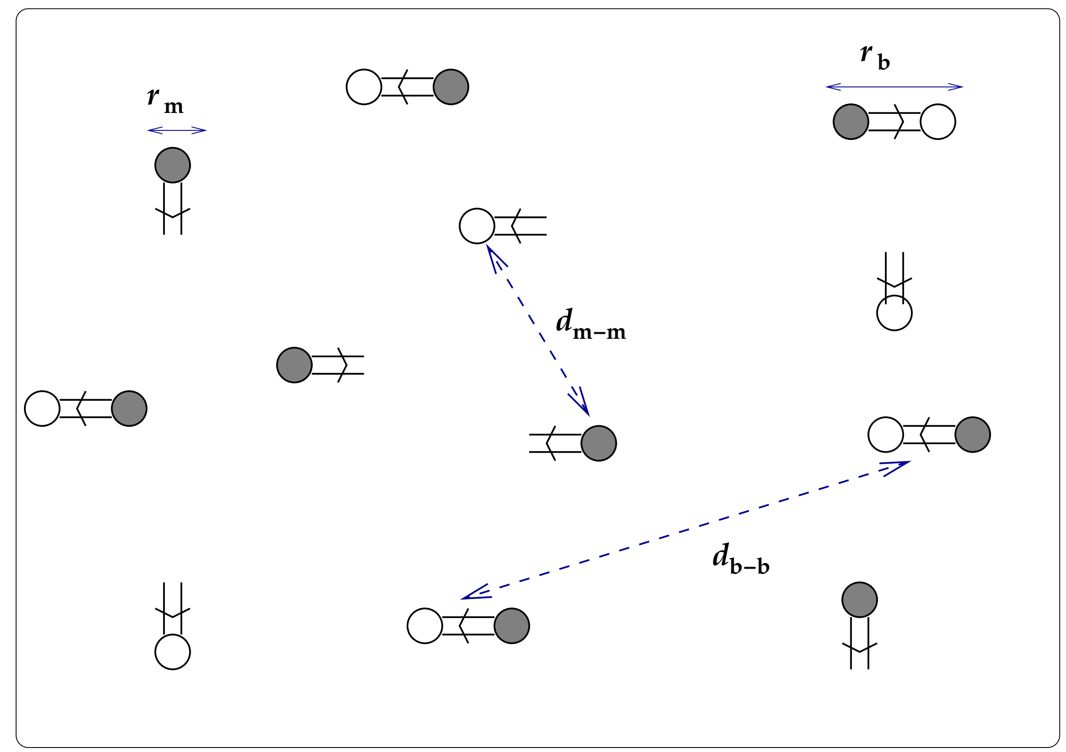
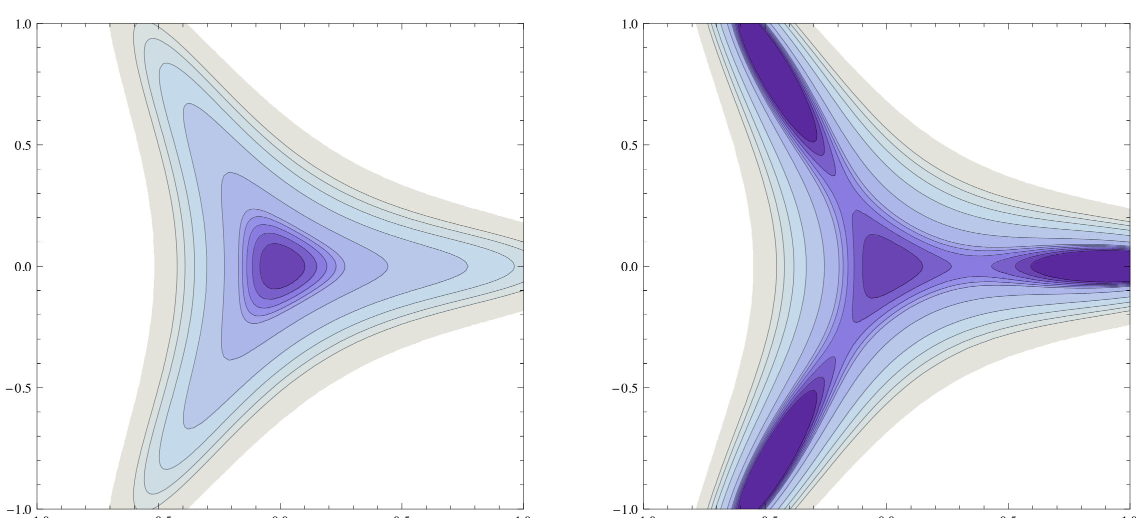
Insightful supersymmetry

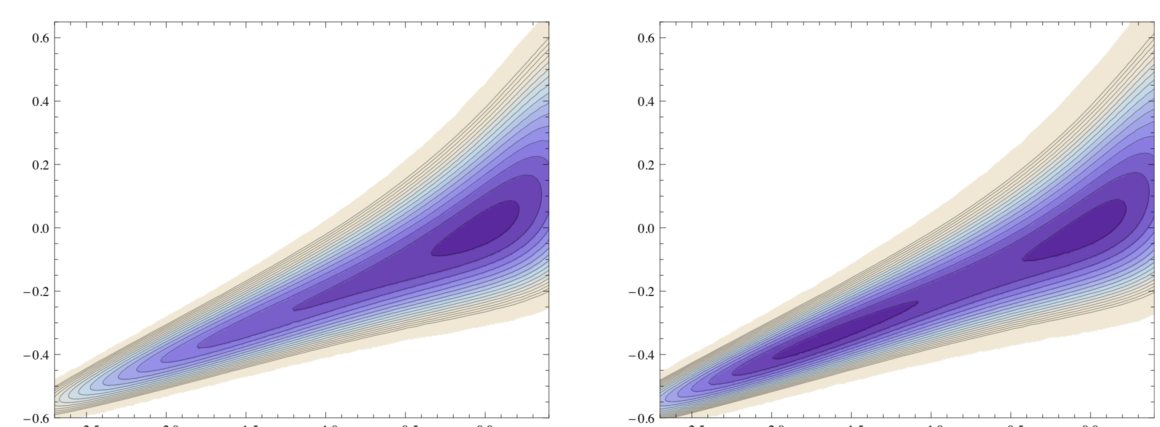
Erich Poppitz

U of Toronto





deconfinement in SU(3)



deconfinement in G(2)

Insightful supersymmetry

Erich Poppitz U of Toronto

in collaboration with

Mithat Ünsal SFSU
Mohamed Anber Toronto - postdoc
Thomas Schäfer NCSU
Scott Collier Toronto - undergrad (now, graduate student at McGill, string theory, with Alex Maloney)
Tin Sulejmanpasic Regensburg - graduate student (QCD/lattice, with Tilo Wettig)
Seth Strimas-Mackey Toronto - still an undergrad
Brett Teeple Toronto - graduate student

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Mithat Ünsal SFSU 0812.2085 0905.0634 0906.5256 0910.1245 0911.0358 1005.3519 1105.3969 1112.6389 1205.0290 1212.1238

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Why the title/abstract I gave?

While the LHC continues the search for variants of weak-scale supersymmetry:
"natural", "compressed", "split" or "flavorful", among others
- and may or may not find evidence for it -
I will discuss another, less direct, less mainstream, and more recent,
use of supersymmetry in particle theory... albeit one that will not seen at the LHC...

main message:

It has been realized that studies of supersymmetric gauge theories in the late 1990's, when properly interpreted, lead to insights whose relevance transcends supersymmetry.

I will illustrate the “insightful” nature of supersymmetry by two examples having to do with the microscopic description of the thermal ***deconfinement*** transition.

