# Lattice chirality and the Eichten-Preskill dream

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hep-lat/0701004  JHEP10(2007)076

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arXiv:0706.1043 [hep-th]  JHEP08(2007)081
arXiv:0801.0587 [hep-lat] IJMPA in press - explanatory/defense from misguided criticism
arXiv:0812.xxxx to appear

# Lattice chirality and the Eichten-Preskill dream

lattice chirality, anomaly matching, and more on the decoupling of mirror fermions

Erich Poppitz

This talk mostly about our more recent work:

with Yanwen Shang

Toronto

arXiv:0706.1043 [hep-th] JHEP08(2007)081

arXiv:0812.xxxx to appear

Thanks to:

- Maarten Golterman and Yigal Shamir for many insightful comments and discussions

- Canadian Institute for Theoretical Astrophysics (CITA) for allowing use of computer cluster for our frivolous games

This talk is about studying the strong dynamics of chiral gauge theories

Note that we are rather ignorant in this regard:

1980s - tumbling, massless composite fermions
(Dimopoulos, Raby, Susskind)

...

1990s - some progress in SUSY chiral gauge theories, aided by the "power of holomorphy"
(Seiberg + ...)

...

2008 - semiclassical studies on $R^3 \times S^1$, role of monopoles, KK monopoles, "bions" and other "oddballs"...
(Ünsal; Shifman, Ünsal +...)

Now, why should we care about strong chiral gauge dynamics?

## *What will the LHC discover?*

resonaances.blogspot.com

# *Blogger Jester says:*

Here are my expectations.
The probabilities were computed using all currently available data and elaborated Bayesian statistics.

*Higgs boson. Probability 80%*
...
*Non-SM Higgs boson. Probability 50%*
...
*New Beyond SM Particles. Probability 50%*
...
*Strong Interactions. Probability 20%*

Nature has repeated this scenario all over again: interactions between fundamental constituents become strong and new collective degrees of freedom emerge. Condensed matter physicists see it everyday in their laboratories. In particle physics, the theory of quarks and gluons known as QCD at low energies undergoes a transition to a confining phase, where it is more adequately described by mesons and baryons. It is conceivable that some of the Standard Model particles also emerge in this manner from a TeV-scale strongly interacting dynamics. **The problem is that we should have already seen the hints of the composite structure in low-energy precision tests, flavor physics and so on, but we see none of that. The reason why the probability for this scenario remains relatively high is our shameful ignorance of strongly interacting dynamics - we might have easily missed something**.

*Dark matter. Probability 5%*
...
*Little Higgs and friends. Probability 1%*
...
*Supersymmetry. Probability 0.1%*
...
*Dragons. Probability $e^{-S(dragon)}$*
...
*Black Holes. Probability $0.1*e^{-S(dragon)}$*
...

- what tools do we currently have to study strong **chiral** gauge dynamics?

| tools one trusts | tools you don't really know whether to trust<br>unless confirmed by other means - experiment, simulations, or the tools on the left - i.e. the chiral equivalent of "voodoo QCD" |
|---|---|
| 't Hooft anomaly matching<br><br>in SUSY aided by "power of holomorphy"<br><br>semiclassical expansions | "MAC"<br><br>~ truncated Schwinger-Dyson equations |

- evidently, there's not much there...

here comes our interest in the lattice:

- lattice gives a nonpertubative definition of the theory and thus provides for some important rigorous results in gauge theories
  - e.g., about continuity between Higgs/confining phases

- the lattice is a controlled, first principle, way to precisely calculate many things in QCD - though not all! - notably, spectra and various matrix elements

can one apply similar methods to chiral gauge theories?

will NOT talk about LHC physics via strong chiral gauge dynamics
will not discuss a potential theory of the real world
e.g., I won't tell you (today) which chiral gauge theory breaks EW symmetry with small S-parameter...

I'd like to tell you - mostly in "pictures" - where the lattice chiral gauge theory problem is at, and about our attempts at progress

I think that it is a theoretically appealing problem, fun to think about
and that doing this may even turn out to be useful - in the (very) long run, of course
many tools come together - both theoretical and "experimental"

## THIS TALK's GOALS and rough **OUTLINE:**

will NOT talk about LHC physics via strong chiral gauge dynamics

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and that doing this may even turn out to be useful - in the (very) long run, of course

many tools come together - both theoretical and "experimental" the approach I'll describe today is a combination of "old" and "new"

will put in larger perspective shortly, motivating the pursuit of this line of thought

*the idea goes like this:*

formulating vectorlike gauge theories (like QCD) on the lattice is not too much of a problem - there are doublers, of course, but we've learned...

so, one can ask a natural question -

can one start with a vectorlike theory, for example:

SU(5) with $\underbrace{\begin{matrix} 5^* \\ 10 \end{matrix}}_{\text{"light"}} \quad \underbrace{\begin{matrix} 5 \\ 10^* \end{matrix}}_{\text{"mirror"}} \Big\}$ all Weyl, L

names :

and then, deform the theory in such a way that

- mirrors decouple from the low-energy spectrum
- the gauge symmetry remains unbroken

?

before attempting to answer - WHY DO WE DO THIS?

## a lightning review of current situation with chiral lattice gauge theories

based on seminal works of

Ginsparg, Wilson (1982); *Callan, Harvey (1985);* D.B. Kaplan (1992); Narayanan, Neuberger (1994); Neuberger (1997); P. Hasenfratz, Laliena, Niedermaier (1997); Luescher (1998); Neuberger (1998),

Luescher has proven (1999-2000) that an exactly gauge invariant lattice action and measure exist **for an* anomaly free chiral gauge theory** based on the Neuberger-Dirac (or "Ginsparg-Wilson") operator D[A]

*... roughly:*

$$Z_{chiral}[A] = e^{i f[A]} \int \Pi \, dc \, d\bar{c} \;\; e^{\bar{c}^k \cdot D[A]_{kp} \cdot c^p}$$

- f[A] must be there, for gauge invariance, locality, smoothness wrt A

*- appropriate f[A] proven to exist for an anomaly free U(1) in finite V; SU(2)xU(1) in infinite V

- outside of perturbation theory, for a general gauge group, there is no explicit formulation of f[A]

fascinating theoretical achievement, but not good for practical use for a general gauge group

## before attempting to answer - WHY DO WE DO THIS?

because the measure ensuring gauge invariance is explicitly not known, one must nonperturbatively tune higher-dimensional gauge field operators to restore gauge invariance - nonperturbative tuning -

- usually considered an anathema

(however, see Golterman and Shamir, 1998+... - gauge-fixed construction, argued that only finite number of tunings )

turning back to gauge invariant formulations, it appears that a formal solution for the measure guaranteeing gauge invariance might be very hard to construct;
we may as well look for a dynamical one:

***attempting to construct a chiral lattice gauge theory by decoupling the mirrors from a vectorlike theory - where the measure is known explicitly - is worthwhile* and of possible practical importance***

---

*there may be other, not thought of yet ways to do this*!*

it was the work of Bhattacharya, Csaki, Martin, Shirman, Terning, (2005) to use warped domain walls + Higgsless ideas in this context that got us started on this ... may one day merit revisiting?

now, back to our question -

- can one start with a vectorlike theory, for example:

| | | | |
|---|---|---|---|
| SU(5) with | $5^*$ | $5$ | all Weyl, L |
| | $10$ | $10^*$ | |
| names : | "light" | "mirror" | |

and then, deform the theory in such a way that

- mirrors decouple from the low-energy spectrum
- the gauge symmetry remains unbroken

?

