Midterm Exam - makeup, PHY 351S, Advanced Classical Mechanics

Wednesday, March 4, 2009

1.

Two particles have masses m and $5m$, respectively. Let them be subject to the same external force $\vec{F} = -\vec{\nabla}U(x, y, z)$ that leads them to perform a closed-orbit motion. What is the ratio of the velocities of the two particles for trajectories of the same energy?

2.

A system consists of three particles of equal mass m. Every pair of particles interacts via a two-body force due to the potential $V(r) = -ge^{-r/a}$, where r is the distance between the two particles. In addition, two of these particles (say 1 and 2) interact with the third due to a generalized velocity-dependent "potential" added to the Lagrangian, $U(\vec{r}, \vec{v}) = -f\vec{v} \cdot \vec{r}$, where $\vec{v}, \vec{r}$ are the relative velocity and distance of the relevant two particles, respectively.

1. Write down the Lagrangian of this system using the center of mass coordinate R and two relative coordinates $\vec{\rho}_1 = \vec{r}_1 - \vec{r}_3$ and $\vec{\rho}_2 = \vec{r}_2 - \vec{r}_3$.
2. What are the Euler-Lagrange equations?
3. Is the angular momentum of this system conserved?
4. Is the energy conserved?

3.

Imagine that a planet's radius is α times smaller than Earth's and that its mass is β times smaller. Find the ratios of the following quantities for this planet to that of the Earth's:

1. The gravitational acceleration g.
2. The minimal speed one needs to throw a body on the surface so it begins to orbit the planet.
3. The minimal speed one needs to throw a body so it escapes to infinity.

4.

A particle of mass m is constrained to move on the surface of a cone of vertical angle 2α, which is placed vertically and with vertex downwards in a gravitational field.

1. What is the Lagrangian?
2. What are the integrals of motion?
3. Reduce the problem to a one-dimensional one and qualitatively describe the motion of the particle.
4. Write down an implicit solution of the equations of motion, allowing in principle to deduce the form of the trajectory.