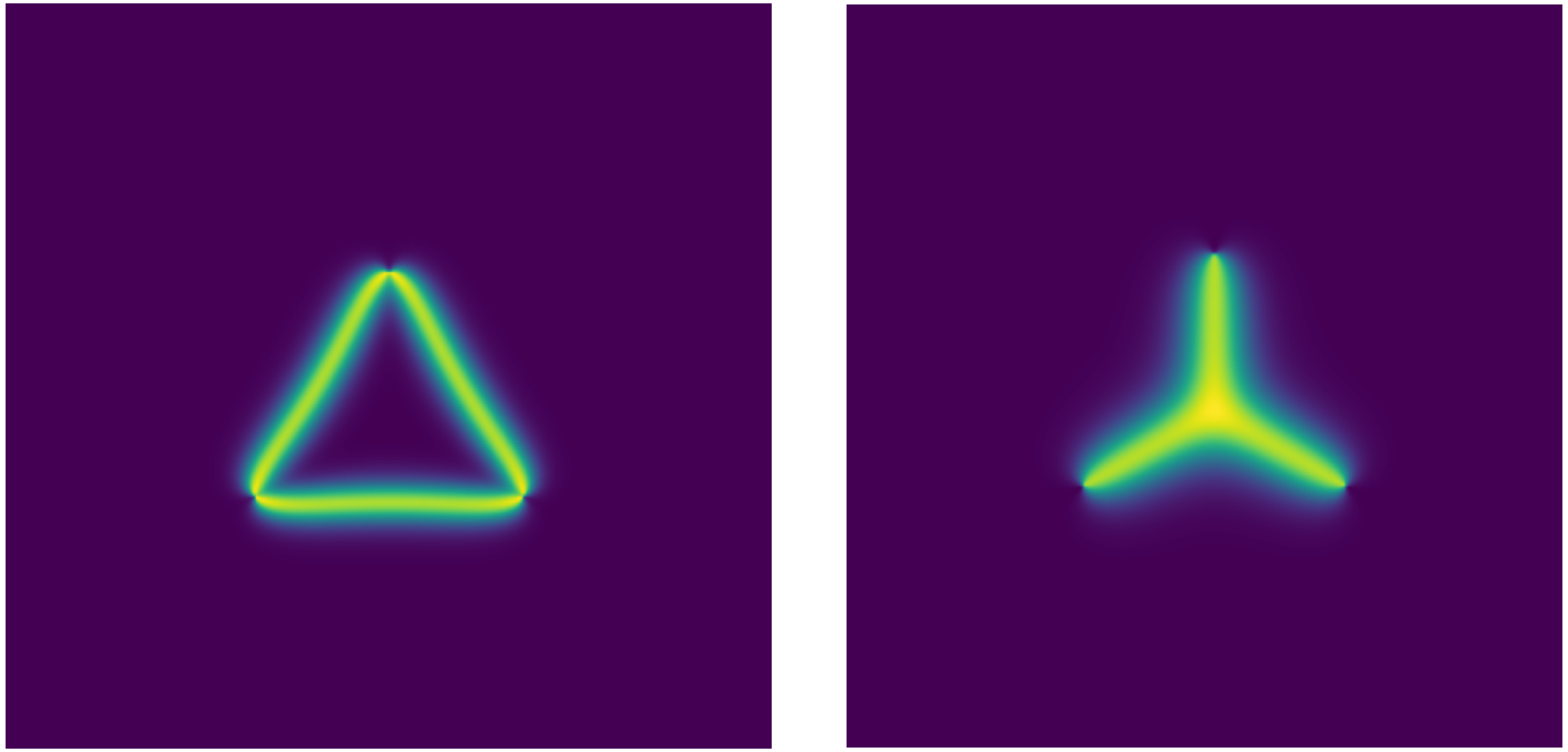
Topological excitations and confinement:

from supersymmetry to the femtouniverse

Erich Poppitz, U. of Toronto



just for fun: “baryons” in the “flatland” [Cox,Wong,EP 2019]

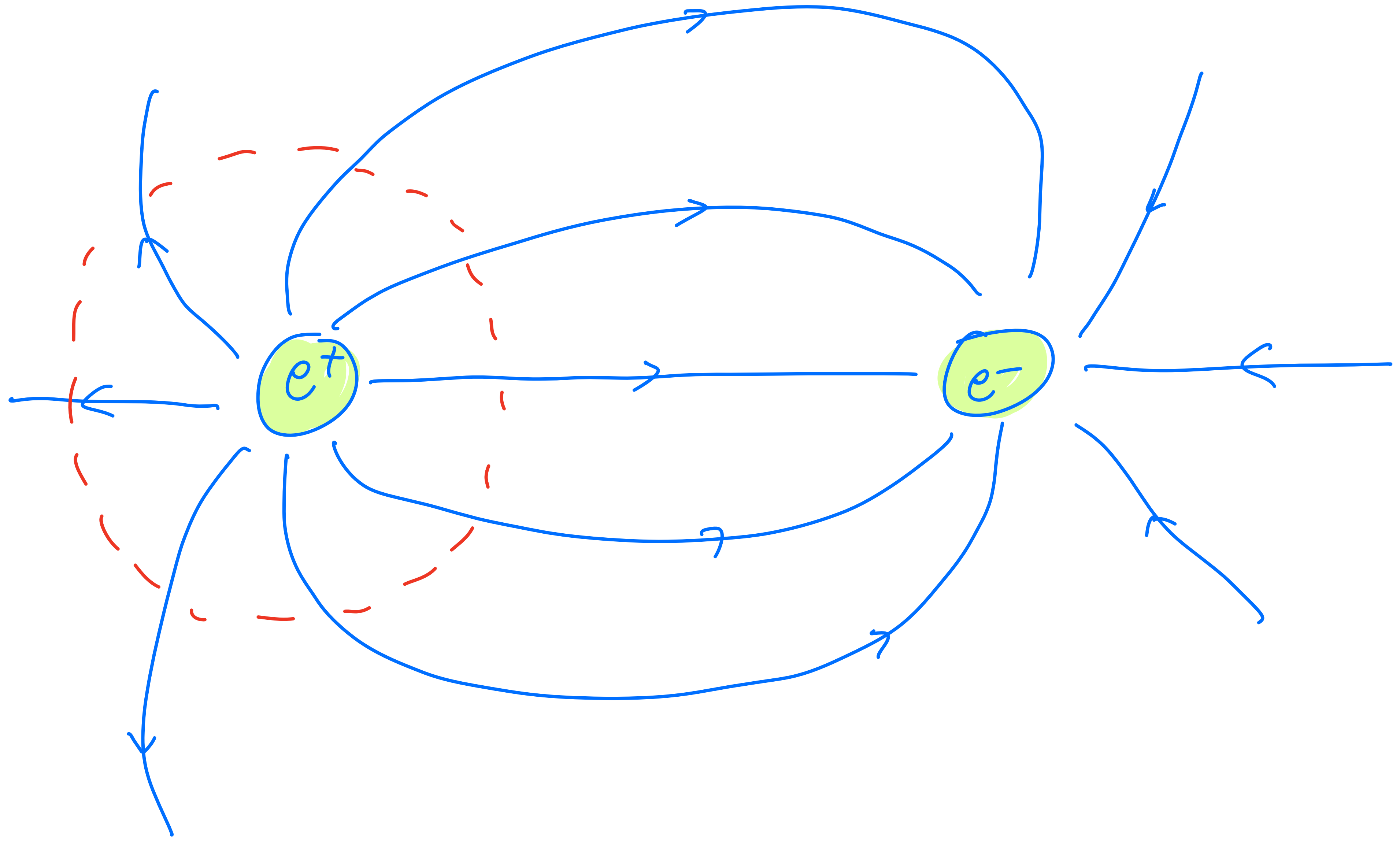
FIU, Miami, February 6, 2026

this talk is about dynamics of 4d Yang-Mills theory

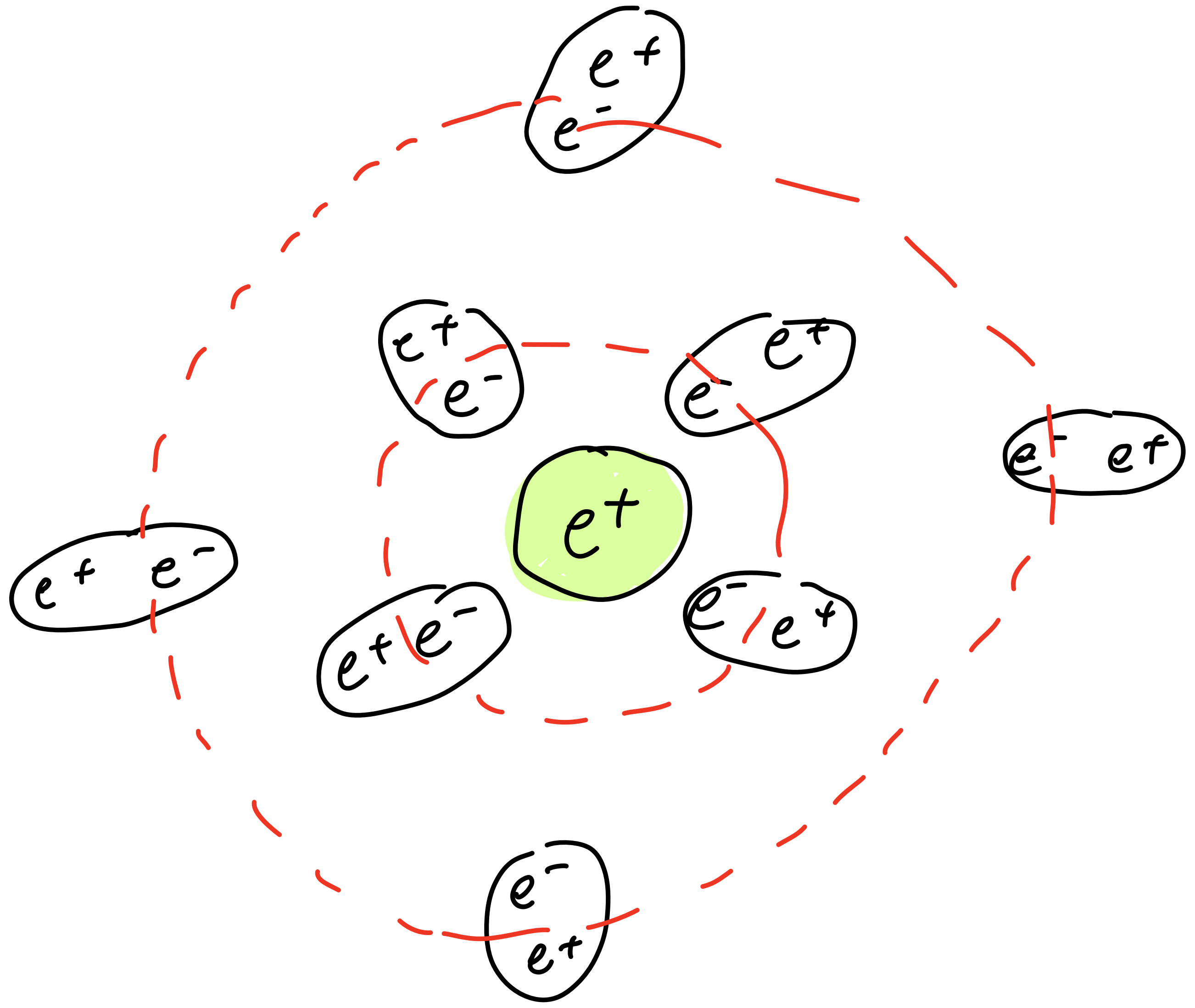
“confinement” is believed to be a property of the strong force, responsible for the formation of “nucleons” and “mesons”… made out of “quarks,” held together by “gluons”

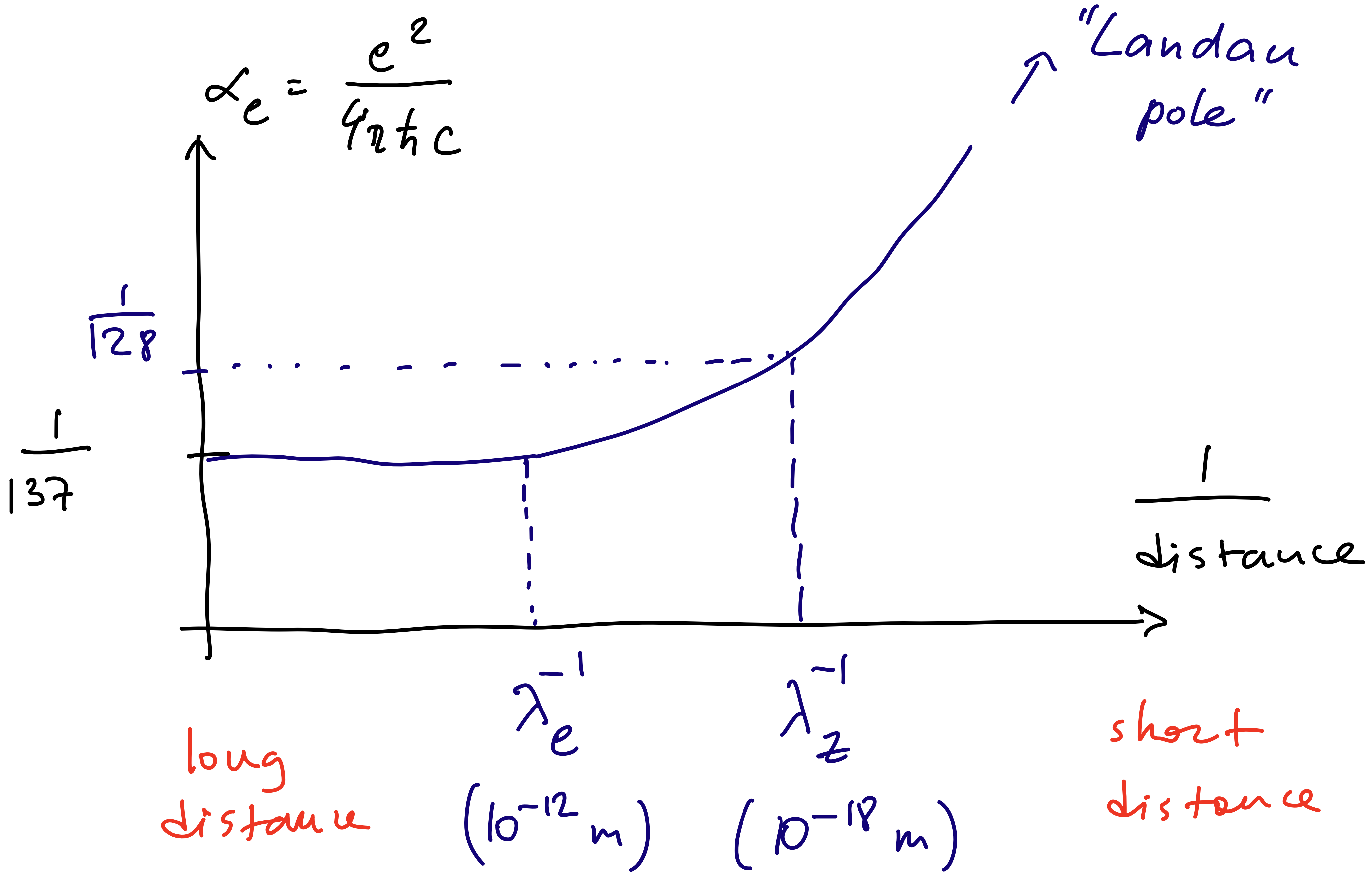
to describe what we mean by “confinement,” contrast with more familiar electromagnetism

Coulomb law, Gauss' law, screening in QED:



$$\text{energy} \sim \frac{e^2}{r}$$





E&M is a U(1) gauge theory:

$(\varphi, \vec{A})$: $\vec{E} = -\vec{\nabla}\varphi - \dot{\vec{A}}$

$\vec{B} = \vec{\nabla} \times \vec{A}$

$(c\varphi, \vec{A}) \equiv A_\mu$

$\mu = 0,1,2,3$

$A_\mu \to A_\mu - \partial_\mu \chi =$

$= e^{i\chi}(A_\mu + i\partial_\mu)e^{-i\chi}$

$$g(\vec{r}, t) \equiv e^{i\chi(\vec{r},t)} \in U(1)$$

photon (gauge) field $\longrightarrow A(\vec{r}, t) \to g(\vec{r}, t)(A(\vec{r}, t) + id)g^{-1}(\vec{r}, t)$

electron wave function $\longrightarrow \Psi(\vec{r}, t) \to e^{i\chi(\vec{r},t)}\Psi(\vec{r}, t) = g(\vec{r}, t)\Psi(\vec{r}, t)$