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- a normal continuum field theorist would say: no!
- a string theorist might say: may be
  if one allows the liberty to think of orbifolding as decoupling of states
- lattice may afford new possibilities:

everybody knows that four-fermi interactions, if taken strong enough, break chiral symmetries

$$\frac{g}{\Lambda^2}(\bar\psi\psi)(\bar\psi\psi) \quad , \quad gN > 8\pi^2$$

as per the NJL "gap equation" *made "believable" via large-N, gN=const, limit, aka "mean field"*

- few continuum people know, however, that if one takes coupling even stronger, the theory enters a "**strong-coupling symmetric phase**," with only massive excitations and unbroken chiral symmetry

- why haven't most people heard about these phases?

    because these phases are a "lattice artifact" - the physics is that of "lattice particles" with small hopping probability

    thus, these "lattice particles" are "heavier than the UV cutoff"
    think of an almost-insulator

I'm not sure who discovered them first

Eichten, Preskill (1986 paper on "Chiral gauge theories on the lattice")

- 4-fermi interactions ... [E-P]

A. Hasenfratz, Neuhaus (1988)

- strong Yukawa case - similar story

E-P story "retold" to suit my current need

$$SU(5) \qquad \underbrace{\begin{matrix}5^*\\10\\1\end{matrix}}_{\text{"light"}} \qquad \underbrace{\begin{matrix}5\\10^*\\1\end{matrix}}_{\text{"mirror"}}$$

$$g_1 \quad 10^* - 5 - 5 - 1$$

$$g_2 \quad 10^* - 10^* - 10^* - 5$$

"picture": strong interactions bind mirrors into vectorlike or singlet composites; these gain mass without breaking gauge symmetry

what makes one think these words are even remotely plausible?

a toy example with SU(4) "chiral" symmetry (the one to be gauged)

$$H_{4\psi} = \sum_x g\left(\psi_a \psi_b \psi_c \psi_d \, \epsilon^{abcd} + hc\right)$$

space lattice only (any dimension); canonical anticommutation relations:

$$\{\psi_a(x), \psi_b^+(y)\} = \delta_{ab}\,\delta_{xy}$$

at g>>1 in lattice units, hopping is negligible:

$$H = \sum_x \underbrace{H_{0\,x}}_{\text{4-fermi}} + \underbrace{H_1}_{\text{hopping} \sim \psi^+ \vec{\alpha}\cdot\vec{\nabla}\psi}$$

to leading order, at every site the same simple 4-fermion QM problem, rename:

$$\psi_{a\,x} \to a_a$$

$$\psi^+_{b\,x} \to a^+_b$$

$$H_0 = g\left(a_a a_b a_c a_d + a_a^+ a_b^+ a_c^+ a_d^+\right)\epsilon^{abcd}$$

| $2^4$ states | SU(4) | F |
|---|---|---|
| $\|0\rangle$ | $\|1\rangle$ | 0 |
| $a^+_a \|0\rangle$ | $\|4\rangle$ | 1 |
| $a^+_a a^+_b \|0\rangle$ | $\|6\rangle$ | 2 |
| $a^+_a a^+_b a^+_c \|0\rangle$ | $\|4^*\rangle$ | 3 |
| $a^+_1 a^+_2 a^+_3 a^+_4 \|0\rangle$ | $\|1'\rangle$ | 4 |

$\langle 1' | H_0 | 1 \rangle \sim g$ ONLY NONZERO

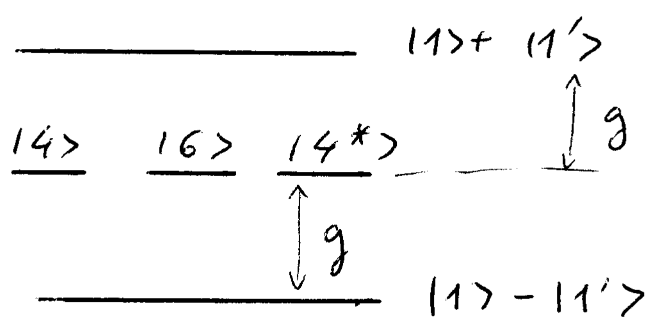
$\langle 4 | H_0 | 1 \rangle = \langle 4 | H_0 | 6 \rangle = \ldots = 0$

$H_o$ conserves F(mod 4); 16 states = |**1**> + |**1’**> + |**4**> + |**4***> + |**6**> under SU(4)

$H_o$ connects only |**1**> (= all fermions empty) and |**1’**> (= all fermions occupied)

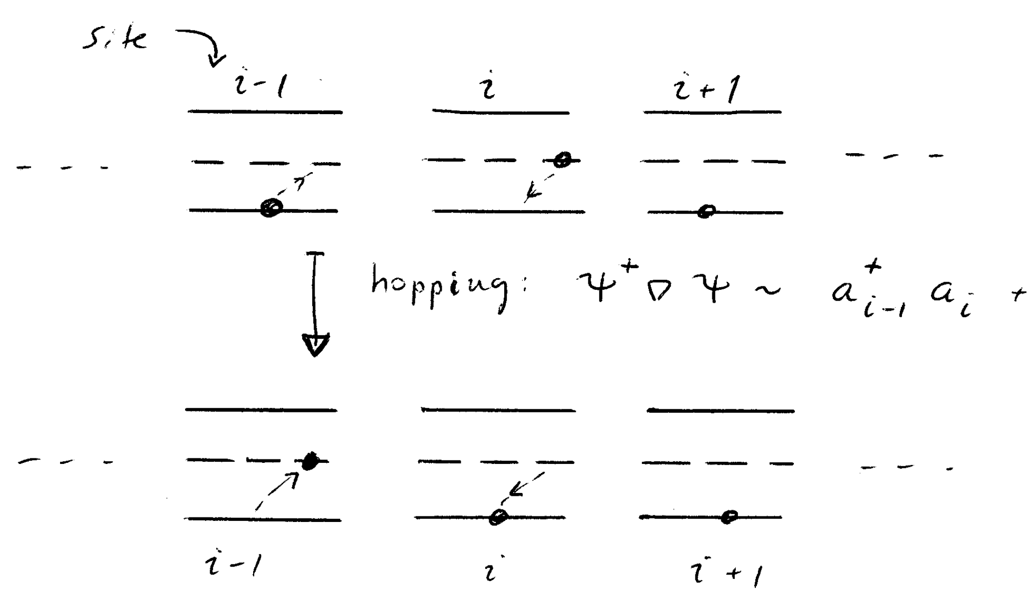
so:

(|**1**>-|**1’**>) has energy -g; (|**1**>+|**1’**>) has energy +g, |**4**>,|**4***>,|**6**> have energy 0.




in the infinite-g limit,
the lattice theory ground
state is unique and an SU(4) - “chiral” - singlet
with a mass gap = g in lattice units

at first order in 1/g, hopping turns on, site-localized states form bands and propagate




propagating states heavy
mass ~ g/a >> 1/a
a - the UV-cutoff

the 1/g (strong-coupling) expansion has finite radius of convergence,
hence this story represents the true ground state of theory, for g sufficiently large

- very much like "static limit" of lattice QCD, but infinite mass limit replaced by infinite four-fermi

- large-g phase same in any dimension provided static limit exists = unique ground state at every site; major differences between dimensions occur at small-g

- like high-T statistical mechanics where disorder always wins - neglect of kinetic terms = uncorrelated fluctuations at neigboring sites, maximum "entropy"

more relevant comments:

- we are not interested in a continuum limit of this "mirror" theory - everything "mirror" is cutoff scale and heavier and decoupled from IR physics... (ideally)

- gauge field appears only in hopping terms and so contributions of heavy "mirror" sector to gauge field action should be ~ 1/g our simple SU(4) exercise, with a bit more group theory, can be repeated for SU(5) of E-P

$$SU(5) \quad \underbrace{\begin{matrix} 5^* \\ 10 \\ 1 \end{matrix}}_{\text{“light”}} \quad \underbrace{\begin{matrix} 5 \\ 10^* \\ 1 \end{matrix}}_{\text{“mirror”}}$$

deformed by

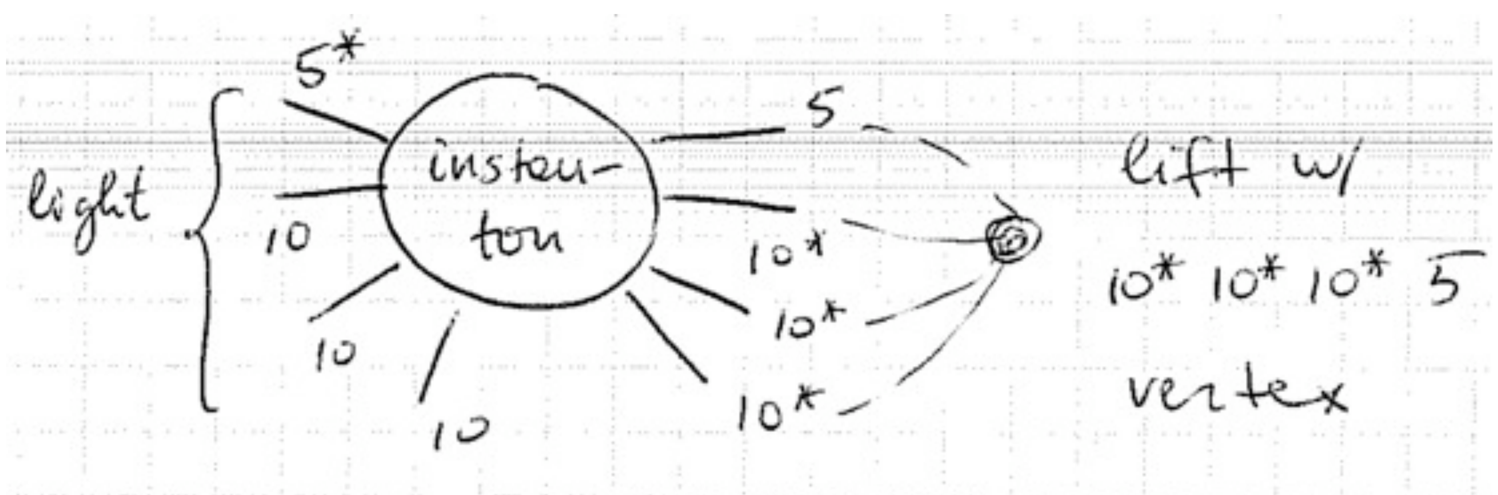
$$g_1 \quad 10^* - 5 - 5 - 1$$
$$g_2 \quad 10^* - 10^* - 10^* - 5$$

btw, singlet needed by E-P to have sensible “static limit” of Euclidean fermion path integral; E-P used Euclidean, not Hamiltonian, strong-couping expansion showing that at infinite-g SU(5) ground state unique and singlet => the name “strong-coupling symmetric phase”

why two 4-fermi terms?