A host of strange ***topological*** molecules will be seen to be the major players in the confinement-deconfinement dynamics.

Interesting connections emerge, between topology, “condensed-matter” ***gases of electric and magnetic charges***, and attempts to make sense of the divergent perturbation series.

I will illustrate the “insightful” nature of supersymmetry by two examples having to do with the microscopic description of the thermal ***1. deconfinement*** transition.

A host of strange ***2. topological*** molecules will be seen to be the major players in the confinement-deconfinement dynamics.

Interesting connections emerge, between topology, “condensed-matter” ***3. gases of electric and magnetic charges***, and attempts to make sense of the divergent perturbation series.

Outline:

1. *deconfinement*

2. *topological* - including “SYM*/thermal YM continuity conjecture”

3. *thermal gases of electric and magnetic charges*

What are 1,2,3?

What do they have in common?

And how did SUSY help?

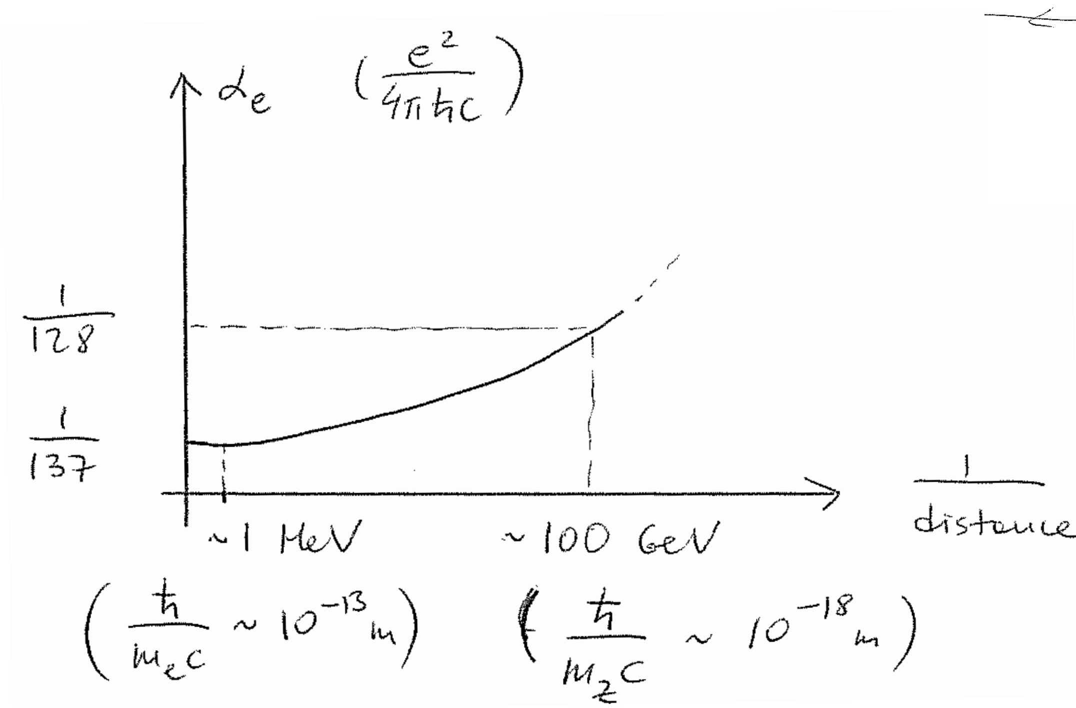
1. deconfinement

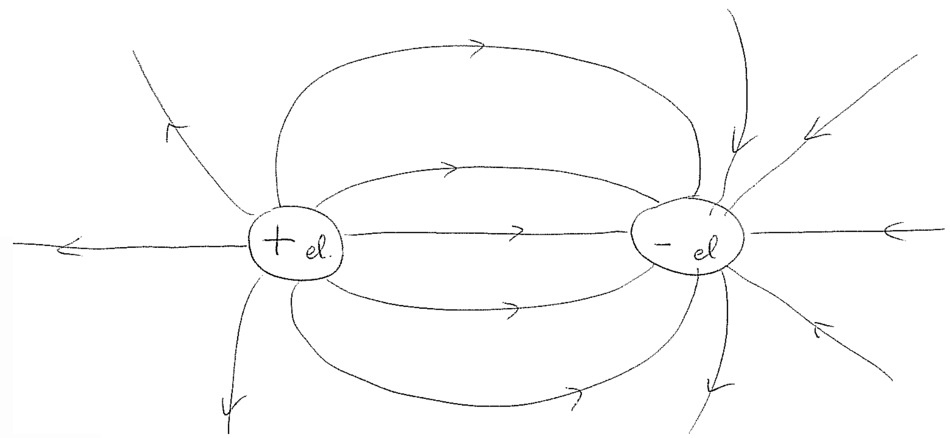
what is it and how do we study it?

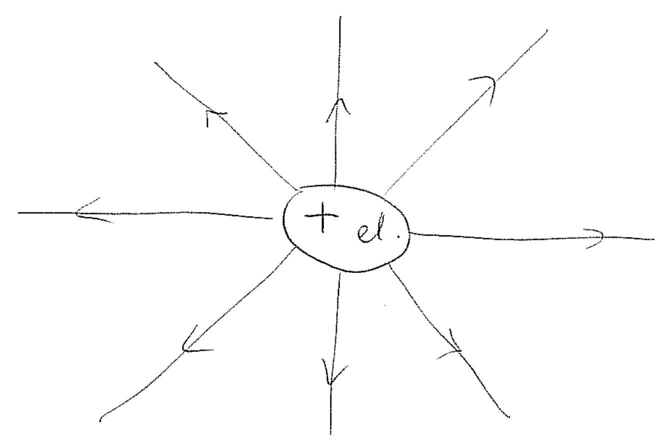
QCD - theory of the strong interactions:
quarks and gluons, discovered in 1970s

asymptotic freedom - antiscreening, reverse of QED

QED:







Coulomb-like field
at long distances

I. deconfinement

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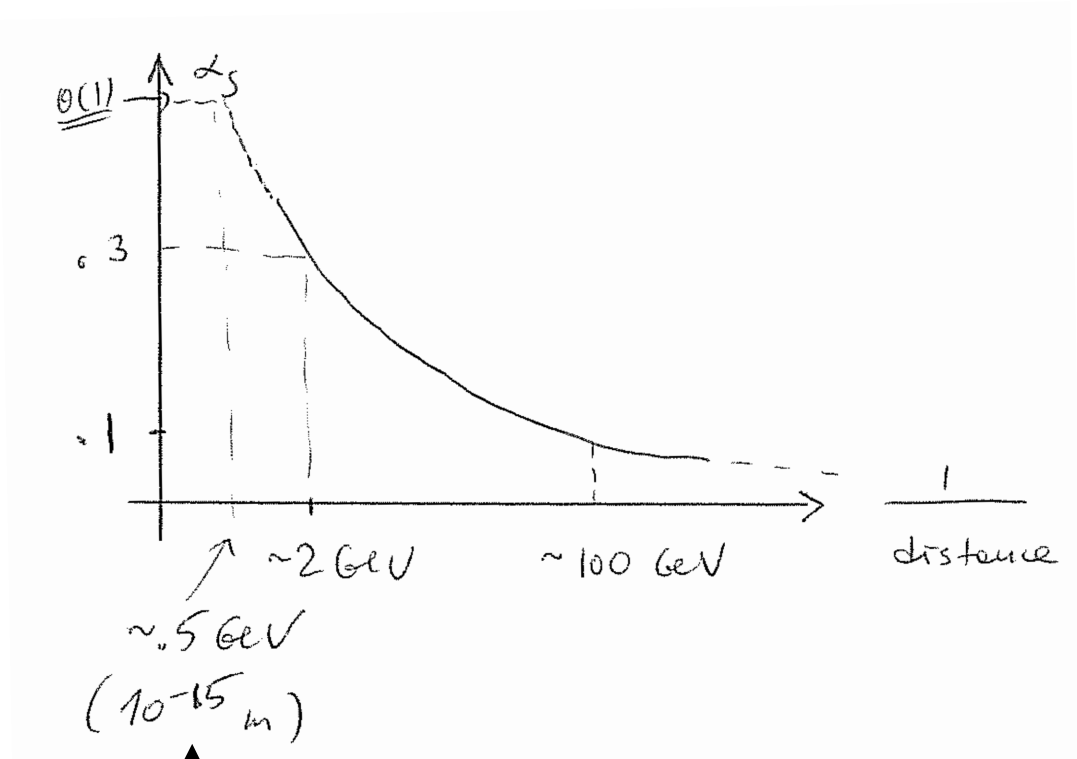
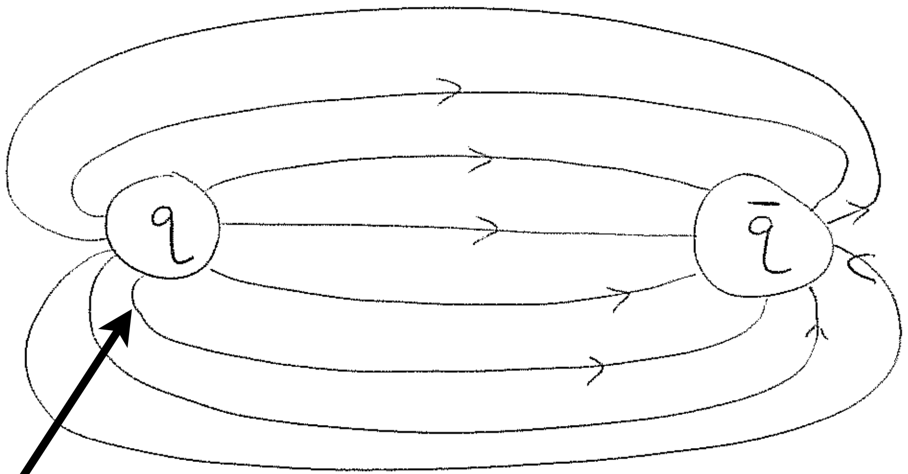
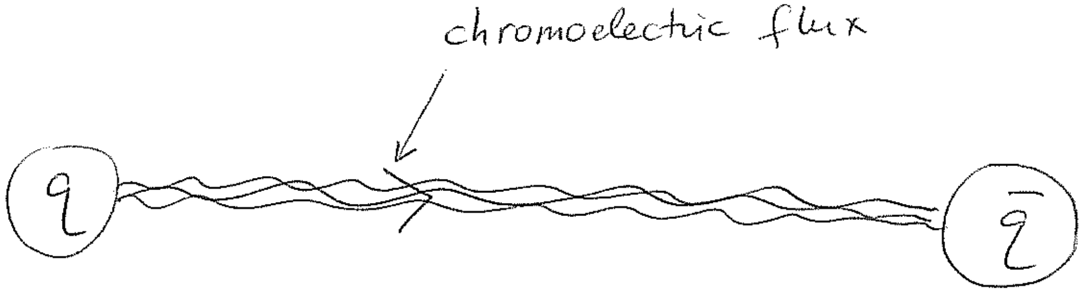
QCD:



strong scale!



Coulomb-like field
at short distances less then inverse strong scale!



$$E_{q\bar{q}} = \sigma \, r_{q\bar{q}}$$

$$[\sigma] = \frac{\text{energy}}{\text{length}} \qquad \sigma \sim (1\ \text{GeV})^2 \frac{1}{\hbar c}$$

(constant force 10^6 N)

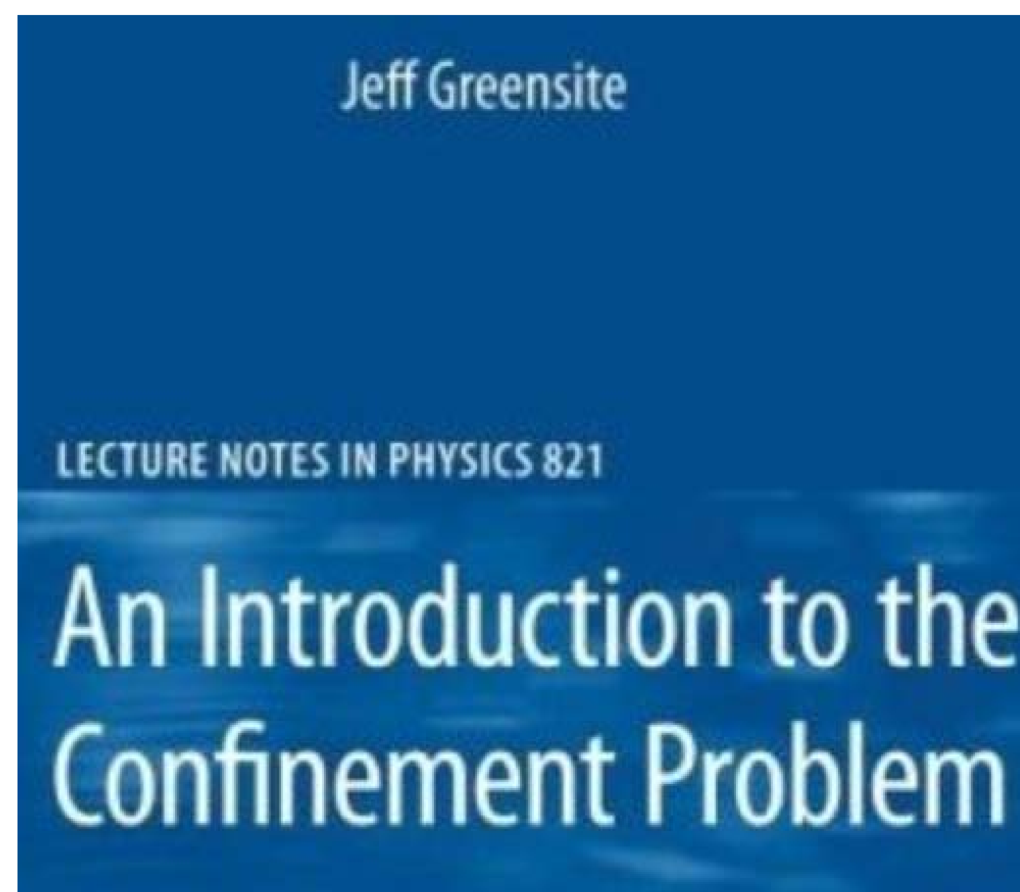
(1 N is the force of Earth's gravity on a mass of about 102 g)

How do the gluons and quarks - the fundamental fields of QCD - give rise to this configuration?

