# Charting a path between semiclassical islands

## (can we relate semiclassical confinement mechanisms?)


Erich Poppitz (Toronto)

*based on works, over the years, with*

Mohamed Anber (Durham)

Andrew Cox and F. David Wandler (Toronto)


*- title borrowed from latest 2211.10347 w/ Wandler*

confinement on $R^4$ is a difficult strong-coupling problem

lattice and large-$N$ offer insight ...but, this talk:

## semiclassics and weak coupling

Polyakov: 1970s monopole-instantons on $R^3$ in $SU(2) \to U(1)$

Ünsal, w/ Yaffe, w/ Shifman...: 2010s (twisted) monopole-instantons on $R^3 \times S^1$ in $SU(N) \to U(1)^{N-1}$ 'dYM', bions QCD(adj)... - or any G SYM!

Tanizaki & Ünsal: 2022 ("wrapped") center vortices on $R^2 \times T^2$ in $SU(N) \to Z_N$ (Greensite et al 1990s,..., González-Arroyo, Pérez et al, 2000s)

weak coupling by separation of scales $vev \gg \Lambda,\ N\Lambda L_{S^1 \ or \ T^2} \ll 1$

not the real world, so should we ignore?

# semiclassics and weak coupling

not the real world, so should we ignore? - NO!

# semiclassics and weak coupling

not the real world, so should we ignore? - NO!

setups offer unique opportunity to analytically study locally-4d nonperturbative physics

*(hope… use "resurgent transseries" to resume series?)*

# semiclassics and weak coupling

not the real world, so should we ignore? - NO!

setups offer unique opportunity to analytically study locally-4d nonperturbative physics

in each case, evidence for continuous connection to expected or known physics on $R^4$

# semiclassics and weak coupling

not the real world, so should we ignore? - NO!

setups offer unique opportunity to analytically study locally-4d nonperturbative physics

in each case, evidence for continuous connection to expected or known physics on $R^4$

studies highlight the importance of objects with fractional topological charges

# semiclassics and weak coupling

not the real world, so should we ignore? - NO!

setups offer unique opportunity to analytically study locally-4d nonperturbative physics

in each case, evidence for continuous connection to expected or known physics on $R^4$

studies highlight the importance of objects with fractional topological charges

use of twisted boundary conditions and relation to generalized anomalies

# semiclassics and weak coupling

not the real world, so should we ignore? - NO!

setups offer unique opportunity to analytically study locally-4d nonperturbative physics

in each case, evidence for continuous connection to expected or known physics on $R^4$

studies highlight the importance of objects with fractional topological charges

use of twisted boundary conditions and relation to generalized anomalies

TWISTED BOUNDARY CONDITIONS:
A NON-PERTURBATIVE PROBE
FOR PURE
NON-ABELIAN GAUGE THEORIES

Pierre van Baal

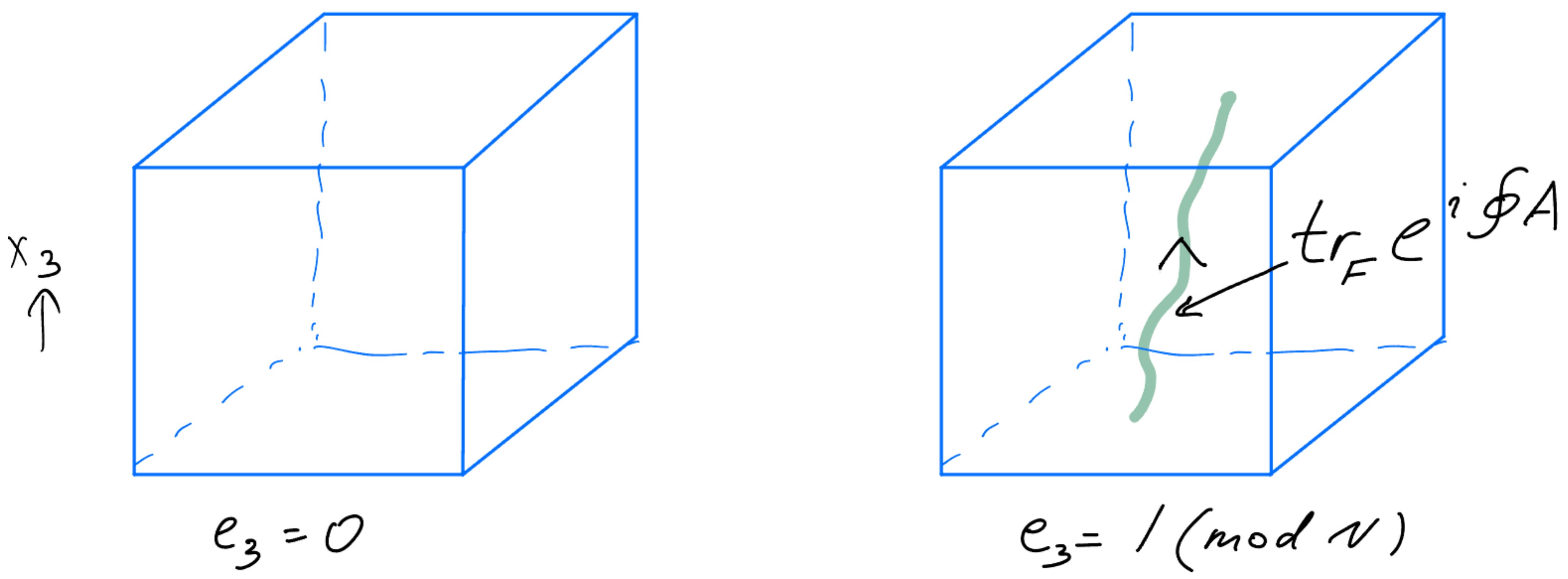
op woensdag 4 juli 1984
des namiddags te 4.15 uur

PROMOTOR: PROF. DR. G. 'T HOOFT

PhD thesis *resurging*, or rising from the ashes…

# now, to this talk… background:

In 2021, w/ Cox & Wandler: 1-form center/0-form anomaly (YM, SYM…) in Hamiltonian on $T^3$ of any size. Anomaly implies exact degeneracies:




$SU(N=2)$ **for simplicity**

$e_3 \in \{0,1\}$ **= eigenvalue of** $\hat{T}_3$

$\hat{T}_3$ **- generator of center symmetry along** $\vec{m} = (0,0,1)$

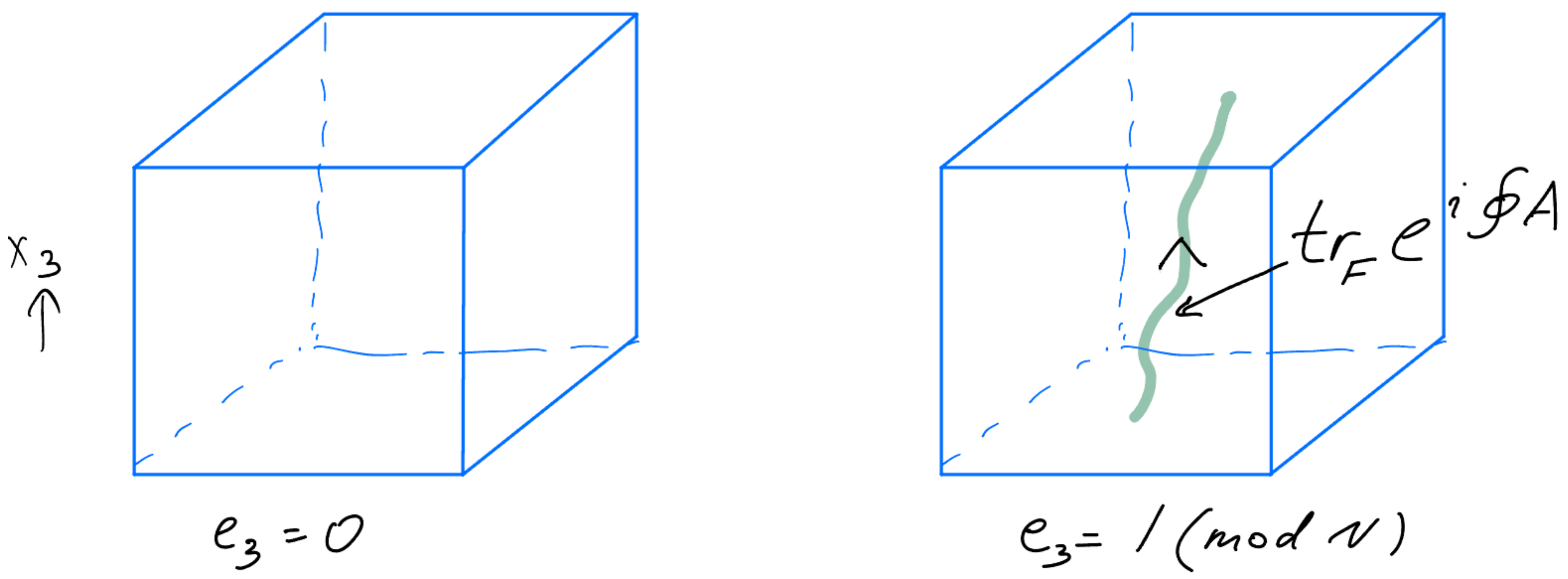
$m_3 = 1$ Hilbert space, with spatial 't Hooft twist $n_{12} = 1$

