

CONFORMALITY OR CONFINEMENT? (IR)RELEVANCE OF TOPOLOGICAL EXCITATIONS

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arXiv:0906.5156,

arXiv:0812.2085 Index theorem for topological excitations on $\mathbb{R}^3 \times S^1$

In collaboration with [E. Poppitz](#)

GOAL

- Develop new methods to study 4d **asymptotically free non-abelian gauge theories**.
- With adequate tools, I expect that there should not be dramatic (difficulty) differences in addressing the dynamics of **YM, QCD-like, chiral or supersymmetric** gauge theories.
- Common lore: **SUSY**: Very friendly, beautiful. **YM**: Formal. **QCD-like**: Hard, leave it to lattice folks! (chiral limit still hard.) **Non-susy chiral gauge theories**: Even lattice does not work in practice.

Motivations

- Dynamics of YM, QCD-like and chiral theories
- Problem of electro-weak symmetry breaking (EWSB), scenarios in TeV scale physics. (where Higgs is a composite) to be tested at LHC.
- **technicolor**: naive scaled-up QCD fails with EW precision data, fails to produce acceptable spectrum. (**walking, conformal...**)
- Theories with interesting long distance behavior, scale invariance.

Conformality or confinement:

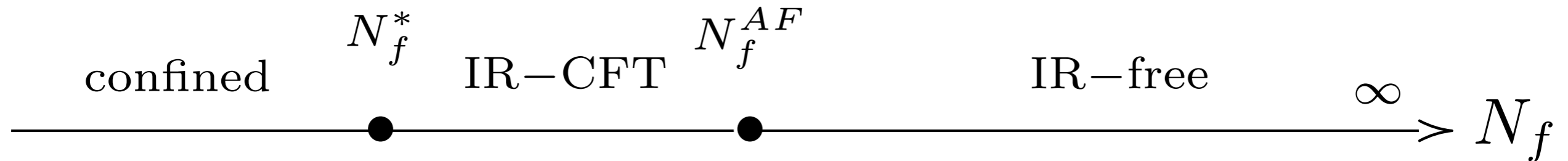
Conceptually, two problems of out-standing importance in gauge theories:

Mechanisms of confinement and conformality:

What distinguishes two theories, one just below the conformal boundary and confines, and the other slightly above the conformal window boundary? In other words, why does a confining gauge theory confine and why does an IR-CFT, with an almost identical microscopic matter content, flows to a CFT?

Lower boundary of conformal window: What is the physics determining the boundary of conformal window?

Phases of non-abelian gauge theories



Outline

- Progress in understanding of confinement mechanism(s) in vector-like and chiral theories
- Generalized QCD (different physical IR behavior, scale invariance)
- Conformality or confinement? (and what determines it?)
- Other applications (chiral gauge theories, susy)

Necessity of new methods.

Non-perturbative gauge theory, chiral or vector-like.

Progress in **SUSY QCD** is well-known (Seiberg, Witten, Intriligator, 90s). However, the dynamics of theories without **elementary scalars** is vastly different from the SUSY examples.

They deserve study of their own.

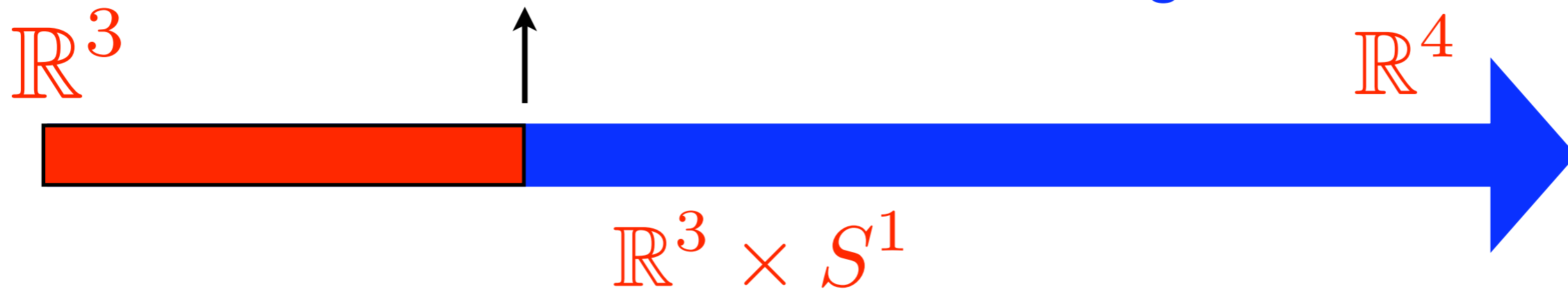
A broad-brush overview of some very recent progress.

$$\mathbb{R}^4 \longrightarrow \mathbb{R}^3 \times S^1 \quad \text{Locally 4d.}$$

Take advantage of circle (as control parameter) AF.

Traditional: thermal compactification

Phase transition: Bad for our goal.



Useful: circle compactification with p.b.c. for fermions or center-stabilizing deformations.

“NEW” METHODS

1) Twisted Partition Function
--circle compactifications--

2) Deformation theory

a small step in the desired direction: One of the two always guarantees that small and large circle physics are connected in the sense of center symmetry and confinement.

“circle compactification” L - circumference of non-thermal circle
why bother?

various “deformations” of 4d field theories have been useful to study aspects of nonperturbative dynamics.

especially true in supersymmetry, where consistency with all calculable deformations play an important role, e.g.:

- circle compactification of $N=2$ 4d SYM

(Seiberg, Witten, 96)

- circle compactification of $N=1$ 4d SYM

(Aharony, Intriligator, Hanany, Seiberg, Strassler, 97;
Dorey, Hollowood, Khoze, Mattis, 99)