1. static limit exists

2. mirror global symmetries, including anomalous ones, must be broken, or else get extra zero modes in instanton - wrong ‘t Hooft vertex




zero modes and lifting by $g_2$-coupling analogy: t-quark decoupling from QCD ‘t Hooft vertex vs non-decoupling from SU(2) ‘t Hooft vertex, no matter how heavy, $\Delta B = 3$

the “E-P dream” was, essentially, to use this* phase to decouple the mirrors

*I am simplifying E-P story - here’s their dream phase diagram:

lambda-axis of EP = my $g_1, g_2$ E. Eichten, J. Preskill / Chiral gauge theories 191

region studied by EP:

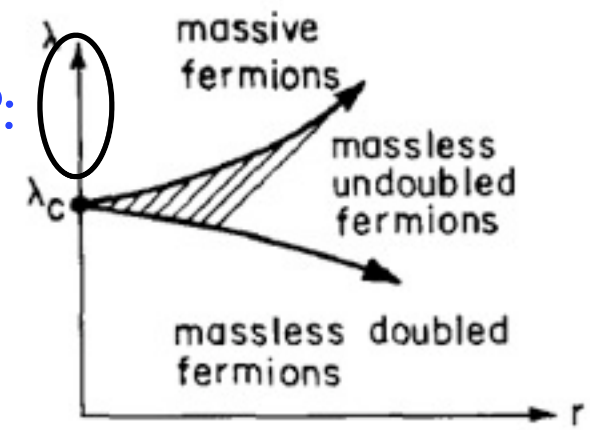
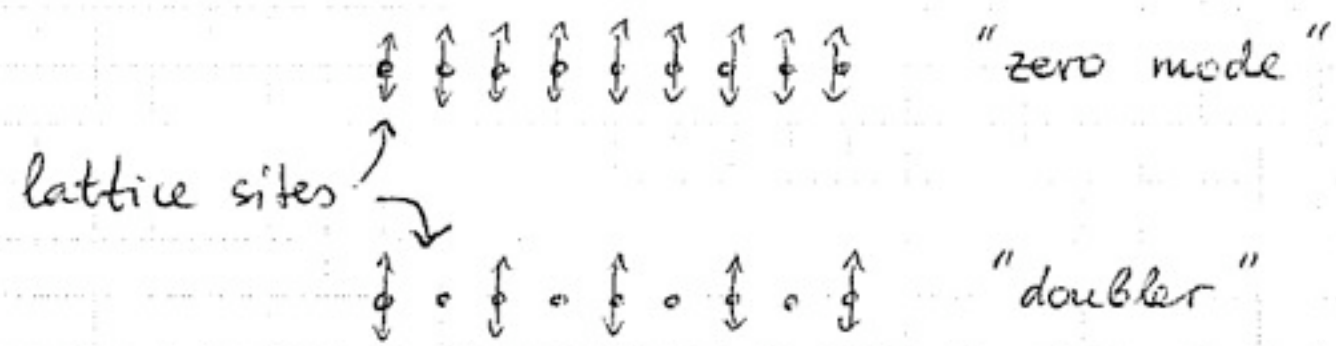



Fig. 3. Phase diagram in the $\lambda - r$ plane, assuming $\lambda_c \neq 0$. Composite fermion states go to threshold along the curves shown. In the shaded region, there is a massless undoubled fermion mode.

even a two-component Weyl field on the lattice, as E-P used, has opposite chirality massless excitations in it, because of fermion doubling

a 1+1dim reminder: spatial lattice Hermitian H...
(remember (de)construction!)

$$i\,\dot{\psi}_i = i\,(\psi_{i+1} - \psi_{i-1})$$




(symmetry)

to avoid, EP introduced "r"-axis:

- more 4-fermi terms, this time with derivatives in them, must break symmetry between "doubler" and "light" modes

- want to tune "r" to make light massless while doublers heavy - were able to only study one region clearly, this is not likely to be very easy...

only one further study, by Golterman, Petcher, Rivas, 1993:

no proof, by all means, but in all regions that they could study using 1/N, strong- and weak-coupling expansions in r, lambda both "mirror" and "light" fermions became heavy at strong-4 fermi /"r"/, while at weak 4-fermi, both "mirror" and "light" were massless, i.e. the theory was vectorlike

EP story is quite complicated by the fact that the strong 4-fermi interactions are felt by both "mirror" (here - doubler) and "light" fermions (no separation existed in 1986)

things would be a lot cleaner if the strong interactions only acted on "mirror" modes - and if one could separate mirror and light already at finite (a,V)

- have to only deal with lambda-axis of EP = my $g_1, g_2$ and avoid need to tune r

- **symmetries** *could be also clearly and unambiguously defined; for example expect that unbroken exact chiral symmetry, if such a thing existed at finite (a,V), would protect the light fermions*

  in EP and similar constructions with Yukawa interactions, no symmetry difference between "light" and "mirror" modes

  - a symmetry distinguishing light from heavy modes, needed to protect the light modes and allow the heavy ones to become massive is expected to "emerge" at some value of "r" (which was never found)

formulating EP in a manifest light/mirror separated way entails re-writing the rules...

## - what makes this possible?

after a series of seminal papers in the 90's (Kaplan, Narayanan/Neuberger, Neuberger, P. Hasenfratz/Laliena/Niedermayer, Luescher, Neuberger) it was realized that there is an exact chirality at any nonzero lattice spacing - massless vectorlike theories can be formulated with explicit chiral symmetry & no doublers

...rediscovering, in 1997, Ginsparg&Wilson's work of 1982!

can be used to define of L and R components of Dirac
- not Weyl, like EP - fermions:

- somewhat complicated, but exact at any (a, N)

- exact chirality transforms, anomaly, Ward identity, "index theorem"

then, one can ask whether the "E-P dream" be resurrected as well?

Bhattacharya, Martin, EP, 2006

important note: Creutz, Rebbi, Tytgat, Xue, 1996, similar proposal using E-P + domain wall - before GW operator and exact chirality - symmetries become exact only as size becomes infinite, so less "pretty," hence more difficult to study theoretically - there was no follow-up work whatsoever!

to explain our proposal and later/current work in more detail requires technicalities

but the structure we arrive at is like this:

Yanwen Shang, EP, 0706.1043

$$Z_{vector}[A] = Z_{light}[A] \times Z_{mirror}[A]$$

$$Z_{light}[A] = \int \Pi\, d\bar{c}\; dc\; e^{\,\bar{c}^k D_{kp}[A] c^p}$$

$$Z_{mirror}[A] = \int \Pi\, d\bar{b}\; db\; e^{\,\bar{b}^k D_{kp}[A] b^p + S_{4fermi}[\bar{b}, b, A]}$$

- light and mirror Z separate explicitly in any A; light fields do not feel strong mirror interaction
- measure is explicitly defined - it is the usual vector theory measure, no ambiguity
- global symmetries, incl. anomalous, are exactly the ones of the target continuum theory
  (e.g., chiral anomaly of light theory arises due to noninvariance of the measure under (lattice) chiral rotations)

some formal differences between anomaly free and anomalous cases, possible to study due to "splitting theorem" of Yanwen Shang, EP, 0706.1043 [hep-th] - useful in many ways!