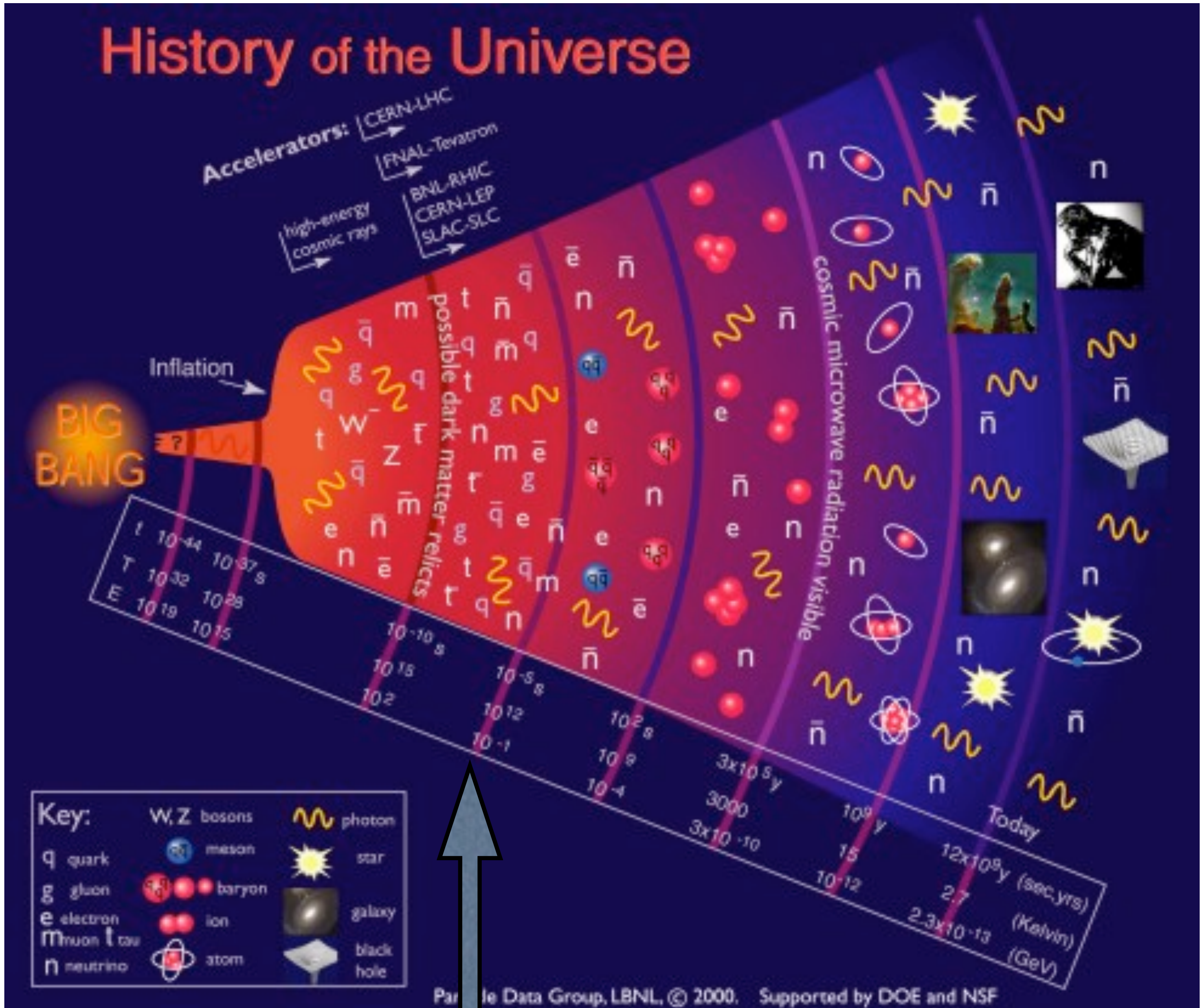
... a "million dollar question" - also, literally...

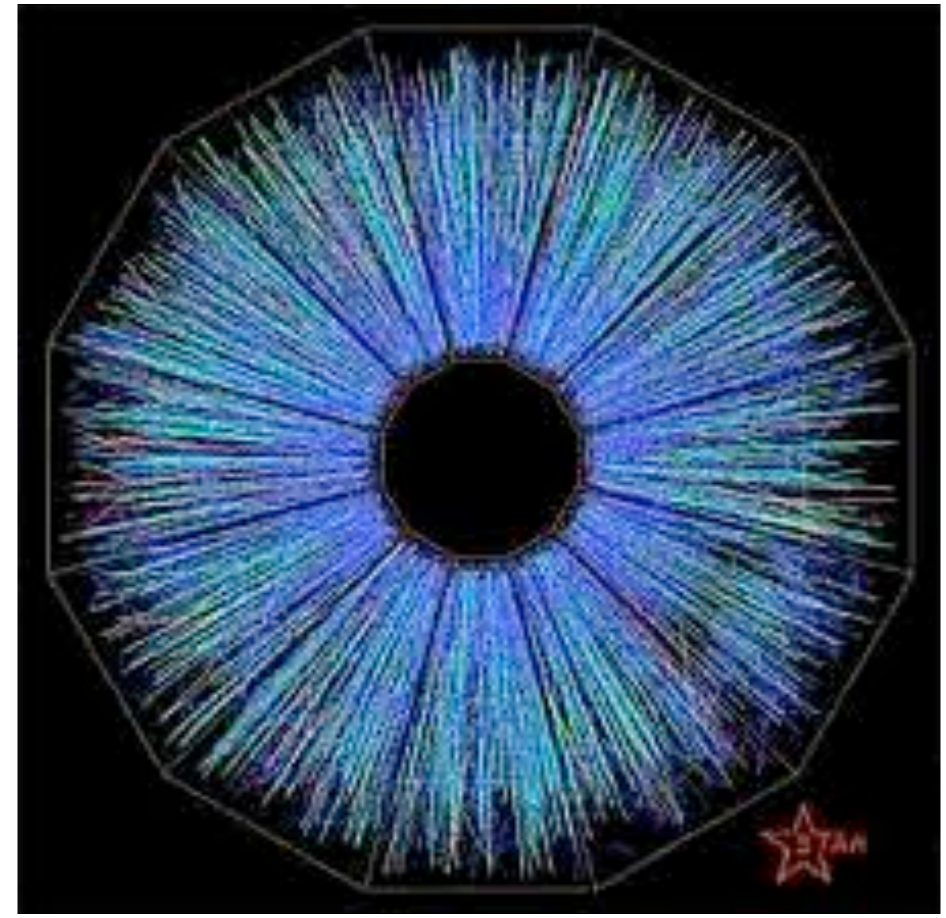
...only one monograph produced, so far

an overview of various existing approaches models, ca. 2010



What happens when quarks and gluons are "heated up"?





$k_B T \sim 100\text{MeV} \quad T \sim 10^{12}\text{K}$

$(10^{-10}\text{s after big bang})$

- quarks and gluons are "liberated" or "deconfined"

Why does deconfinement occur? - a picture and an estimate...

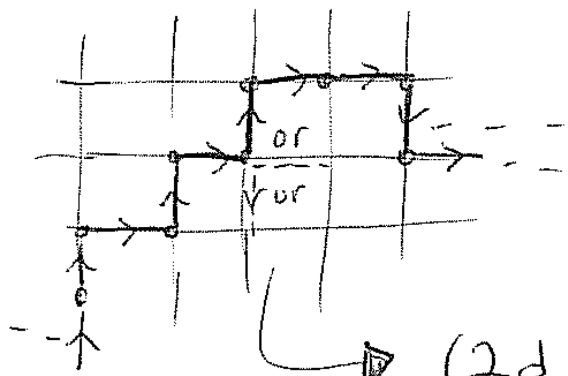
assume YM theory confines, hence it is a theory of chromoelectric fluxes

energy of a flux tube of length L

$$E \sim L\sigma$$

entropy of a flux tube of length L

$$S \sim k_B \log(2d-1)^{L\sqrt{\sigma}}$$



$(2d-1)^{\#\text{links}}$ (or "bits" of string")