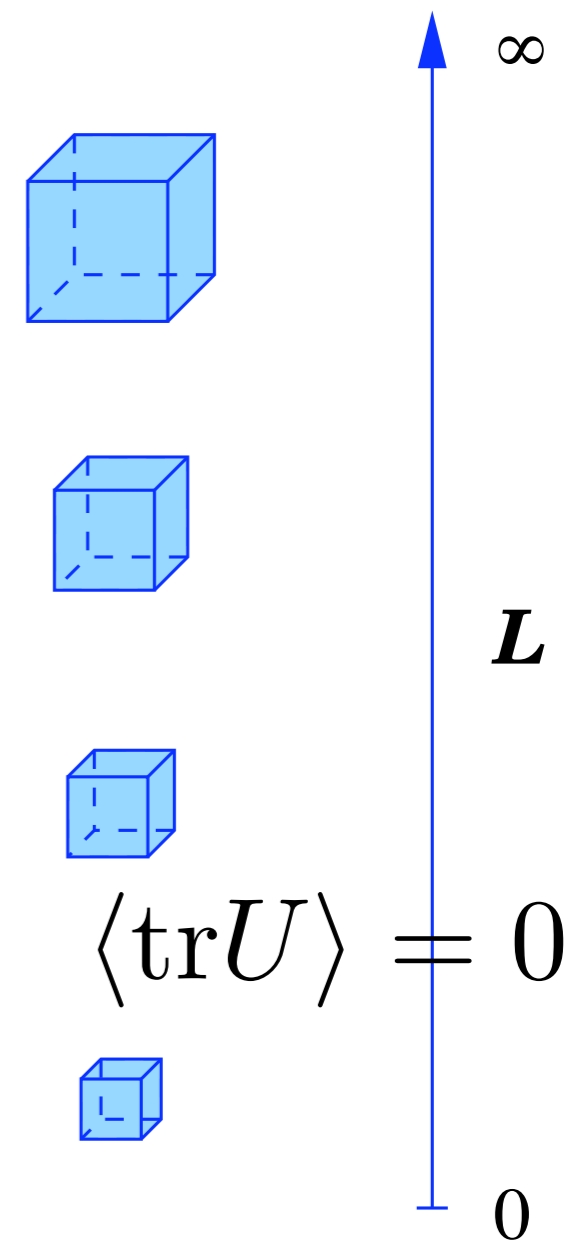
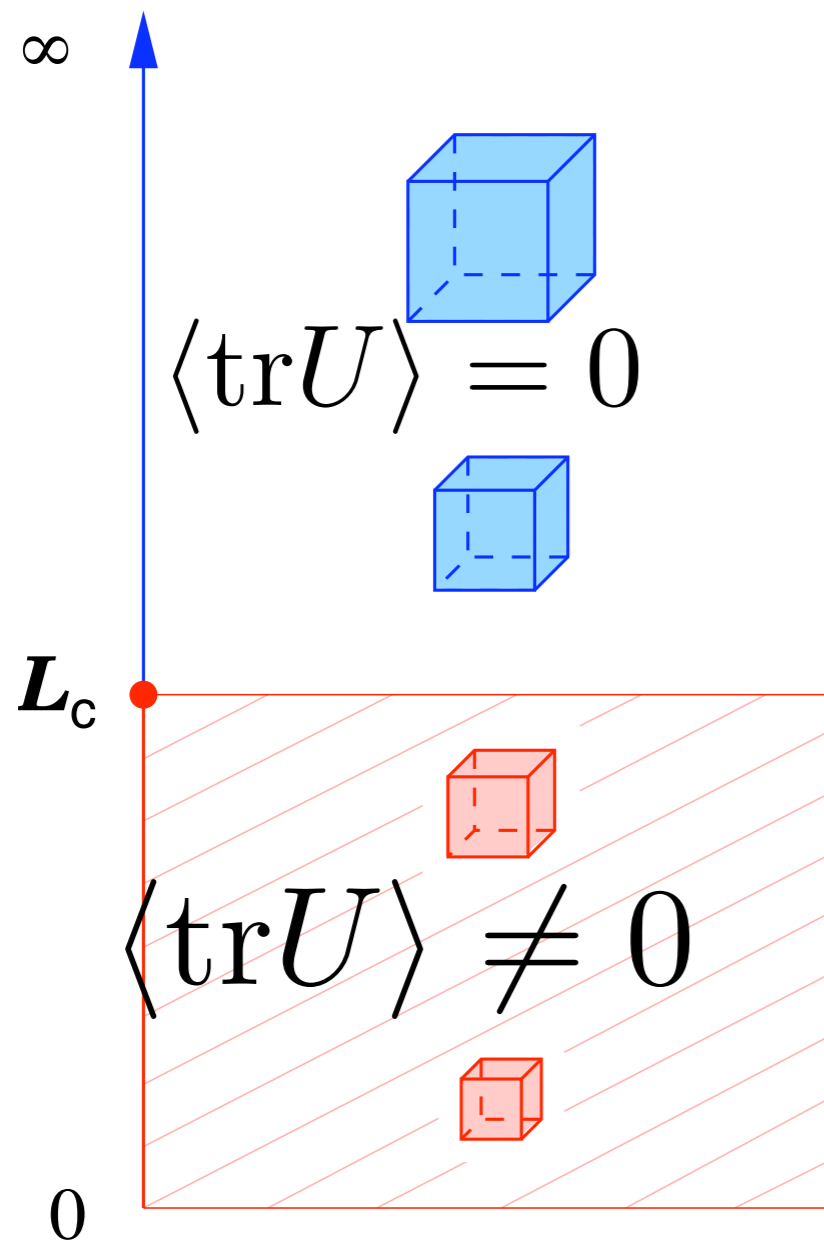
in the supersymmetric case, using holomorphy, one argues that with supersymmetric b.c. there is a smooth 4d limit

for nonsupersymmetric theories, its utility is understood recently.

Order Parameter/center symmetry

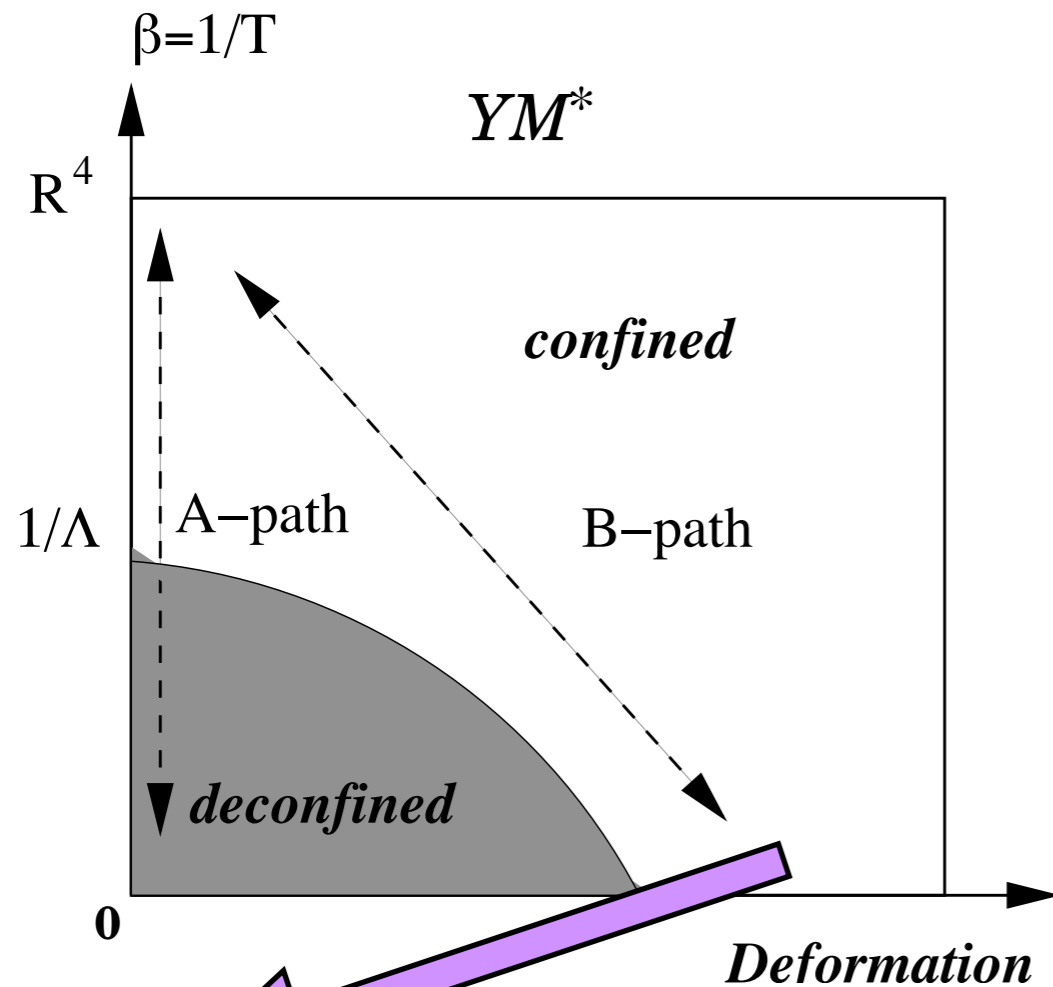
ordinary Yang–Mills

deformed Yang–Mills



What is the relation between ordinary and deformed theory?