- Z_mirror/Z_light separation singular in A if anomalous mirror rep (full Z_vector smooth)
- Z_mirror is a globally smooth function of A iff anomaly free
- "splitting theorem" encodes on the lattice the fact that anomalies are independent of the action, but property of fermion representation

**should we be opening the champagne, then?**

I didn't tell all:

c, b not usual local fermion variables, slight nonlocality having to do with implementing exact lattice chirality should we be opening the champagne, then?

not yet - a "few" questions remain to be answered first:

1 with the slightly nonlocal Yukawa/4-fermi mirror interactions, is it still true that a "strong coupling symmetric phase" exists?
are the mirrors heavy?

2 in typical models, there is more than one strong Yukawa/4-fermi interaction - needed to break all classical mirror global symmetries - and there can be a nontrivial phase structure as their ratios change (not necessarily a problem, but another issue to understand)

3 what happens if one tries to decouple an anomalous mirror representation?

1,2, and 3 can be addressed with background nondynamical gauge fields only, but NEED TO USE NUMERICS; no simple analytic strong-coupling expansion as in original models with non-exactly chiral fermions
- beauty has a price; however, in a matter of principle, we can stay in 2d at first

adding dynamical gauge fields brings in a new set of questions, for example:

4 with dynamical gauge fields included, is the long-distance theory unitary?
we have defined a complex Euclidean partition function: different treatment of conjugate mirror fermion variables through the different chiral projectors

5 suppose all checks above are fine - apart from gaining intellectual satisfaction, what can we now learn about strong chiral gauge dynamics?
can we calculate with $T_{\text{simulation}} < e^{S(\text{dragon})}$?

## but we are (slowly) learning:

- with the slightly nonlocal Yukawa/4-fermi mirror interactions, is it still true that a "strong coupling symmetric phase" exists?

yes, in the 2d models studied
Joel Giedt, EP, hep-lat/0701004

yes, in the 4d model studied
P. Gerhold, K. Jansen, arXiv:0707.3849[hep-lat]
(no Majorana type coupling due to different motivation;
unlifted "mirror" zero modes quite easy to predict and spot)

are the mirrors heavy?
- it depends... this talk

## but we are (slowly) learning:

2 in typical models, there is more than one strong Yukawa/4-fermi interaction - needed to break all classical mirror global symmetries - and there can be a nontrivial phase structure as their ratios change (not necessarily a problem, but another issue to understand)

there is a nontrivial phase structure in the 2d model (vectorlike Schwinger model at strong chirally invariant Yukawa) studied

reaching symmetric phase at strong coupling does not appear to require fine-tuning (a large region in coupling space)

Joel Giedt, EP, hep-lat/0701004

**but we are (slowly) learning:**

3 what happens if one tries to decouple an anomalous mirror representation?

Z_mirror/Z_light split has something to do with it
important to differentiate between options
- massless mirror fermion, Green-Schwarz field, nonunitarity???

Yanwen Shang, EP, arXiv:0706.1043[hep-th] + in progress

4 with gauge fields included, is the long-distance theory unitary?

we have defined a complex Euclidean partition function: different treatment of conjugate mirror fermion variables through the different chiral projectors

*not obvious,* but some indications

this talk...

using GW chiral projectors, split kinetic term and define Yukawa/4-fermi only in terms of "mirror" components:

$$\bar{\Psi}_q D_q \Psi_q = \bar{\Psi}_{q,+} D_q \Psi_{q,+} + \bar{\Psi}_{q,-} D_q \Psi_{q,-}$$

"simple" case:

massless Schwinger model + singlet massless fermion + strong mirror interaction

will also call it "1-0 model":

$$\begin{aligned} S &= S_{light} + S_{mirror} \\ S_{light} &= (\bar{\psi}_+, D_1 \psi_+) + (\bar{\chi}_-, D_0 \chi_-) \end{aligned}$$

$$\boxed{\begin{aligned} S_{mirror} &= (\bar{\psi}_-, D_1 \psi_-) + (\bar{\chi}_+, D_0 \chi_+) \\ &+ \; y \left\{ (\bar{\psi}_-, \phi^* \chi_+) + (\bar{\chi}_+, \phi \psi_-) + h \left[ (\psi_-^T, \phi \gamma_2 \chi_+) - (\bar{\chi}_+, \gamma_2 \phi^* \bar{\psi}_-^T) \right] \right\} \\ S_\kappa &= \frac{\kappa}{2} \sum_x \sum_{\hat{\mu}} \left[ 2 - ( \phi_x^* \, U_{x,x+\hat{\mu}} \, \phi_{x+\hat{\mu}} + \text{h.c.} ) \right] \end{aligned}}$$

mirror theory action

Joel Giedt, EP (2007), simulated mirror partition function at A=0:

$$S_{mirror} = \cancel{-(\overline{\psi}_- \cdot D_1 \cdot \psi_-) - (\overline{\chi}_+ \cdot D_0 \cdot \chi_+)}$$ *y=infinity - drop kinetic term*

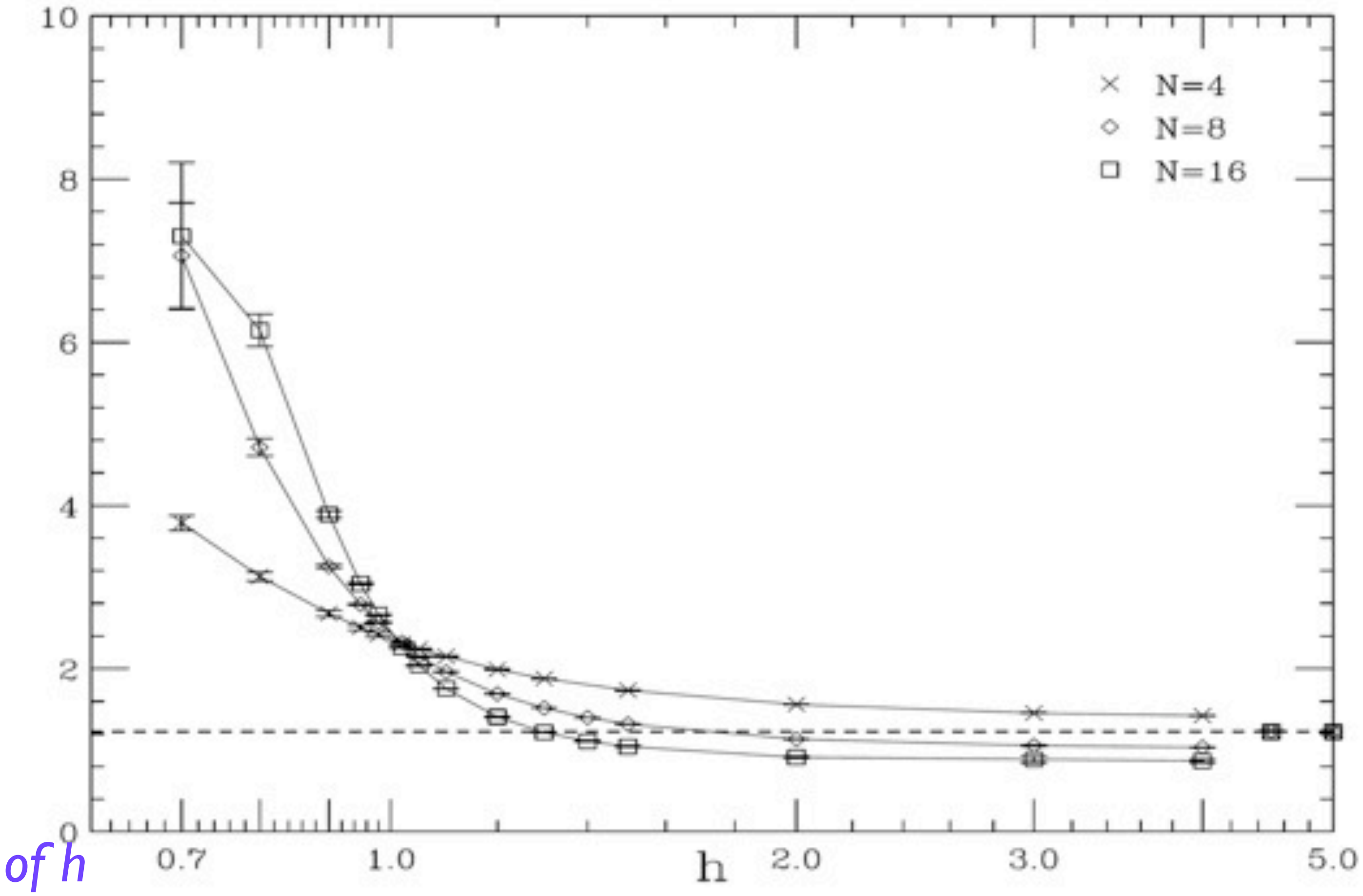
$$+\, y \left\{ (\overline{\psi}_- \cdot \phi^* \cdot \chi_+) + (\overline{\chi}_+ \cdot \phi \cdot \psi_-) + h \left[ (\psi_-^T \cdot \phi \gamma_2 \cdot \chi_+) - \left( \overline{\chi}_+ \cdot \gamma_2 \cdot \phi^* \cdot \overline{\psi}_-^T \right) \right] \right\}$$

$$S_\kappa = \frac{\kappa}{2} \sum_x \sum_{\hat\mu} [2 - (\, \phi_x^* \, U_{x,x+\hat\mu} \, \phi_{x+\hat\mu} + \text{h.c.}\, )]$$

$$\kappa < \kappa_c \sim 1$$

Giedt, EP, 2007

$$\phi(x) = e^{i\alpha(x)}$$




$$\chi = \int d^2x \, < \phi^*(x) \, \phi(0) > \;\sim\; \frac{1}{m_\phi^2}$$

$$\left(\begin{matrix}\text{long}\\ \text{range}\\ \text{order}\end{matrix}\right) \sim \int d^2x \, \frac{1}{|x|^{2-\eta}} \;\sim\; L^\eta$$