$$F = E - TS \sim L\sigma - k_B T L\sqrt{\sigma}\log(2d-1)$$

Z diverges at Tc $\quad k_B T_c \sim \sqrt{\sigma} \sim 100\mathrm{MeV}$

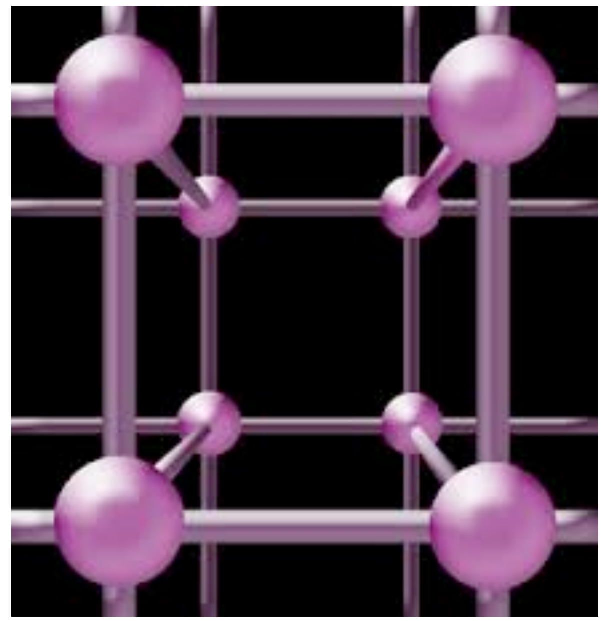
above Tc entropy dominates strings "melt" (or "condense"), confinement lost...

... despite "success" - this is a "picture", quite far from a "theory" (QCD)

"picture" becomes a "theory" - but of compact lattice U(1)
at strong coupling in the Villain representation [Polyakov; Susskind 1970s]

How do people actually study deconfinement?

experiment: real or lattice, *i.e. numerical*

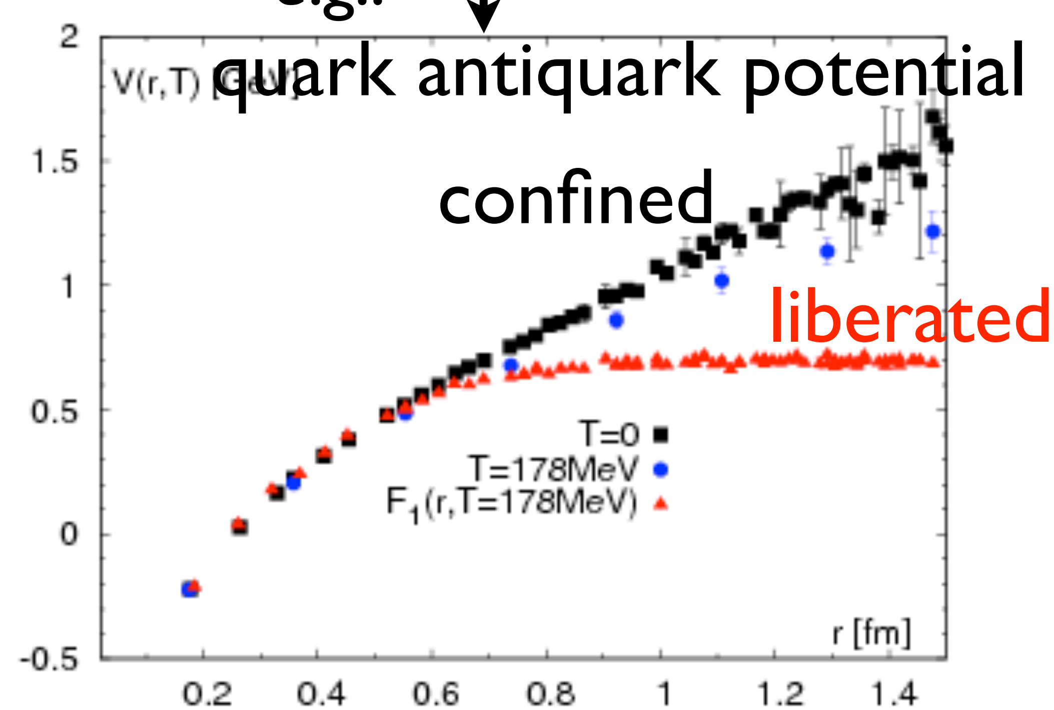


+

- description of hydrodynamic flow
- equation of state...

e.g.:

quark antiquark potential



Is there a place/need/opportunity for any analytical work here?

a "picture", a "model", or a "theory"

e.g., two slides ago

Models, in the best of cases, are designed to fit (some subset of) data from lattice field theory numerical results, e.g.: *Pisarski et al./ Diakonov, Petrov/Zhitnitnsky,Parnachev/Shuryak, Sulejmanpasic-Faccioli/FRG approach...*

When dealing with "messy" stuff, these have their place - but there may be dangers lurking if taken too seriously. Often, "voodoo QCD" characterization justified...

BJ?(via Ken Intriligator):
"...never know if you're right, until confirmed by some other means..."

Lattice QCD, is, of course, a "theory", whose use in the continuum limit requires numerics.

Are there any theoretically-controlled first-principles calculations that allow analytic studies?

Lattice QCD, is, of course, a "theory", whose use in the continuum limit requires numerics.

Are there any theoretically-controlled first-principles calculations that allow analytic studies?

There are a only a few of these.

None of them captures all features of real QCD.

So why do we care?

Before answering, recall some facts about thermal theories.

Thermal partition function is (without fermions):

$$Z(\beta) = \mathrm{tr}[e^{-\beta H}], \qquad \beta = 1/T = \text{radius of } S^1 \qquad \mathbb{R}^3 \times S^1$$

L - size of S[1]

$\mathbb{R}^3$ $\qquad$ $\mathbb{R}^4$

high-T:
Quark Gluon Plasma

Λ^{-1}

low-T:
Hadronic Confined Phase

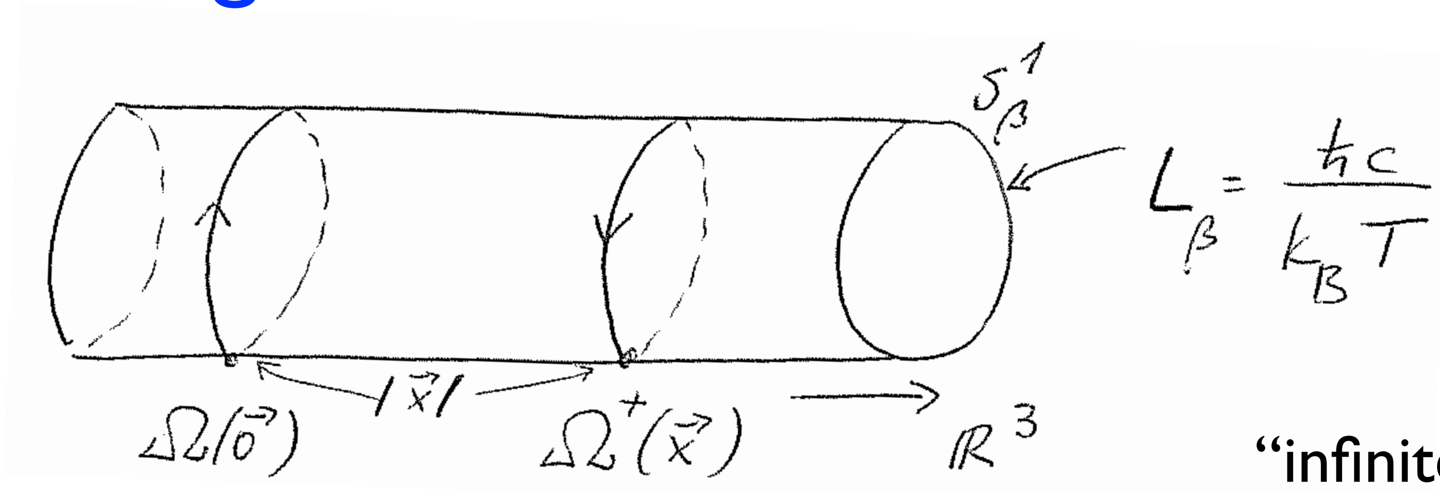
L

strong scale!