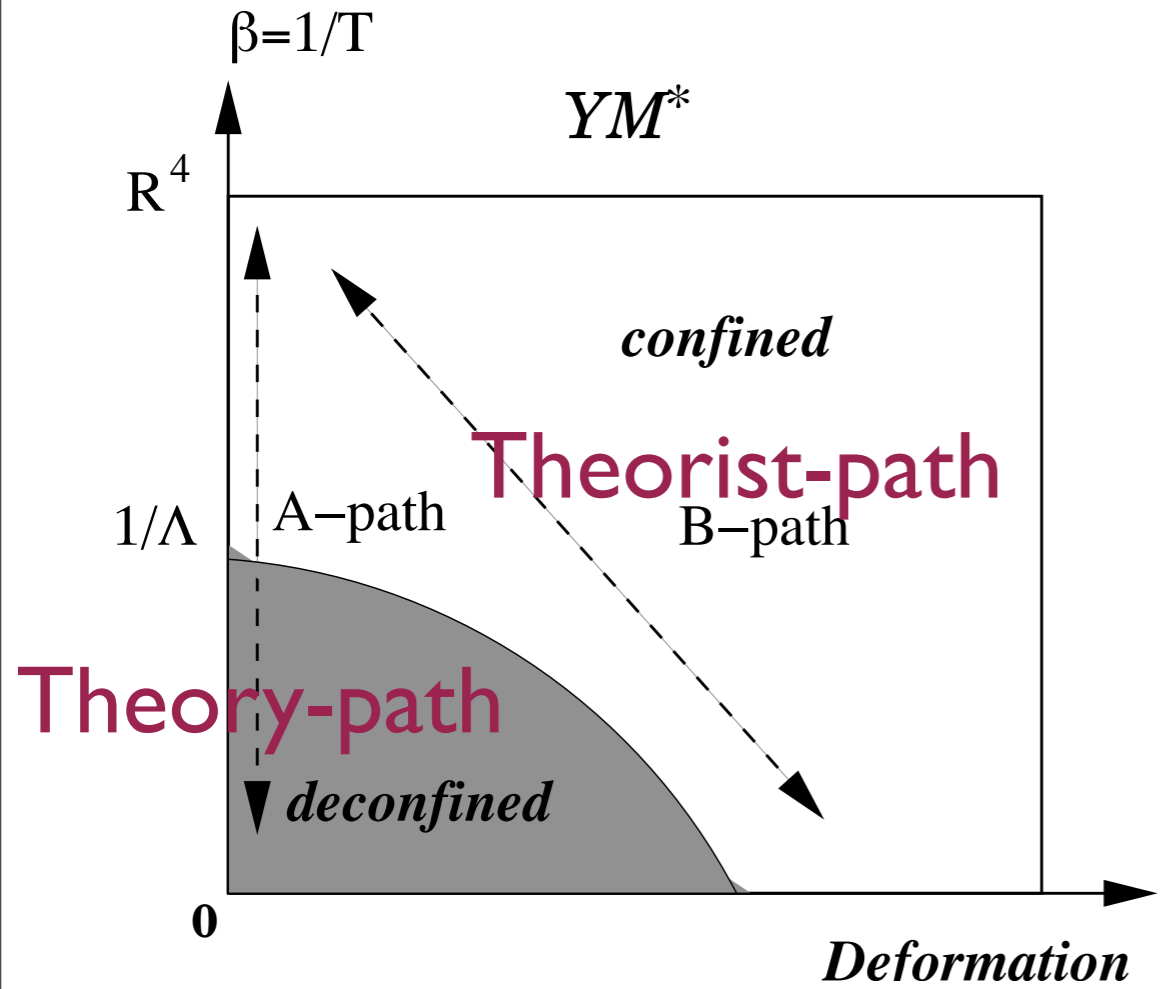
Raison d'être of deformation theory at finite N



Smooth connection to the target theory. A new method to avoid singularities.

One can show the **mass gap** and linear confinement (similar to Seiberg-Witten and Polyakov solutions). Although the region of validity does not extend to large circle, it is continuously connected to it with no gauge invariant order parameter distinguishing the two regimes.

Deformed YM theory at finite N Shifman-MU, Yaffe-MU



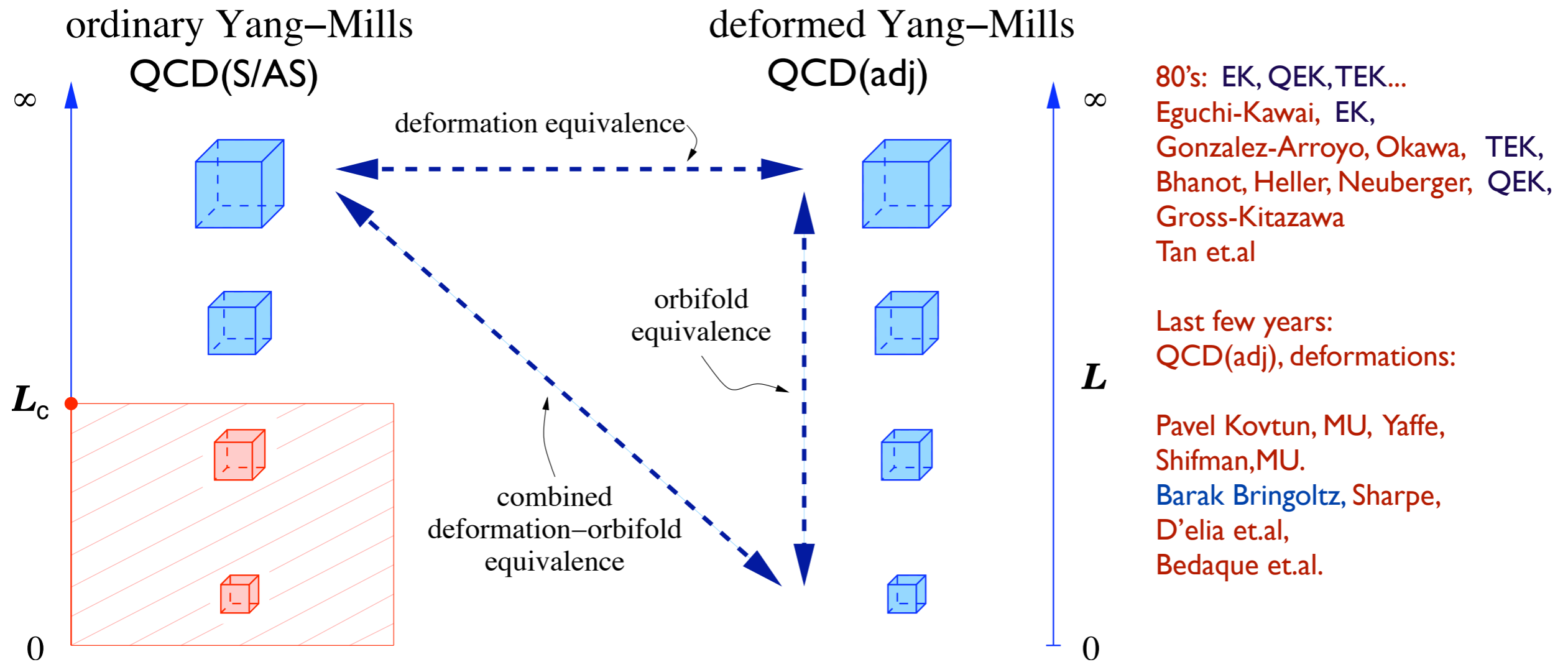
$$S^{YM^*} = S^{YM} + \int_{R^3 \times S^1} P[U(\mathbf{x})]$$

$$P[U] = A \frac{2}{\pi^2 \beta^4} \sum_{n=1}^{\lfloor N/2 \rfloor} \frac{1}{n^4} |\text{tr}(U^n)|^2$$

Lattice studies by Ogilvie, Myers, Meisinger backs-up the smoothness conjecture.

Ogilvie, Myers also independently proposed the above deformations to study phases of partially broken center.

Raison d'être of deformation theory at infinite N



At large N, **volume independence** is an **exact** property, a theorem.
Solution of small volume theory implies the solution of the theory on R⁴.
 First working example of EK-reduction (25 years after the birth of idea)
 QCD(Adj) with pbc: The most insightful/friendly QCD-like theory.

