UNIVERSITY OF TORONTO
Faculty of Arts and Science

APRIL/MAY 2009 EXAMINATIONS

PHY 351H1 S

Duration - 3 hours.

Aids allowed: one 8 $\frac{1}{2}$"×11" sheet of paper, double-sided, hand- or computer- written

I. A rigid cylinder of radius R and mass μ has a moment of inertia I around an axis going through the center of mass and parallel to the central axis of the cylinder. The cylinder is homogeneous along its central axis, but not in the radial and angular directions. Thus, its center of mass is displaced a distance a ($0 < a < R$) from the central axis. The cylinder can roll without slipping on a plane and dissipation due to friction is assumed to be negligible. The plane is inclined on an angle θ with respect to the horizontal and the central axis of the cylinder is parallel to the horizontal plane.

1. Express the speed of the center of mass of the cylinder in terms of its angular velocity, angle of rotation, and the geometric parameters a, R, I.

   6 points

2. Find the kinetic energy of the cylinder.

   5 points

3. Find the potential energy of the cylinder.

   6 points

4. Find the Euler-Lagrange equation describing the motion of the cylinder.

   5 points

5. Discuss qualitatively the motion of the cylinder for $\theta = 0$ and $\theta \neq 0$ and describe how the two cases differ.

   8 points

Total marks for **I.***: 30 points*

II. A particle moves in a circular orbit in the Kepler potential, $U(r) = -\frac{k}{r}$. Show that if k suddenly decreases to half its original value, the particle's orbit becomes parabolic.

Total marks for **II.***: 20 points*

III. A particle of mass m is subject to a potential force $U(|\vec{r}|)$. Consider a frame of reference uniformly rotating around $\vec{r} = 0$ with angular velocity $\vec{\Omega}$. Find the Lagrangian and then the Hamiltonian of the particle in the rotating frame of reference. Explain the meaning of any $\vec{\Omega}$-dependent terms in the Hamiltonian you obtain.

Total marks for **III.***: 20 points*

IV. Consider the scattering of a particle of mass m and energy E in an attractive potential well described by a potential $U(\vec{r}) = -V_0$ if $|\vec{r}| < R$ and $U(\vec{r}) = 0$ if $|\vec{r}| > R$.

1. Find the differential scattering cross section and sketch its angular dependence.

   22 points

2. What is the total scattering cross section? Explain.

   8 points

Total marks for **IV.***: 30 points*

Total marks for the exam 30 + 20 + 20 + 30 = 100

Total number of pages = 2