- *strong coupling symmetric phase exists, for a range of h*

- *probing charged scalar and fermion mirror spectrum with local (elementary or composite) operators showed no evidence for massless charged mirror states (many plots in Giedt, EP, 2007)*

*taken at face value, have a puzzle: have we "decoupled" an anomalous representation w/out gauge breaking?*

...after gauging: S_light = chiral Schwinger model, anomalous + unbroken gauge symmetry... so:

a.) we must have missed something - what?

b.) maybe taking large-y somehow leads to nonunitary theory - after all we start from a complex $Z_{\text{Euclidean}}$ , no H...?

$$\Pi_{\mu\nu}(x,y) \equiv \left.\frac{\delta^2 \ln Z[A]}{\delta A_\mu(x)\delta A_\nu(y)}\right|_{A=0}$$

since Z[A] of 1-0 theory is gauge invariant:

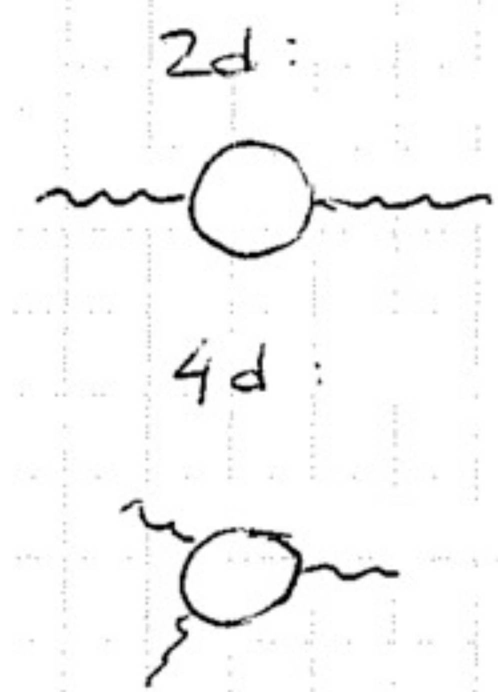
$$\nabla^*_\mu \Pi_{\mu\nu} = 0$$

since Z[A] splits into light + mirror for arbitrary A:

$$\ln Z[A] = \ln Z_+[A] - \ln J[A] + \ln Z_-[A]$$

and since split of Z into light + mirror is locally smooth in A, polarization operator splits, too:

$$\Pi_{\mu\nu}(x,y) = \Pi^{+/J}_{\mu\nu}(x,y) + \Pi^-_{\mu\nu}(x,y)$$




but the light theory has a chiral charged fermion, hence its polarization operator is not transverse:

$$\nabla^*_\mu \Pi^{+/J}_{\mu\nu} \sim \frac{\delta}{\delta A_\nu}\left(\epsilon_{\alpha\beta} F^{\alpha\beta}\right)$$

but since the total polarization operator is transverse, it must be that the mirror one is also not transverse:

$$\nabla^*_\mu \Pi^-_{\mu\nu} = - \nabla^*_\mu \Pi^{+/J}_{\mu\nu}$$

These are very usual considerations in continuum.

*It is a remarkable consequence GW and exact lattice chirality that they can be exactly transcribed, with all i's and pi's (and some extras like N,a + ...) to a lattice of a finite size.*

Could easily do in 4d, but 3pt function.

details are of great interest (to me) but I'll spare you... see Shang, EP, to appear thus, in low-momentum limit, Fourier transform of the imaginary part of the mirror polarization operator we're in Euclidean, anomaly is in ImLogZ, which obeys:

$$i q^\mu \tilde{\Pi}^{-}_{\mu\nu}(q) = \frac{1}{2\pi} \epsilon_{\nu\lambda} q^\lambda + \ \mathcal{O}(q^2)$$

should have some nonlocal contribution

- **in a unitary, Lorentz invariant theory, means also real part should be nonlocal**
- **poles in real part of polarization operator mean massless charged particles, so mirror should have light states**

this condition on the mirror dynamics (remember, gauge coupling=0) is **exact:**

- independent on the strength of the mirror 4-fermi, Yukawa, etc. couplings - never used explicit form of mirror action!

- true for any volume, lattice spacing

- analogous to ‘t Hooft anomaly matching in theories with strong IR dynamics

  - so, strong non-gauge mirror dynamics has to comply with it

  - as usual, anomaly matching does not tell us what the pole is from, a Goldstone boson or massless fermion, and one needs to study the dynamics

- *it is interesting to better understand how our attempt to decouple one chirality of the Schwinger model fails*

  - a check on unitarity - could've imagined a nonlocal Im-part and a local Re-part

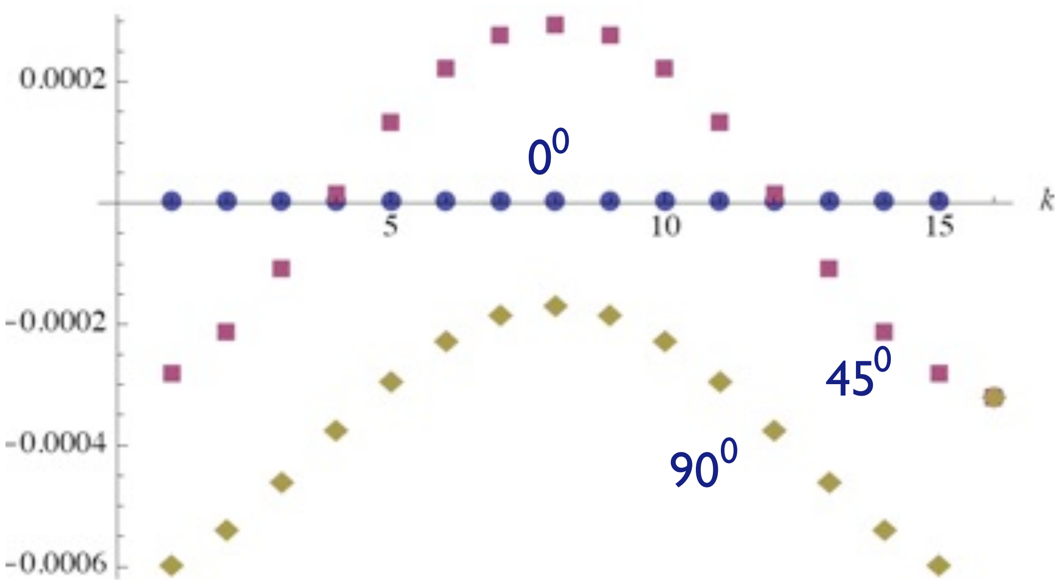
  - relatively cheap exercise, if a bit long to set up