$|E, e_3 = 1\rangle_{m_3=1}$ **degenerate w/** $|E, e_3 = 0\rangle_{m_3=1}$ **for all $E$, any size $T^3$, at $\theta = \pi$**

( **due to anomaly:** $\hat{T}_3\,\hat{P} = (-)^{m_3}\,\hat{P}\,\hat{T}_3^{-1} \quad [\hat{P}, \hat{H}_{\theta=\pi}] = [\hat{T}_3, \hat{H}_{\theta=\pi}] = 0$ )

# now, to this talk… background:

In 2021, w/ Cox & Wandler: 1-form center/0-form anomaly (YM, SYM…) in Hamiltonian on $T^3$ of any size. Anomaly implies exact degeneracies:




dynamical check via semiclassics

$T^3_{small}$ "femtouniverse" (van Baal 1984)

$S^1_{size\to\infty} \times T^2_{small}$ (Tanizaki, Ünsal 2022)

$m_3 = 1$ Hilbert space, with spatial 't Hooft twist $n_{12} = 1$

two states only have $E < L^{-1}$ (L=small size)

$$A^{(0)} = 0 \to |0\rangle$$

$$A^{(1)} = iT_3 dT_3^{-1} \to |1\rangle = \hat{T}_3|0\rangle$$

$\xrightarrow[\hat{T}_3 \text{ center}]{\text{eigenstates of}}$

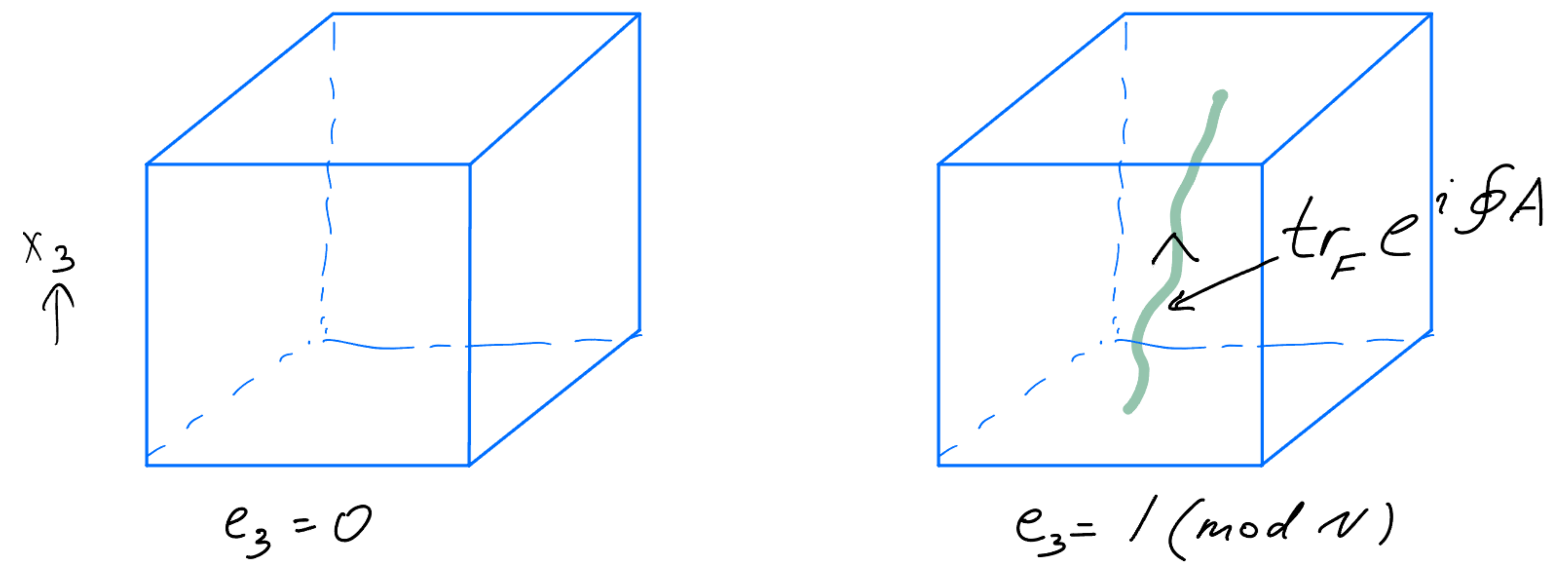
$$|E_{cl} = 0, e_3 = 0\rangle = |0\rangle + |1\rangle$$

$$|E_{cl} = 0, e_3 = 1\rangle = |0\rangle - |1\rangle$$

(Witten 1982, $tr(-1)^F$, $E_{cl} = 0$)

# now, to this talk… background:

In 2021, w/ Cox & Wandler: 1-form center/0-form anomaly (YM, SYM…) in Hamiltonian on $T^3$ of any size. Anomaly implies exact degeneracies:




dynamical check via semiclassics

$T^3_{small}$ "femtouniverse" (van Baal 1984)

$S^1_{size\to\infty} \times T^2_{small}$ (Tanizaki, Ünsal 2022)

$m_3 = 1$ Hilbert space, with spatial 't Hooft twist $n_{12} = 1$

two states only have $E < L^{-1}$ (L=small size)

$$A^{(0)} = 0 \rightarrow |0\rangle$$

$$A^{(1)} = iT_3 dT_3^{-1} \rightarrow |1\rangle = \hat{T}_3 |0\rangle$$

(Witten 1982, $tr(-1)^F$, $E_{cl} = 0$)