a static quark probe

$$\Omega = \mathrm{tr}\,\mathcal{P}\exp[i\int_{S^1} A_4 dx^4]$$

Wilson/Polyakov loop



$\bar{q}$ at $\vec{x}$ $\qquad$ q at $\vec{0}$

"infinite F_quark"

$$\langle \Omega^\dagger(\vec{x})\ \Omega(0)\rangle \sim e^{-\frac{V(|\vec{x}|)}{T}}$$

confined $\quad e^{-\frac{\sigma|\vec{x}|}{T}} \underset{x\to\infty}{\longrightarrow} 0$

deconfined $\quad e^{-\frac{e^{-m_e|\vec{x}|}}{|\vec{x}|T}} \underset{x\to\infty}{\longrightarrow} 1$

hence

$\langle\Omega\rangle = 0$ confined

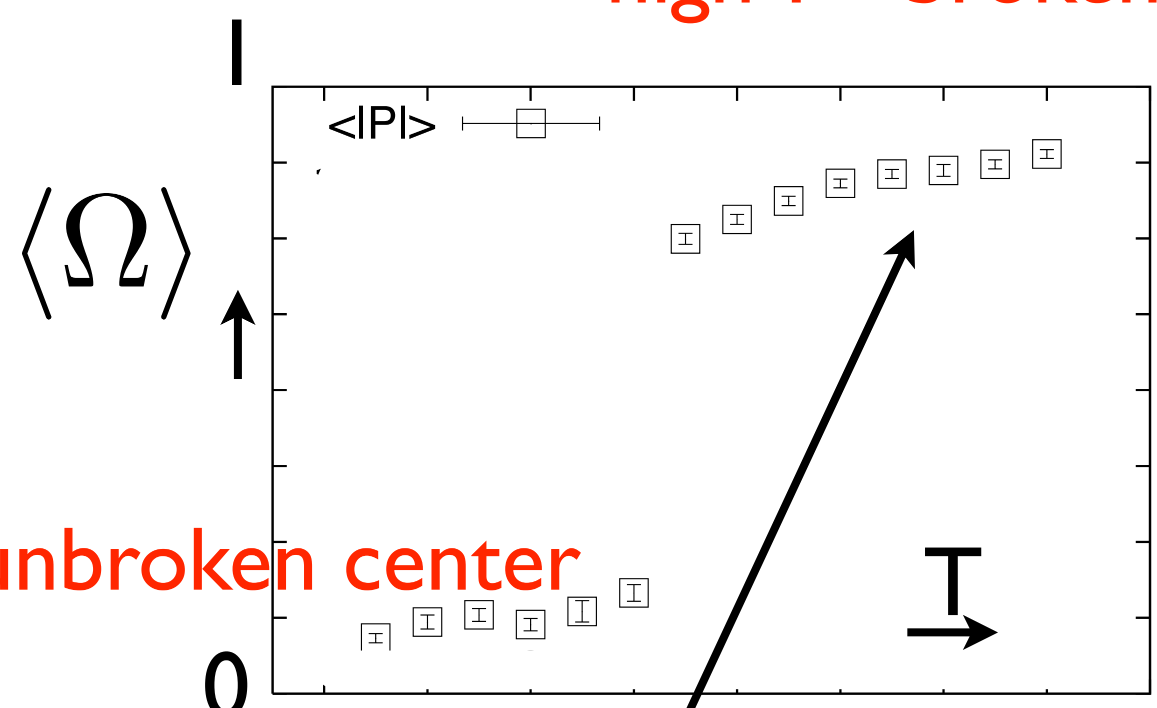
$\langle\Omega\rangle \neq 0$ deconfined

in SU(N) theory without fundamentals, deconfinement =
breaking of global Z_N center symmetry ["gauge transform" periodic up to center]

$$\Omega_{\text{fund}} \xrightarrow{z \subset Z_N} z\Omega_{\text{fund}}$$

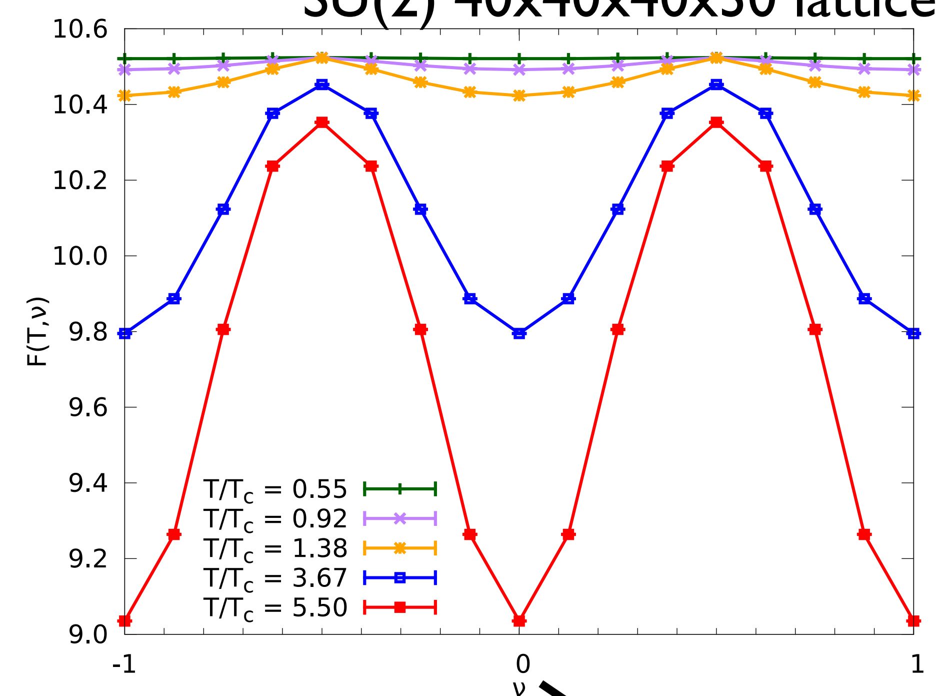
$$\Omega = \mathrm{tr}\,\mathcal{P}\exp[i\int_{S^1} A_4 dx^4]$$

high T - broken center



low T - unbroken center

e.g. 1205.4768
SU(2) 40x40x40x30 lattice



$T \gg T_c$ behavior has been understood for 30 years

[Gross, Pisarski, Yaffe, 1981]

High-T perturbation theory good, gives one-loop V(pert), favors center-broken vacuum, e.g.

$$V_{\text{pert.}}(\Omega) = -\frac{2}{\pi^2\beta^4}\sum_{n=1}^{\infty}\frac{1}{n^4}|\mathrm{tr}\Omega^n|^2(1+O(g^2)).$$

high-T:
coinciding
eigenvalues

$$\Omega = \frac{1}{2}\mathrm{Tr}\begin{pmatrix} e^{i\pi\nu} & 0 \\ 0 & e^{-i\pi\nu}\end{pmatrix}$$

Gross, Pisarski, Yaffe, 1981

"*It is hardly surprising that we cannot explore the transition, as the temperature is lowered, from the unconfined to the confined phase using solely weak coupling techniques*"

Nonetheless, it is of interest to find examples where one could study deconfinement by reliable analytical techniques (*"why bother?"*):

- because we can, it is great fun, and it is beautiful.

