Midterm Exam, PHY 354S, Advanced Classical Mechanics
Friday, March 11, 2011
One double-sided $8'' \times 11''$ aid sheet allowed. No calculators.

1. A particle of mass m is constrained to move on the surface of a fixed sphere of radius R. The sphere is placed in a uniform homogeneous gravitational field $\vec{g}$.

1. What is the Lagrangian describing the motion of the particle?

   8 points

2. What are the integrals of motion? What are the symmetries responsible?

   8 points

3. Is this system integrable? Can you use the integrals of motion to find the trajectories "in quadratures"? (And if you think that the answer is "yes," please do so!)

   10 points

4. Describe qualitatively the kind of trajectories that the particle can have.

   10 points

2. Imagine that the only force acting on a charged particle is due to the electrostatic field of the charged objects described below. In each case, enumerate all conserved quantities you can think of.

1. Four charged particles fixed in space.

   8 points

2. A uniformly charged sphere pierced, through its center, by an infinitely long charged wire.

   6 points

3. An infinitely long charged tube with a triangular cross-section.

   5 points

4. An infinitely long charged tube with a circular cross-section.

   5 points

5. An infinitely long charged helix.

   8 points

3. Two particles have masses m and $4m$, respectively. Let them be subject to the same external force $\vec{F} = -\vec{\nabla}U(x, y, z)$, which leads them to perform bounded motion. What is the ratio of the velocities of the two particles for trajectories of the same energy?

12 points

4. A closed system of $n + 1$ particles consists of one particle of mass M and n particles of equal masses m. Find the Lagrangian in the center of mass frame, showing that the problem reduces to that of the motion involving n particles.

20 points

`Total number of points:  36 + 32 + 12 + 20 = 100.`