$\xrightarrow[\hat{T}_3 \text{ center}]{\text{eigenstates of}}$

Q=1/2 instantons

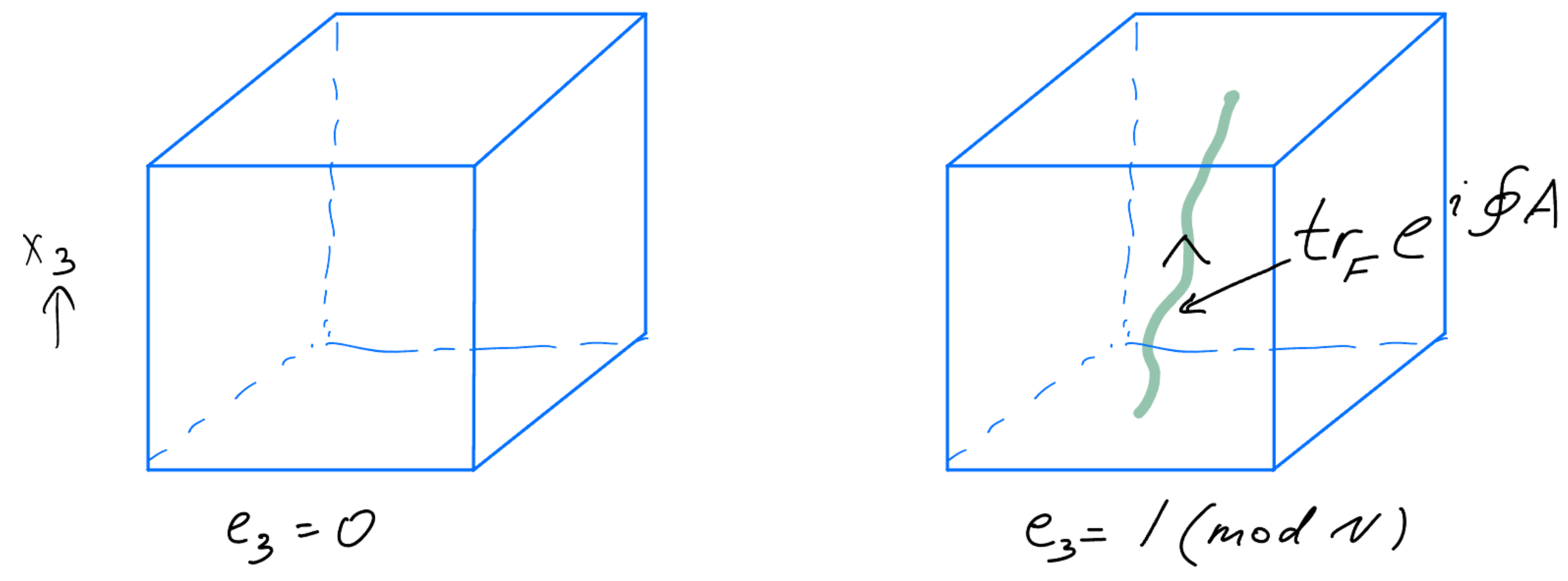
$$|E_{cl} = 0, e_3 = 0\rangle = |0\rangle + |1\rangle$$

$$|E_{cl} = 0, e_3 = 1\rangle = |0\rangle - |1\rangle$$

(van Baal [t Hooft?] 1984, 1999)

# now, to this talk… background:

In 2021, w/ Cox & Wandler: 1-form center/0-form anomaly (YM, SYM…) in Hamiltonian on $T^3$ of any size. Anomaly implies exact degeneracies:




$m_3 = 1$ Hilbert space, with spatial ‘t Hooft twist $n_{12} = 1$

dynamical check via semiclassics

$T^3_{small}$ “femtouniverse” (van Baal 1984)

$S^1_{size\to\infty} \times T^2_{small}$ (Tanizaki, Ünsal 2022)

Q=1/2 instantons

(González-Arroyo, Pérez… 1990s)

$$E(e_3) - E_{pert.} = -\, c\, \frac{e^{-\frac{8\pi^2}{2g^2}}}{L} \cos\left(\pi e_3 - \frac{\theta}{2}\right)\Bigg|^{e_3\in\{0,1\}\ degenerate\ at\ \theta=\pi}_{only\ valid\ in\ m_3=1\ background}$$

(van Baal [t Hooft?] 1984, 1999)

# now, to this talk… background:

In 2021, w/ Cox & Wandler: 1-form center/0-form anomaly (YM, SYM…) in Hamiltonian on $T^3$ of any size. Anomaly implies exact degeneracies:

deformed YM (Ünsal-Yaffe 2008)

$R^3 \times S^1_{small}$

$$\rho(k) - \rho_{pert.} = -c' \frac{e^{-\frac{8\pi^2}{2g^2}}}{L^4} \cos\left(\pi k - \frac{\theta}{2}\right)\Bigg|_{no\ m_3\ to\ speak\ of!}^{k=0,1}$$

monopole-instantons Q=1/2

(Kraan, van Baal; Yi, Lee 1990s)

$$E(e_3) - E_{pert.} = -c \frac{e^{-\frac{8\pi^2}{2g^2}}}{L} \cos\left(\pi e_3 - \frac{\theta}{2}\right)\Bigg|_{only\ in\ m_3=1\ background}^{e_3\in\{0,1\}}$$

Q=1/2 instantons

(González-Arroyo, Pérez… 1990s)

$T^3_{small}$ "femtouniverse"

$S^1_{size\to\infty} \times T^2_{small}$

## Similarity suggests that these semiclassical limits are related:

## can we relate semiclassical confinement mechanisms?

answer: "not yet"… **will simply share some observations (+ wishlist, confusions)**

deformed YM (Ünsal-Yaffe 2008)

$R^3 \times S^1_{small}$

$$\rho(k) - \rho_{pert.} = -c' \frac{e^{-\frac{8\pi^2}{2g^2}}}{L^4} \cos\left(\pi k - \frac{\theta}{2}\right)\Bigg|^{k=0,1}_{no\ m_3\ to\ speak\ of!}$$

monopole-instantons Q=1/2

(Kraan, van Baal; Yi, Lee 1990s)

$$E(e_3) - E_{pert.} = -c \frac{e^{-\frac{8\pi^2}{2g^2}}}{L} \cos\left(\pi e_3 - \frac{\theta}{2}\right)\Bigg|^{e_3 \in \{0,1\}}_{only\ in\ m_3=1\ background}$$

Q=1/2 instantons

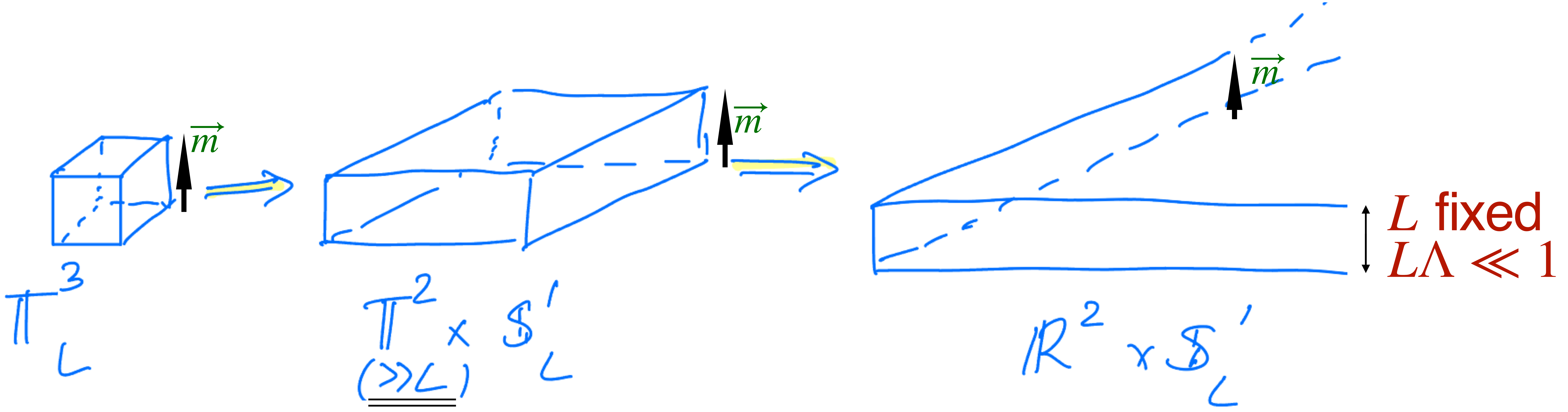
(González-Arroyo, Pérez… 1990s)

$T^3_{small}$ "femtouniverse"

