Parity & chirality

in Newtonian mechanics $\vec{r} \to -\vec{r}$
is a symmetry of the fundamental equations;
same in E&M:

$$\ddot{\vec{r}} \sim \vec{E} + q\,\vec{v} \times \vec{B}$$

$$\vec{r} \to -\vec{r} \qquad \vec{v} \to -\vec{v}$$

$$\vec{E} \to -\vec{E} \qquad \vec{B} \to \vec{B}$$

"$\vec{E}$ = vector" "$\vec{B}$ = axial vector"

i.t.o. φ & $\vec{A}$, we have

$$\vec{r} \to -\vec{r}$$
$$\varphi \to \varphi$$
$$\vec{A} \to -\vec{A}$$

clearly

$$\mathcal{L}_{int.} \sim q(\varphi - \vec{v}\cdot\vec{A})$$

is then "P - invariant"

$$\mathcal{L}_{E.M.} \sim E^2 - B^2$$ => also "P - invariant"

($\vec{E}\cdot\vec{B}$ = "P - odd" - but total derivative!)

(2)

often formulated as "mirror symmetry"
(Kelvin)

instead of
$$x \to -x$$
$$y \to -y \quad : P$$
$$z \to -z$$

imagine mirror @ $x = 0$, so do
$$x \to -x$$
$$y \to y \qquad \text{"mirror"}$$
$$z \to z$$

"mirror" $=$ P + (rotn on π around x)

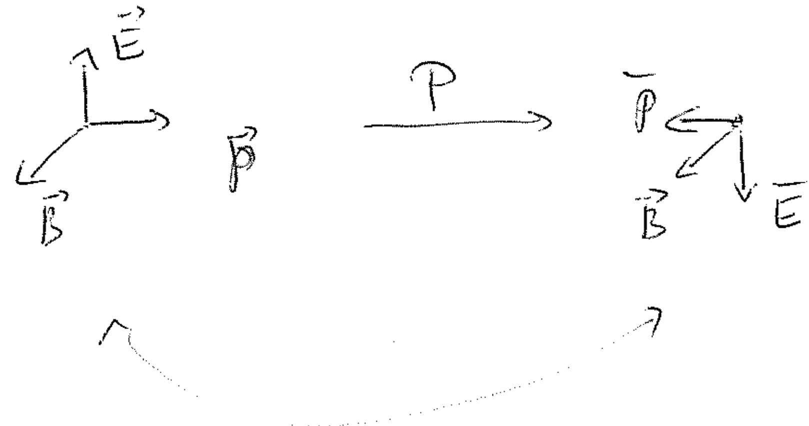
(so long as rotations are a symmetry, totally equivalent)

"chiral object" $\equiv$ one that can not be superimposed on its mirror image via continuous transforms (rotation, translation)

(Kelvin)

(e.g.  & r. hand.
l.h. (don't dare to draw)
)

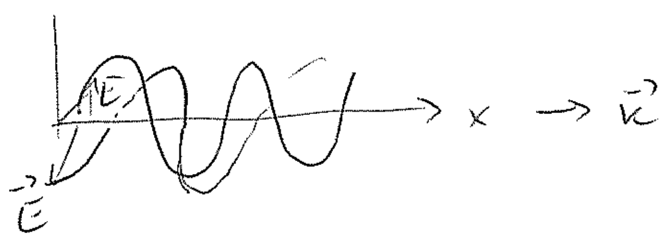
plane <u>linearly polarized</u> wave



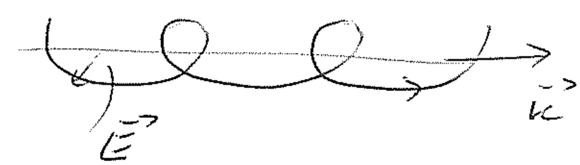
(these can be superimposed by rotation)

On the other hand, circularly polarized waves are chiral (l.h. $\xleftrightarrow{P}$ r.h.) a.