U(1) is an "Abelian" group: $g_1 g_2 = g_2 g_1$

QCD is an SU(3) gauge theory, looks same as above:

$$A(\vec{r}, t) \to g(\vec{r}, t)(A(\vec{r}, t) + id)g^{-1}(\vec{r}, t)$$

$$\Psi(\vec{r}, t) \to g(\vec{r}, t)\Psi(\vec{r}, t)$$

$$g(\vec{r}, t) \in SU(3)$$

QCD is an SU(3) gauge theory, looks same as above:

$$A(\vec{r},t) \to g(\vec{r},t)(A(\vec{r},t) + id)g^{-1}(\vec{r},t)$$

$$\Psi(\vec{r},t) \to g(\vec{r},t)\Psi(\vec{r},t) \longleftarrow$$ quark wave function $\in \mathbb{C}^3$ **of (complex) dimension 3**

$$g(\vec{r},t) \in SU(3)$$

$$A(\vec{r},t) \longrightarrow$$ gluon (gauge) field

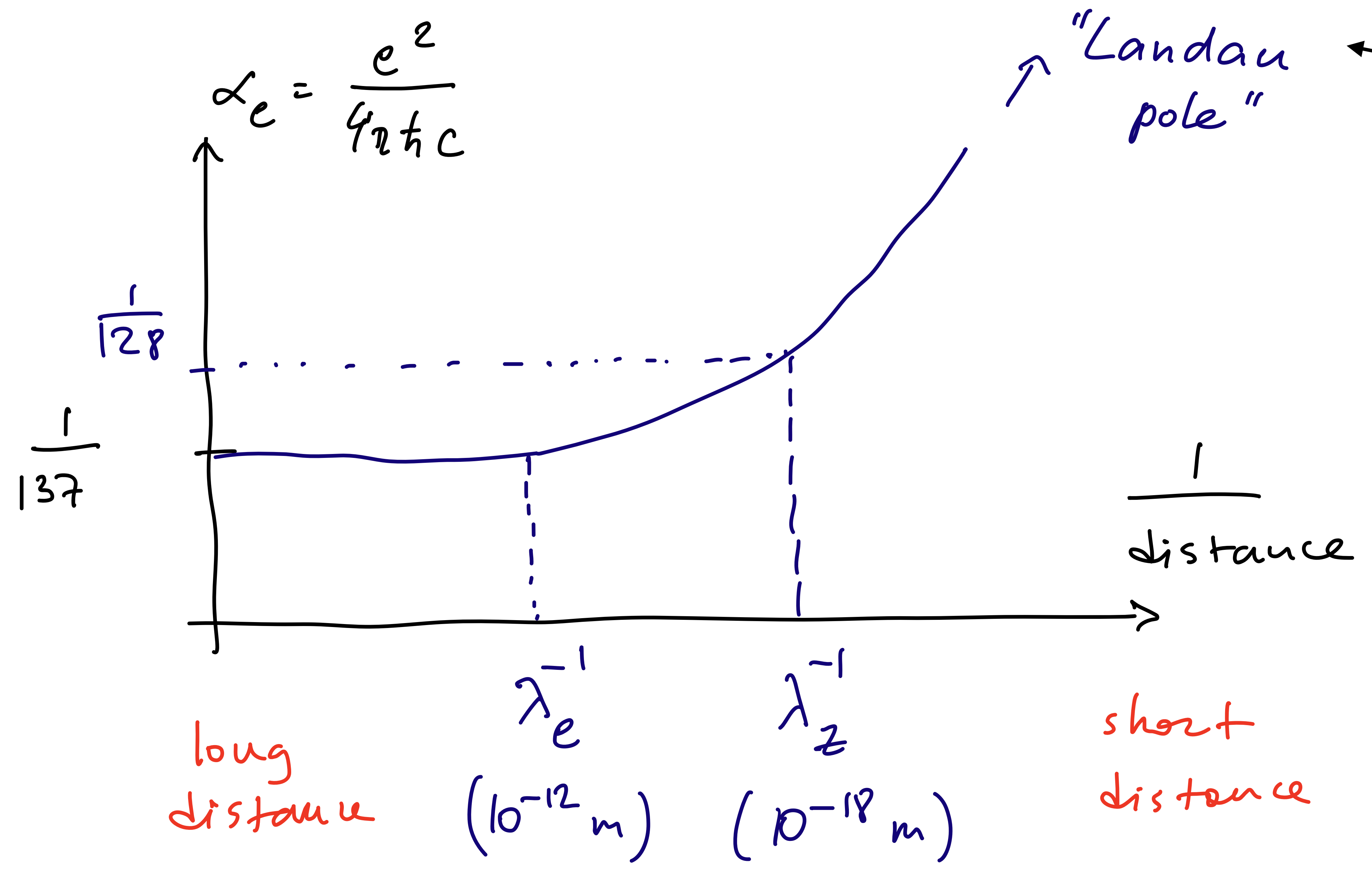
3x3 traceless Hermitean
$\in$ Lie algebra of SU(3)
of (real) dimension 8

SU(3)= 3x3 special unitary matrices: rotations 3 $\mathbb{C}$ dim

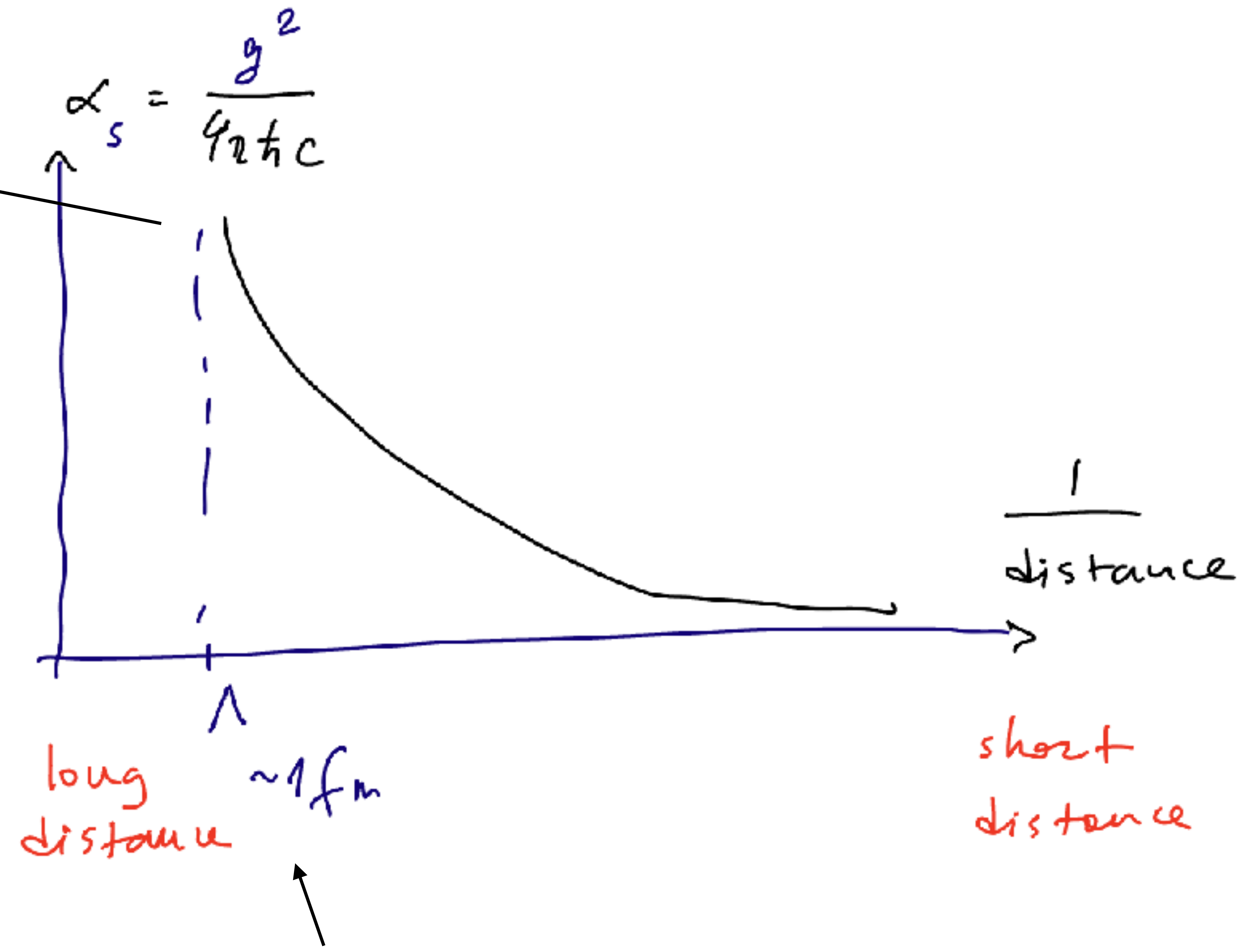
SU(3) is a "non-Abelian" group: $g_1 g_2 \neq g_2 g_1$

$\Longrightarrow$ e.g. color E-field $\overrightarrow{E} = \frac{\partial}{c\partial t}\overrightarrow{A} - \overrightarrow{\nabla} A_0 + i[A_0, \overrightarrow{A}]$ **not linear wrt A!**

U(1): screening

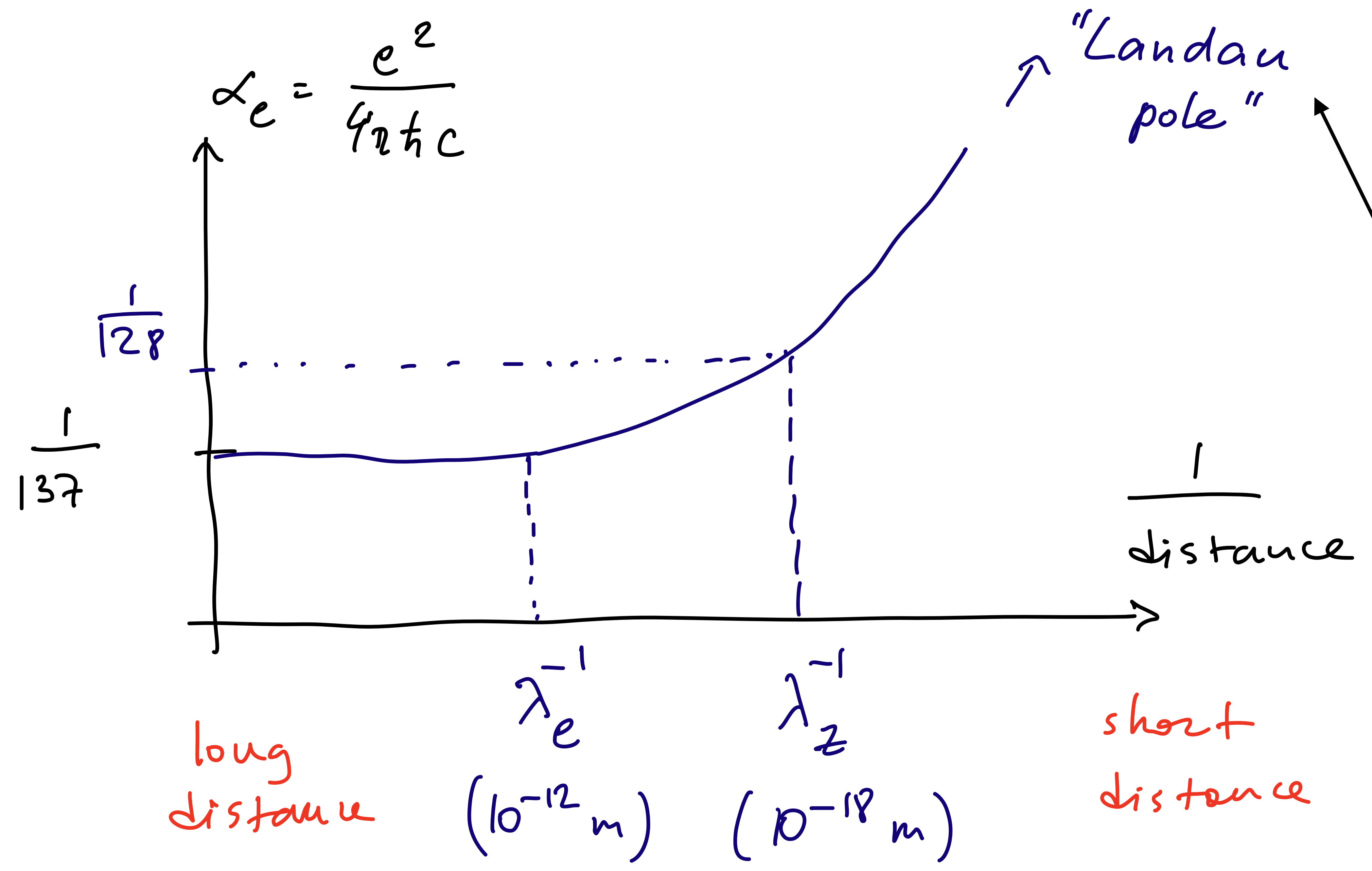


SU(3): anti-screening



Λ : **strong coupling scale**

U(1): screening



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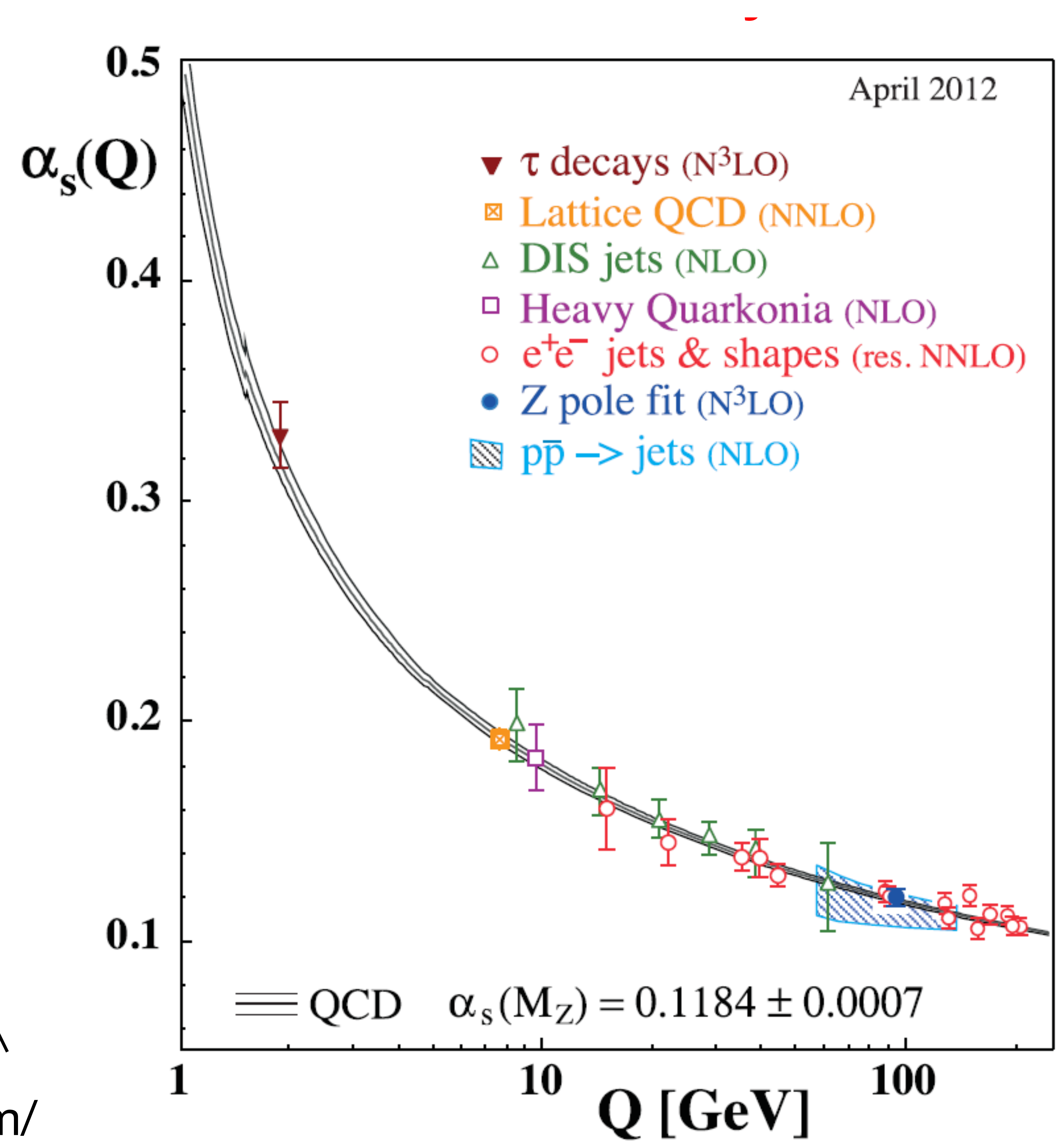


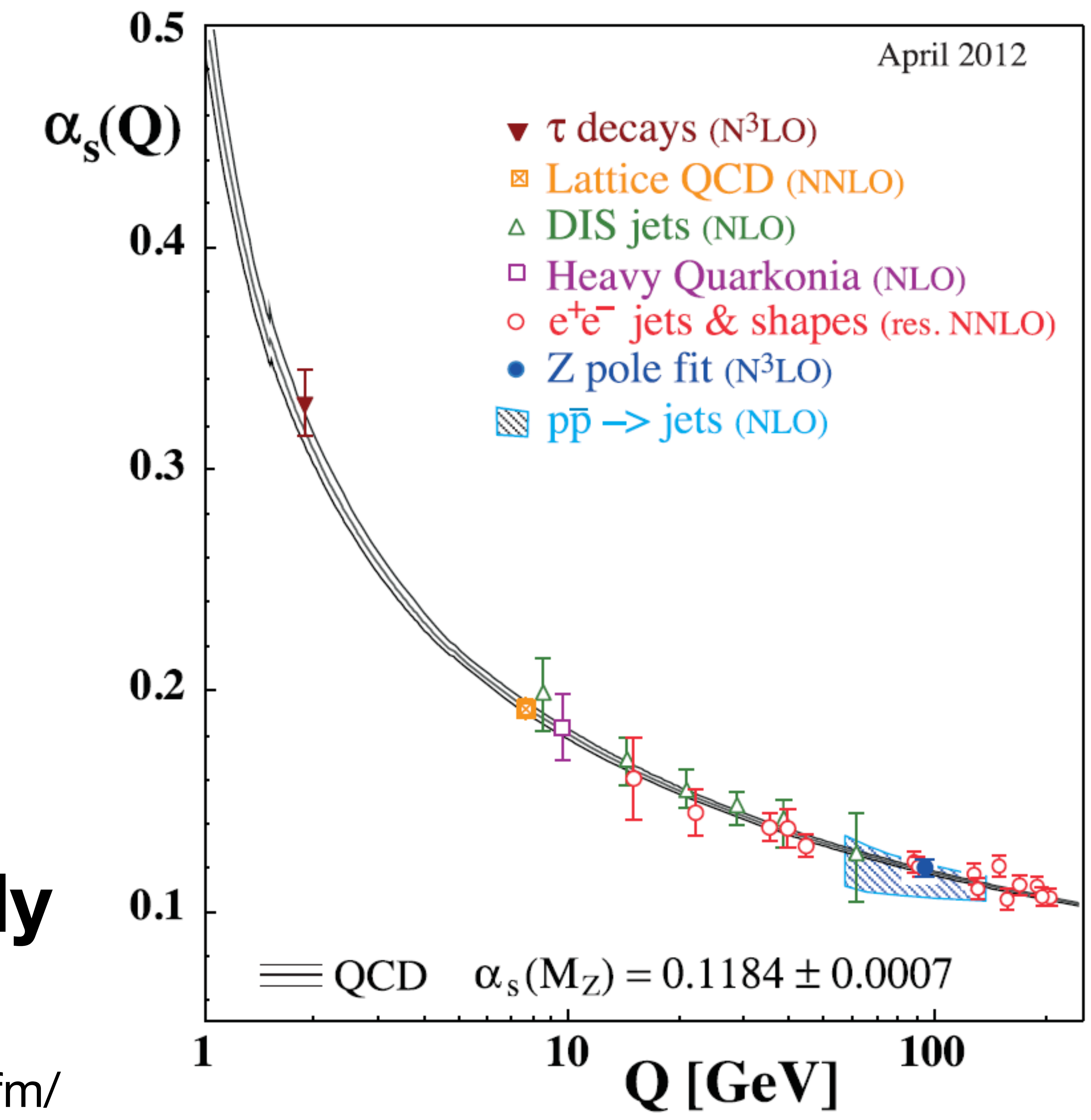
1 fm/
197.3 MeV

SU(3): anti-screening

anti-screening, or asymptotic freedom, understood since 1970s [Gross, Wilczek, Politzer]