$S^1_{size \to \infty} \times T^2_{small}$

## observations (+ wishlist, confusions)

femtouniverse … dYM w/flux … dYM



## observations (+ wishlist, confusions)

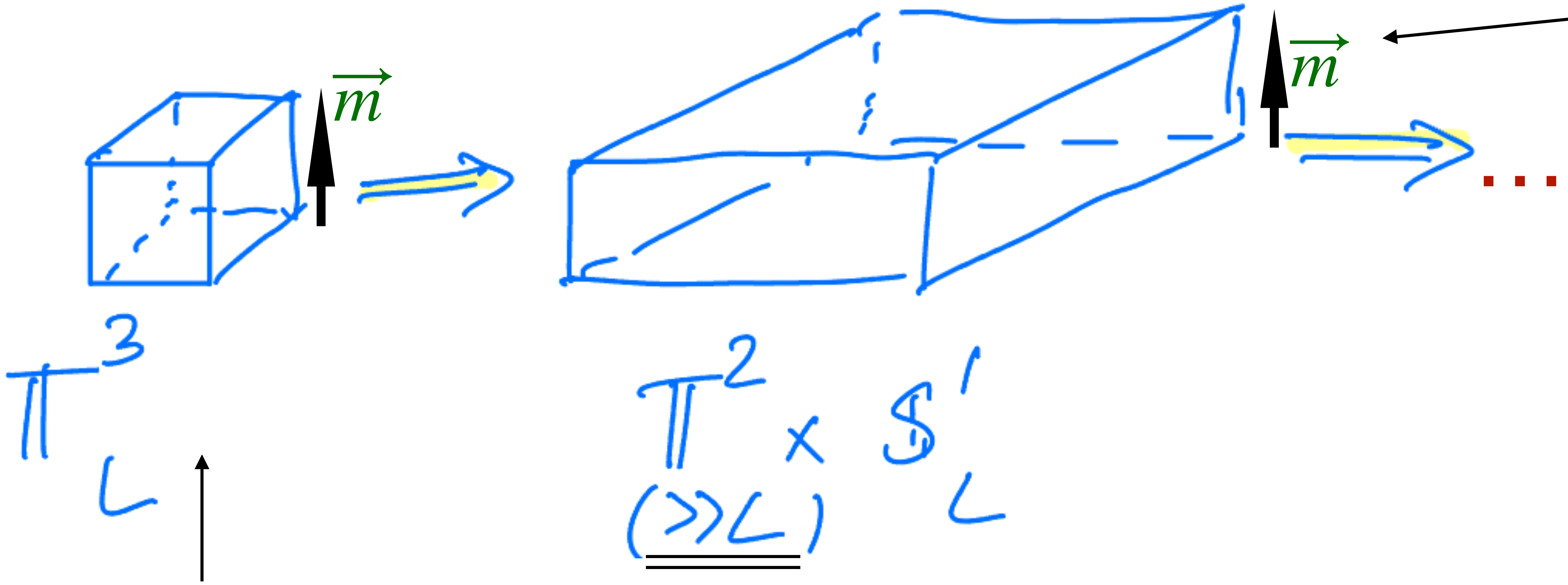
femtouniverse … dYM w/flux … dYM

(G.-Arroyo, Pérez, Okawa 2013; Ünsal 2020)

$$A_\pm = \frac{\sigma^3}{2}\left(-\frac{2\pi x^2}{L_1 L_2}dx^1 \pm \frac{\pi}{L}dx^3\right)$$

obeys twist b.c. + minimize V_dYM

+ correct $L_1 L_2 \to \infty$ limit




$$A^{(0)} = 0 \quad \to |0\rangle$$

$$A^{(1)} = iT_3 dT_3^{-1} \to |1\rangle = \hat{T}_3|0\rangle$$

$$trW_3 = \pm 2$$

## observations (+ wishlist, confusions)

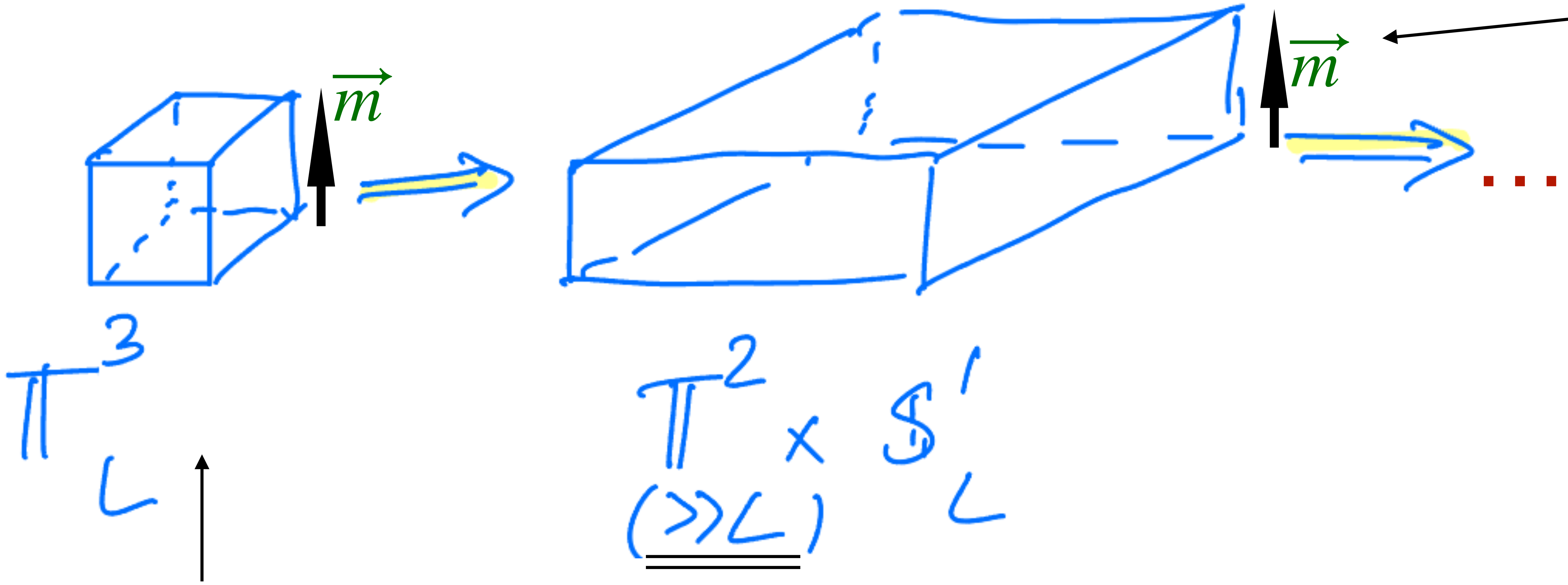
# femtouniverse … dYM w/flux … dYM

(G.-Arroyo, Perez, Okawa 2013; Ünsal 2020)

$$A_{\pm} = \frac{\sigma^3}{2}\left(-\frac{2\pi x^2}{L_1 L_2}dx^1 \pm \frac{\pi}{L}dx^3\right)$$

