UNIVERSITY OF TORONTO
Faculty of Arts and Science
APRIL 2011 EXAMINATIONS
PHY 354H1 S
Duration - 3 hours.
Aids allowed: one 8 $\frac{1}{2}$"×11" sheet of paper, double-sided, hand- or computer- written

I. Carefully enumerate all conditions that would make it possible for a particle of mass m moving in a central potential $U(r) = \frac{\alpha}{r^2}$ to fall into the center of the potential. Would such a fall take a finite or infinite time?

Total marks for **I.***: 10 points*

II. Consider the family of orbits in a central potential for which the total energy is constant (but the orbits have different values of angular momentum). If a stable *circular* orbit exists at $r = r_0$ with angular momentum M_0, argue that any other orbits of the family with $r \approx r_0$ must have angular momentum $M < M_0$.

Total marks for **II.***: 20 points*

III. Find the Hamiltonian of a heavy symmetric top with one fixed point (either derive from the Lagrangian, or simply write down). Write the Hamiltonian equations of motion and show that these equations can be solved "in quadrature."

Total marks for **III.***: 25 points*

IV. A particle of mass m and charge q is moving in a constant homogeneous magnetic field B in the z direction (for definitiveness, use the $A_x = -By$ gauge). Write down the Largangian and the Hamiltonian of the particle in a frame rotating with a uniform angular velocity Ω around the z axis. Find the special value of Ω for which the form of the Hamiltonian is simplest (be free to have physical intuition guide you; in the end, explain why the result should have been expected).

Total marks for **IV.***: 20 points*

V. A homogeneous cone (of mass density ρ, height h, and opening angle at the tip $\pi/6$) is rolling on a plane on its side with its tip at a fixed position. There is no sliding and no dissipation of energy due to friction. Express the kinetic energy of the cone as a function of ρ, h, and the appropriate dynamical variables.

Total marks for **V.***: 25 points*

Total marks for the exam 10 + 20 + 25 + 20 + 25 = 100

Total number of pages = 1