

UNIVERSITY OF TORONTO  
Faculty of Arts and Science  
APRIL 2011 EXAMINATIONS  
PHY 450H1 S

Duration - 3 hours.

Aids allowed: one 8  $\frac{1}{2}$ "  $\times$  11" sheet of paper, double-sided, hand- or computer- written

**I.** Consider a wave in vacuum, which is the superposition of two monochromatic plane waves with opposite wave vectors and of the same frequency, amplitude, and linear polarization (e.g., take  $\vec{E}$  for the two waves to be collinear).

1. Write down the general form of the electric field of this wave.
2. Determine the magnitude and direction of the Poynting vector of the superposed waves.
3. What is the average of the Poynting vector over a period? Explain the result.

*Total marks for I.: 20 points*

**II.** A particle of charge  $q$  is moving along a circle of radius  $a$  with constant speed  $v$ . Do not assume that  $v \ll c$ . For definiteness, let the circle lie in the  $x$ - $y$  plane and be centered at  $x = y = 0$  and let the particle be at  $x = a, y = 0$  at  $t = 0$ .

1. Find the electromagnetic four-vector potentials  $A^i$  near the center of the circle and use them to find the electric and magnetic fields at the center. What is your expectation for the  $v \rightarrow 0$  limit and does the result agree with it?
2. Focus now on the fields in the radiation zone. Use your knowledge of the radiation field of an electric dipole to sketch the intensity of the emitted radiation as a function of direction. (*In answering this part, preferably use written arguments and diagrams, not mathematical expressions.*)

*Total marks for II.: 25 points*

**III.** An incident plane electromagnetic wave is reflected off a wall. Let the angle of incidence be  $\theta$  and the reflection coefficient be  $R$  (the coefficient of reflection is simply the ratio of the intensity of the reflected and incident waves; as for any wave, the angles of incidence and reflection are equal). Find the pressure and any tangential forces that the wave exerts on the wall, proceeding in any order with the steps below:

1. Use your physical intuition.
2. Use the Maxwell stress tensor.

*Total marks for III.: 30 points*

**IV.** A photon bounces off an electron at rest. The photon's energy-momentum four-vector in the electron's rest frame is  $p^i = (\hbar\omega, \hbar c\vec{k})$ . Let the momentum  $\vec{k}'$  of the scattered photon be at an angle  $\theta$  with respect to  $\vec{k}$ . Find the frequency of the scattered photon as a function of  $\theta$  and other relevant parameters.

*Total marks for IV.: 25 points*

*Total marks for the exam 20 + 25 + 30 + 25 = 100*

*Total number of pages = 1*