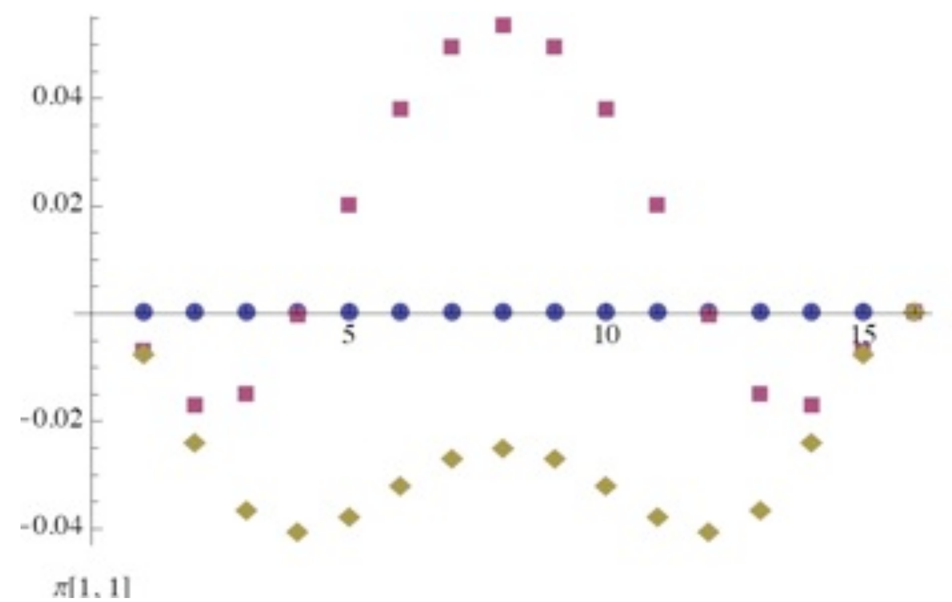
  - tools developed to express mirror polarization operator in terms of mirror correlators likely useful in future, e.g. for studying anomaly-free case (more expensive, of course, but still g=0 good first step)

  - hope it will teach us something on strong mirror dynamics, a la EP, with GW fermions

real part of polarization operator of free fermion, using GW operator:

$$\frac{1}{2}\,\delta_\nu \mathrm{Tr} D^{-1} \delta_\mu D$$




in continuum, loop of massless particle in 2d:

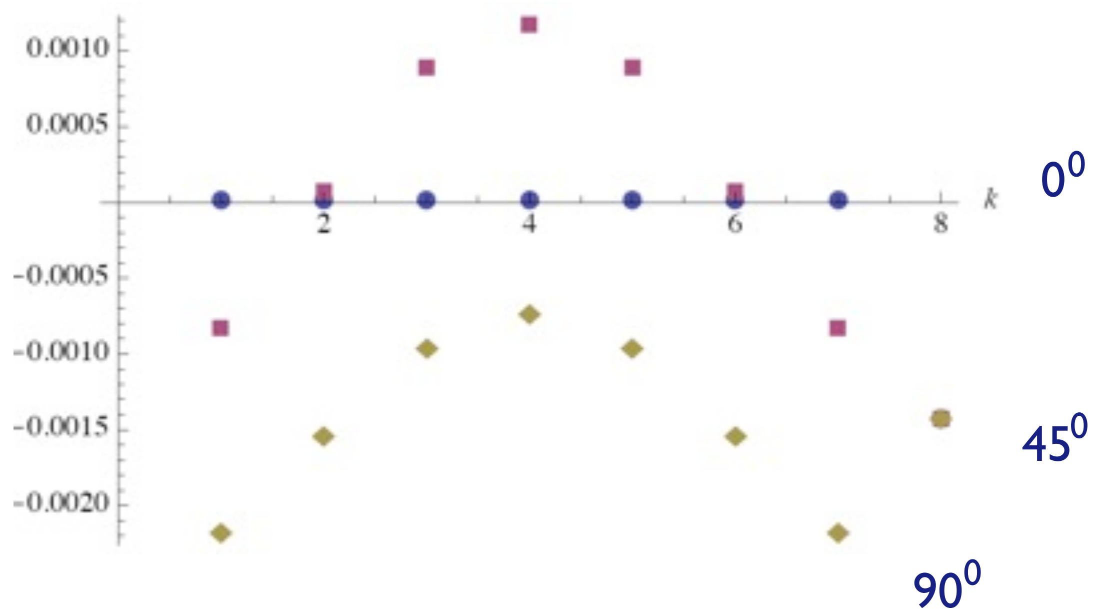
$$\Pi_{11} \sim 1 - \frac{k_1^2}{k_1^2 + k_2^2}$$

loop of cutoff-scale mass particle

this is not Monte-Carlo but exact sum over loop momenta with Mathematica
16x16 lattice

works well even on 8x8 lattice!

$$\frac{1}{2}\,\delta_\nu \mathrm{Tr} D^{-1} \delta_\mu D$$

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must use Monte-Carlo, as mirror theory is a strongly-coupled nonlinear system!

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two steps involved:

1. find expression of polarization operator in terms of mirror correlation functions
expressed in terms of variables of integration... long and tedious, but now we know how to do it for any theory

2. use MC to calculate polarization operator...

- use expansions in terms of chiral eigenvectors to define Z(mirror)

- use splitting theorem (crucial!) to find second derivative of LogZ(mirror) wrt gauge field = polarization operator w, v are eigenvectors of $\hat{\gamma}_5$ at A=0 (known functions of momentum)

<...> are mirror-theory correlators $\overline{\alpha}^i_- \beta^j_+ \; \alpha^j_- \; \overline{\beta}^j_+$ · are the mirror Grassmann variables of integration

$$
\begin{aligned}
\Pi^-_{\mu\nu}\big|_{y=\infty} &= \delta_\nu J^w_\mu \\
&- y \, \langle \overline{\alpha}^i_- \beta^j_+ \; (w^\dagger_i \cdot \delta_\nu(\hat{P}_+ \delta_\mu \hat{P}_+) \cdot \phi^* v_j) \rangle - yh \, \langle \overline{\alpha}^i_- \overline{\beta}^j_+ \; (u^\dagger_j \gamma_2 \cdot \phi^* \cdot \delta_\nu(\delta_\mu \hat{P}^T_+ \cdot \hat{P}^T_+) \cdot w^*_i) \rangle \\
&+ y^2 \, \langle \Big( \overline{\alpha}^i_- \beta^j_+ (w^\dagger_i \cdot \delta_\mu \hat{P}_+ \cdot \phi^* v_j) + h \overline{\alpha}^i_- \overline{\beta}^j_+ (u^\dagger_j \gamma_2 \cdot \phi^* \cdot \delta_\mu \hat{P}^T_+ \cdot w^*_i) \Big) \times \\
&\qquad \Big( \overline{\alpha}^k_- \beta^l_+ (w^\dagger_k \cdot \delta_\nu \hat{P}_+ \cdot \phi^* v_l) + h \overline{\alpha}^k_- \overline{\beta}^l_+ (u^\dagger_l \gamma_2 \cdot \phi^* \cdot \delta_\nu \hat{P}^T_+ \cdot w^*_k) \Big) \rangle^C \\
&+ \frac{\kappa}{2} \, \langle (\phi^* \cdot \delta_\nu \delta_\mu U \cdot \phi) + \text{h.c.} \rangle \\
&+ \frac{\kappa^2}{4} \, \langle [(\phi^* \cdot \delta_\mu U \cdot \phi) + \text{h.c.}] \, [(\phi^* \cdot \delta_\nu U \cdot \phi) + \text{h.c.}] \rangle^C \\
&- \frac{y\kappa}{2} \, \Big[ \langle [(\phi^* \cdot \delta_\mu U \cdot \phi) + \text{h.c.}] \Big[ \overline{\alpha}^i_- \beta^j_+ (w^\dagger_i \cdot \delta_\nu \hat{P}_+ \cdot \phi^* v_j) + h \overline{\alpha}^i_- \overline{\beta}^j_+ (u^\dagger_j \gamma_2 \cdot \phi^* \cdot \delta_\nu \hat{P}^T_+ \cdot w^*_i) \Big] \rangle^C \\
&\qquad + (\mu \leftrightarrow \nu) \Big] \; .
\end{aligned}
$$

result is really y-independent as y factors canceled by 1/y from fermion "propagators"

feed to the code - written by Joel Giedt, 2006/7, modifications for many processors by Yanwen Shang, 2008
- and then to the computer

run it - note lots of momentum sums (i,j...) and "disconnected diagrams" in the lattice-QCD lingo
very demanding even on 8x8 lattices

~300 processors of CITA cluster run for 5 hours to get Pi on 8x8 for a given value of $h$, $\kappa$