obeys twist b.c. + minimize V_dYM

+ correct $L_1 L_2 \to \infty$ limit




$$trW_3 = 0 \ \underline{\text{but}} \ \ trF_{12}W_3 \sim \pm\frac{1}{L_1 L_2}$$

$$A^{(0)} = 0 \qquad \to |0\rangle$$

$$A^{(1)} = iT_3 dT_3^{-1} \to |1\rangle = \hat{T}_3|0\rangle$$

$$trW_3 = \pm 2$$

$$A^{(0)} = A_+ \to |0\rangle$$

$$A^{(1)} = A\_ \to |1\rangle = \hat{T}_3|0\rangle$$

## observations (+ wishlist, confusions)

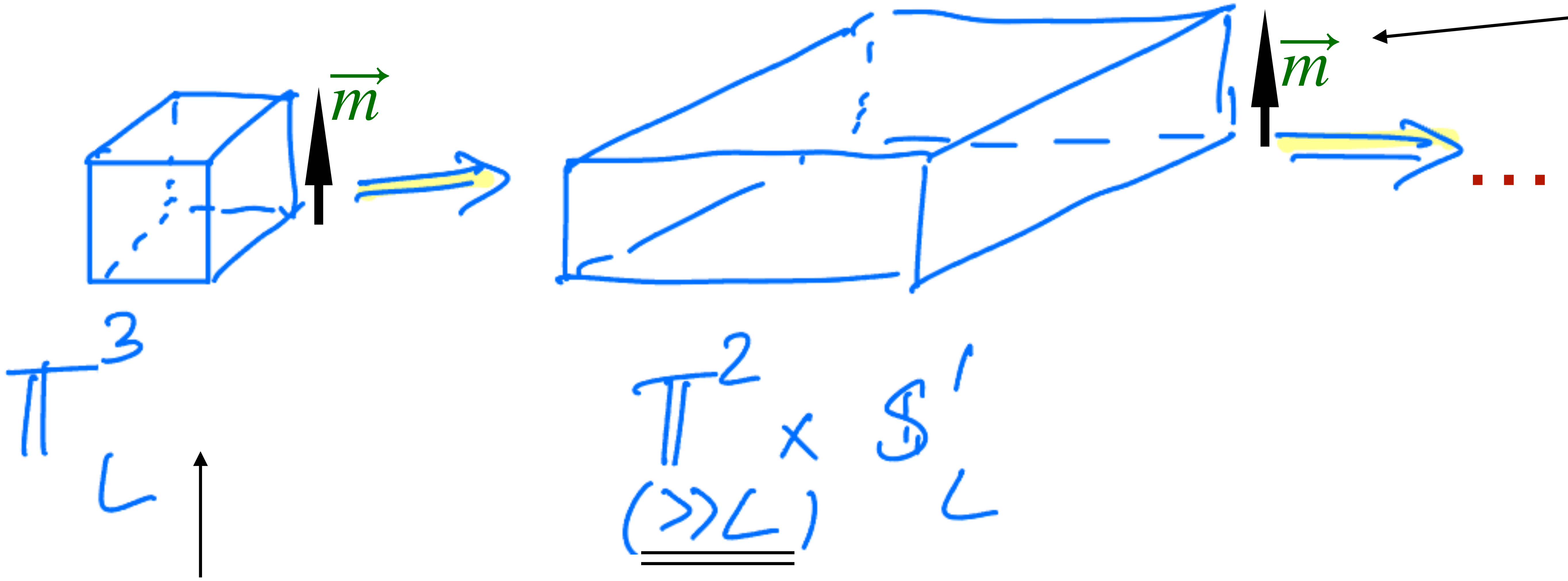
femtouniverse … dYM w/flux … dYM

(G.-Arroyo, Pérez, Okawa 2013; Ünsal 2020)

$$A_\pm = \frac{\sigma^3}{2}\left(-\frac{2\pi x^2}{L_1 L_2}dx^1 \pm \frac{\pi}{L}dx^3\right)$$

obeys twist b.c. + minimize V_dYM

+ correct $L_1 L_2 \to \infty$ limit




$$A^{(0)} = 0 \quad \to |0\rangle$$

$$A^{(1)} = iT_3 dT_3^{-1} \to |1\rangle = \hat{T}_3 |0\rangle$$

$$trW_3 = \pm 2$$

$trW_3 = 0$ but $trF_{12}W_3 \sim \pm \dfrac{1}{L_1 L_2}$

$$A^{(0)} = A_+ \to |0\rangle$$

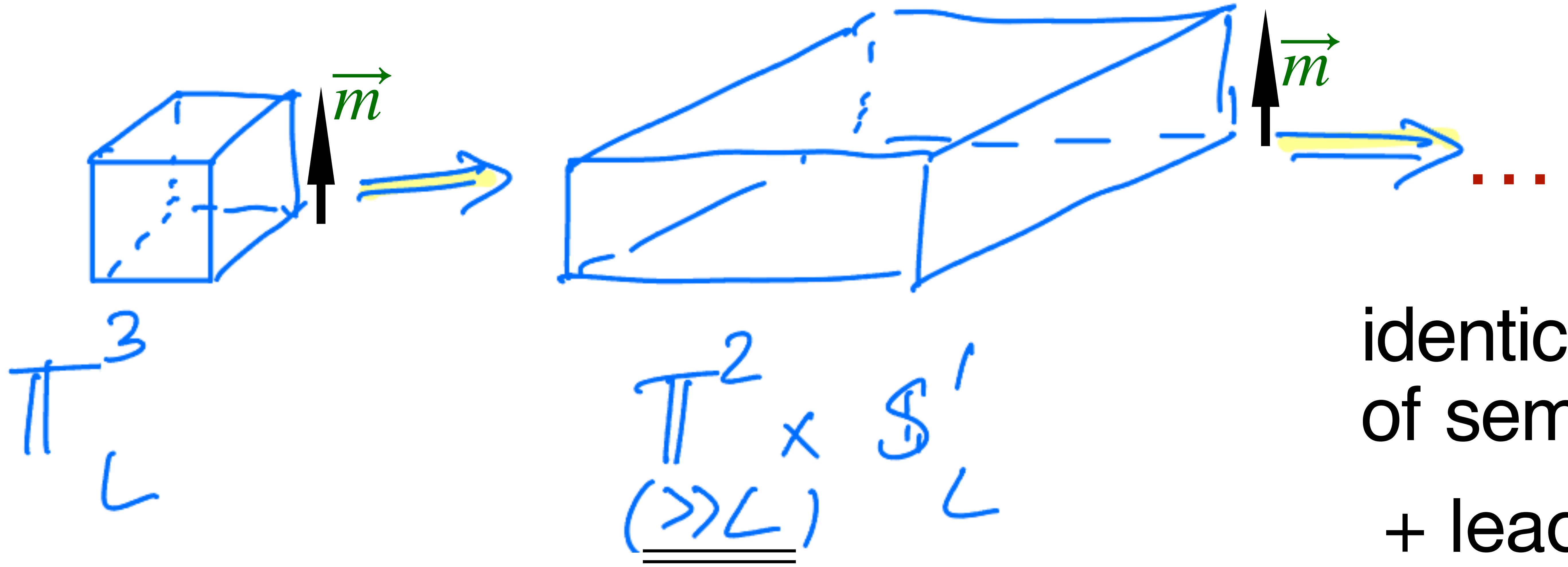
$$A^{(1)} = A_- \to |1\rangle = \hat{T}_3 |0\rangle$$

( w/ Wandler 2211: different gauges etc…

same at $\theta = \pi$ $\quad \hat{T}_3\, \hat{P} = (-)^{m_3}\, \hat{P}\, \hat{T}_3^{-1}$ )

## observations (+ wishlist, confusions)

femtouniverse … dYM w/flux … dYM




$$\rho(k) - \rho_{pert.} = -\, c' \, \frac{e^{-\frac{8\pi^2}{2g^2}}}{L^4} \cos\left(\pi k - \frac{\theta}{2}\right)\Bigg|^{k=0,1}_{no\ m_3!}$$

$$E(e_3) - E_{pert.} = -\, c \, \frac{e^{-\frac{8\pi^2}{2g^2}}}{L} \cos\left(\pi e_3 - \frac{\theta}{2}\right)\Bigg|^{e_3 \in \{0,1\}}_{only\ in\ m_3=1\ background}$$

