = 1 - \frac{1}{3} = 0.67$$

The product $g_1 g_2 = 0.67$ satisfies stability for the cavity $0 < g_1 g_2 < 1$.

The Gaussian beam matches the mirror curvature at either end, with formula:

$$R(z) = z + \frac{z_0^2}{z}$$

R=∞ at flat mirror, z=0 there; R(1m)=3m, so

$$z_0(oscillator) = \sqrt{3-1} = \sqrt{2}$$

Then the beamwaist on the flat mirror is:

$$w_0 = \sqrt{\frac{z_0 \lambda}{\pi}} = 0.67\,\mathrm{mm}$$

where we've chosen an example wavelength $\lambda = 1$ μm. The far-field divergence angle is:

$$\theta = \frac{\lambda}{\pi w_0} = \frac{w_0}{z_0} = 0.47\,\mathrm{mrad}$$

b) The second cavity has these parameters:

$$5\,\mathrm{m} = R(1.25\,\mathrm{m}) = 1.25\,\mathrm{m} + \frac{z_0^2}{1.25\,\mathrm{m}}$$

$$\Rightarrow z_0 = 2.165\,\mathrm{m}, \quad w_0 = 0.83\,\mathrm{mm}$$

The two flat mirrors with the output of the oscillator and input to the regen are facing each other 4 m apart. We need to find where to put a lens and with what focal length so that we can relay the flat-wavefront beamwaist from the output flat mirror to the *right diameter* flat-wavefront beamwaist at the input to the regen.

From that point, this can be done the hard way or the not-so-hard way: the hard way is to write the general ray-matrix for an unknown lens an unknown distance x along the 4 m separation:

$$M = \begin{bmatrix} 1 & (4-x) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix}$$

and then use the new $ABCD$ matrix in a fractional-linear transformation

$$q_{regen} = T(q_{osc}) = \frac{Aq_{osc} + B}{Cq_{osc} + D}$$

to map the beam-parameter q_{osc} at the first flat mirror onto the required parameter q_{regen} on the second flat mirror, using:

$$q = \frac{i\lambda}{\pi w_0^2} = -iz_0$$

Since both these q's are imaginary-valued, the real part of $T(q_{\text{osc}})$ must vanish and you can get formulae for x and f.

However, it's always easier to split the problem up in a *physical* sense, if it will make two decoupled equations.

Here, we do that by noting that when crossing the thin lens the beam does not change diameter. So the position of the lens must be where the beam of the oscillator is the same size w as the beam of the regen, working backwards from the flat input mirror. That is, for each cavity separately we use:

$$w^2(z) = w_0^2\left(1 + \frac{z}{z_0^2}\right)$$

to get:

$$w_0^2\left(1 + \frac{z^2}{z_0^2}\right) = w^2(z) = w^2(-4+z) = w_0^2\left(1 + \frac{(-4+z)^2}{z_0^2}\right)$$

where blue is for the oscillator parameters, red for the regen. This can be rewritten as the quadratic equation:

$$0 = \left(\frac{w_0^2}{z_0^2} - \frac{w_0^2}{z_0^2}\right)z^2 - 8\frac{w_0^2}{z_0^2}z + \left(w_0^2 - w_0^2 + 16\frac{w_0^2}{z_0^2}\right)$$

which for our parameters becomes

$$0 = 7.8\times 10^{-8} z^2 + 1.17\times 10^{-6} z - 2.59\times 10^{-6}$$

and the lens goes at z=1.95 m, measured from the output flat mirror of the oscillator.

The lens must then change the curvature of the beam from positive (expanding) to negative (converging) to go to the second beamwaist. We find the curvatures at this location:

$$R(1.95\,\text{m}) = z + \frac{z_0^2}{z} = 1.95 + \frac{\left(\sqrt{2}\right)^2}{1.95} = 2.98\,\text{m}$$

$$R(-4\,\text{m} + 1.95\,\text{m}) = (-4+1.95) + \frac{2.17^2}{(-4+1.95)} = -4.34\,\text{m}$$

The relation for the focal length is then

$$\frac{1}{R_2} = \frac{1}{R_1} - \frac{1}{f}$$

which gives a focal length of 1.77 m.

c) The oscilloscope will show the beat-frequency between the two transverse modes of the regen if the detector sees just a spot in the beam (and doesn't average the whole thing). The frequencies of different transverse modes are given by:

$$\nu_{qmn} = \frac{c}{2L}\left(q + \frac{1}{\pi}(m+n+1)\cos^{-1}\sqrt{g_1 g_2} \right)$$

For the regen, $g_2 = 1\text{–}1.25/5\text{=}0.75$, and $\cos^{-1}\sqrt{g_1 g_2}$ =0.72. The frequency difference of transverse modes is smaller than the difference between longitudinal modes, and the net difference between TEM_{00} and TEM_{10} modes is

$$\Delta\nu = \nu_{q10} - \nu_{q00} = \frac{c}{2L}\left(\frac{1}{\pi}\cos^{-1}\sqrt{g_1 g_2} \right) = 27.5\,\mathrm{MHz}$$

This shows up as a modulation in the photodiode signal with a period around 36ns, which is about 4-5 roundtrips.