- because, we believe that understanding an analytically calculable regime is always good, likely to give insight into important aspects of the physics and into how QFT "works"

- because pushing a calculable regime to (or beyond) borders of its validity can be useful; resulting models can be compared, e.g. with lattice (e.g. work of Shuryak, Sulejmanpasic; lattice work...)

Several ways to do this have been found in the past 30 years:

1. Gauge-gravity duality [many, after Witten 1998, ...]

pro: semiclassical string theory provides a weak-coupling description of strongly-coupled gauge theory

deconfinement=Hawking-Page

useful macroscopically (especially out-of-equilibrium)

con: comes with extra baggage - non decoupling KK modes;

no asymptotic freedom;

microscopic connection ?

2. $S^1 x S^3$ compactifications [Aharony, Marsano, Minwalla, Papadodimas, van Raamsdonk, 2003-5]

thermal

non-thermal

pro: at small S^3, a weakly coupled matrix model

low-T: Vandermonde repulsion of EVs

high-T: pert. attraction of Polyakov loop EVs

con: thermodynamic limit means large-N transition only

These authors rejected the possibility of finding a weak-coupling transition at infinite volume...

such a description has been found:

3. $R^2 \times S^1 \times S^1$ compactifications

non-thermal

thermal

[Simic, Unsal 2010
Unsal 2012

Anber, EP, Unsal 2011
Anber, Collier, EP 2012
Anber, Collier, Strimas-Mackey, Teeple, EP 2013]

"deformed" pure-YM

"QCD(adj)" = YM with many massless adjoint Weyl fermion

(~ large-N limit of QCD with fundamental quarks via some large-N "orientifold" equivalences...)

pro: at small S^1, map 4d thermal gauge theory to a 2d spin system - "affine" XY spin models related to cond. mat. systems: e.g., 2d triangular lattice crystal melting for SU(3)(adj) - or more general new stat-mech models

con: abelianized, L< infinity

nonetheless (I think) fascinating systems:

2d "gases" of el. and m. charged particles, with Aharonov-Bohm interactions, inheriting the symmetries of their respective 4d gauge theories and showing a deconfinement transition [far from all is understood!]

In the process of unraveling the above map, SUSY played a crucial role...

- to be explained later; note the nf=1 adjoint theory is N=1 SYM -

4. $R^3 \times S^1$ compactifications of SYM*

(non-) thermal

[Schaefer, Unsal, EP 1205.0290, 1212.1238
Anber 1302.2641; Sulejmanpasic, EP 1307.1317;
early remarks in Unsal, Yaffe 1006.2101]

DEFINITIONS:

1
super YM = "SYM" = YM + massless quark, a triplet of SU(2), aka "gaugino"

fields: gauge bosons + gauginos; Z_4 chiral symmetry

2
SYM* = SYM + mass for the triplet quark, i.e. with a "gaugino mass" m

supersymmetry and Z_4 chiral symmetry explicitly broken by m

we study SYM* on $\mathbb{R}^3 \times S^1_L$ with periodic (supersymmetric, non-thermal) boundary condition for gaugino

there are only two parameters to vary: L and m

Z_2 center symmetry- S^1_L

the theory is asymptotically free with a strong scale! Λ

$\frac{m}{\Lambda}$ ΛL

4. $R^3 \times S^1$ compactifications of SYM*

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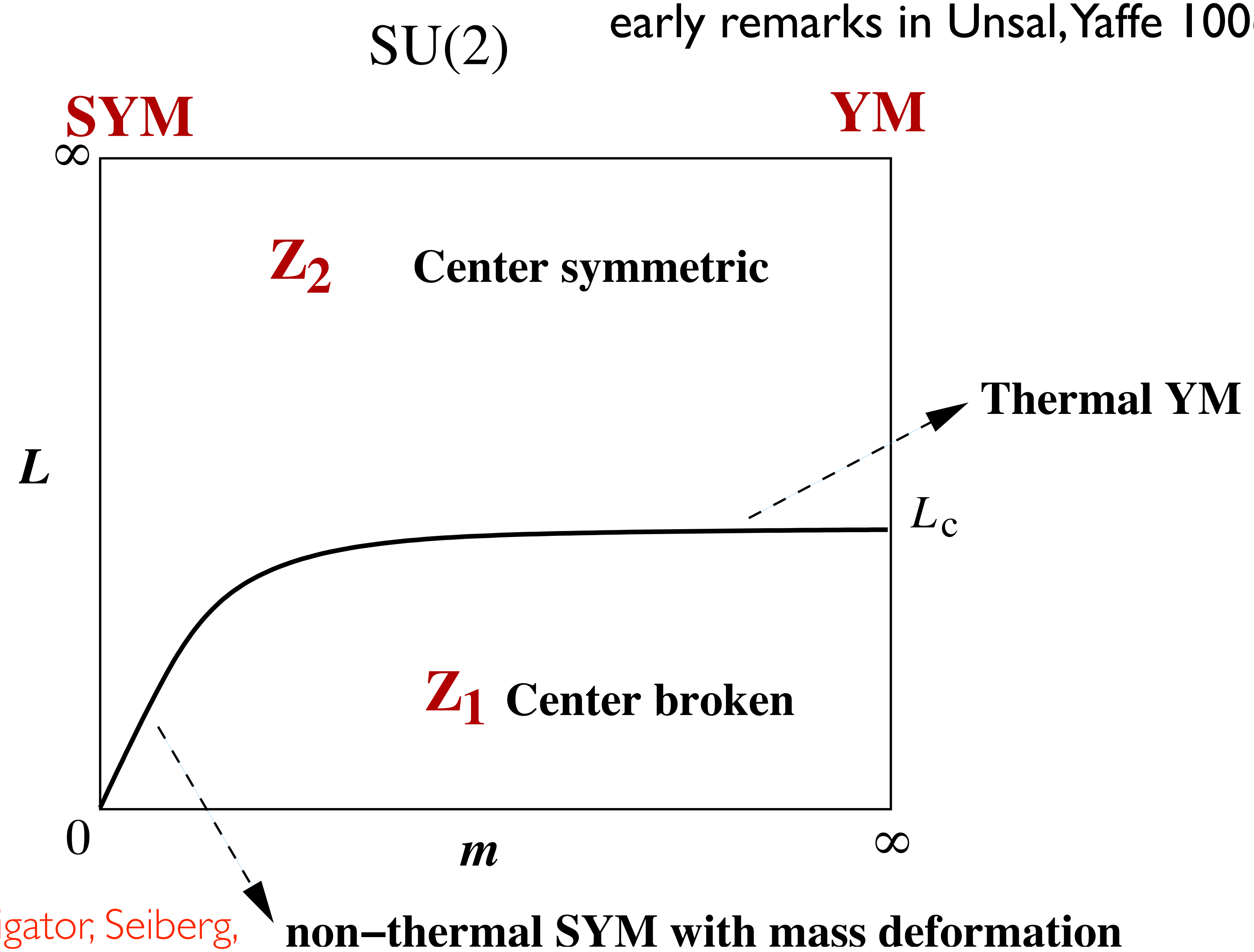

size of circle

gaugino mass

SYM on R3xS1:
Seiberg,Witten 1996
Aharony, Hanany, Intriligator, Seiberg, Strassler 1997
Davies, Hollowood, Khoze 1999
important relevant details of instanton calculation only recent
EP, Schaefer, Unsal, 2012