identical symmetries
of semiclassical vacua

+ leading order semiclassics w/ $Q = 1/2$

$$trW_3 = \pm 2$$

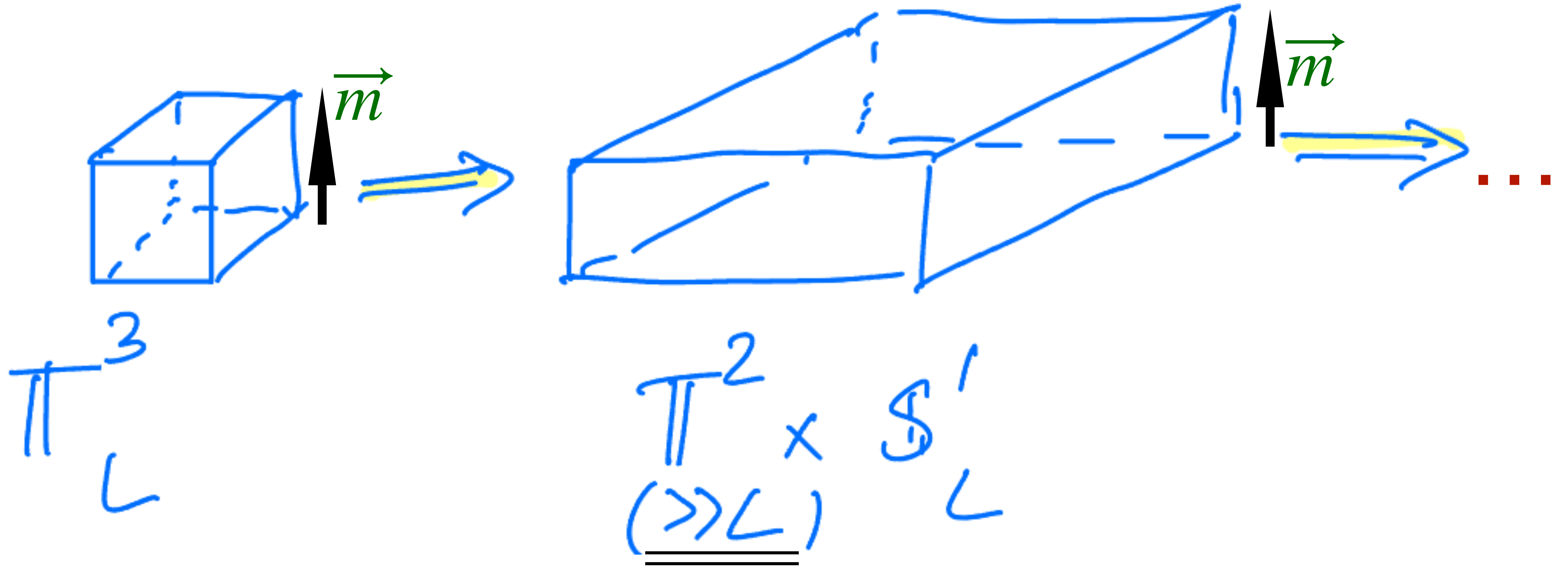
$$|1\rangle = \hat{T}_3 |0\rangle$$

Q=1/2 instantons

$$trF_{12}W_3 \sim \pm \frac{1}{L_1 L_2}$$

## observations (+ wishlist, confusions)

femtouniverse … dYM w/flux … dYM




Q=1/2 monopole-instantons
on $R^3 \times S^1_L$ ("Coulomb branch")

*relate…?*

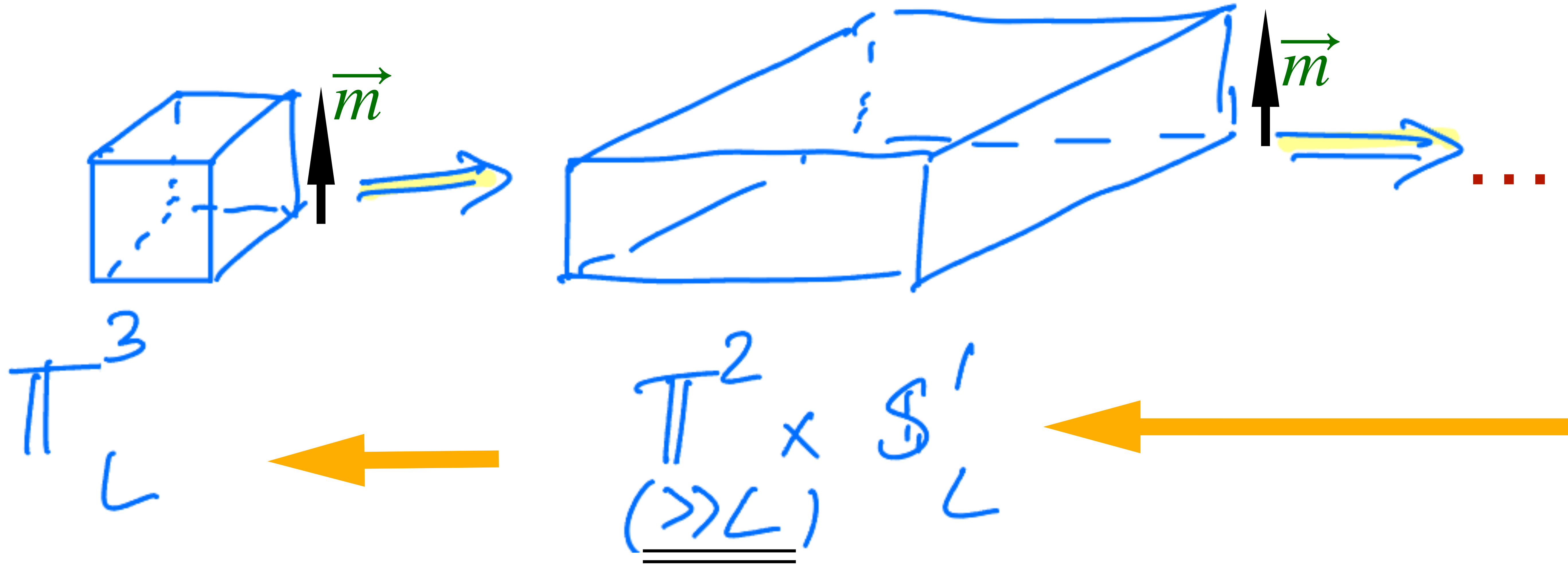
Q=1/2 instantons
on $T^2_{large} \times S^1_L \times R$

$$trW_3 = \pm 2 \qquad |1\rangle = \hat{T}_3 |0\rangle \qquad trF_{12}W_3 \sim \pm \frac{1}{L_1 L_2}$$

Q=1/2 instantons
on $T^3 \times R$ (no analytic!)

## observations (+ wishlist, confusions)

femtouniverse … dYM w/flux … dYM




confinement by
Q=1/2 monopole-instantons
on $R^3 \times S^1_L$ ("Coulomb branch")

Q=1/2 instantons
on $T^2_{large} \times S^1_L \times R$

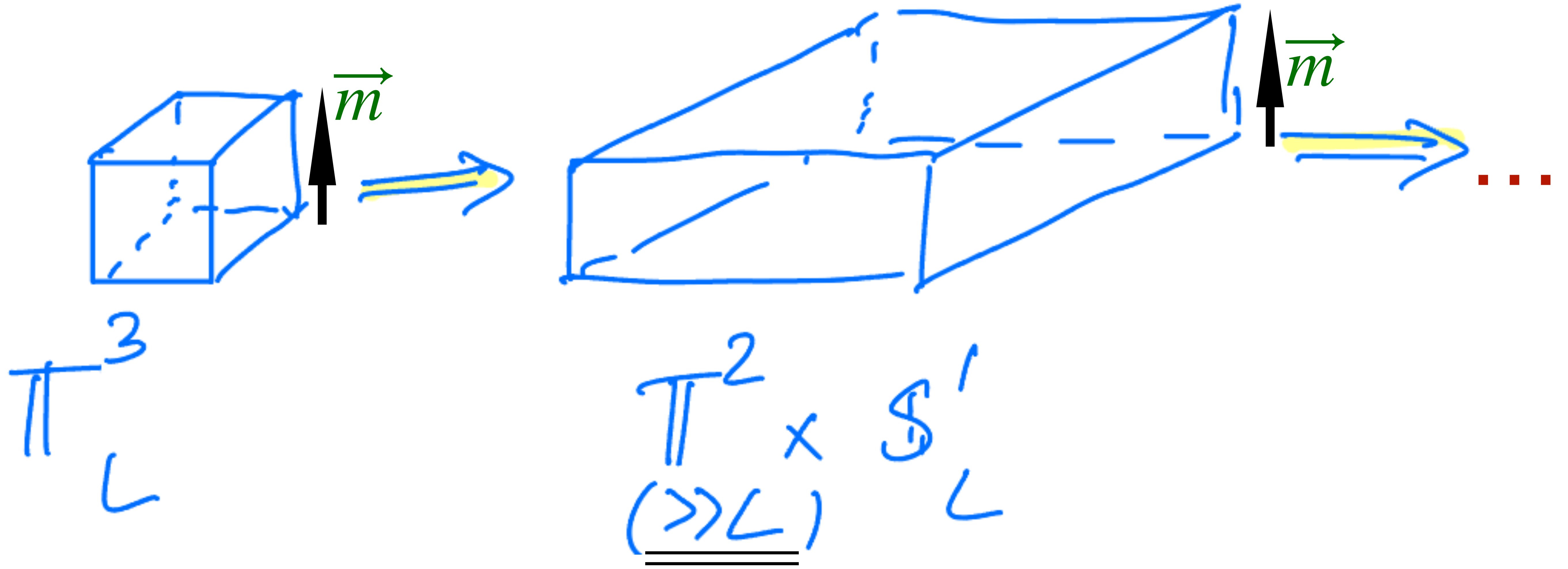
Q=1/2 instantons
on $T^3 \times R$ (no analytic!)