SU(N) QCD(adj)

$$S = \int_{R^3 \times S^1} \frac{1}{g^2} \text{tr} \left[\frac{1}{4} F_{MN}^2 + i \bar{\lambda}^I \bar{\sigma}^M D_M \lambda_I \right] \quad \text{short distance}$$

Center Z_{N_c}

Chiral $(SU(n_f) \times Z_{2N_c n_f}) / Z_{n_f}$

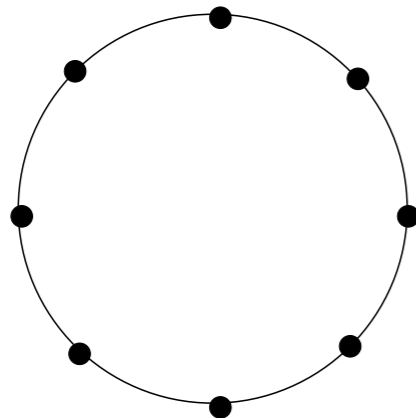
Solve it by using **twisted partition function**.

techicolor: minimal walking for 4 flavors?

AF-boundary: 5.5 flavors

Spatial Wilson line/non-thermal Polyakov loop

With deformation or pbc for adjoint fermions,
eigenvalues repel. Minimum at



$$U = \text{Diag}(1, e^{i2\pi/N}, \dots, e^{i2\pi(N-1)/N})$$

$$\langle \text{tr}U \rangle = 0$$

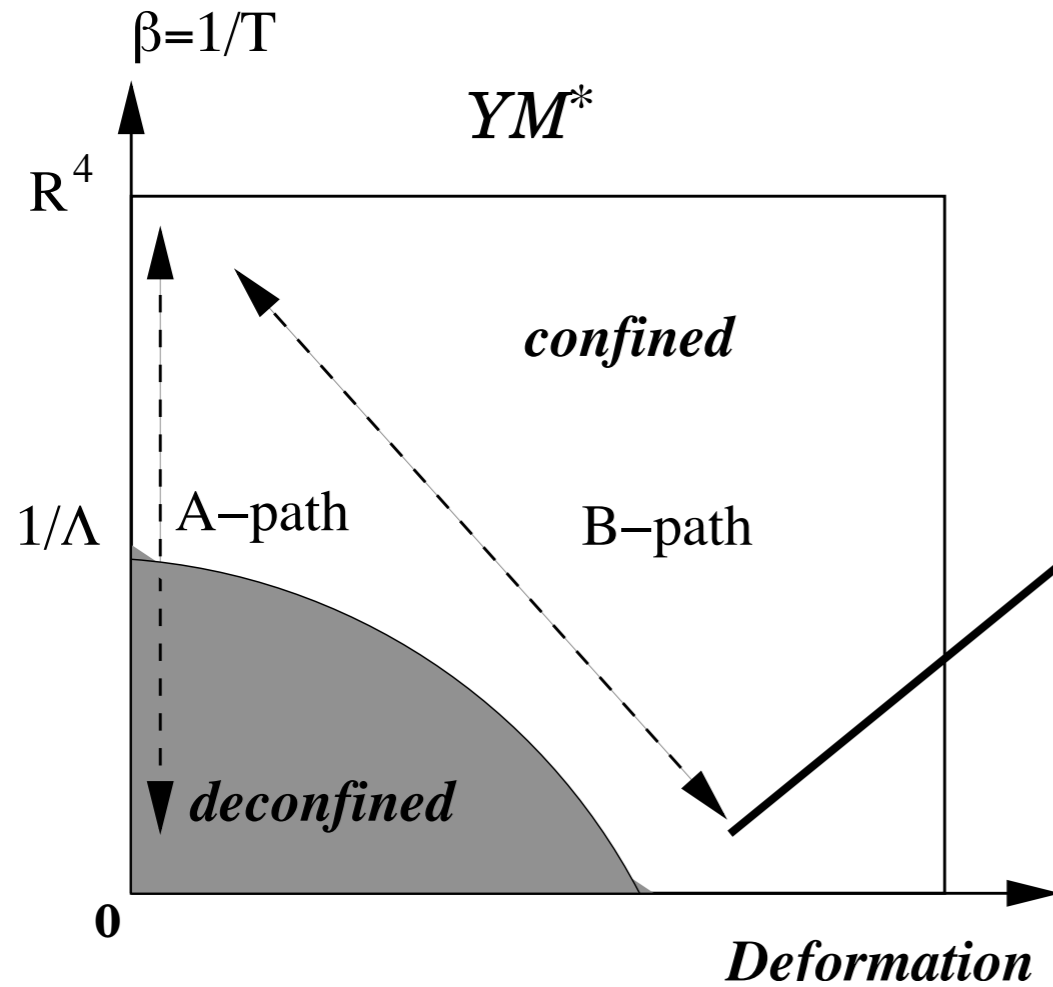
At weak coupling, the fluctuations are small, a “Higgs regime”

$$SU(N) \rightarrow [U(1)]^{N-1}$$

Georgi-Glashow model with **compact** adjoint Higgs field.

Compactness implies N types of monopoles, rather than N-1.

Abelian Dualities



Small volume theory becomes solvable in the same sense as Polyakov model or Seiberg-Witten theory by **abelian duality**.

Reminder: Abelian duality and Polyakov model

Free Maxwell theory is dual to the free scalar theory.

$$F = *d\sigma$$

The masslessness of the dual scalar is protected by a **continuous shift symmetry**

$$U(1)_{\text{flux}} : \sigma \rightarrow \sigma - \beta$$

Topological current vanishes by Bianchi identity.

Noether current of dual theory:

$$\mathcal{J}_\mu = \partial_\mu \sigma = \frac{1}{2} \epsilon_{\mu\nu\rho} F_{\nu\rho} = F_\mu$$

Its conservation implies the absence of magnetic monopoles in original theory

$$\partial_\mu \mathcal{J}_\mu = \partial_\mu F_\mu = 0$$

Proliferation of monopoles

The presence of the monopoles in the original theory implies reduction of the continuous shift symmetry into a discrete one. Polyakov mechanism.

$$\partial_\mu \mathcal{J}_\mu = \partial_\mu F_\mu = \rho_m(x)$$

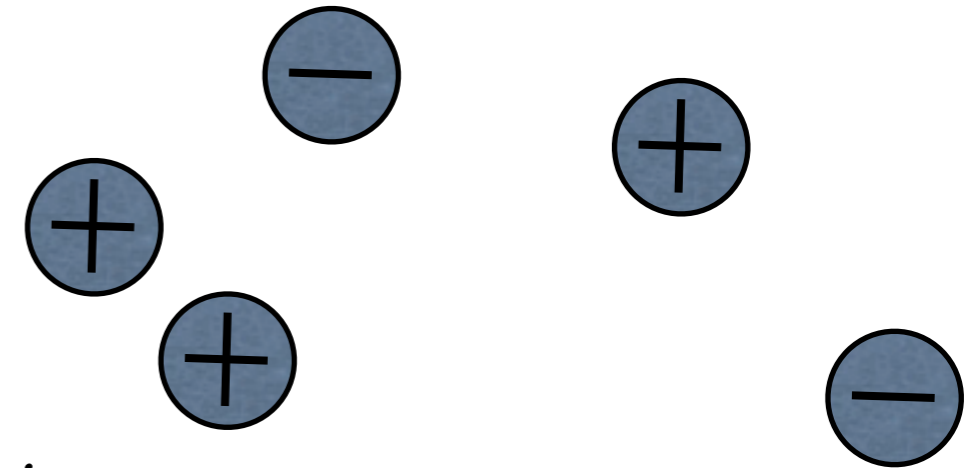
The dual theory

$$L = \frac{1}{2} (\partial\sigma)^2 - e^{-S_0} (e^{i\sigma} + e^{-i\sigma})$$

Physics of Debye mechanism

Discrete shift symmetry: $\sigma \rightarrow \sigma + 2\pi$

$U(1)_{\text{flux}}$ if present, forbids (magnetic) flux carrying operators.