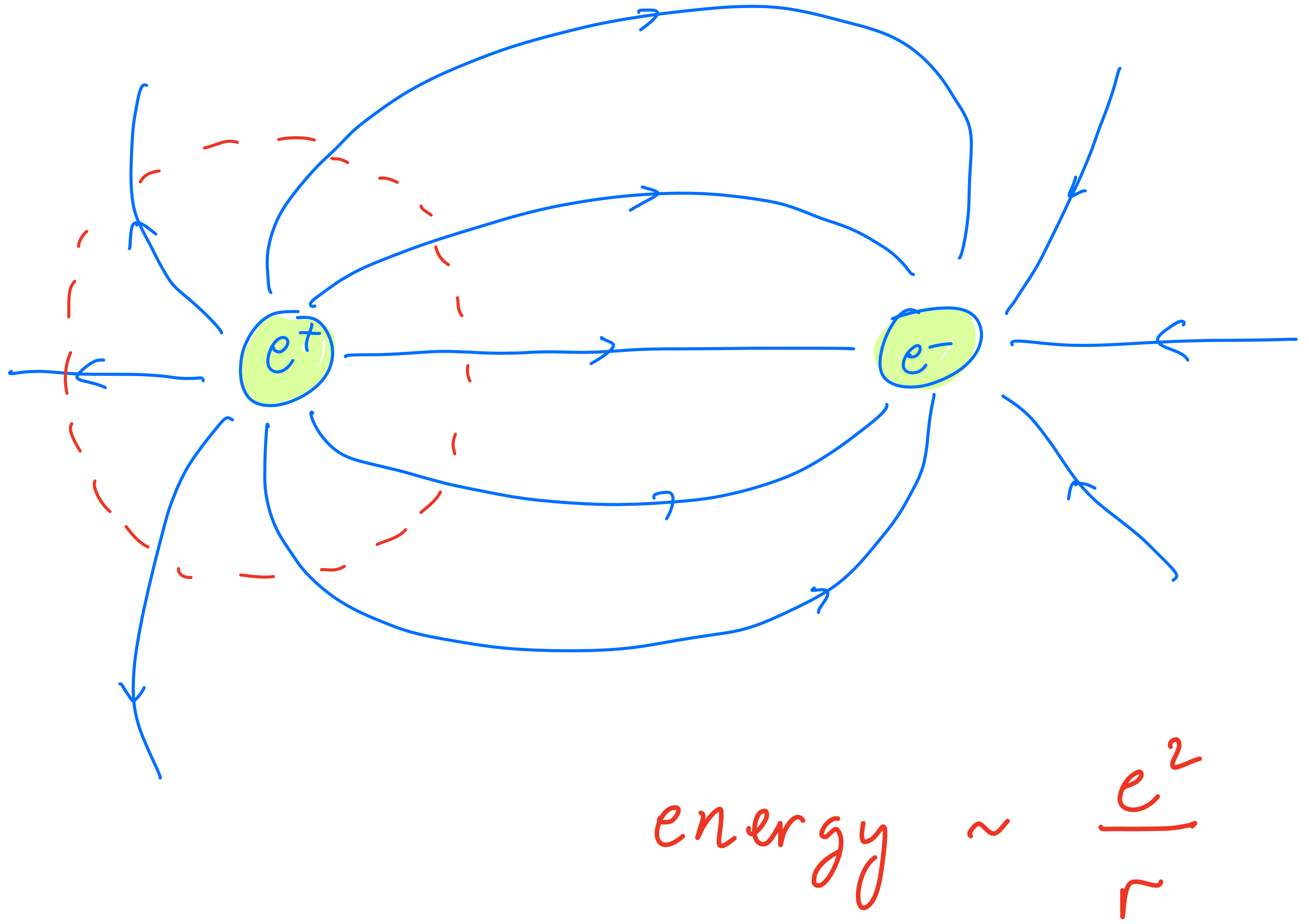
perturbative result, measured

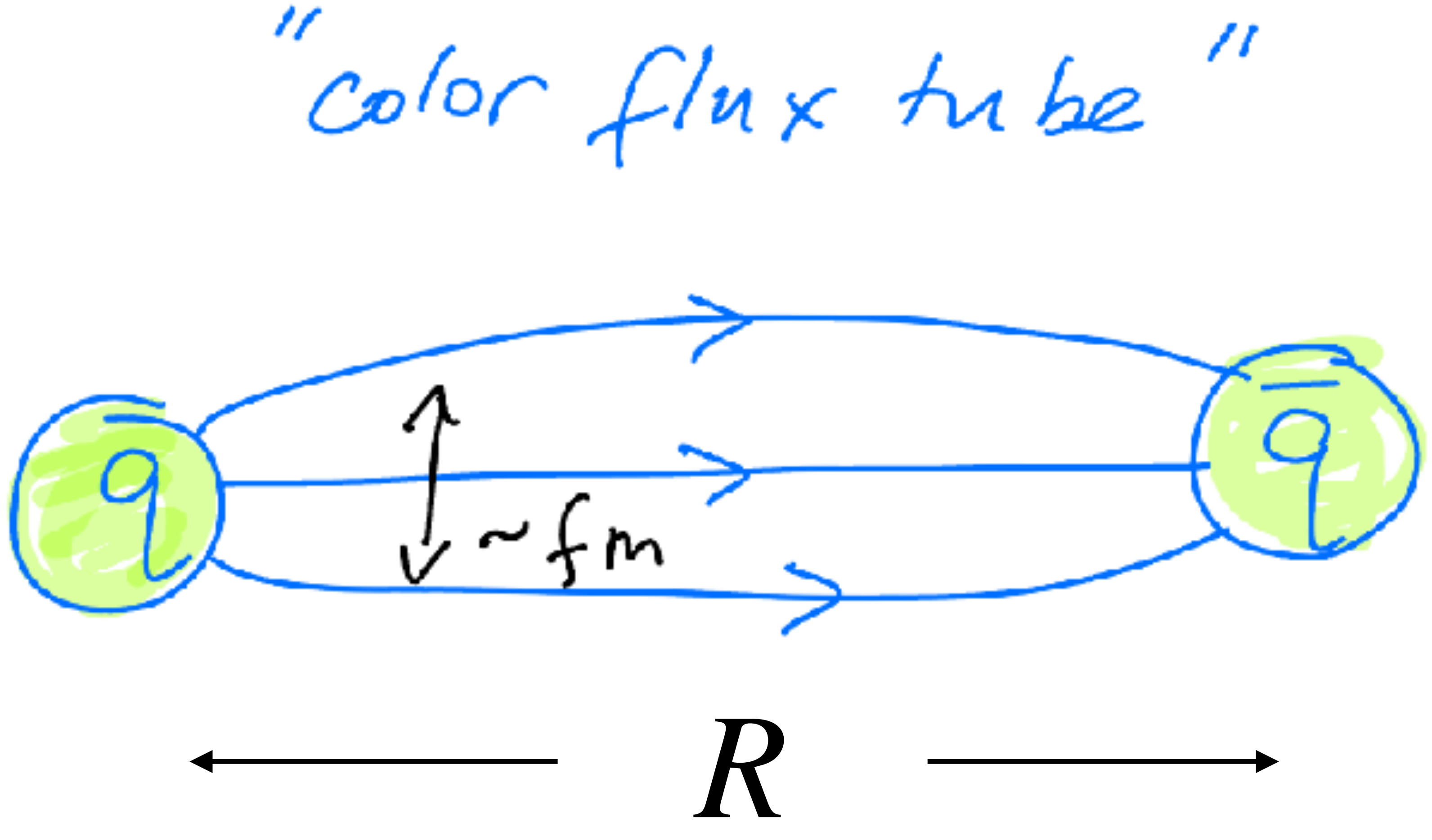
strong coupling "Landau pole" at 1fm means perturbation theory breaks down at long distances, **hence consequences of QCD can not be computed analytically**



1 fm/
197.3 MeV

a striking consequence of the **strong** force is **confinement**, for which there is much evidence, experimental and numerical…





$$E_{q\bar{q}} = \Sigma R$$

"confining string tension"

force = constant, not $1/R^2$

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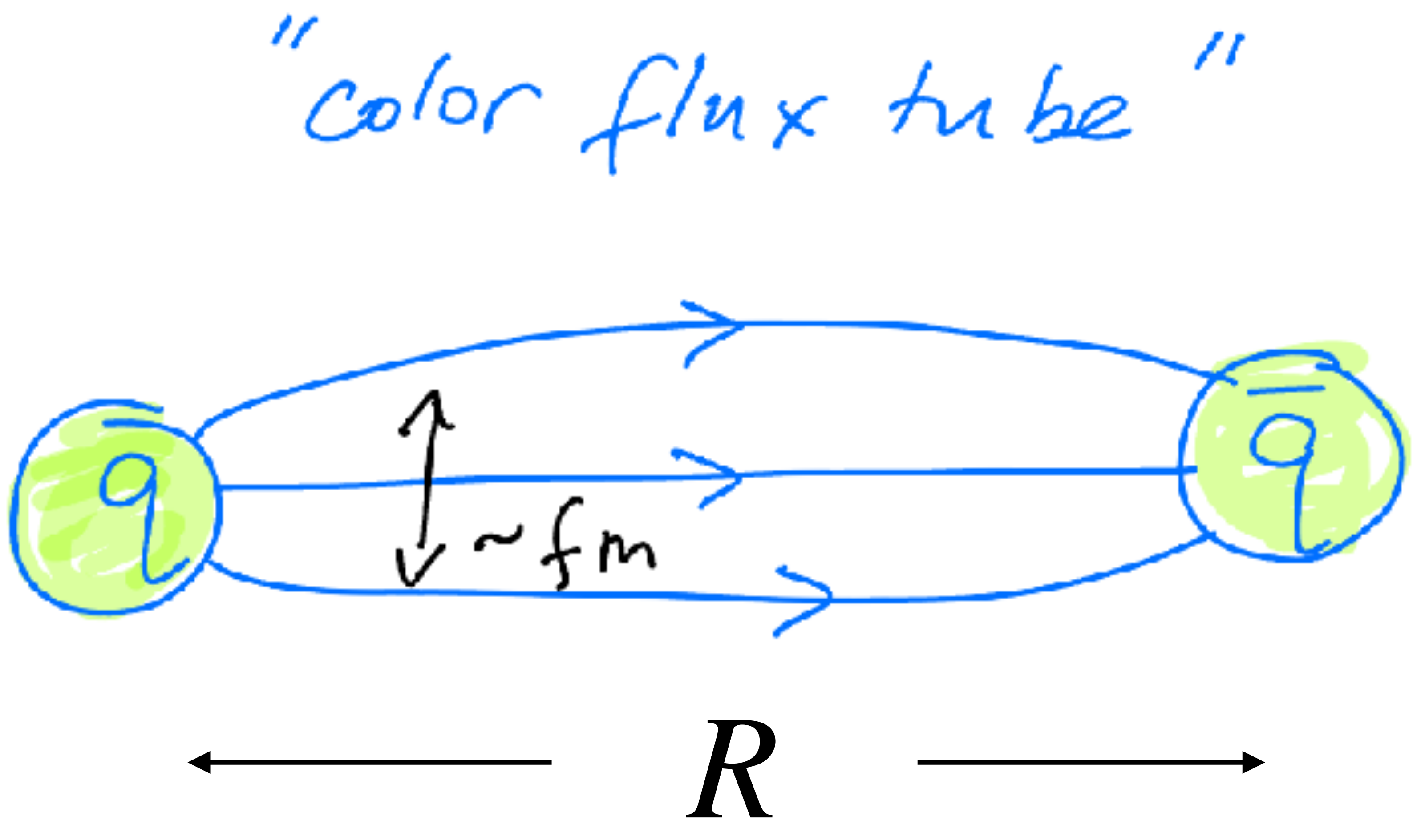
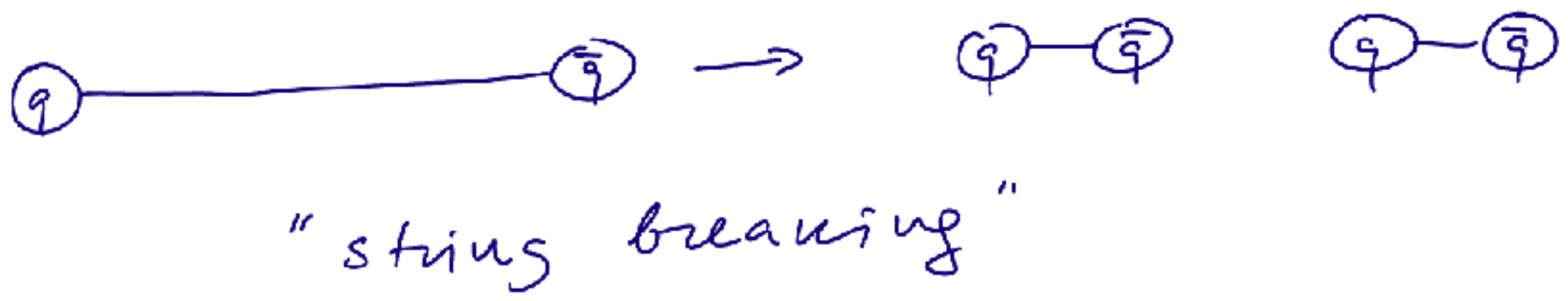


now,

if $m_q < \infty$, as $\Sigma R > 2 m_q c^2$



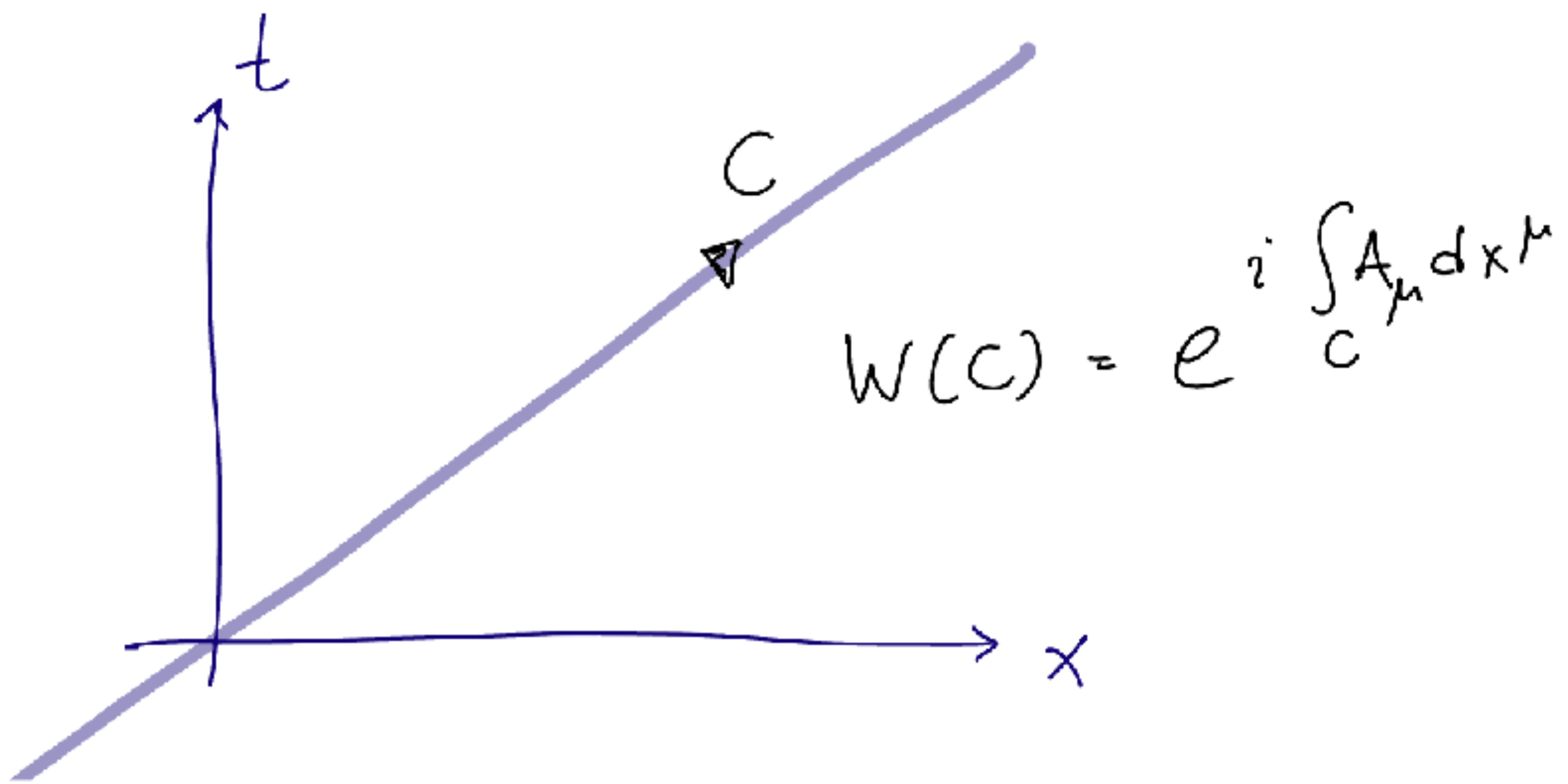
$$E_{q\bar{q}} = \Sigma R$$

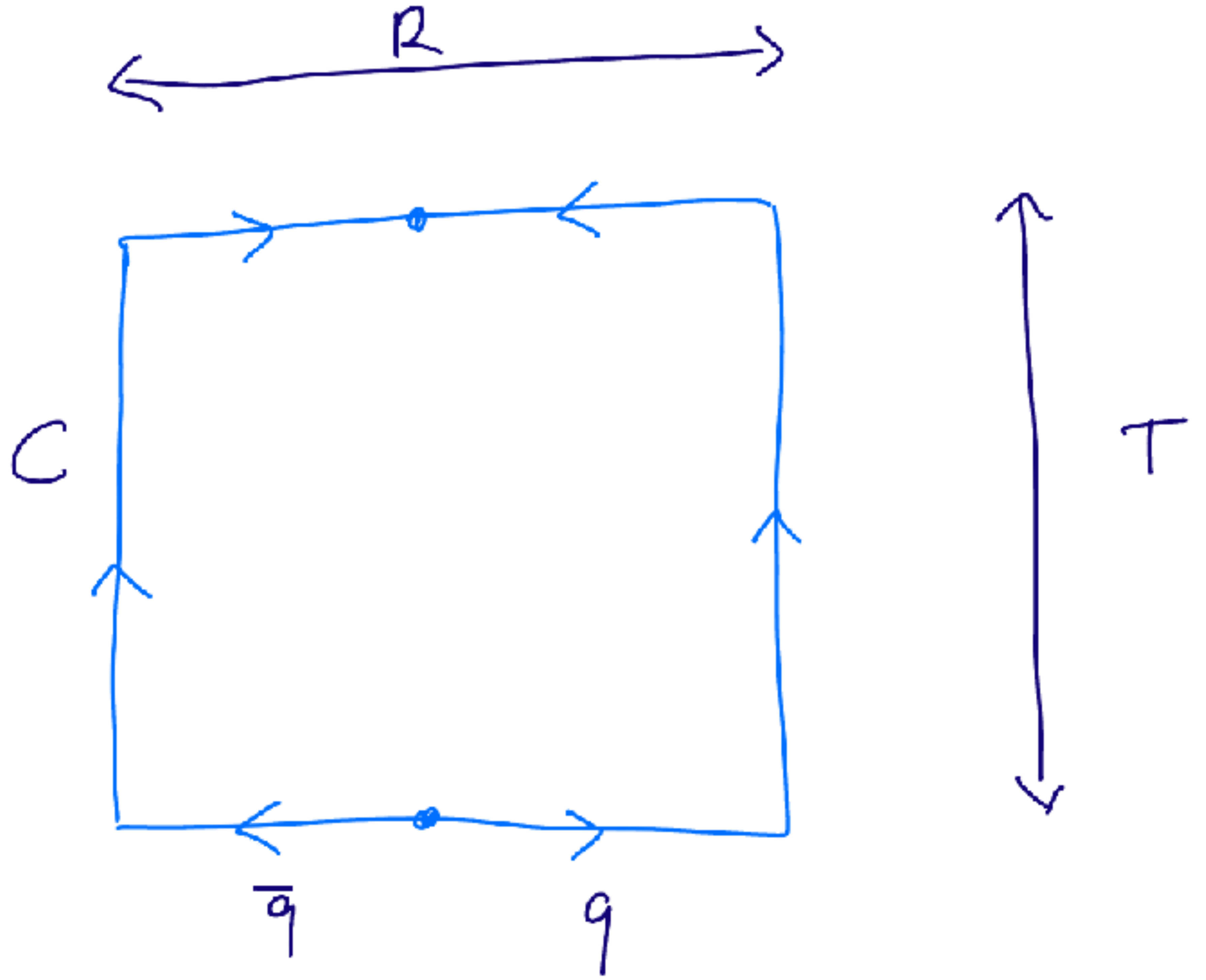
"confining string tension"

