UNIVERSITY OF TORONTO
Faculty of Arts and Science
APRIL 2010 EXAMINATIONS
PHY 354H1 S
Duration - 3 hours.
Aids allowed: one 8 $\frac{1}{2}$"×11" sheet of paper, double-sided, hand- or computer- written

I. Determine the effective cross-section for a particle to "fall" to the centre of the field $U(r) = -\frac{\alpha}{r^2}$.

Total marks for **I.**: *22 points*

II. Consider a molecule consisting of N atoms arranged on a straight line. Let the atoms have masses m_a $(a = 1, \ldots N)$ and let the distance between the a-th and b-th atom be l_{ab}. Show that the principal moments of inertia are:

$$I_1 = I_2 = \frac{1}{\mu} \sum_{a \neq b} m_a m_b l_{ab}^2 \ , \quad I_3 = 0 \ ,$$

where $\mu = \sum_a m_a$ is the total mass of the molecule.

Total marks for **II.**: *20 points*

III. Answer these questions as briefly as you can:

1. The length of the arm of a harmonic pendulum is increasing in time. Does the energy of the pendulum increase or decrease?

   5 points

2. Why are the Euler angles the most useful parameterization of orientation?

   4 points

3. Is the angular momentum of a satellite orbiting the Earth conserved?

   4 points

4. How does the escape velocity from a planet depend on its mass and radius?

   4 points

5. For what central potentials in three spatial dimensions are the bound orbits closed?

   3 points

6. A free rigid body is constrained so that one point on the body, say P, is fixed. What are the integrals of motion? In particular, what if the body is symmetric with respect to an axis going through P?

   5 points

7. What is the ratio of the Coriolis forces on a particle moving north at the same latitude and at the same speed on two planets of different mass, radius, and angular velocity of rotation around their axes?

   5 points

Total marks for **III.**: *30 points*

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IV. A rigid cylinder of radius R and mass μ has a moment of inertia I around an axis going through the center of mass and parallel to the central axis of the cylinder. The cylinder is homogeneous along its central axis, but not in the radial and angular directions. Thus, its center of mass is displaced a distance a $(0 < a < R)$ from the central axis. The cylinder can roll without slipping on the inside of a cylindrical surface of radius ρ, assume that $\rho > R$. Dissipation due to friction is assumed to be negligible. The central axis of the cylinder is parallel to the horizontal plane (needless to say, there is a homogeneous gravitational field present). See the figure below.

1. Express the speed of the center of mass of the cylinder in terms of its angular velocity, angle of rotation, and the various geometric parameters of the problem.

   6 points

2. Find the kinetic energy of the cylinder.

   5 points

3. Find the potential energy of the cylinder.

   5 points

4. Find the Euler-Lagrange equation describing the motion of the cylinder.

   5 points

5. Describe qualitatively the motion of the cylinder.

   7 points

Total marks for **IV.**: *28 points*

Total marks for the exam 22 + 20 + 30 + 28 = 100

Total number of pages = 2