Topological excitations in QCD(adj)

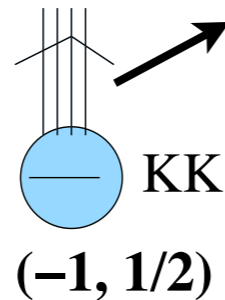
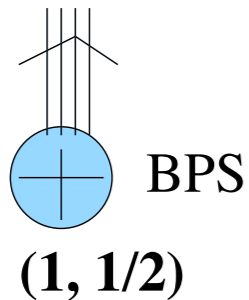
$$\left(\int_{S^2} F, \int_{R^3 \times S^1} F \tilde{F} \right)$$

Magnetic
Monopoles

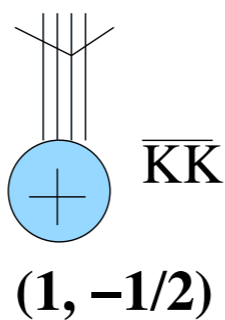
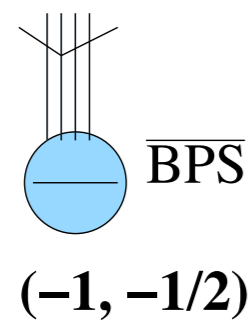
Magnetic
Bions

relevant index theorems
Callias 78
Nye-A.M.Singer, 00
E. Weinberg 80
Poppitz, MU 08
Atiyah-M.I.Singer 75

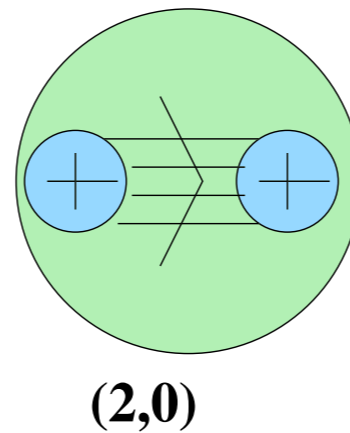
fermionic zero modes



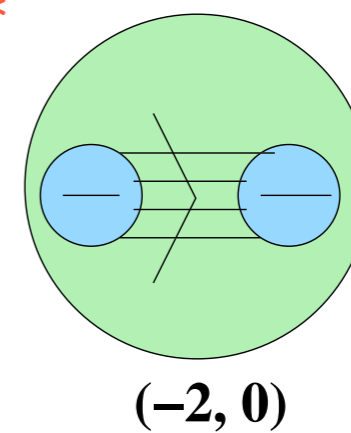
$$e^{-S_0} e^{i\sigma} \det_{I,J} \psi^I \psi^J,$$



$$e^{-S_0} e^{i\sigma} \det_{I,J} \bar{\psi}^I \bar{\psi}^J$$



$(\mathbb{Z}_2)_*$



$$e^{-2S_0} (e^{2i\sigma} + e^{-2i\sigma})$$

Discrete shift symmetry : $\sigma \rightarrow \sigma + \pi$ $\psi^I \rightarrow e^{i\frac{2\pi}{8}} \psi^I$

Crucial earlier work: van Baal et.al. and Lu, Yi, 97

Dual Formulation of QCD(adj)

$$L^{\text{dQCD}} = \frac{1}{2}(\partial\sigma)^2 - b e^{-2S_0} \cos 2\sigma + i\bar{\psi}^I \gamma_\mu \partial_\mu \psi_I + c e^{-S_0} \cos \sigma (\det_{I,J} \psi^I \psi^J + \text{c.c.})$$



magnetic bions
Non-selfdual



magnetic monopoles
Self-dual

Same mechanism in N=1 SYM.

Also see Hollowood, Khoze, ... 99

Earlier in N=1 SYM, the bosonic potential was derived using supersymmetry and SW-curves, F, M theories, field theory methods. However, the physical origin of it remained elusive till this work.

$$m_\sigma \sim \frac{1}{L} e^{-S_0(L)} = \frac{1}{L} e^{-\frac{8\pi^2}{g^2(L)N}} = \Lambda(\Lambda L)^{(8-2N_f^W)/3},$$

Proliferation of magnetic bions

To the surprise of the past

Theory	Confinement mechanism on $R^3 \times S^1$	Index for monopoles $[\mathcal{I}_1, \mathcal{I}_2, \dots, \mathcal{I}_N]$ Nye-Singer, E. Poppitz, MU	Index for instanton $\mathcal{I}_{\text{inst.}}$ Atiyah-Singer	(Mass Gap) ²
YM	monopoles	$[0, \dots, 0]$	0	e^{-S_0}
QCD(F)	monopoles	$[2, 0, \dots, 0]$	2	e^{-S_0}
SYM/QCD(Adj)	magnetic bions	$[2, 2, \dots, 2]$	$2N$	e^{-2S_0}
QCD(BF)	magnetic bions	$[2, 2, \dots, 2]$	$2N$	e^{-2S_0}
QCD(AS)	bions and monopoles	$[2, 2, \dots, 2, 0, 0]$	$2N - 4$	e^{-2S_0}, e^{-S_0}
QCD(S)	bions and triplets	$[2, 2, \dots, 2, 4, 4]$	$2N + 4$	e^{-2S_0}, e^{-3S_0}
$SU(2)$ YM $I = \frac{3}{2}$	magnetic quintets	$[4, 6]$	10	e^{-5S_0}
chiral $[SU(N)]^K$	magnetic bions	$[2, 2, \dots, 2]$	$2N$	e^{-2S_0}
$AS + (N - 4)\bar{F}$	bions and a monopole	$[1, 1, \dots, 1, 0, 0] + [0, 0, \dots, 0, N - 4, 0]$	$(N - 2)AS + (N - 4)\bar{F}$	$e^{-2S_0}, e^{-S_0},$
$S + (N + 4)\bar{F}$	bions and triplets	$[1, 1, \dots, 1, 2, 2] + [0, 0, \dots, 0, N + 4, 0]$	$(N + 2)AS + (N + 4)\bar{F}$	$e^{-2S_0}, e^{-3S_0},$

More refined data

Table 1: Topological excitations which determine the mass gap for gauge fluctuations and chiral symmetry realization in vectorlike and chiral gauge theories on $\mathbf{R}^3 \times \mathbf{S}^1$.

To the surprise of the past

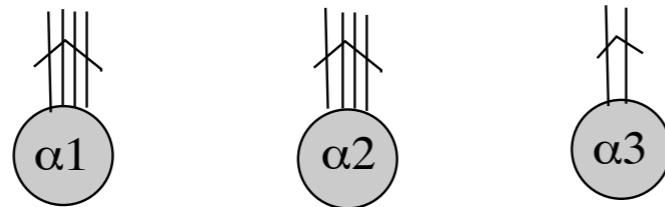
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YM LGY, MU, 08	monopoles	$[0, \dots, 0]$	0	e^{-S_0}
QCD(F) MS, MU, 08	monopoles	$[2, 0, \dots, 0]$	2	e^{-S_0}
SYM/QCD(Adj) MU, 07	magnetic bions	$[2, 2, \dots, 2]$	$2N$	e^{-2S_0}
QCD(BF) MS, MU, 08	magnetic bions	$[2, 2, \dots, 2]$	$2N$	e^{-2S_0}
QCD(AS) MS, MU, 08	bions and monopoles	$[2, 2, \dots, 2, 0, 0]$	$2N - 4$	e^{-2S_0}, e^{-S_0}
QCD(S) EP, MU, 09	bions and triplets	$[2, 2, \dots, 2, 4, 4]$	$2N + 4$	e^{-2S_0}, e^{-3S_0}
$SU(2)$ YM $I = \frac{3}{2}$ EP, MU, 09	magnetic quintets	$[4, 6]$	10	e^{-5S_0}
chiral $[SU(N)]^K$ MS, MU, 08	magnetic bions	$[2, 2, \dots, 2]$	$2N$	e^{-2S_0}
$AS + (N - 4)\bar{F}$ MS, MU, 08	bions and a monopole	$[1, 1, \dots, 1, 0, 0] + [0, 0, \dots, 0, N - 4, 0]$	$(N - 2)AS + (N - 4)\bar{F}$	$e^{-2S_0}, e^{-S_0},$
$S + (N + 4)\bar{F}$ EP, MU, 09	bions and triplets	$[1, 1, \dots, 1, 2, 2] + [0, 0, \dots, 0, N + 4, 0]$	$(N + 2)AS + (N + 4)\bar{F}$	$e^{-2S_0}, e^{-3S_0},$