these are superpositions of plane polarized w/ $\pi/2$ phase shift :



so we have



(see §48 L&L)

(and an opposite handedness one)

E&M (& QED) "vectorlike"

not "chiral" (E.O.M. P-inv)

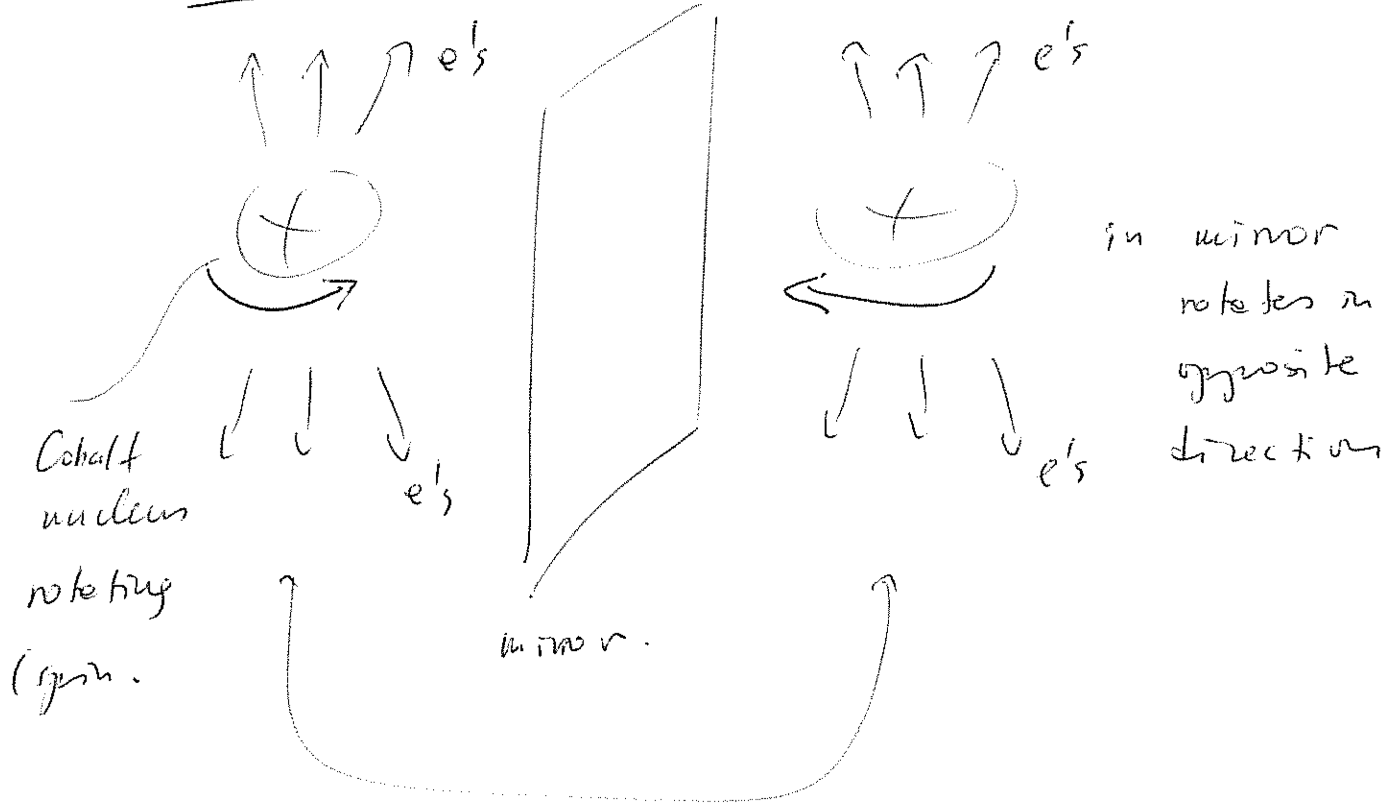
weak interactions "chiral" ⇒ L & R different properties →