force = constant, not $1/R^2$

hence, can't isolate a quark…

as a first step to define **confinement** we take $m_q = \infty$ i.e. consider the pure gauge theory with infinitely heavy quark sources





static quark sources, distance R apart

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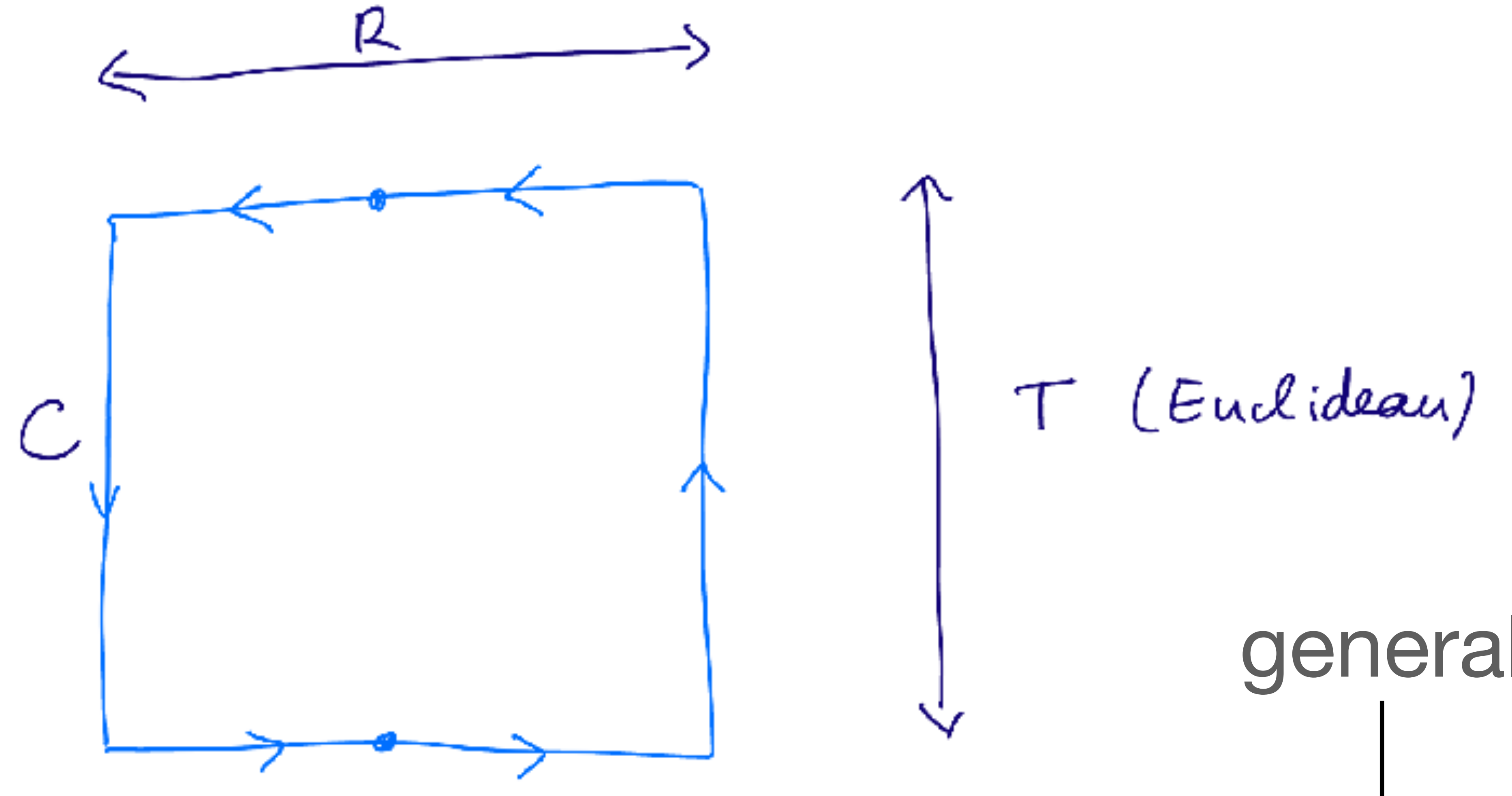


$$W(C) = \mathrm{tr}\, \mathcal{P}\, e^{i \oint_C A_\mu dx^\mu}$$

general → $E_{q\bar{q}}(R)$; confining → $\Sigma R T$

$$\langle W(C) \rangle \underset{T \to \infty}{\sim} e^{-E_{q\bar{q}}(R)\, T} = e^{-\Sigma R T}$$

$$\longrightarrow \langle W(C) \rangle \sim e^{-\Sigma \cdot \mathrm{Area}}$$

explain confinement = explain area law

[K. Wilson, 1970’s]

explain confinement = explain area law

(part of) one of the yet unsolved Clay Institute “Millennium Problems”


CMI

Home — Millennium Problems — Yang-Mills & the Mass Gap

Unsolved

Yang-Mills & the Mass Gap

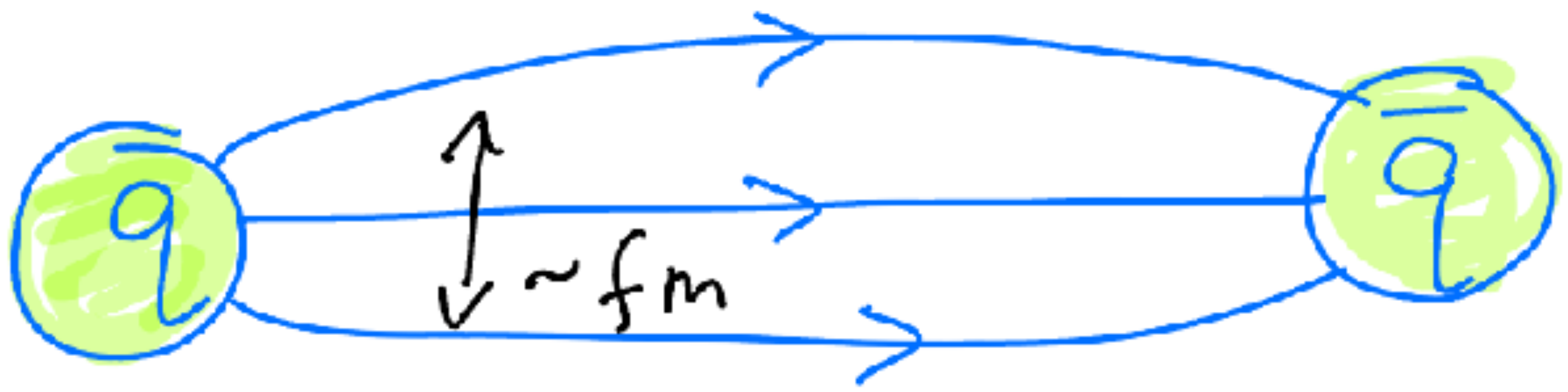
Experiment and computer simulations suggest the existence of a “mass gap” in the solution to the quantum versions of the Yang-Mills equations. But no proof of this property is known.

$m^2 \sim \Sigma$

in what follows:

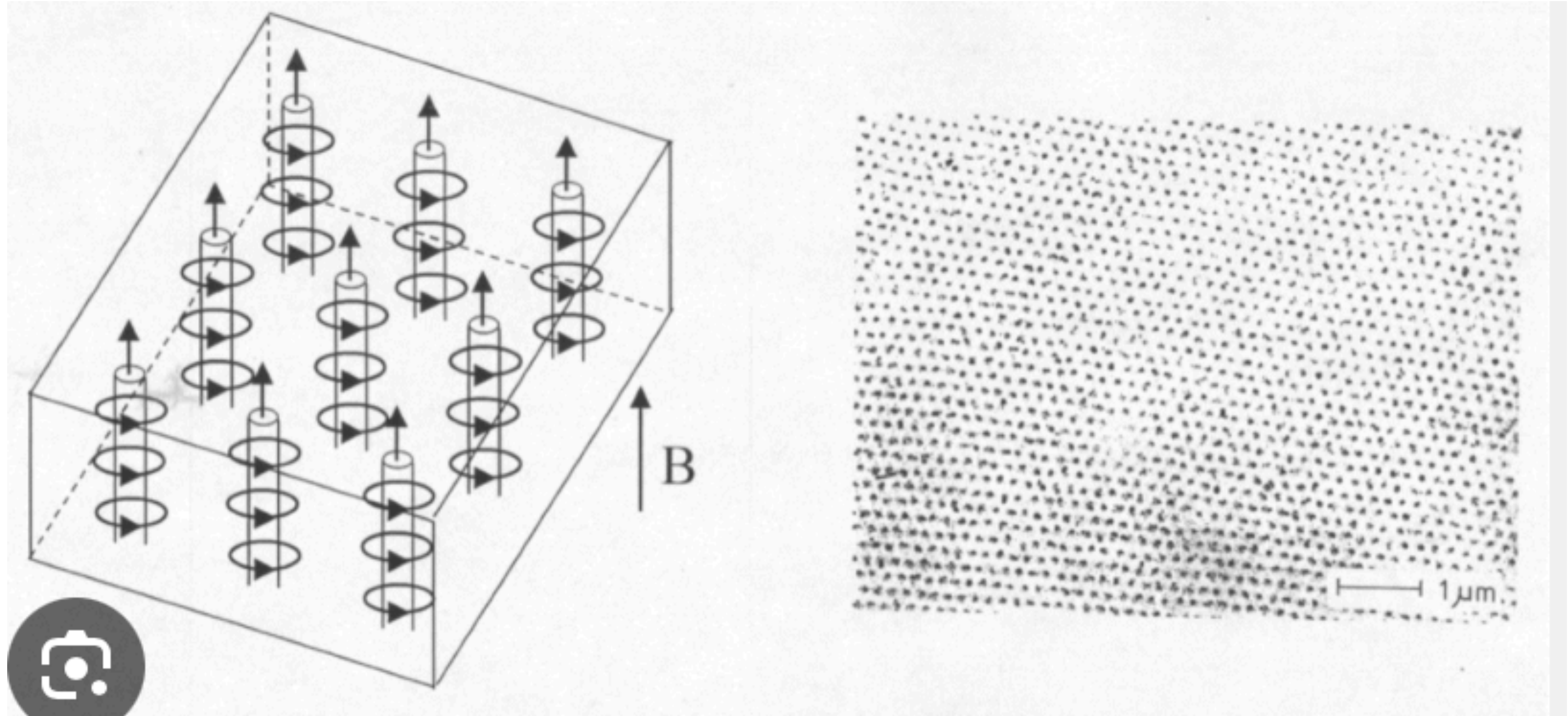
describe recent (and not so recent) developments of qualitative and quantitative ideas of confinement in theories ***believed/shown to be continuously related to* YM**…

Mandelstam (1970’s) noted that the picture of a “color electric flux tube”

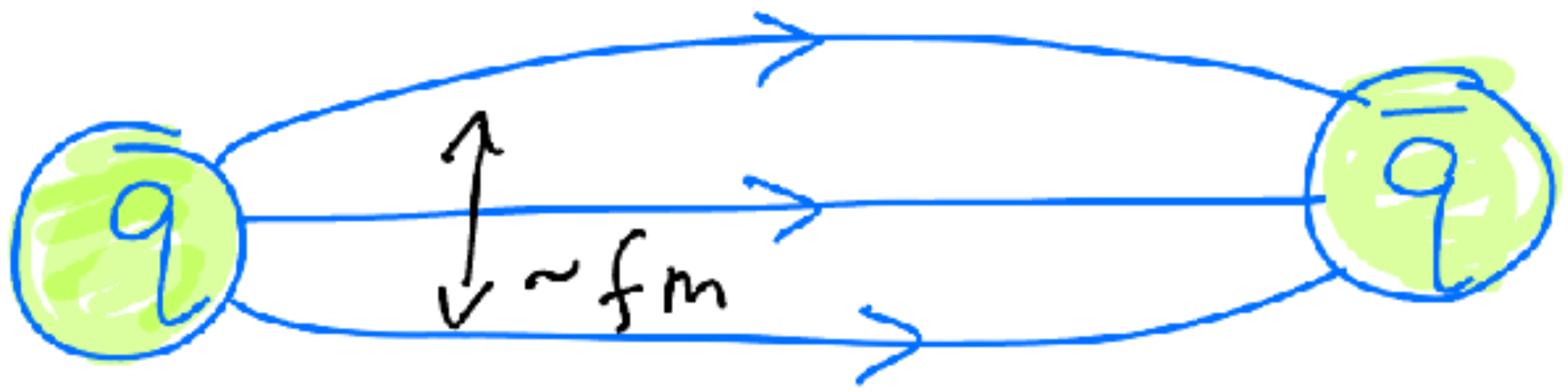


was similar to what happens in superconductors (SC)…

- SC vacuum is a condensate of electric charges “Cooper pairs,” photon massive
- magnetic field is expelled (London penetration depth λ_L) and can only penetrate in magnetic flux tubes, or Abrikosov vortices, of tension proportional to the superconducting gap, whose magnetic flux is quantized



Mandelstam (1970's) noted that the picture of a "color (electric) flux tube"



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-> if magnetic monopoles existed, this would imply their confinement in superconductors!

"Abrikosov (magnetic) flux tube"

confinement

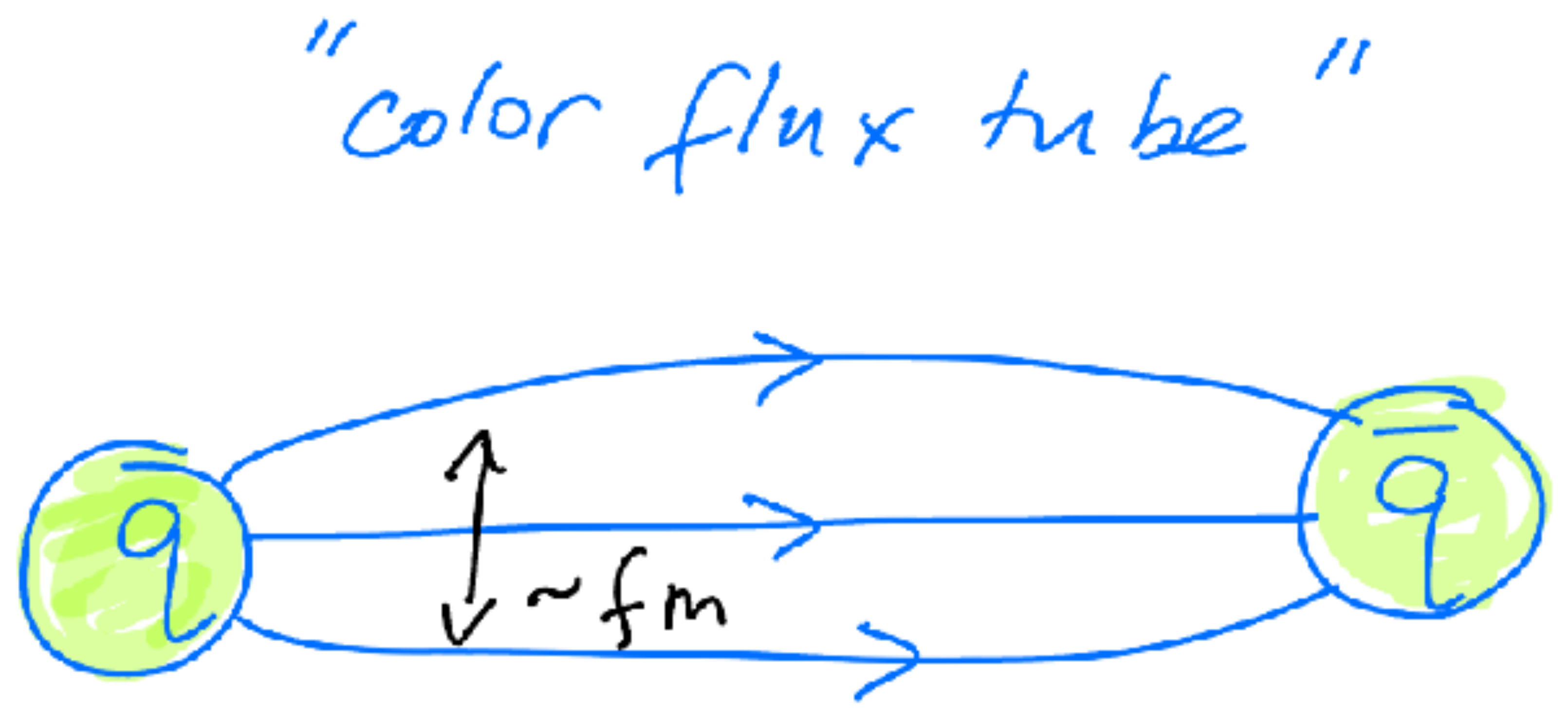
? ||

dual superconductivity

confinement

? ||

dual superconductivity



picture would suggest that

- YM vacuum is a condensate of magnetic charges "??", (dual) gluon massive
- color electric field is expelled and can only penetrate in vortices/confining strings

questions linger, however…

…the superconductor is a U(1) gauge theory, while YM is nonabelian…?

…the U(1) flux tube picture is semiclassical, weak coupling, while YM is strongly coupled…?