confinement by
center vortices on $S^1_{large} \times T^2 \times R$

*this is the goal… but for now:*

## observations (+ wishlist, confusions)

femtouniverse … dYM w/flux … dYM

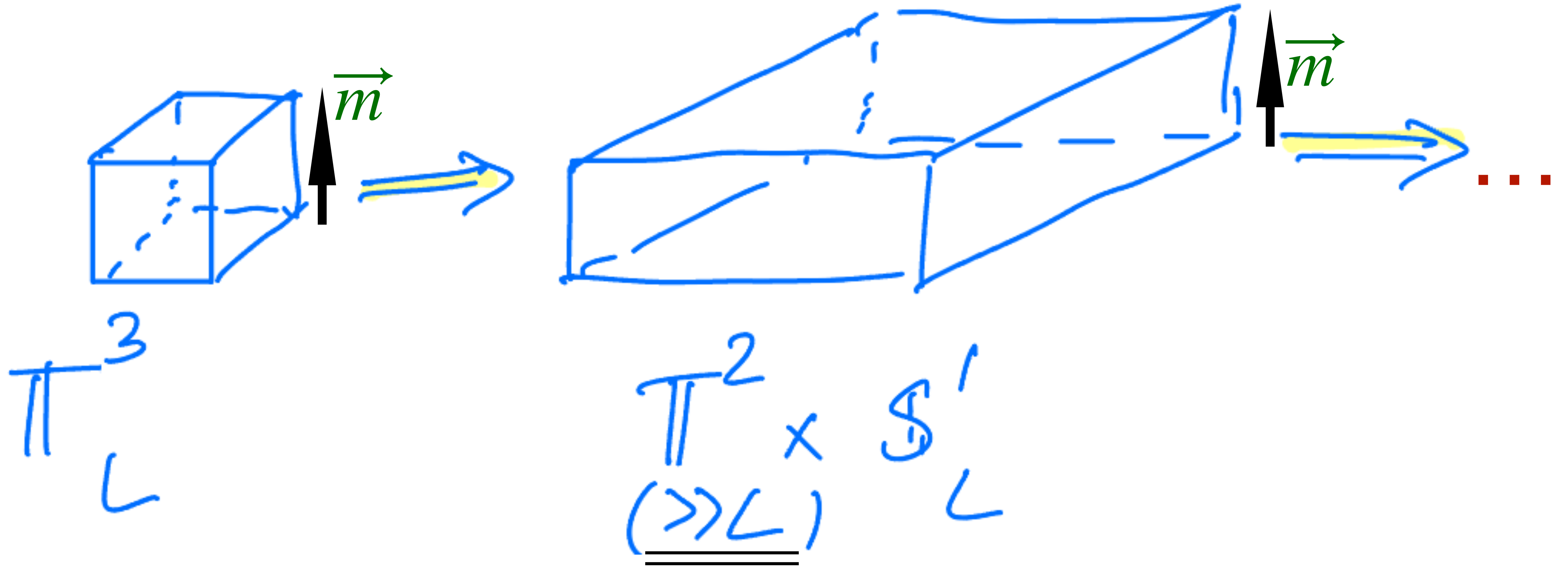



*small 1st step*: study (d)YM vacua as $T^2_{large}$ varies (UV complete)

$$A_\pm = \frac{\sigma^3}{2}\left(-\frac{2\pi x^2}{L_1 L_2}dx^1 \pm \frac{\pi}{L}dx^3\right) \qquad A_\pm = \frac{\sigma^3}{2}\left(-\frac{2\pi x^2}{L_1 L_2}dx^1 \pm \frac{W}{L}dx^3\right)$$

## observations (+ wishlist, confusions)

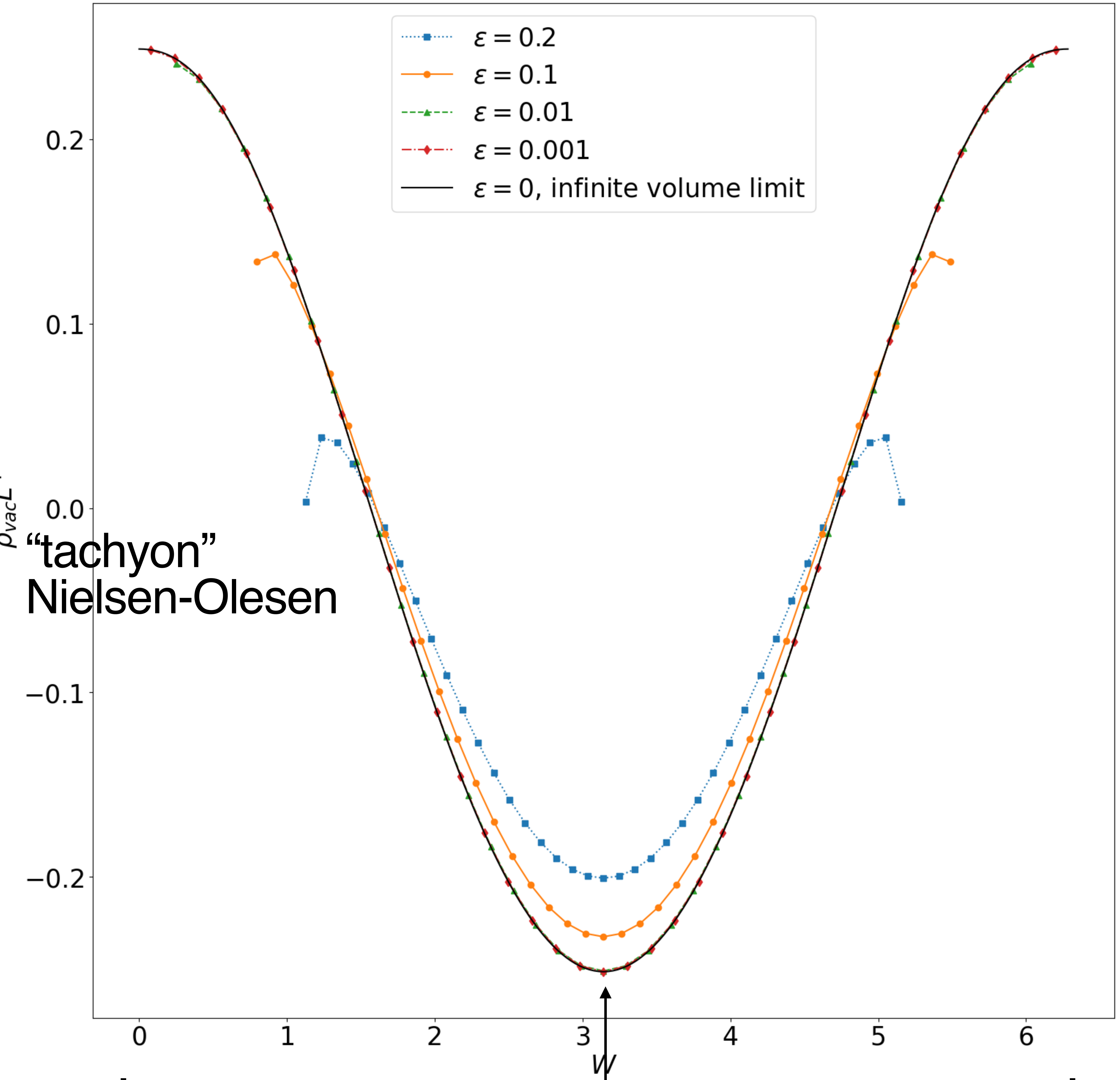
femtouniverse … dYM w/flux … dYM




vary $\varepsilon = \frac{L^2}{L_1 L_2}$ "GPY" potential for modulus W

$$A_\pm = \frac{\sigma^3}{2}\left(-\frac{2\pi x^2}{L_1 L_2}dx^1 \pm \frac{\pi}{L}dx^3\right) \qquad A_\pm = \frac{\sigma^3}{2}\left(-\frac{2\pi x^2}{L_1 L_2}dx^1 \pm \frac{W}{L}dx^3\right)$$