More refined data

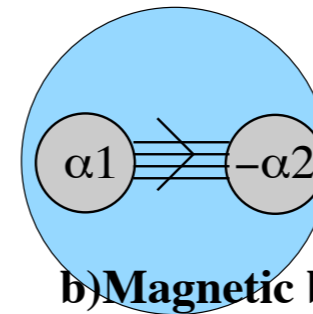
Table 1: Topological excitations which determine the mass gap for gauge fluctuations and chiral symmetry realization in vectorlike and chiral gauge theories on $\mathbf{R}^3 \times \mathbf{S}^1$.

QCD(S) topological excitations/confinement

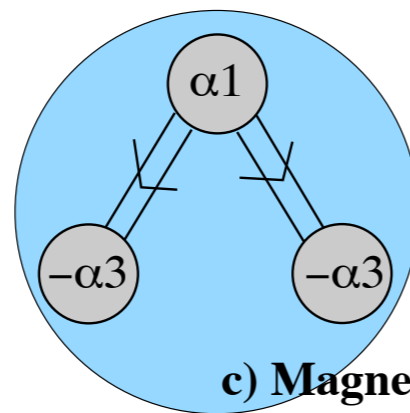
$$[\mathcal{I}_1, \mathcal{I}_2, \mathcal{I}_3] = [4, 4, 2], \quad \mathcal{I}_{\text{inst}} = \sum_{i=1}^3 \mathcal{I}_i ,$$



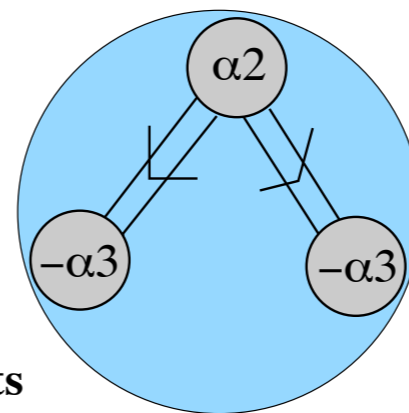
a) Magnetic monopoles



b) Magnetic bion



c) Magnetic triplets



Conformality or confinement:

Conceptually, two problem of out-standing importance in gauge theories:

Mechanisms of confinement and conformality:

What distinguishes two theories, one just below the conformal boundary and confines, and the other slightly above the conformal window boundary? In other words, why does a confining gauge theory confine and why does an IR-CFT, with an almost identical microscopic matter content, flows to a CFT?

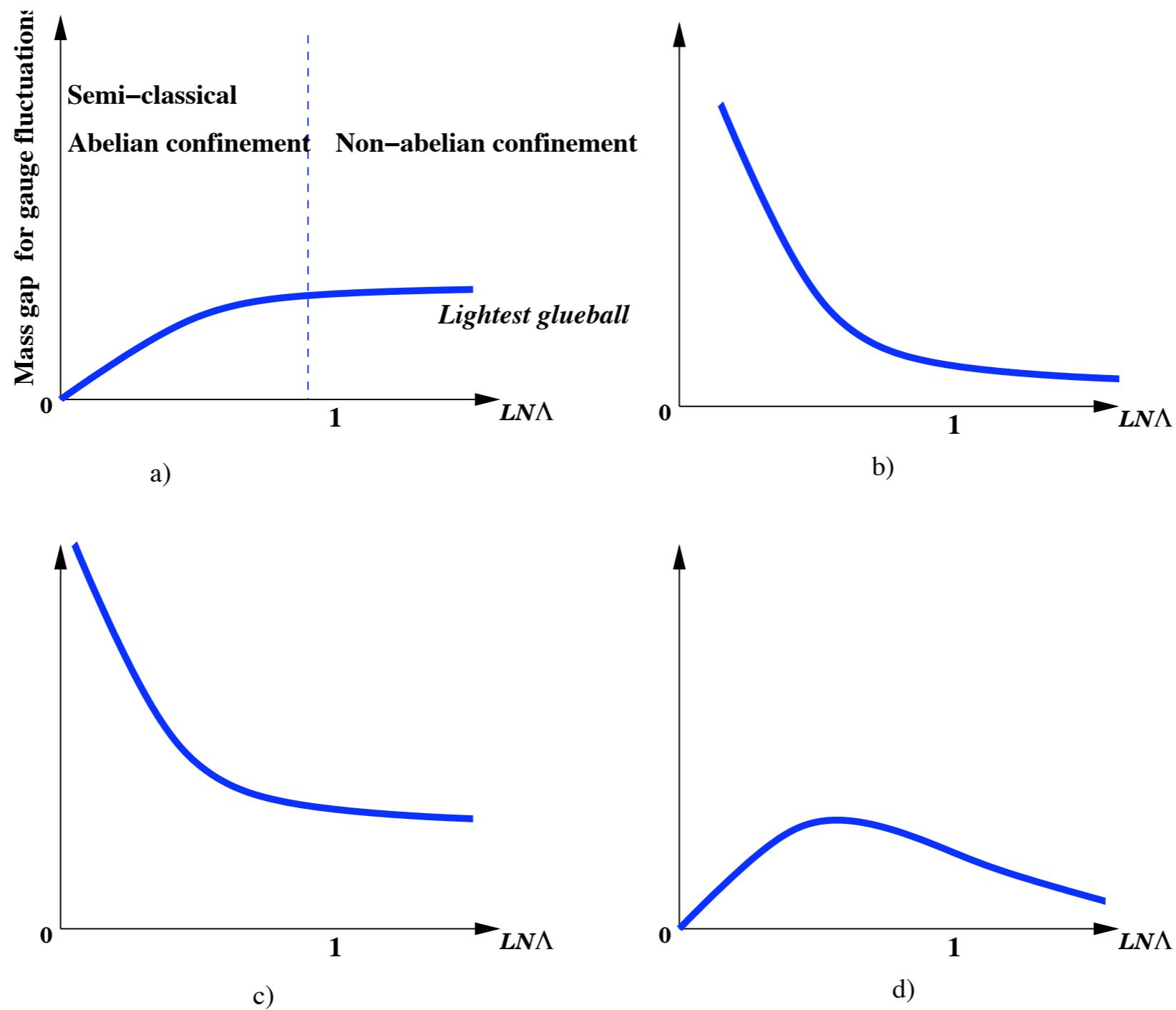
Lower boundary of conformal window: What is the physics determining the boundary of conformal window?