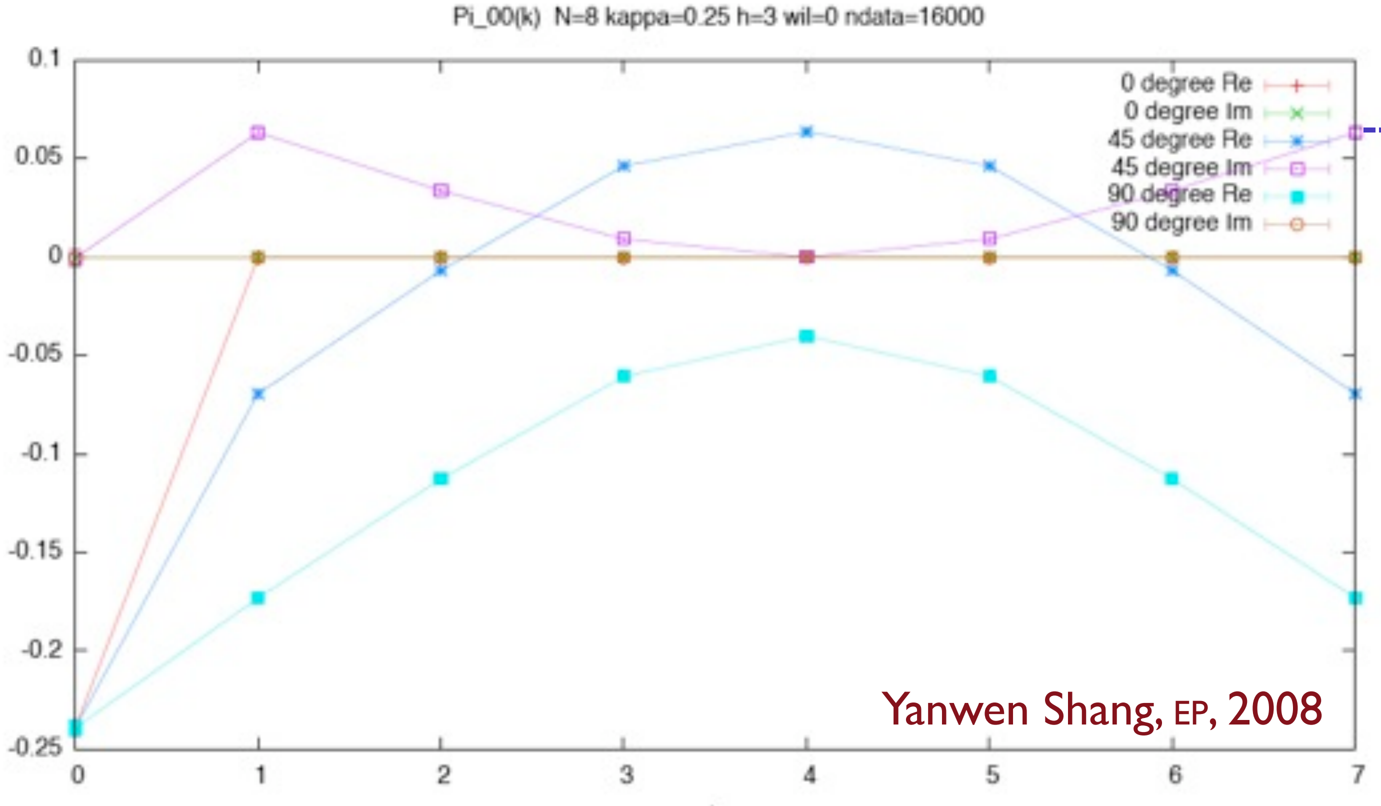
## what comes out?

after numerous checks and balances - incl. check that the divergence of imaginary part = exactly what 't Hooft predicts...

lots of plots

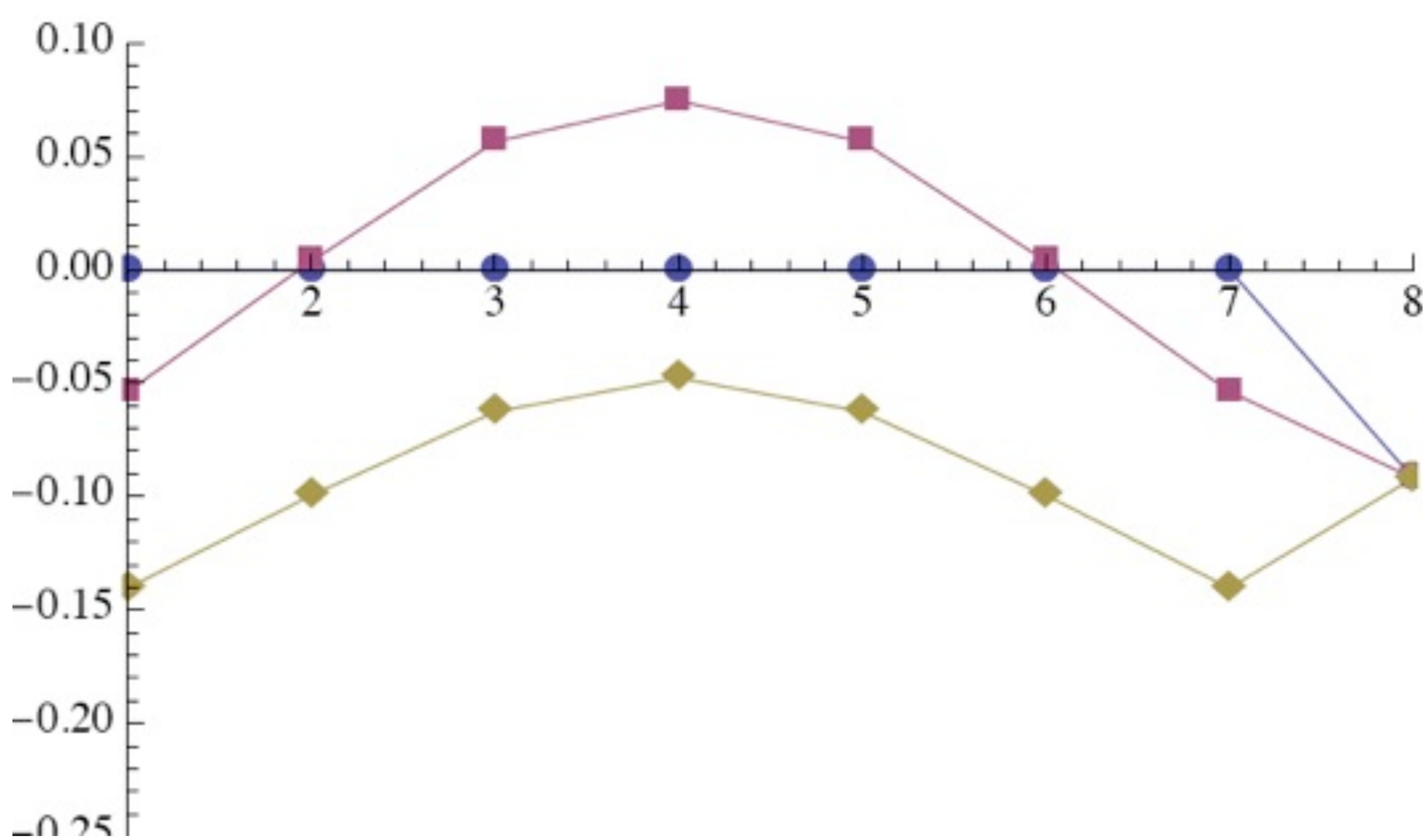
but first,  in words:

- for h>1: real part of mirror polarization operator - probing number of massless charged modes - is like that of one charged massless fermion (this result is independent on h, so long as h>1)