…what are these magnetic monopoles supposed to fill the YM vacuum…?

for a long time dual SC was thought to be just that, a "picture"… until some supersymmetric theories were understood by **Seiberg and Witten, 1990's**

- it turns out Mandelstam's dual SC picture becomes very concrete in these nonabelian YM theories, as I will qualitatively describe for SU(2) YM:

supersymmetric (SUSY) theories: #boson = #fermion

$\mathcal{N} = 2$ *SYM:* A_μ, ψ, ϕ*: gauge + Dirac spinor* ψ *+* $\mathbb{C}$ *scalar* ϕ *all triplets of SU(2)*

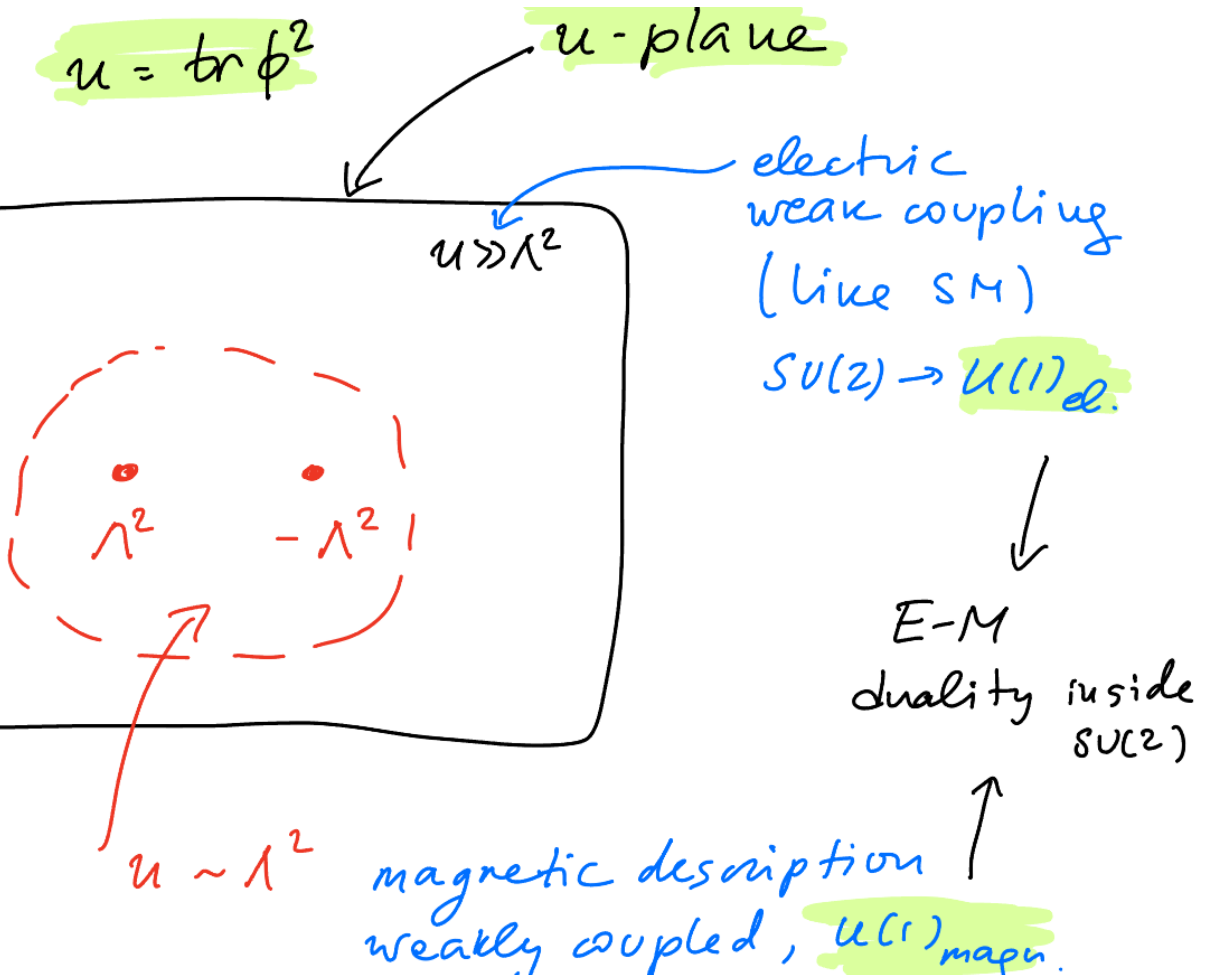
full $\mathcal{N} = 2$ *supersymmetry = a single gauge coupling, no mass terms*

$\mathcal{N} = 2$ *SYM ->* $\mathcal{N} = 1$ *SYM:* $m(\psi_L, \phi)$ *->* $\mathcal{N} = 0$ *YM:* $m(\psi_L, \phi)$ *+*$m'(\psi_R)$

$\mathcal{N} = 2$ **SYM -> *"moduli space" of vacua:*** $\langle \mathrm{Tr}\phi^2 \rangle = u, SU(2) \to U(1)_{el.}$

now, $A_\mu + \phi$*: theory has 't Hooft-Polyakov monopoles as solitonic solutions*

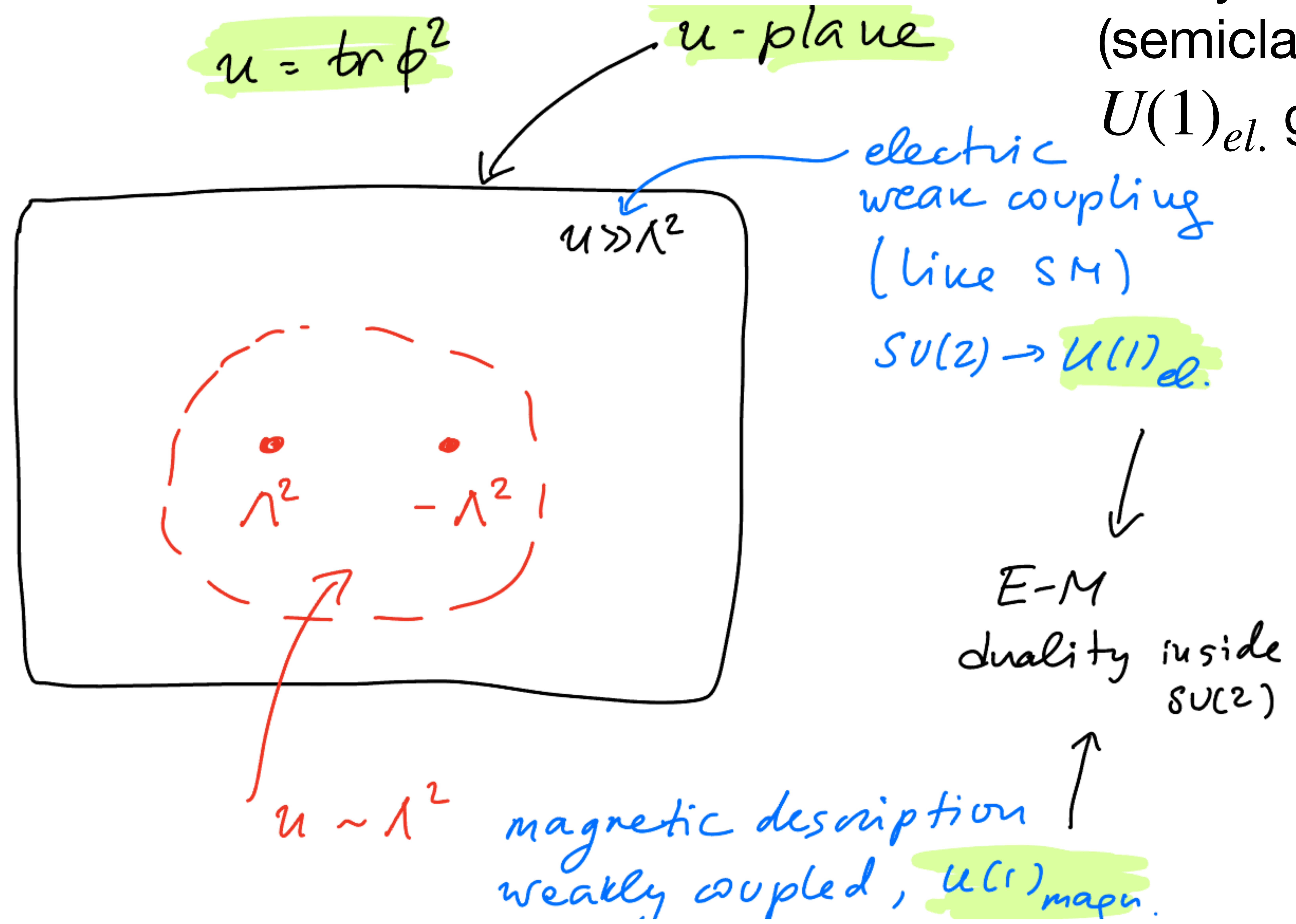
$\mathcal{N} = 2$ SYM ->



$\mathcal{N} = 2$ SYM ->

heavy solitonic (semiclassical) monopoles

$U(1)_{el.}$ gauge theory



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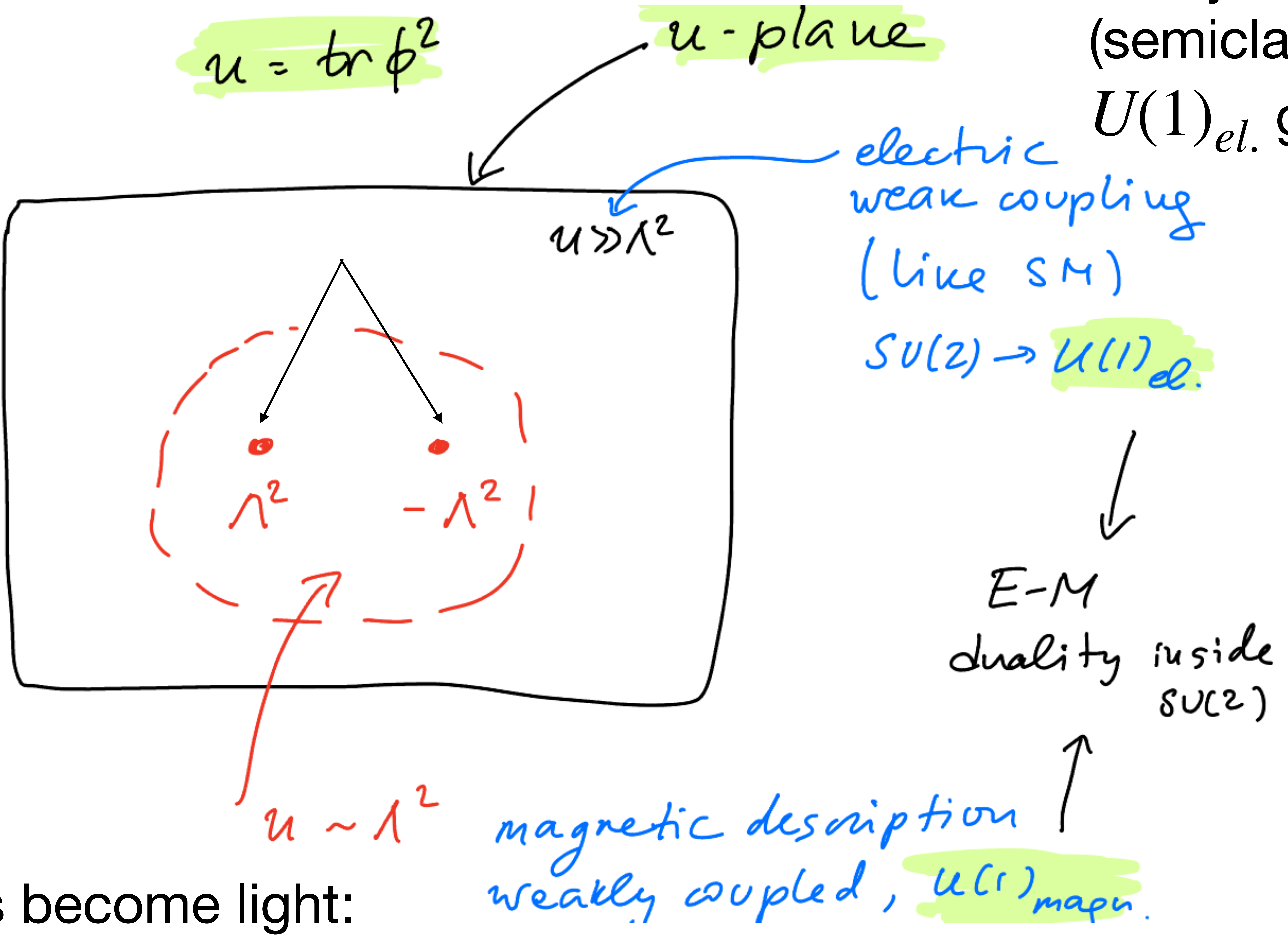
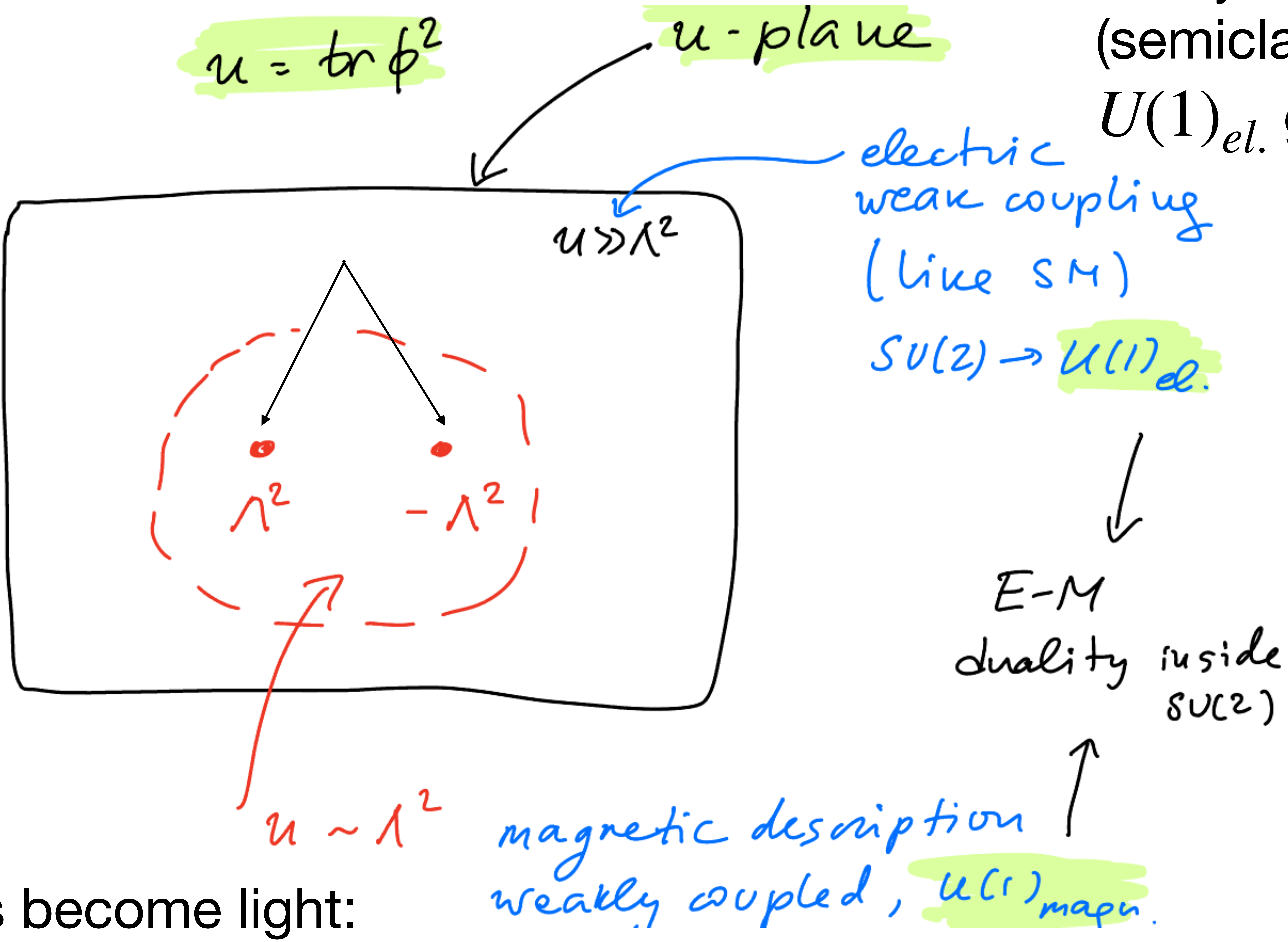


monopoles become light:

fundamental fields charged under $U(1)_{magn.}$ gauge theory, <u>massless at two points</u>

$\mathcal{N} = 2$ SYM ->

heavy solitonic (semiclassical) monopoles $U(1)_{el.}$ gauge theory



notice that while UV theory is $SU(2)$,nonabelian, IR is abelian: $U(1)_{el./magn.}$, so long as all SUSY intact… but:

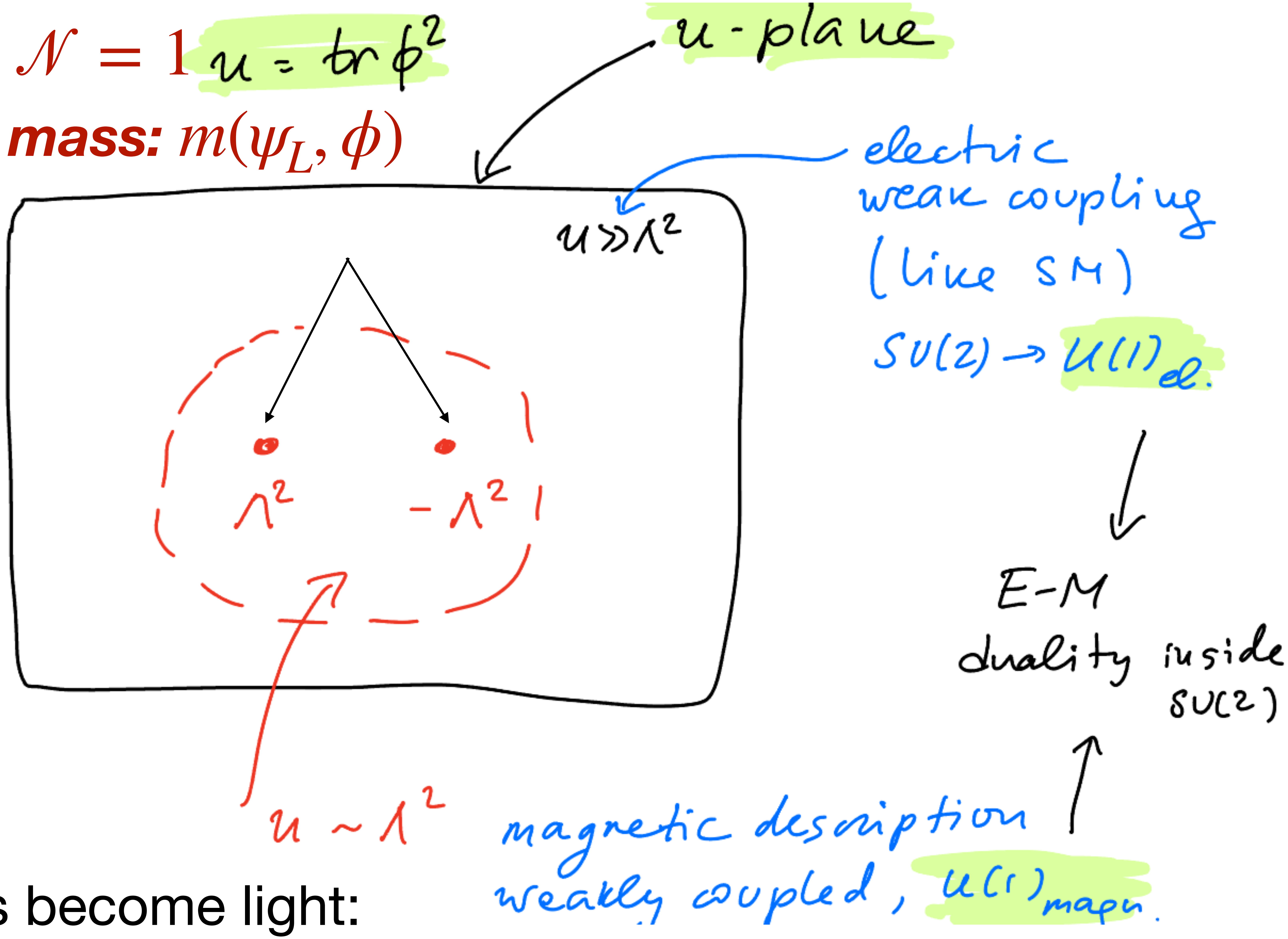
monopoles become light:

fundamental fields charged under $U(1)_{magn.}$ gauge theory, <u>massless at two points</u>

finally, break

$\mathcal{N}=2 \rightarrow \mathcal{N}=1$

by adding mass: $m(\psi_L, \phi)$



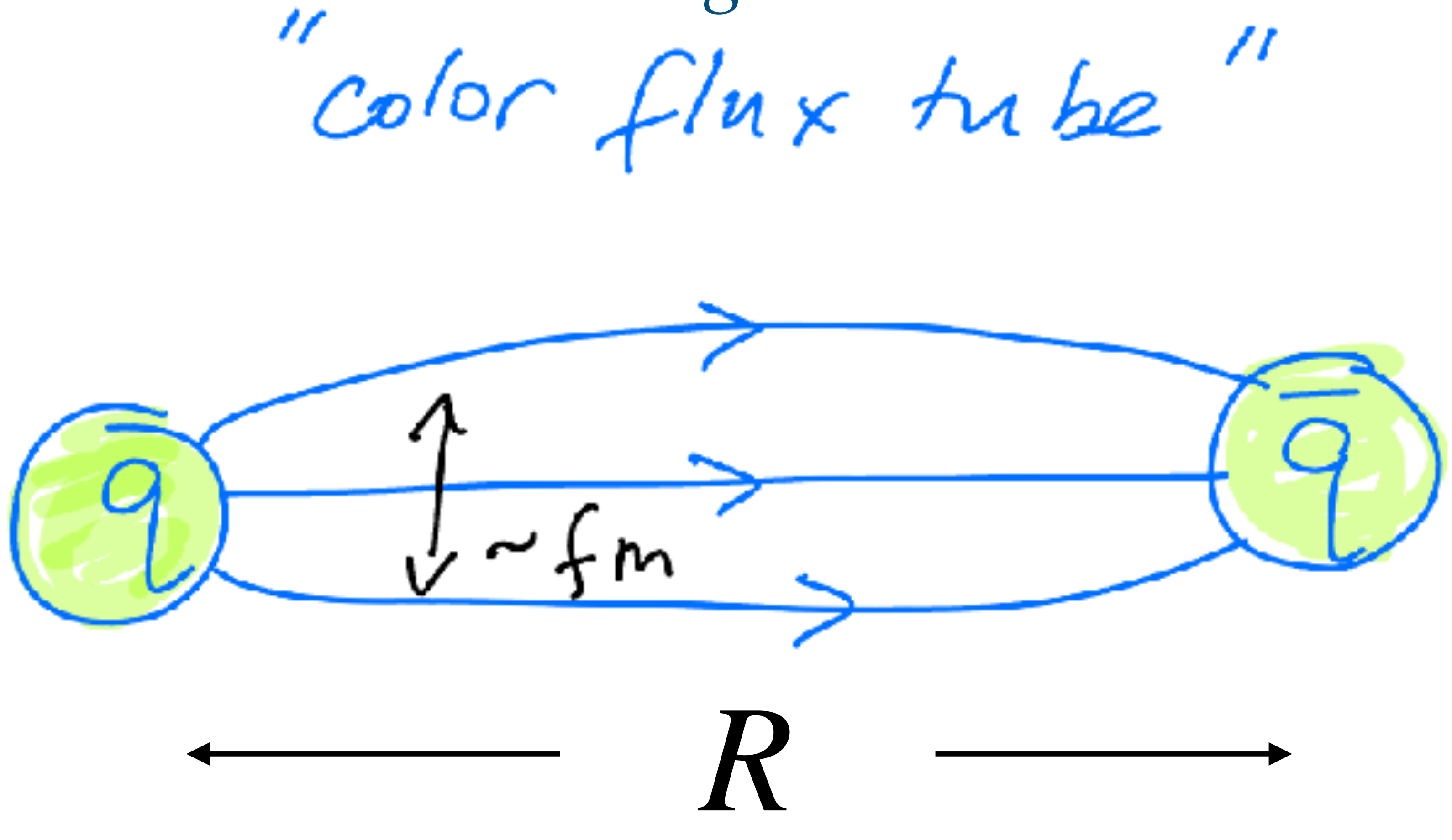
at $m \neq 0$:
two vacua only!
massless monopoles/ dyons condense -> dual SC picture:
electric charges confined by Abrikosov vortices in dual $U(1)_{magn.}$

monopoles become light:

fundamental fields charged under $U(1)_{magn.}$ gauge theory, <u>massless at two points</u>

Seiberg-Witten: break $\mathcal{N}=2 \to \mathcal{N}=1$ by $m(\psi_L, \phi)$

$U(1)_{magn.}$



$$E_{q\bar{q}} = \Sigma R$$

"confining string tension"

force = constant, not $1/R^2$

SUMMARY:

Mandelstam dual SC picture holds within some nonabelian QFT, related to pure YM theory (m, m')

flux tube made of "dual U(1)", established by "power of SUSY" within Abelian effective theory

relation to fundamental SU(2) variables not explicit

currently out of reach to microscopic (lattice) study

continuous connection to pure YM conjectured, as $m, m' \to \infty$***, but not shown in any way***

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continuous connection to pure YM conjectured, as
$m, m' \to \infty$, but not shown in any way

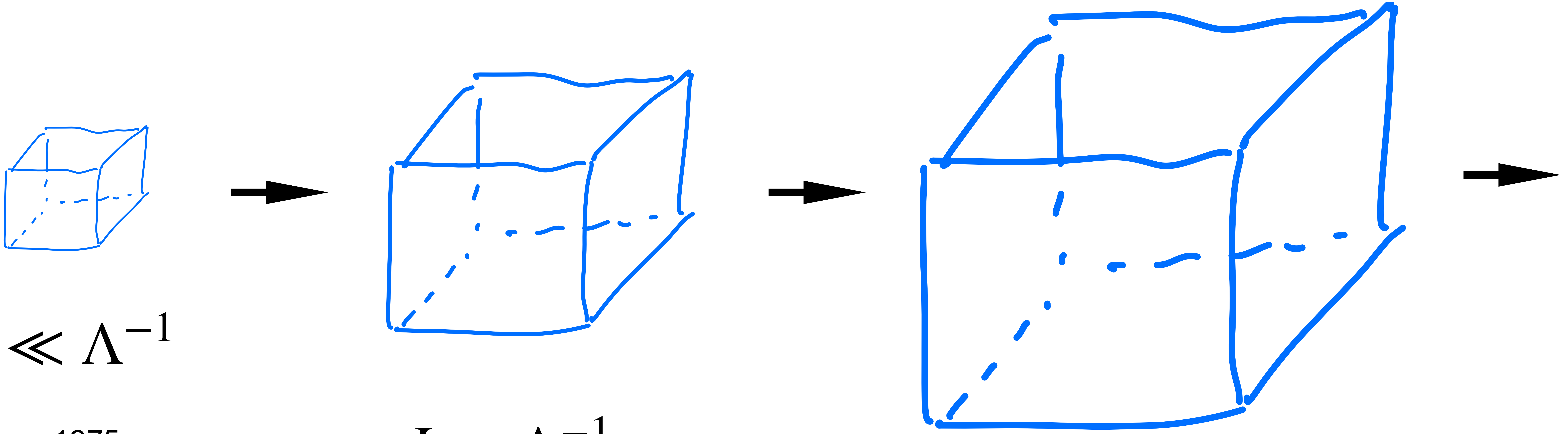
we can, therefore, ask:

is there any set up, not reliant on SUSY and its power, where we can establish confinement analytically?

such that, one can show,* at the very least using lattice, *continuous connection to pure YM theory?

-> YES! (under much scrutiny lately)

the femtouniverse and its twisted variants: 1975-2026…



$$L \ll \Lambda^{-1} \qquad L \sim \Lambda^{-1} \qquad L \gg \Lambda^{-1} \qquad \dots \mathbb{R}^3$$

Bjorken, 1975:
use small spatial box to keep weak coupling!

Lüscher, 1982: calculated spectra w/ **periodic b.c.**

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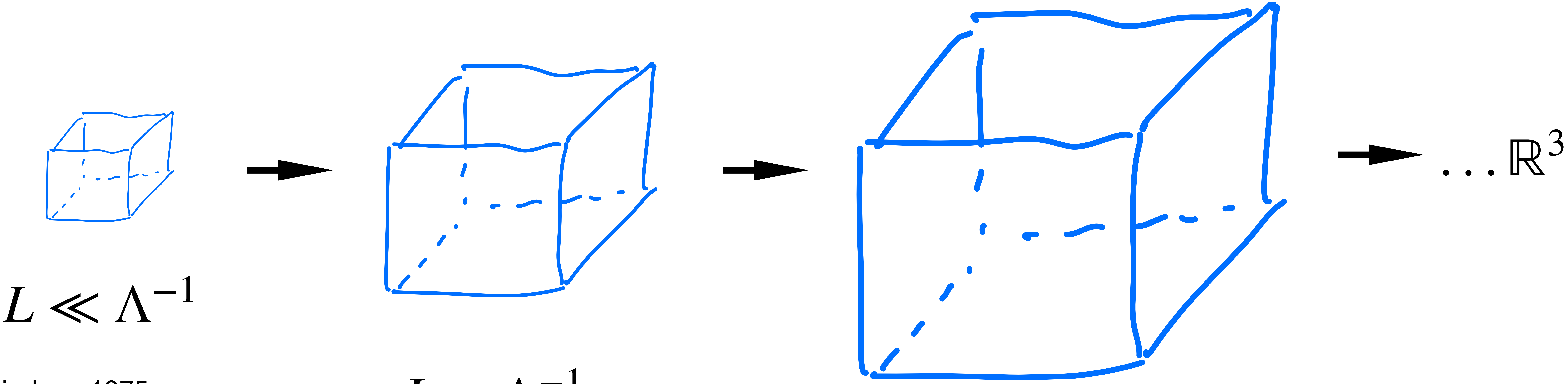

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regarding confinement, however, periodic b.c. femtouniverse has a drawback:

the small box theory does not confine <— deconfining phase transition at $L \sim \Lambda^{-1}$

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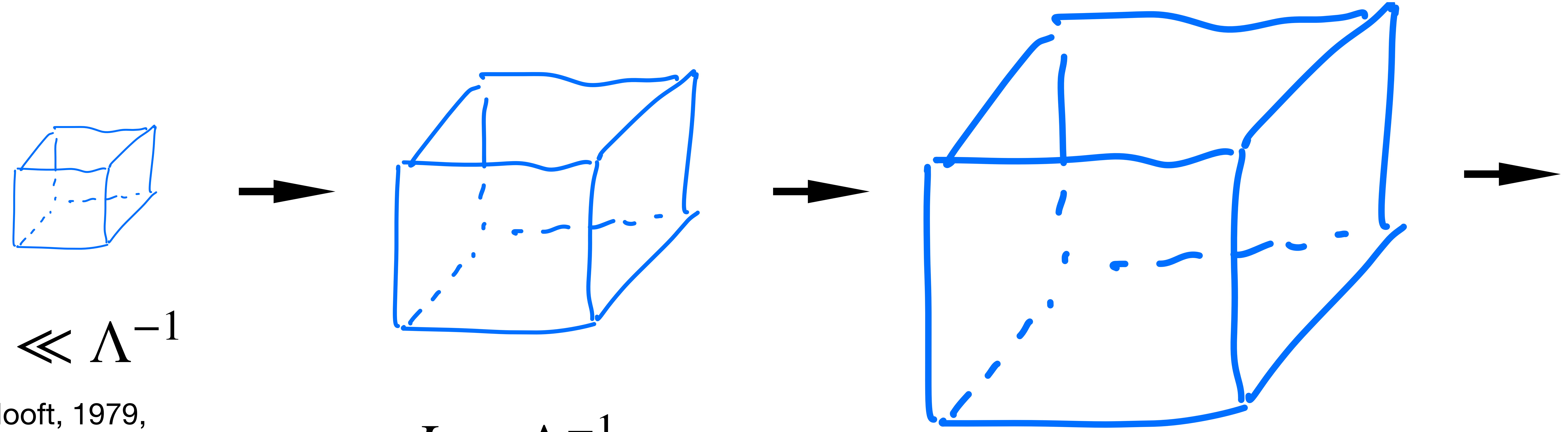


$\ldots \mathbb{R}^3$

$L \ll \Lambda^{-1}$

't Hooft, 1979,
van Baal, 1980s
Witten, 1982

"twisted b.c."

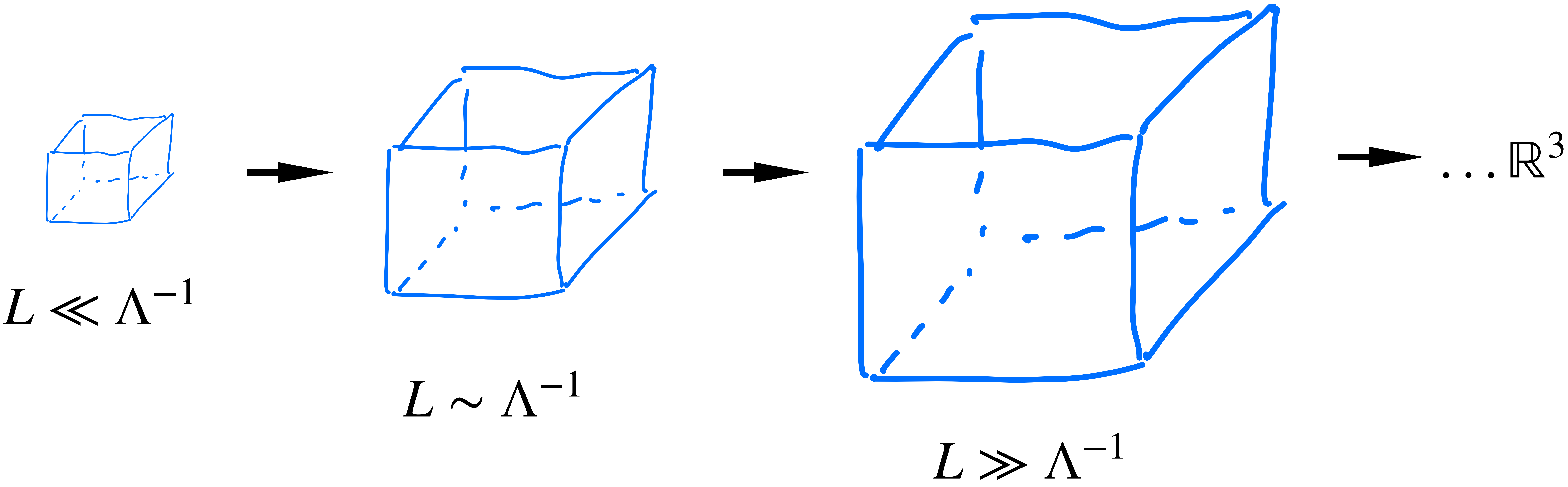
$L \sim \Lambda^{-1}$

$L \gg \Lambda^{-1}$

"center symmetry twists of the gauge bundle" *'t Hooft*

"background two-form gauge field for the one-form $\mathbb{Z}_N$ symmetry" *modern "generalized symmetry" language*

the femtouniverse and its twisted variants: 1975-2026...



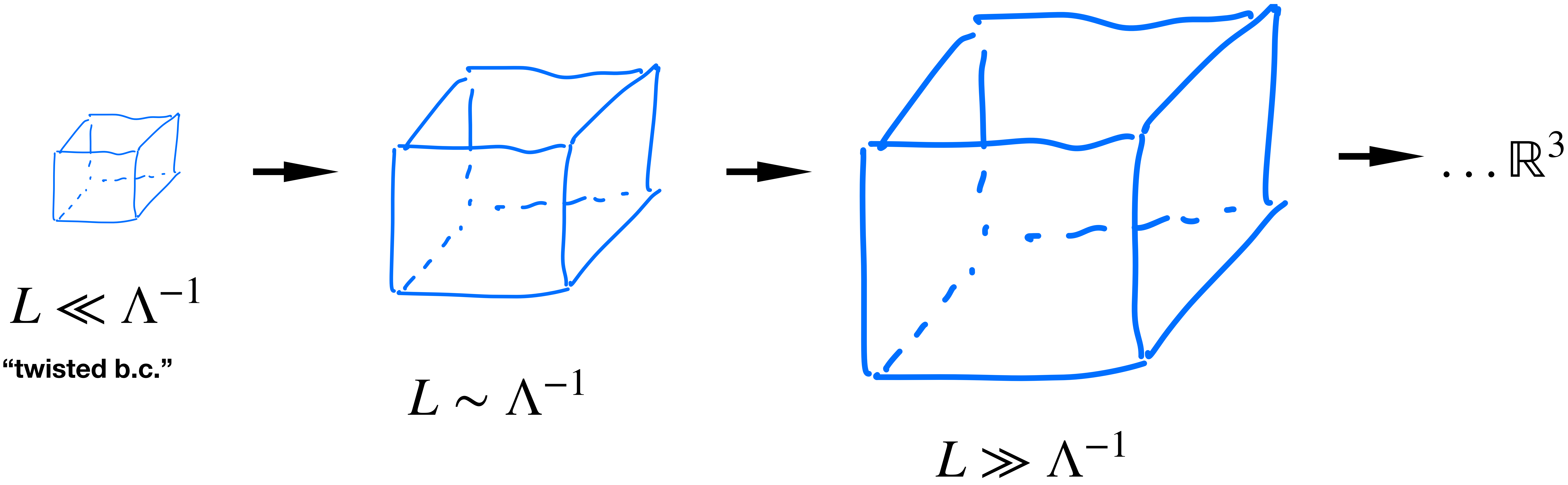
"twisted b.c."