# observations (+ wishlist, confusions)




"tachyon"
Nielsen-Olesen

not-…

semiclassical

not-…

## dYM:

$$\varepsilon = \frac{L^2}{L_1 L_2}$$

"GPY" potential for W, on $T^2_{flux} \times S^1_L$

*from 2211.10347 w/ Wandler*

*skip technicalities… spectrum, etc.*

*(as opposed to GPY, no analytic form for $\varepsilon > 0$)*

$$A_\pm = \frac{\sigma^3}{2}\left(-\frac{2\pi x^2}{L_1 L_2} dx^1 \pm \frac{W}{L} dx^3\right)$$

$W = \pi$ appears (meta)stable up to $L_1 \sim L$

$L_1 \sim L_2 \sim L$ suggests validity of two semiclassical descriptions overlap

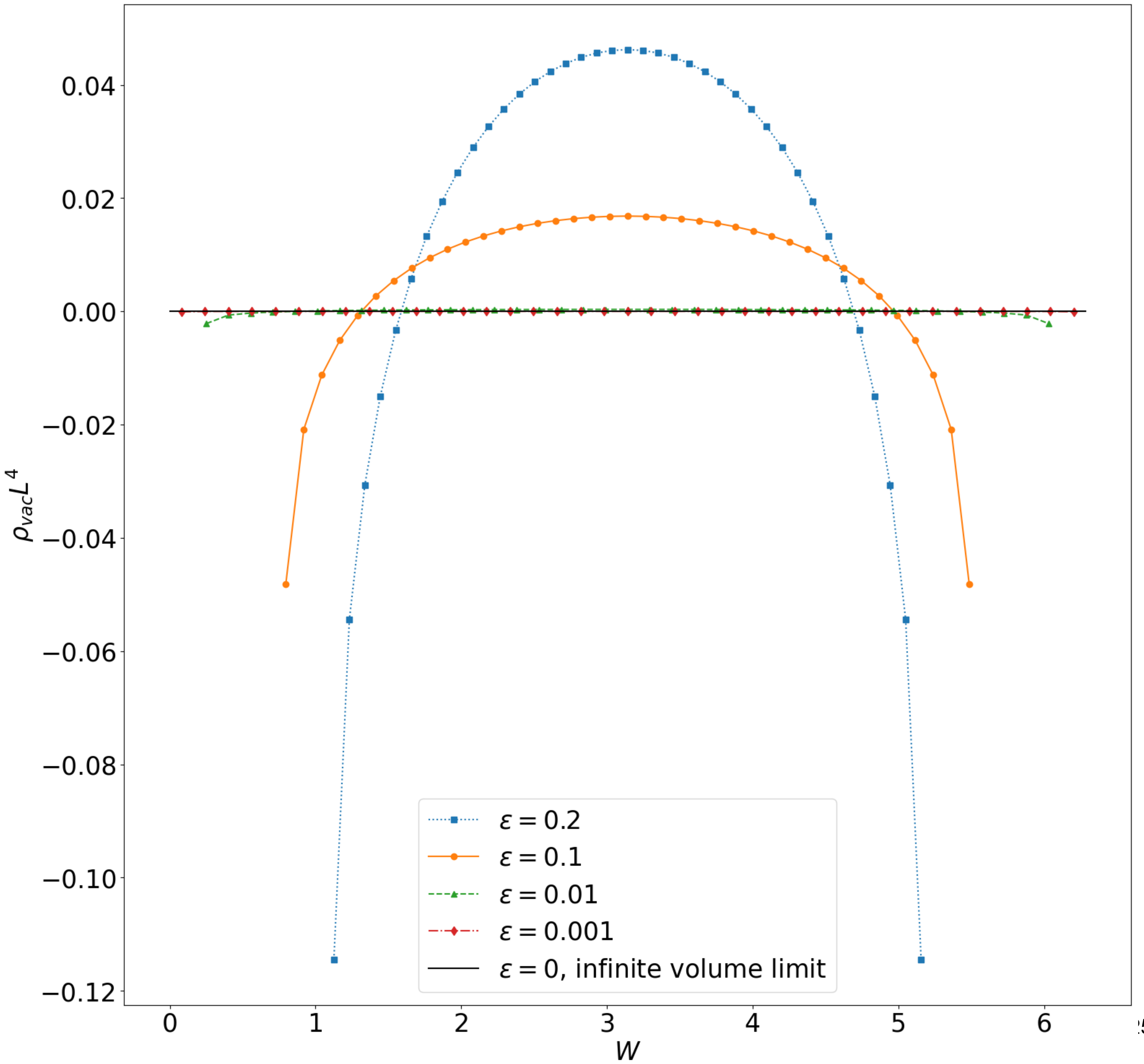
(useful…? future)

# observations (+ wishlist, confusions)

SYM:

$\varepsilon = \frac{L^2}{L_1 L_2}$

*from 2211.10347 w/ Wandler*




not unexpected, non-SUSY background

$$A_\pm = \frac{\sigma^3}{2}\left(-\frac{2\pi x^2}{L_1 L_2} dx^1 \pm \frac{W}{L} dx^3\right)$$

at $T^2_{flux,large} \times S^1_L$ Coulomb branch lifted

Witten's configurations $A = 0$ *and* $A = i T_3 d T_3^{-1}$
- the only classical SUSY states w/ 't Hooft b.c.

semiclassics on
$T^2_{flux,infinite} \times S^1_L = R^2 \times S^1_L$
but not on any finite size $T^2$ … ?

## observations (+ wishlist, confusions)

BEGIN WITH SYM, WHERE I HAVE ONLY QUESTIONS:

SYM offers best-understood semiclassics! = instanton calculus

most detailed semiclassical studies of confining strings, domain walls, and anomalies on $R^3 \times S^1$

e.g. 2011-21 w/ M. Anber, M. Bub, A. Cherman, S. Collier, A. Cox, T. Schäfer, S. Strimas-Mackey, T. Sulejmanpašić, B. Teeple, M. Ünsal, F.D. Wandler, S. Wong

→ Is there a way to "salvage" semiclassics in SUSY at large $T^2_{flux}$, at least?? (SW, anyone?)

SYM for any G on the circle: confinement due to r+1 monopole-instantons with $Q = (\frac{1}{c_2}, \ldots, \frac{1}{c_2}, 1 - \frac{r}{c_2})$.

't Hooft twists in $G \neq SU(N)$ do not produce such charges (instead …1/2,1/3,1/4)

What is the relation, if any !?

More questions re. SYM w/ twists + semiclassics: Anber, EP 2210.13568, see talk at "SUSY-50"!

## observations (+ wishlist, confusions)

ON THE LESS CONFUSING SIDE:

Argued that symmetry realization and Q=1/2 tunneling (1/N for SU(N)) are responsible for the similarities between semiclassical results in femtouniverse/dYM/"Tanizaki-Ünsal" limits.

It should be possible to relate the various self-dual Q=1/N configurations in the regimes where semiclassics valid. Regimes where different semiclassics overlap exist.

There are also earlier numeric studies utilizing various twists and size limits, and likely future ones are needed. (González-Arroyo, Pérez, Montero, van Baal 1999)

Better understand fractional $Q = \frac{m}{N}$ solutions in SU(N) with twists and their dynamical implications.

(work with Anber, in progress).