Map the problem to the mass gap for gauge fluctuations:

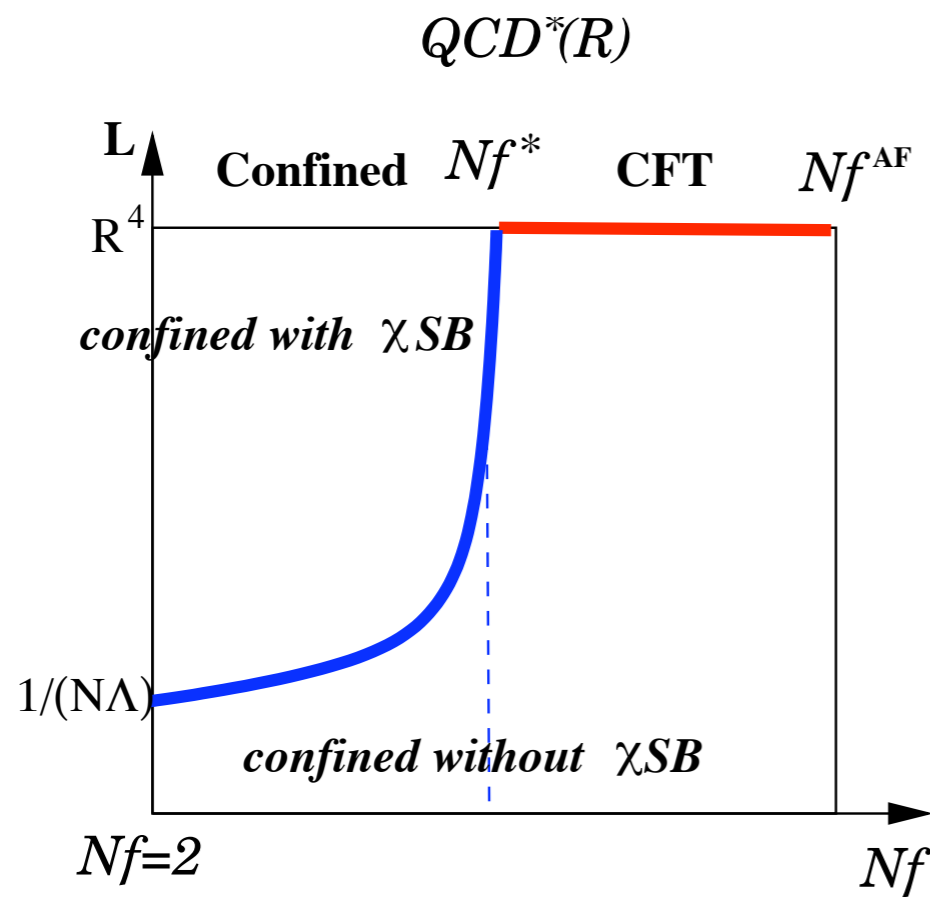
$$m_{\text{gauge fluc.}}^{-1}(\mathbf{R}^4) = \begin{cases} \text{finite} & N_f < N_f^* & \text{confined} \\ \infty & N_f^* < N_f < N_f^{AF} & \text{IR - CFT} \end{cases}$$

A priori, not a smart strategy.

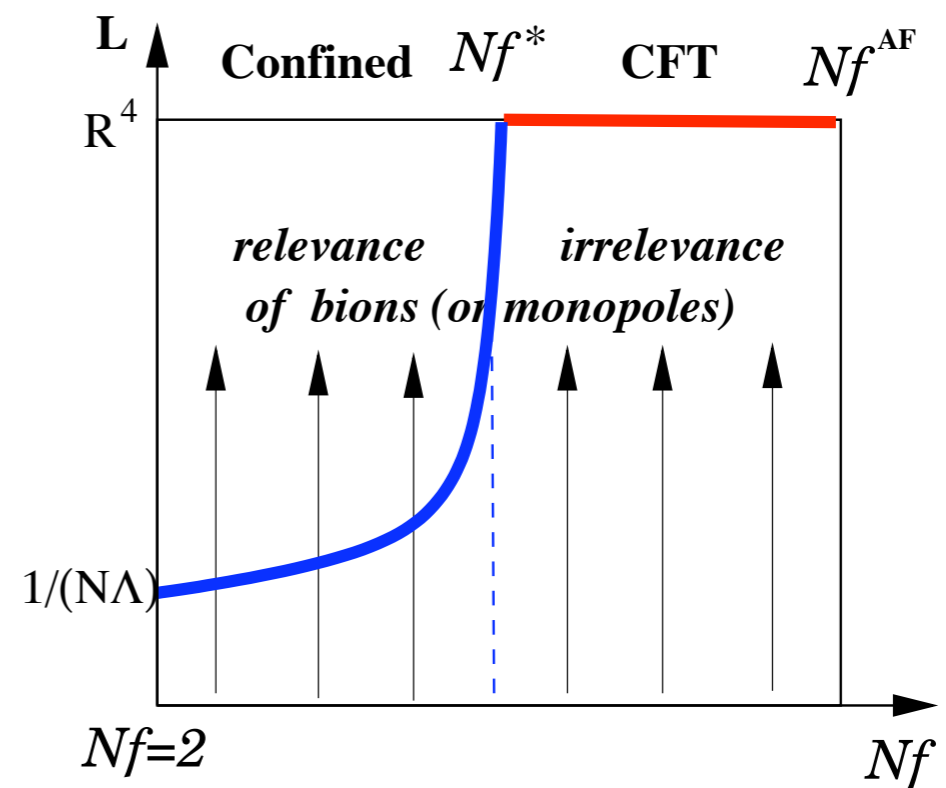
Mass gap for gauge fluctuations



Main Idea of our proposal



a)



b)

(IR)relevance of topological excitations

Conjecture: (E. Poppitz and M.Ü)

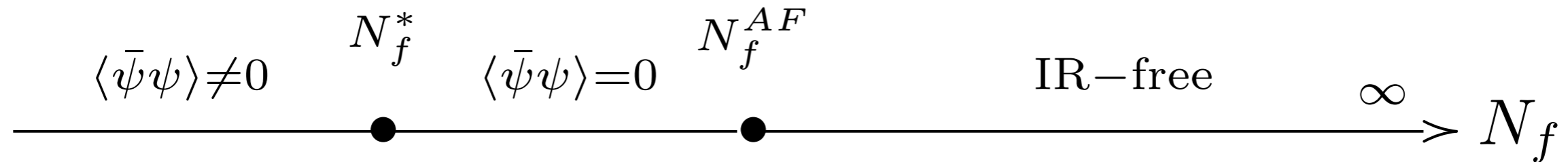
In a given center symmetric gauge theory on $\mathbf{R}^3 \times \mathbf{S}^1$, if the topological excitations become non-dilute and the mass gap in the semiclassical window increases with increasing L , this theory eventually flows to a confining gauge theory on \mathbf{R}^4 with a finite mass gap. If the mass gap for gauge fluctuations decreases with increasing L , diluting the topological excitations in the semiclassical regime, such theories will (most likely) flow to IR-CFT on \mathbf{R}^4 , with a vanishing mass gap; if, on the other hand, χSB is triggered outside of the semiclassical regime, when $L\Lambda \sim 1$, they will flow to confining theories on \mathbf{R}^4 .

Crucial data: Index theorem on $\mathbf{R}^3 \times \mathbf{S}^1$, the knowledge of mechanism of confinement, and one-loop beta function.

QCD(F/S/AS/Adj): Estimates and comparisons

Below, I will present the estimates based on this idea and compare it various other approaches. In particular:

- 1) Truncated SD (ladder, rainbow) approximation. M.E.Peskin, Chiral Symmetry And Chiral Symmetry Breaking, Les Houches, 1982 (Up-to date review)



QCD(F): Appelquist, Lane, Mahanta, and Miransky
Two-index cases: Sannino, Dietrich.

- 2) NSVZ-inspired conjecture: Sannino, Rytov.

Crucial data for 1) and 2): Two-loop (or conjectured all orders) beta function, anomalous dimension of fermion bilinear.

Caveat: chiral gauge theories.

QCD(S/AS/Adj): Estimates and comparisons

N	Deformation theory (bions)	Ladder (SD)-approx.	NSVZ-inspired: $\gamma = 2/\gamma = 1$	N_f^{AF}
3	2.40	2.50	1.65/2.2	3.30
4	2.66	2.78	1.83/2.44	3.66
5	2.85	2.97	1.96/2.62	3.92
10	3.33	3.47	2.29/3.05	4.58
∞	4	4.15	2.75/3.66	5.5

Table 1: Estimates for lower boundary of conformal window in QCD(S), $N_f^* < N_f^D < 5.5 \left(1 - \frac{2}{N+2}\right)$.