"Madrid group" 1980s-1990s-...
González-Arroyo, García Pérez + ...

stress: this is pure YM theory, no extra fields to decouple

- *in SU(2) YM showed that confinement occurs at all L*
- *showed that on $\mathbb{R} \times \mathbb{T}^3$, at small-L, area law due to* **fractional instantons**
- *showed continuous,* w/out phase transition, *connection to the large-L limit (lattice)*

the femtouniverse and its twisted variants: 1975-2026…



more recent work showed that, with appropriate twists,
the continuous connection to $\mathbb{R}^{1,3}$ continues in various asymmetric $\mathbb{T}^3$ limits

Ünsal, Ünsal-Yaffe + 2007, 2008+…

González-Arroyo, Bergner + … 2000 … 2025

Cox, Wandler, EP… 2021, 2022

Ünsal et al, Tanizaki et al 2020, 2022, 2024, 2025

Anber, EP, et al. 2022-2025…

twisted variants of the femtouniverse as per 2026:

$$\mathbb{R} \times \mathbb{T}^3 \to \mathbb{R}^2 \times \mathbb{T}^2 \to \mathbb{R}^3 \times \mathbb{S}^1_*$$

fractional instantons — **center vortices** — **monopole-instantons**

topological objects responsible for area law for Wilson loop

- in each case, if size of compact space is $L \ll \Lambda^{-1}$ semiclassics holds

- common feature is that in all these cases objects responsible for area law have fractional topological charge

- in addition, these topological excitations continuously morph into each other as successive radii decompactified

this is recent/ongoing work by groups showed above… will only give flavor, one example

a dilute gas of fractional instantons has been analytically shown to cause confinement in pure YM in several theoretically-controlled semiclassical settings:

$\mathbb{R} \times \mathbb{T}^3_{n_{12}}$

't Hooft twisted b.c. in 1-2 plane

$\mathbb{R}^2 \times \mathbb{T}^2_{n_{12}}$

- in each case, the size L of the compact space is small, i.e. $LN\Lambda \ll 1$

- the 't Hooft twist (n_{12}) or the adjoint fermions ($_*$) ensure that no deconfinement phase transition takes place at small L

$\mathbb{R}^3 \times \mathbb{S}^1_*$

need adjoint fermions (or double-trace deformation)

a dilute gas of fractional instantons has been analytically shown to cause confinement in pure YM in several theoretically-controlled semiclassical settings:

at a scale $1/(LN) \gg \Lambda$

$\mathbb{R} \times \mathbb{T}^3_{n_{12}}$ →

$\mathbb{R}^2 \times \mathbb{T}^2_{n_{12}}$ →

$SU(N) \to Z_N$

IR: 1d/2d Z_N TQFT **+ fractional instantons** or **center vortices**

[IR: "empty" + topological solitons (instantons)]

$\mathbb{R}^3 \times \mathbb{S}^1_*$ ⟶ $SU(N) \to U(1)^{N-1}$

IR: 3d U(1) with no light charged matter **+ monopole-instantons**

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$\mathbb{R}^2$ - theory (ignore $\mathbb{T}^2$ modes, gapped $1/(LN)$) has N classically degenerate ground states

- degeneracy lifted by tunneling (fractional instantons/center vortices)

- this leads to area law! - **two ways to show…**

a.)slick

b.)less so…

a.)slick

$\mathbb{R} \times \mathbb{T}^3_{n_{12}}$

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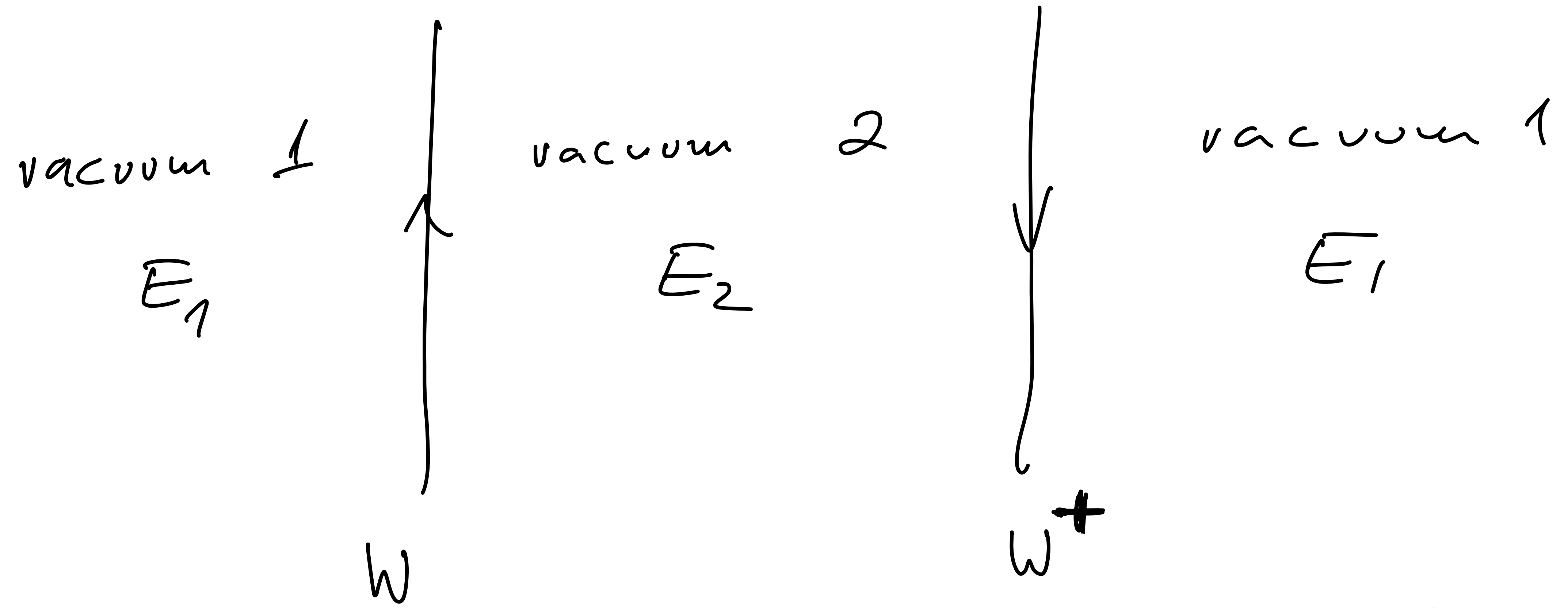

① $E_2 > E_1$ due to tunneling

② W separates vacua k & k+1

(for a slick generalized symmetry perspective: Nguyen, Sulejmanpasic, Unsal, 2401)

$\mathbb{R} \times \mathbb{T}^3_{n_{12}}$

$\mathbb{R}^2 \times \mathbb{T}^2_{n_{12}}$

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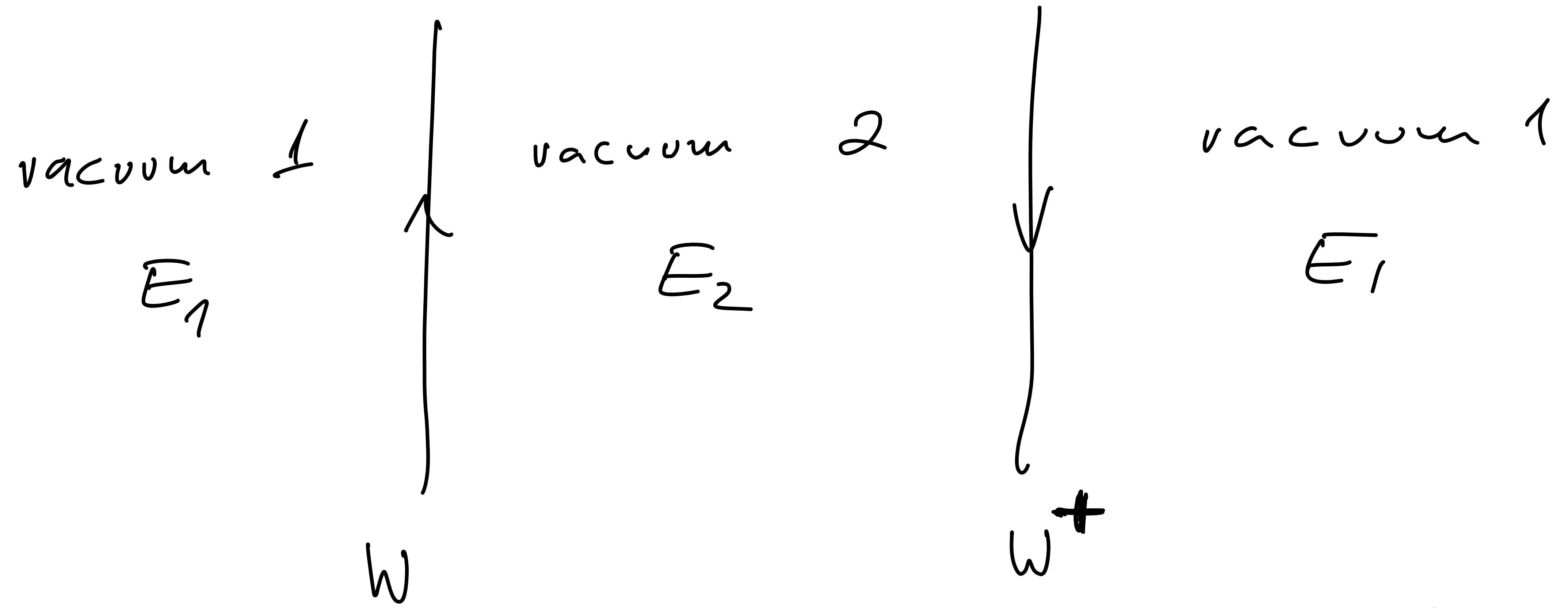
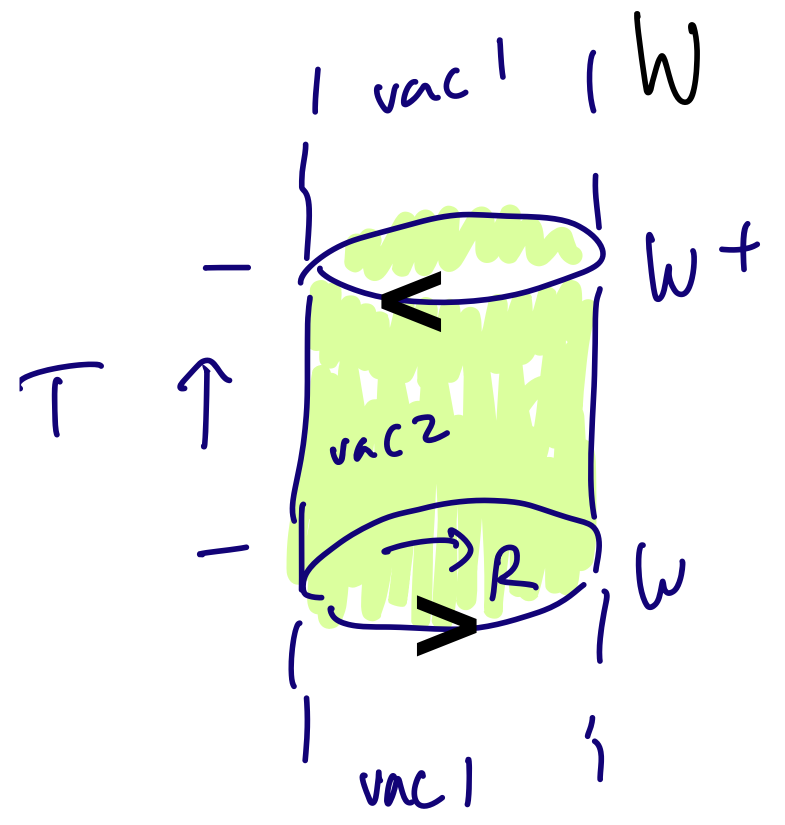
- degeneracy lifted by tunneling (fractional instantons/center vortices)

$\mathbb{R}^2$ Wilson loop is codim 1 ("fills" space or time)



① $E_2 > E_1$ due to tunneling

② W separates vacua k & k+1



$$\langle W(0)\, W^+(T) \rangle \sim e^{-TR(E_2 - E_1)}$$

a.)slick

a dilute gas of fractional instantons has been analytically shown to cause confinement in pure YM in several theoretically-controlled semiclassical settings:

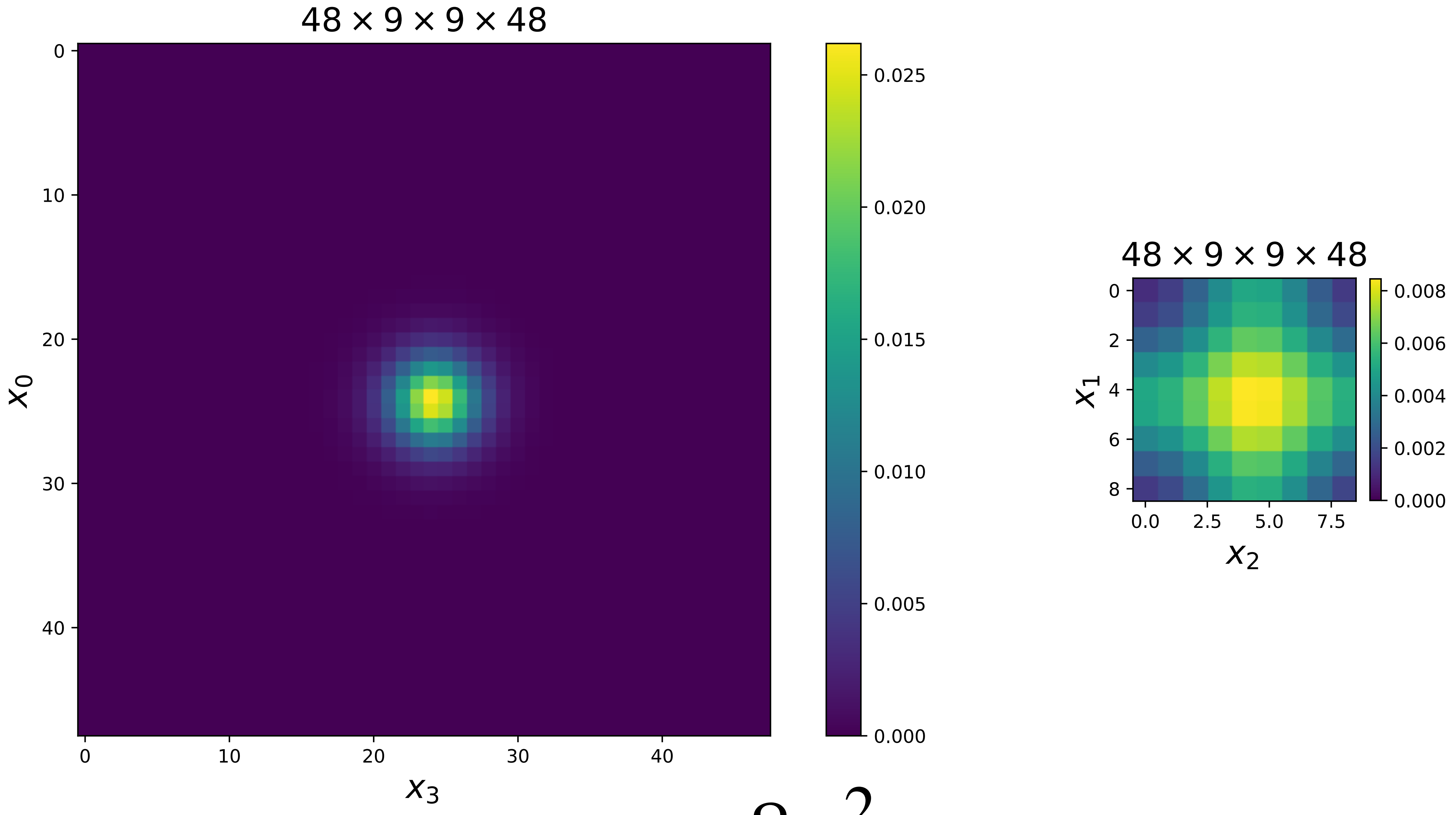
$\mathbb{R} \times \mathbb{T}^3_{n_{12}}$

$\mathbb{R}^2 \times \mathbb{T}^2_{n_{12}}$

$\mathbb{R}^3 \times \mathbb{S}^1_*$

EX.: center vortices

localized in $\mathbb{R}^2$, spread in (wrapped around) $\mathbb{T}^2$



size: $L_{\mathbb{T}^2}$ **action:** $\dfrac{8\pi^2}{g^2 N}$ **top. charge:** $\pm\dfrac{1}{N}$

no analytical solutions known; pic. taken from F.D. Wandler 2406.07636 …

b.)less so…

a dilute gas of fractional instantons has been analytically shown to cause confinement in pure YM in several theoretically-controlled semiclassical settings:

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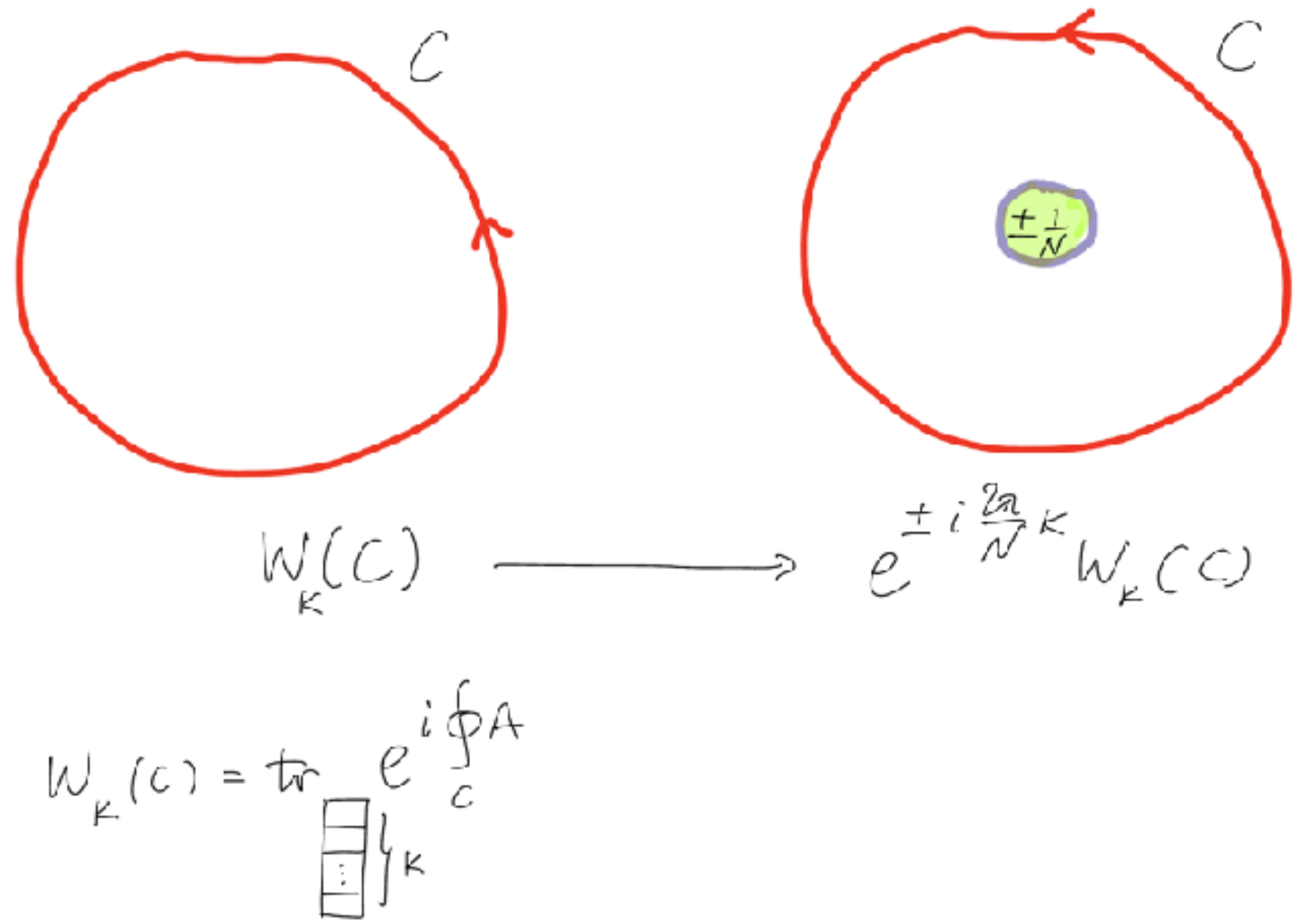