----never mind this curve

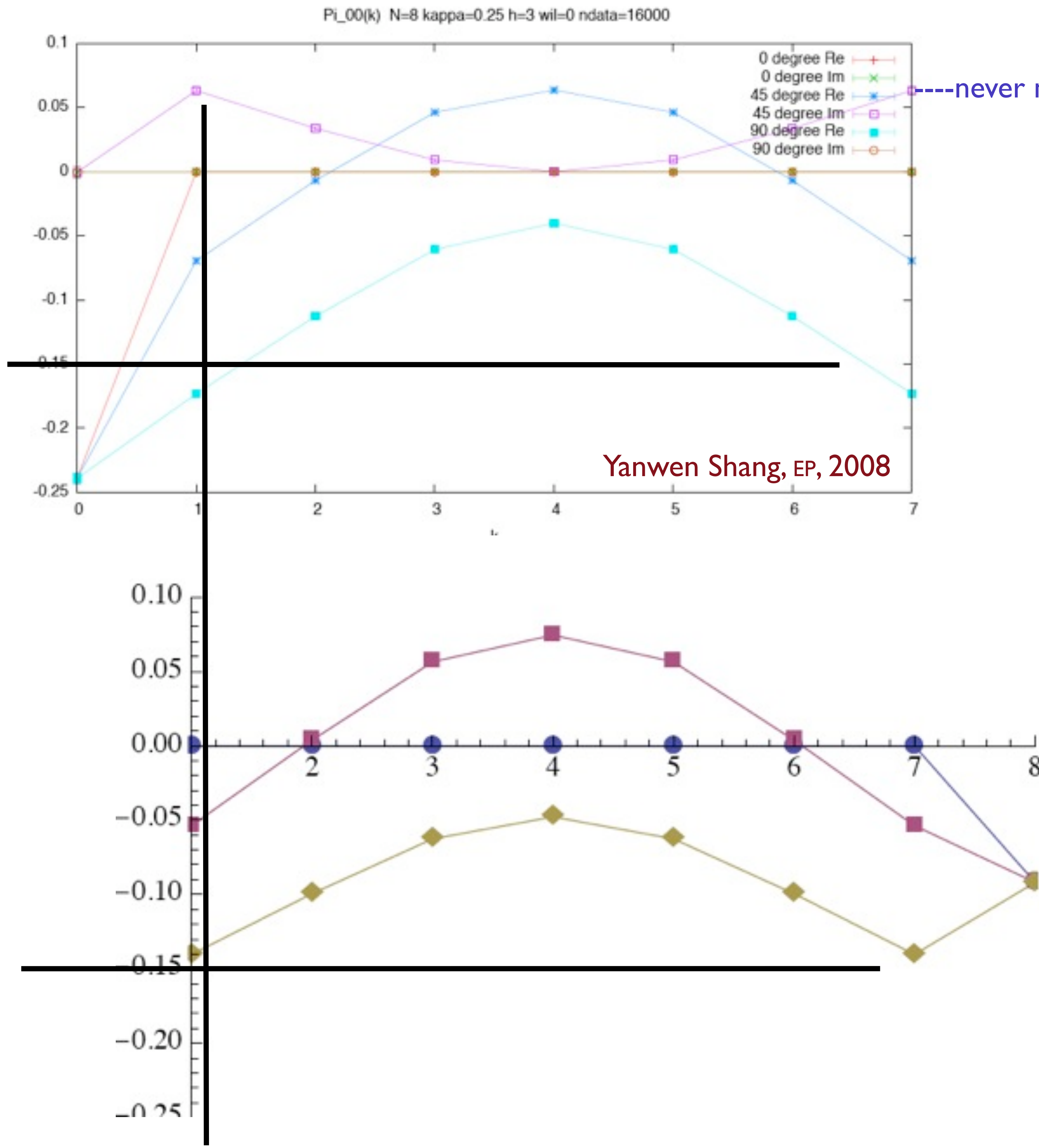
this it the Monte-Carlo calculation of the mirror polarization operator at y=infinity, h=3, 8x8 lattice, disordered phase, 16000 field configurations
(error bars are almost invisible)




looking very much like the one from Mathematica

*I have not fudged anything!*

different value at k=0 has to do with Wilson line needed to avoid singularity when computing loop, not with error bars of MC

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**Monte-Carlo “proof” of ‘t Hooft anomaly matching at strong mirror coupling**

## what comes out?

- for h>1: real part of mirror polarization operator - probing number of massless charged modes - is like that of **one charged massless fermion** (this result is independent on h, so long as h>1)

- for h<1, h close to 0: real part of mirror polarization operator like that of **three charged massless fermions** (since anomaly same for all h, must be one - chirality and a +/- chirality pair)

- can not interpolate through h=1, huge sign problem for h~0.7 *about where a KT-like transition lies*

all this is in the disordered-scalar phase, small \kappa
large-\kappa - "broken phase" scalar Green-Schwarz field, massive gauge boson

- so, 't Hooft is obeyed, it seems, by having, for h>1, the minimal number of massless charged mirror fermions required by anomaly

- however, for h=0, there are more massless charged states, at y=infinity, than required by anomaly cancellation (extra massless vectorlike pair due to extra symmetry)

**can we understand this complicated strongly-coupled soup?**
*(some analytic ideas, not ripe yet, not for all h...)*

**what lessons are we learning?**

- so anomaly matching works; *mirror dynamics is "smart" - and appears unitary - both Re and Im nonlocal*

- it appears light mirror modes may be not local wrt original variables
  - explains why sourcing with local charged operators didn't reveal them?

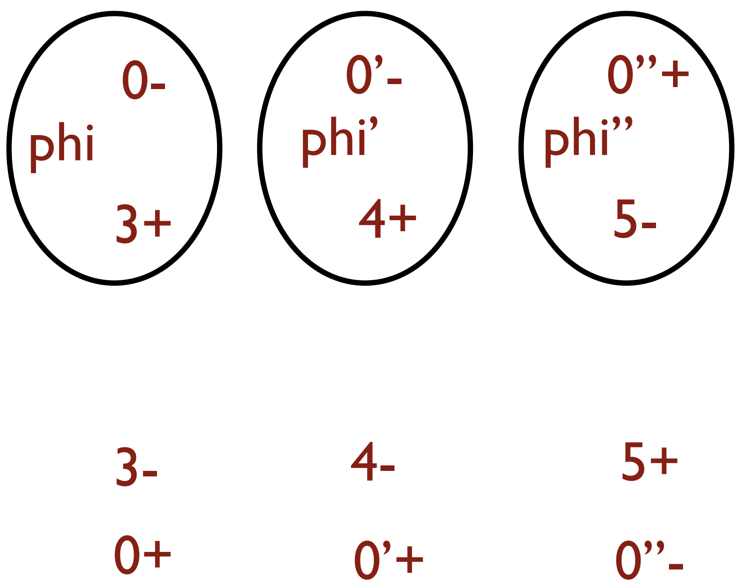
- polarization operator is a better (universal) probe of charged spectrum of strongly-interacting mirror than local charged fermion and scalar (composite) operators *[S-parameter, once again...]*
  *in principle, in 4d must also check consistency of 3pt functions - harder*

- Majorana couplings crucial - recall initially motivated by breaking mirror global symmetries
  without them massless spectrum at y=infinity always 3 doubler modes

**finally, the question on everybody's mind: but what about anomaly free models?**

best I can say for now is it will depend on how mirror is implemented (symmetries again)
in principle, 't Hooft implies conditions even in that case

- consider "345 model," an anomaly-free 2d theory with 3- 4- 5+ charged fermions
  3 is charge; - is chirality, etc.




this implementation is bound to have massless mirrors:

three disjoint copies of our 1-0 model (3-0, 4-0 and 5-0 models)

three exact global symmetries appear when g=0
couple light/mirror fermions
imagine gauging each one & argue by 't Hooft

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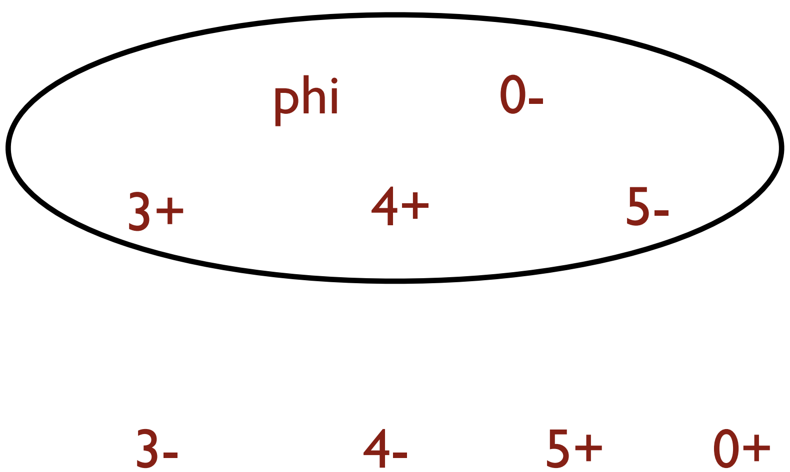


so change mirror implementation

break all global symmetries that involve mirror fields by allowing cross couplings between 3,4,5 mirrors (0- needed for static limit)

now at g=0 only global symmetries are the three light chiral U(1)s

- so, we have no reason to think that there will be massless mirror states for all values of the couplings, now that there's no symmetry reason for this

**but if there are no reasons, beyond anomaly matching, then why should there always be massless mirror modes?**

- unless we, or anybody else willing to think about this, comes up with a general argument why there always should be massless states, only future "experiment" will tell for sure

- "experiment" is quite doable, as *no gauge fields are involved, and most of the groundwork is done*
  345 (or 11112) chiral models in 2d are an obvious first try...

there are interested people who can and say they'll do it

## SUMMARY

the
“**decoupling of mirror fermions via strong-coupling symmetric phases**” idea, combined with “**exact lattice chirality**” leads to a proposed formulation of chiral lattice gauge theories, which is:

a.) exactly gauge invariant

b.) has explicit definition of path integral action and measure
so one can study it numerically

c.) has the correct - anomalous or not - Ward identities
of the continuum target theory

*but*

d.) requires (more) numerical + analytic work to study *and, most importantly,*

e.) ***we have not seen reasons to give up -***

*we don't know if we have succeeded or "not failed", yet!*

I have not failed. I've just found 10,000 ways that won't work.

Thomas A. Edison