N	Deformation theory (bions)	Ladder (SD)-approx.	NSVZ-inspired: $\gamma = 2/\gamma = 1$	N_f^{AF}
4	8	8.10	5.50/7.33	11
5	6.66	6.80	4.58/6.00	9.16
6	6	6.15	4.12/5.5	8.25
10	5	5.15	3.43/4.58	6.87
∞	4	4.15	2.75/3.66	5.50

Table 2: Estimates for lower boundary of conformal window in QCD(AS), $N_f^* < N_f^D < 5.5 \left(1 + \frac{2}{N-2}\right)$.

N	Deformation theory (bions)	Ladder (SD)-approx.	NSVZ-inspired: $\gamma = 2/\gamma = 1$	N_f^{AF}
any N	4	4.15	2.75/3.66	5.5

Table 3: Estimates for lower boundary of conformal window in QCD(adi), $N_f^* < N_f^D < 5.5 \left(1 + \frac{2}{N-2}\right)$.

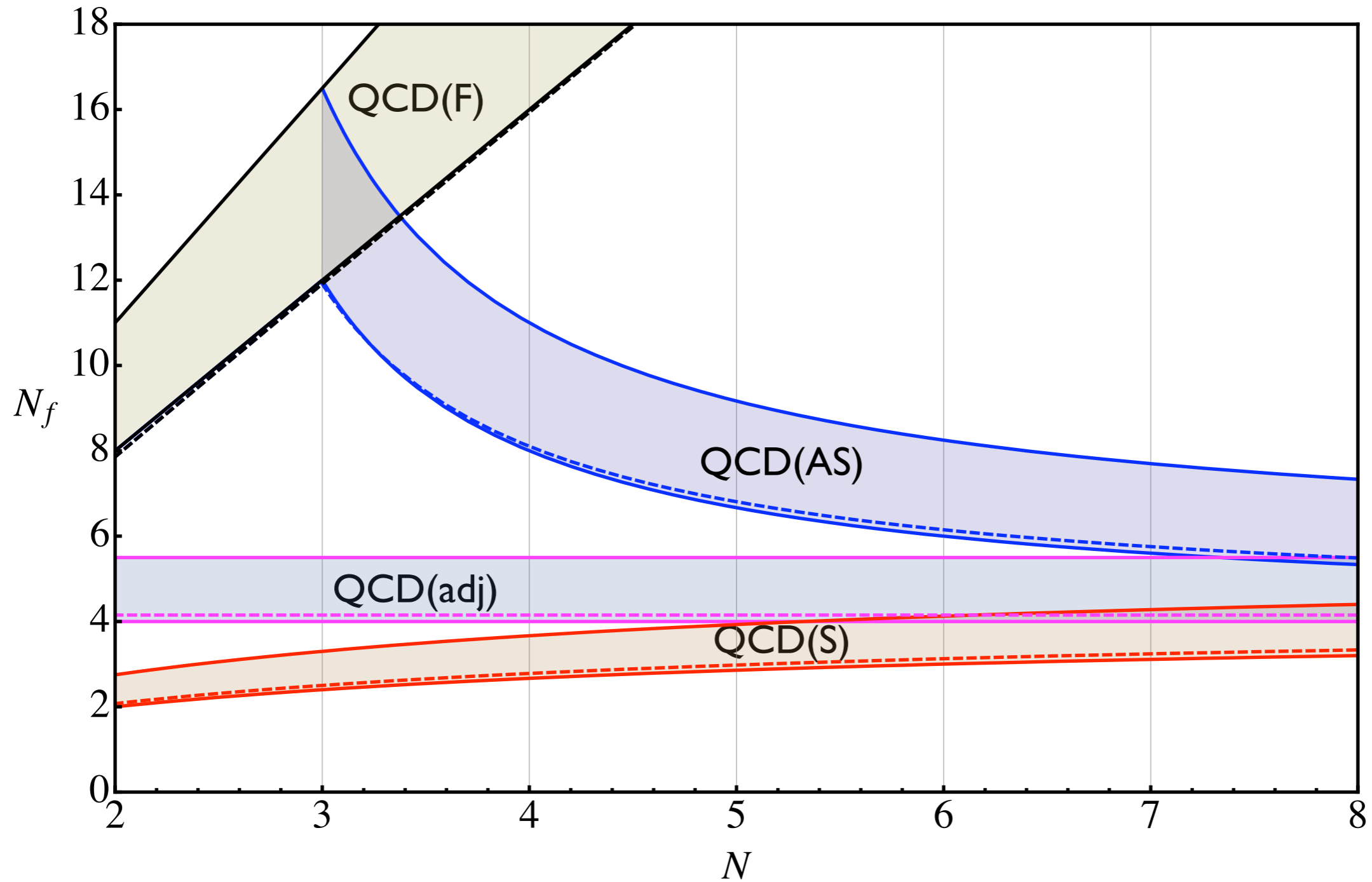
QCD(F)

N	D.T. 1a/1c	Ladder (SD)-approx.	Functional RG	NSVZ-inspired: $\gamma = 2/\gamma = 1$	N_f^{AF}
2	5/8	7.85	8.25	5.5/7.33	11
3	7.5/12	11.91	10	8.25/11	16.5
4	10/16	15.93	13.5	11/14.66	22
5	12.5/20	19.95	16.25	13.75/18.33	27.5
10	25/40	39.97	n/a	27.5/36.66	55
∞	$2.5N/4N$	$4N$	$\sim (2.75 - 3.25)N$	$2.75N/3.66N$	$5.5N$

Table 1: Estimates for lower boundary of conformal window for QCD(F), $N_f^* < N_f^D < 5.5N$

The above estimates from DT are for class a and a+c, respectively.

Estimates of the conformal window from deformation theory



Dashed line: **Truncated SD (ladder, rainbow)** approximation.

QCD(F): Miransky, Appelquist, Lane, Mahanta.....

Two-index: Sannino et.al.

BUT WHY?

How can we relate perturbation theory to non-pert. physics?

$$\gamma(g^2) = \frac{3}{2} \frac{(g^2 N)}{8\pi^2} [1 + O(g^2 N)].$$

Anomalous dimension of fermion bilinear

If $\gamma(L) < 1$, no χ SB

Monopole action: $S_0(L) = \frac{8\pi^2}{g^2(L)N}$

$$\begin{aligned} \gamma(L) \ll 1 &\Rightarrow S_0(L) \gg 1 \Rightarrow e^{-S_0} \ll 1, & \text{dilute gas of monopoles and bions} \\ \gamma(L) \sim 1 &\Rightarrow S_0(L) \sim 1 \Rightarrow e^{-S_0} \sim 1, & \text{non-dilute} \end{aligned}$$

means non-abelian confinement cannot set-in.

$$\gamma(\bar{\psi}\psi)S_0 = \gamma(g^2(L))S_0(g^2(L)) \sim 1$$

Needs refinement, but it seems to be on the right path.

Conclusions

- There is now a window through which we can look into non-abelian gauge theories and understand their internal goings-on. Whether the theory is chiral, pure glue, or supersymmetric is immaterial. We always gain a semi-classical window (in some theories smoothly connected to R4 physics.)
- Deformation theory is complementary to lattice gauge theory. Sometimes lattice is more powerful, and sometime otherwise. Currently, DT is **the only dynamical framework** for chiral gauge theories. It may also be more useful in the **strict** chiral limit of vector-like theories.
- We learned the existence of a large class of new non-self dual topological excitations through this program in the last two years.
- We understand mechanism of conformality and confinement in gauge theory. This is tied with the (IR)relevance of topological excitations.