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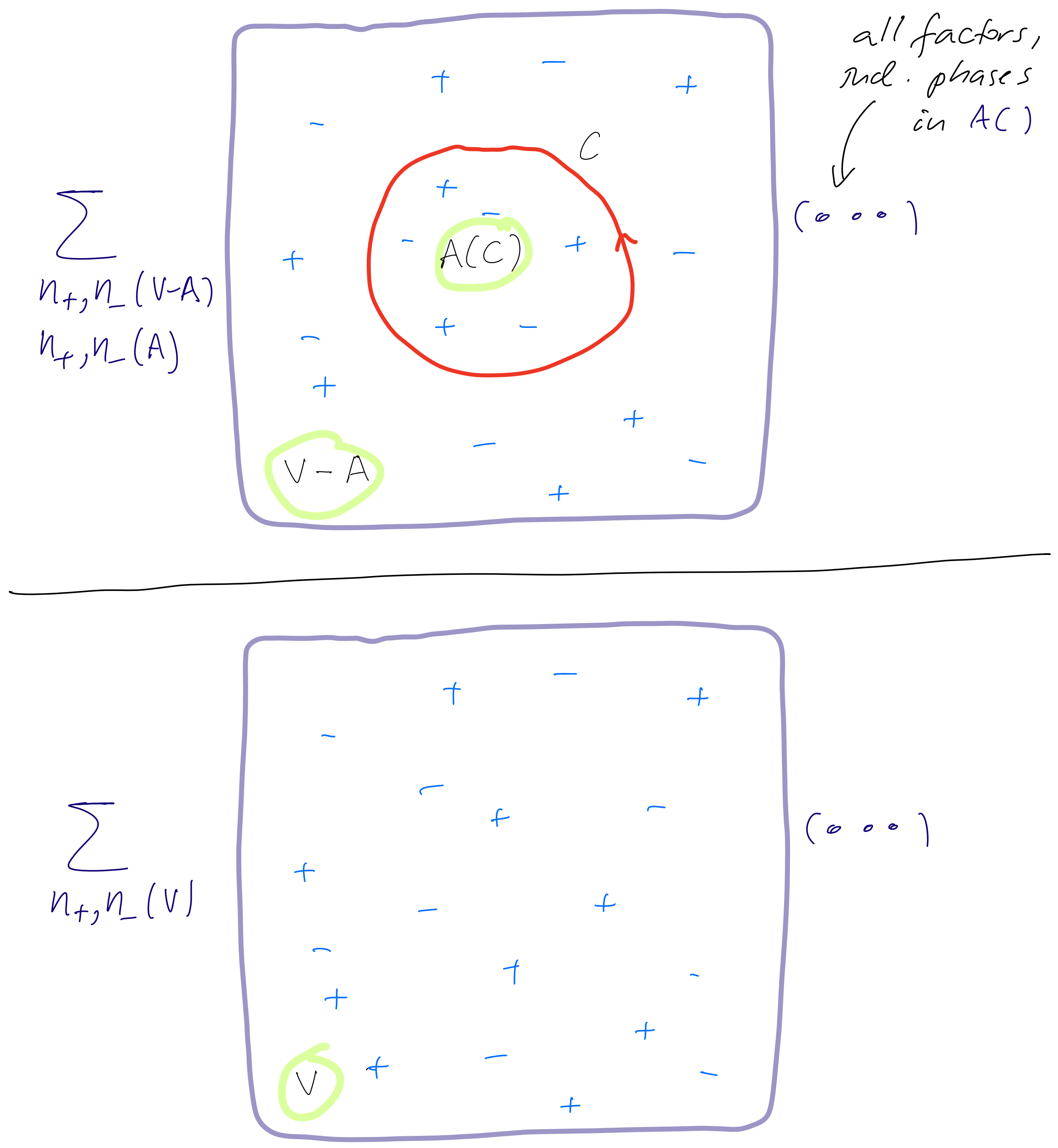
dilute gas of center vortices:

b.)less so…

$$\mathbb{R}^2 \times \mathbb{T}^2_{n_{12}}$$

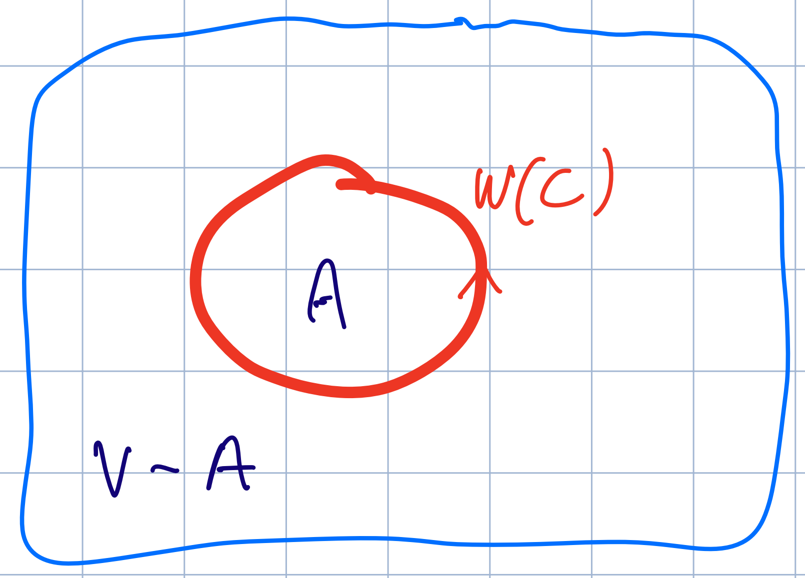
$$\langle W_k(C) \rangle =$$



…
skipping detail of evaluation
…

b.)less so…

here, sum over all topological sectors $\mathbb{Z}/N, \forall\ \mathbb{Z}$, is taken
[infinite volume limit… see details in Anber, EP 2025]



$$Z[A] = \sum_{n_2 \bar{n}_2} \sum_{n_1 \bar{n}_1} \left(\frac{A}{L^2}\right)^{n_1+\bar{n}_1} \frac{1}{n_1!\,\bar{n}_1!} e^{-S_0(n_1+\bar{n}_1)} e^{i\frac{2\pi k+\theta}{N}(n_1-\bar{n}_1)} \times$$

$$\times \left(\frac{V-A}{L^2}\right)^{n_2+\bar{n}_2} \frac{1}{n_2!\,\bar{n}_2!} e^{-S_0(n_2+\bar{n}_2)} e^{i\frac{\theta}{N}(n_1-\bar{n}_1)}$$

$$= e^{\frac{A}{L^2} e^{-S_0}\left(e^{i\frac{2\pi k+\theta}{N}} + e^{-i\frac{2\pi k+\theta}{N}}\right)} \times$$

$$\times e^{\frac{V-A}{L^2} e^{-S_0}\left(e^{i\frac{\theta}{N}} + e^{-i\frac{\theta}{N}}\right)} =$$

$$= e^{e^{-S_0}\left(\frac{2A}{L^2}\cos\frac{2\pi k+\theta}{N} + \frac{2(V-A)}{L^2}\cos\frac{\theta}{N}\right)}$$

$$= e^{-e^{-S_0}\frac{2A}{L^2}\left(\cos\frac{\theta}{N} - \cos\frac{2\pi k+\theta}{N}\right) + e^{-S_0}\frac{2V}{L^2}\cos\frac{\theta}{N}}$$

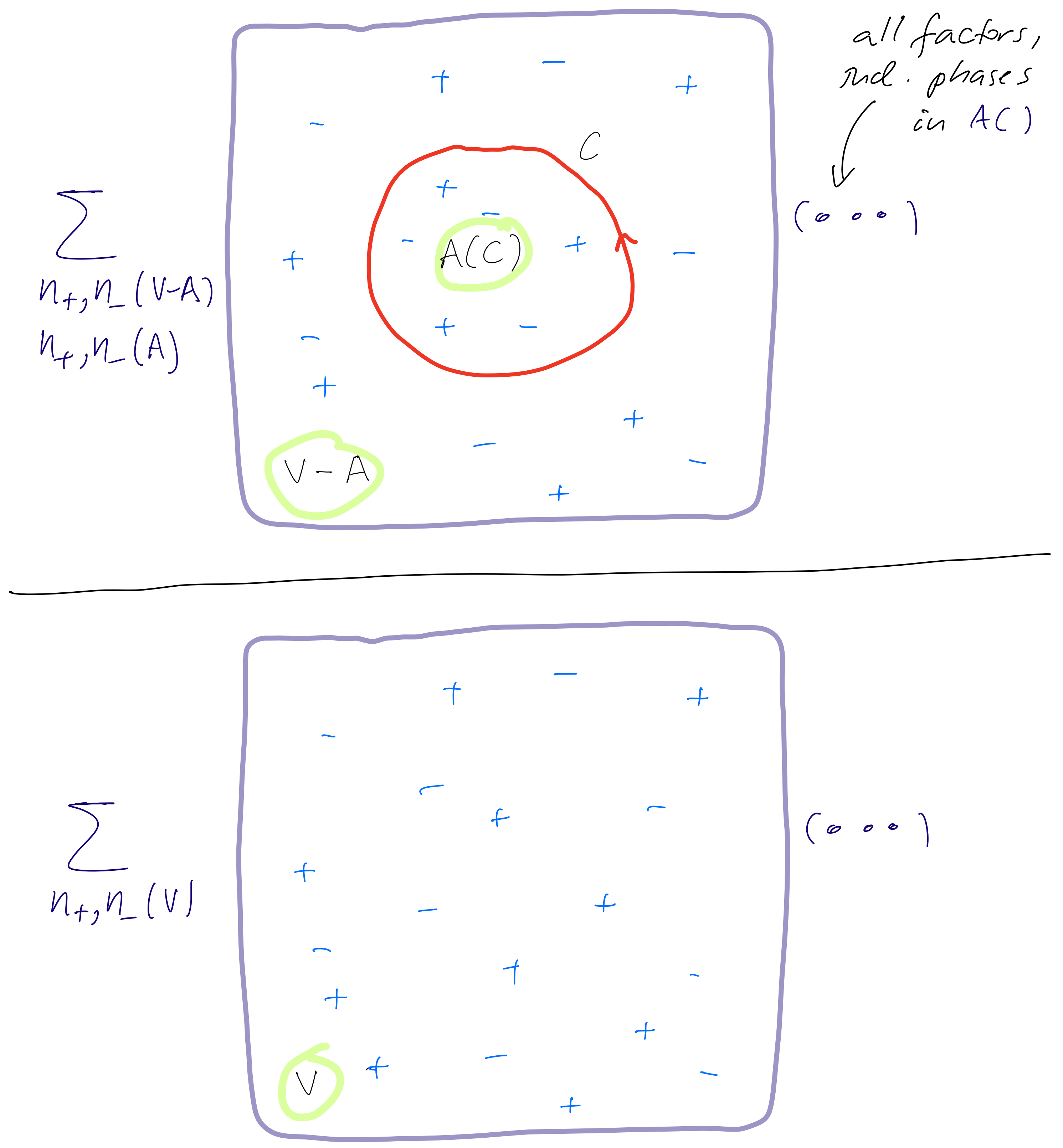
$$\frac{Z[A]}{Z[0]} = e^{-A\frac{2e^{-S_0}}{L^2}\left(\cos\frac{\theta}{N} - \cos\frac{2\pi k+\theta}{N}\right)} \qquad \theta \in [-\pi, \pi]$$

…
skipping detail of evaluation
…

dilute gas of center vortices:

b.)less so…

$$\langle W_k(C)\rangle =$$



$$\mathbb{R}^2 \times \mathbb{T}^2_{n_{12}}$$

k-string tension

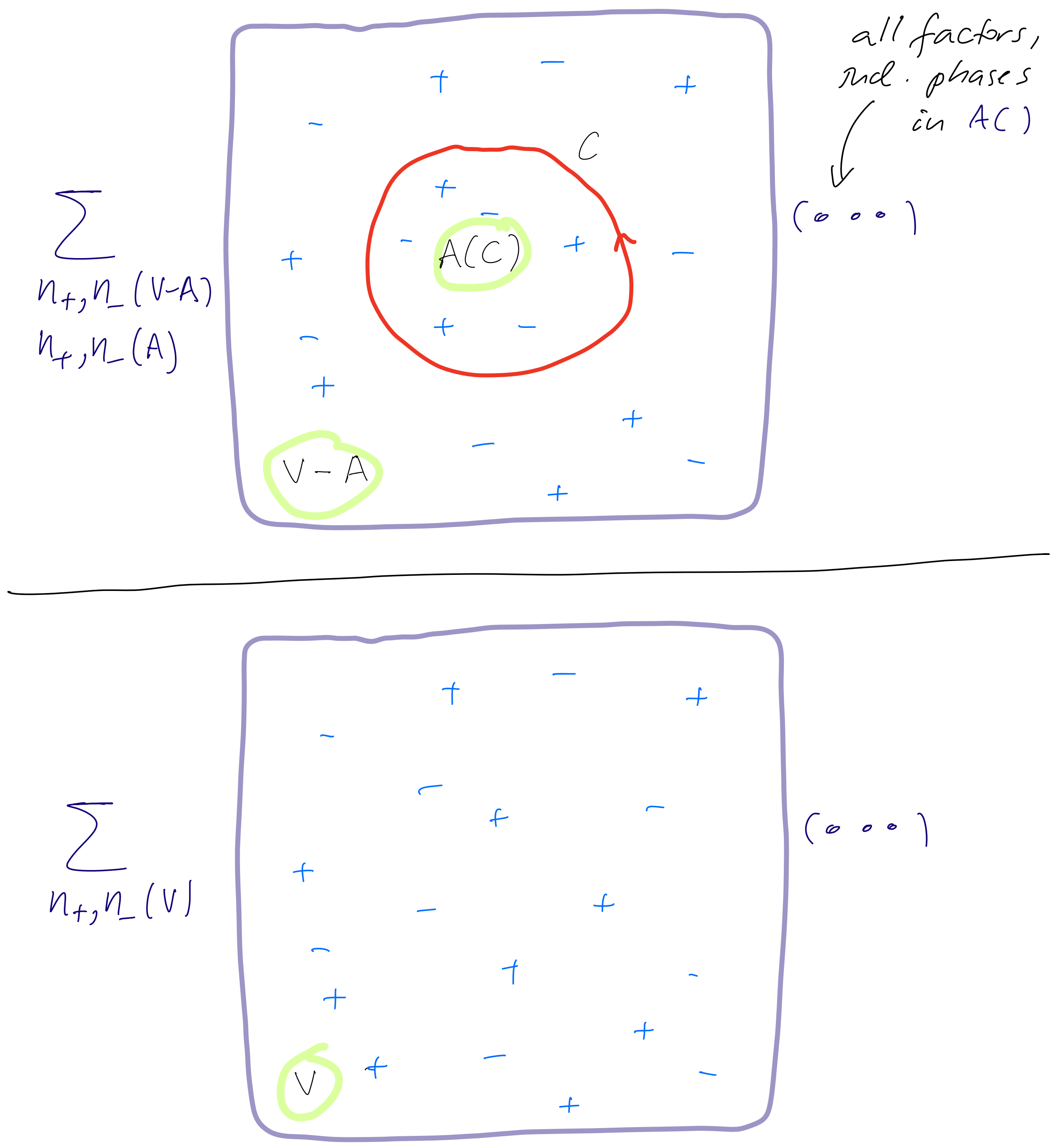
$$\langle W_k(C)\rangle = e^{-A(C)\,\frac{\#}{L^2} e^{-\frac{8\pi^2}{g^2 N}}\left[\cos\frac{\theta}{N} - \cos\frac{2\pi k+\theta}{N}\right]}\Bigg|_{\theta\in(-\pi,\pi]}$$

dilute gas of center vortices:

b.)less so…

$$\mathbb{R}^2 \times \mathbb{T}^2_{n_{12}}$$

$$\langle W_k(C) \rangle =$$



a semiclassical realization of 4d "center vortex" lattice mechanism DelDebbio, Faber, Giedt, **Greensite**, Olejnik 1997

see his book

k-string tension

$$\langle W_k(C) \rangle = e^{-A(C)\, \frac{\#}{L^2} e^{-\frac{8\pi^2}{g^2 N}} \left[\cos\frac{\theta}{N} - \cos\frac{2\pi k + \theta}{N} \right]} \Bigg|_{\theta \in (-\pi, \pi]}$$

area law is shown similarly in the other cases…

twisted variants of the femtouniverse as per 2026:

$$\mathbb{R} \times \mathbb{T}^3 \rightarrow \mathbb{R}^2 \times \mathbb{T}^2 \rightarrow \mathbb{R}^3 \times \mathbb{S}^1_*$$

↙ **fractional instantons** ↓ **center vortices** ↘ **monopole-instantons**

TIME TO SUMMARIZE/CONCLUDE…

SUMMARY:

1 **confinement is a nonperturbative, strong coupling phenomenon, which so far has defied analytic treatment in pure YM theory on $\mathbb{R}^4$**

2 **it can be demonstrated in highly SUSY theories, using the "power of SUSY"**
Ward identities, holomorphy, complex elliptic curves

- relation to microscopic UV nonabelian theory not explicit
- on approaching pure-YM limit analytic control lost
- hard to study on lattice

3 **can be shown semiclassically in partially compactified theories with twisted b.c.**

- on approaching large-volume limit analytic control lost
- $\mathbb{R}^4$ limit can be studied on lattice

rich structure of topological excitations: all appear related, many details and mathematically intriguing features remain to be uncovered