Cobalt $\xrightarrow{\text{"}\beta\text{ decay"}}$ Nickel + electron + antineutrino

27 protons → 28 protons

33 neutrons → 32 neutrons

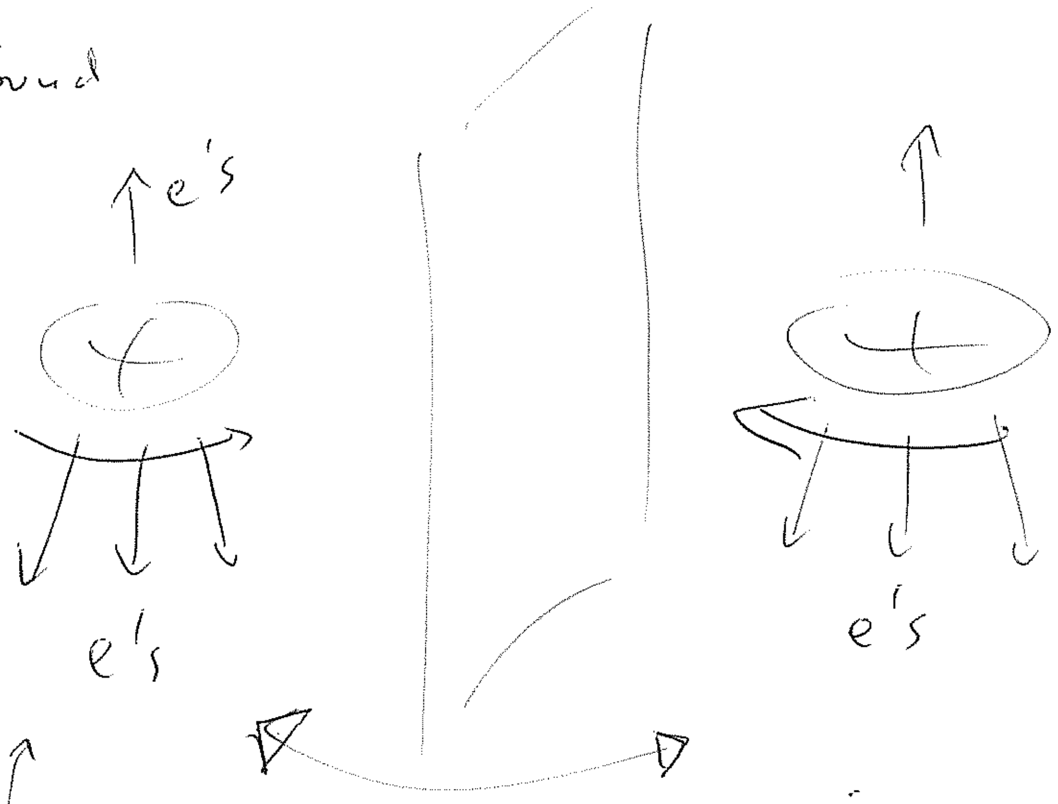
If P was a symmetry, we'd have:



the point is that in both original & mirror, electrons come out symmetrically w.r.t spin of Cobalt nucleus, so there's no difference w/ mirror image (as per Kelvin).

But, M^{me} Wu (1957)

found



more e's along spin of Co

mirror image is <u>different</u>, can't be superimposed w/ original.

<u>Cause:</u> at the level of Eq. of Motion, there's no $L \Leftrightarrow R$ symmetry in weak interaction (very different from Maxwell.)

But, a <u>tiny</u> effect ("weak" interactions)

You may ask, how is this different

from having a l.h. polarized wave

$\updownarrow P$

r.h. polarized wave, ... ?

The point of symmetries vs. non-symmetries of E.O.M. is that an initial state which is P-symmetric(*) can evolve into a state which is not P-symmetric (i.e. the one on p. 6)

(*) as the state of Co^{57} before decay, it has a definite parity

(within Maxwell's theory this will never occur, as laws are P-inv.)