4. $R^3 \times S^1$ compactifications of SYM*

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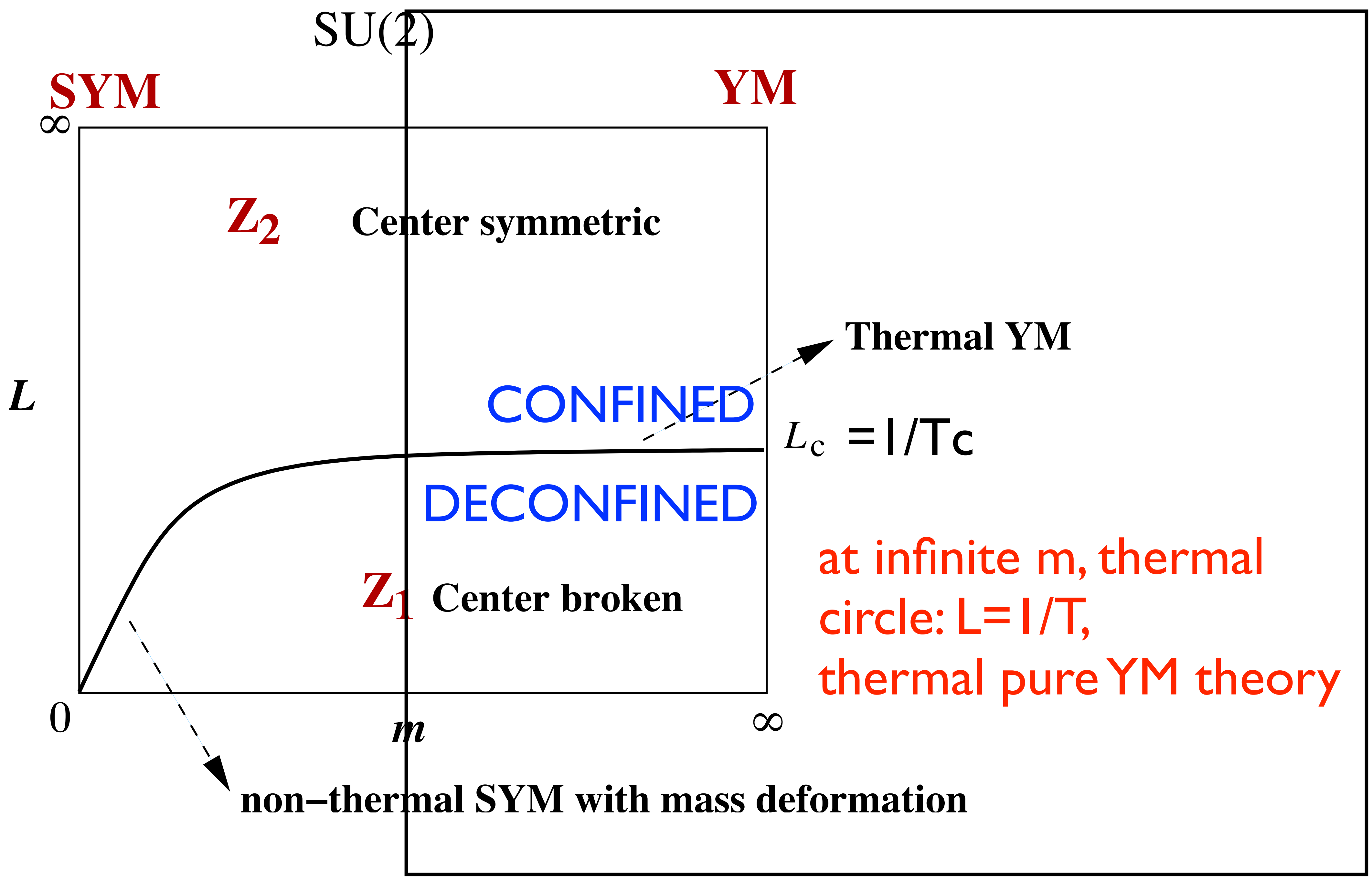
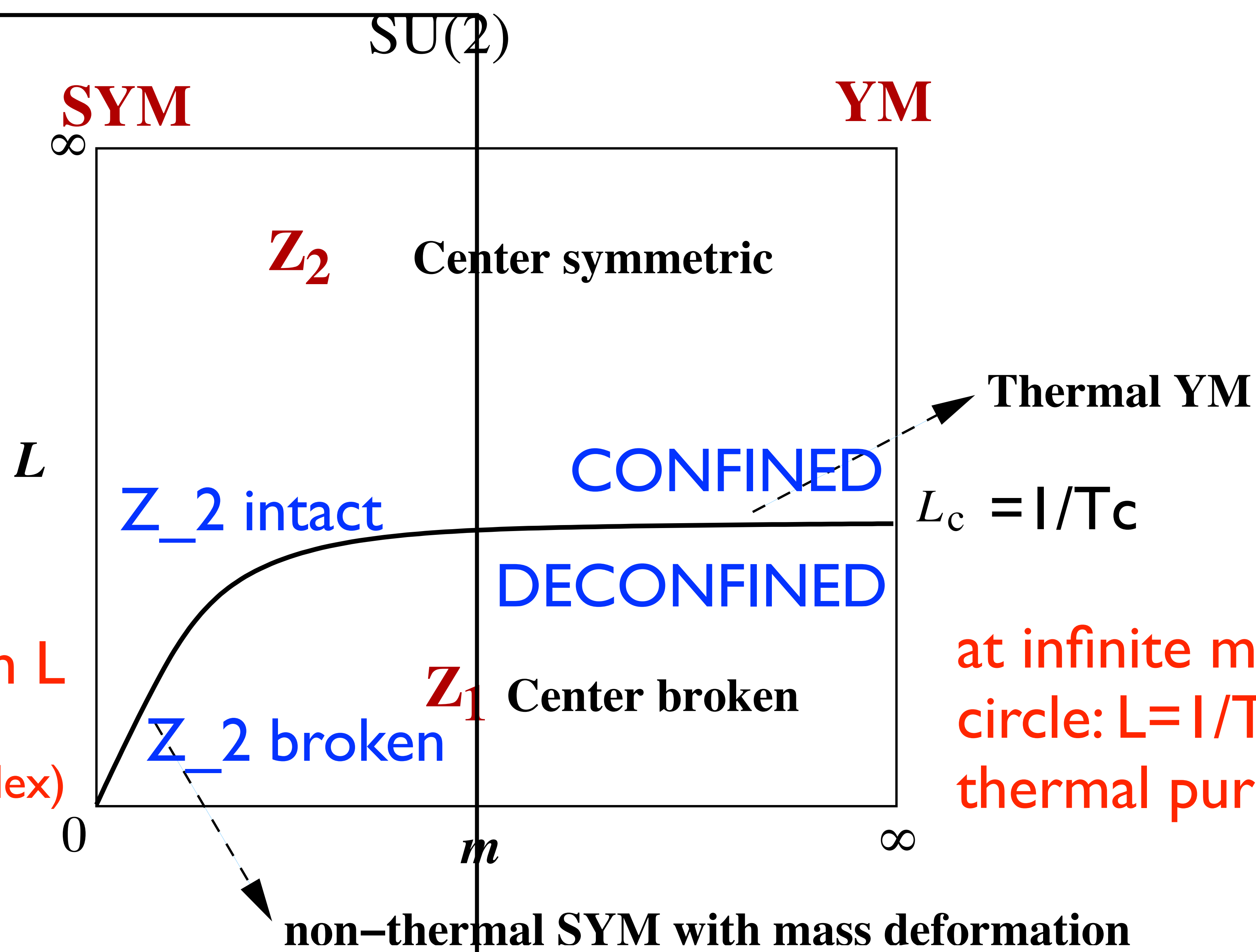


at infinite m, thermal circle: L=1/T, thermal pure YM theory

thermal deconfinement transition, e.g., from lattice experiment

4. $R^3 \times S^1$ compactifications of SYM*

(non-) thermal

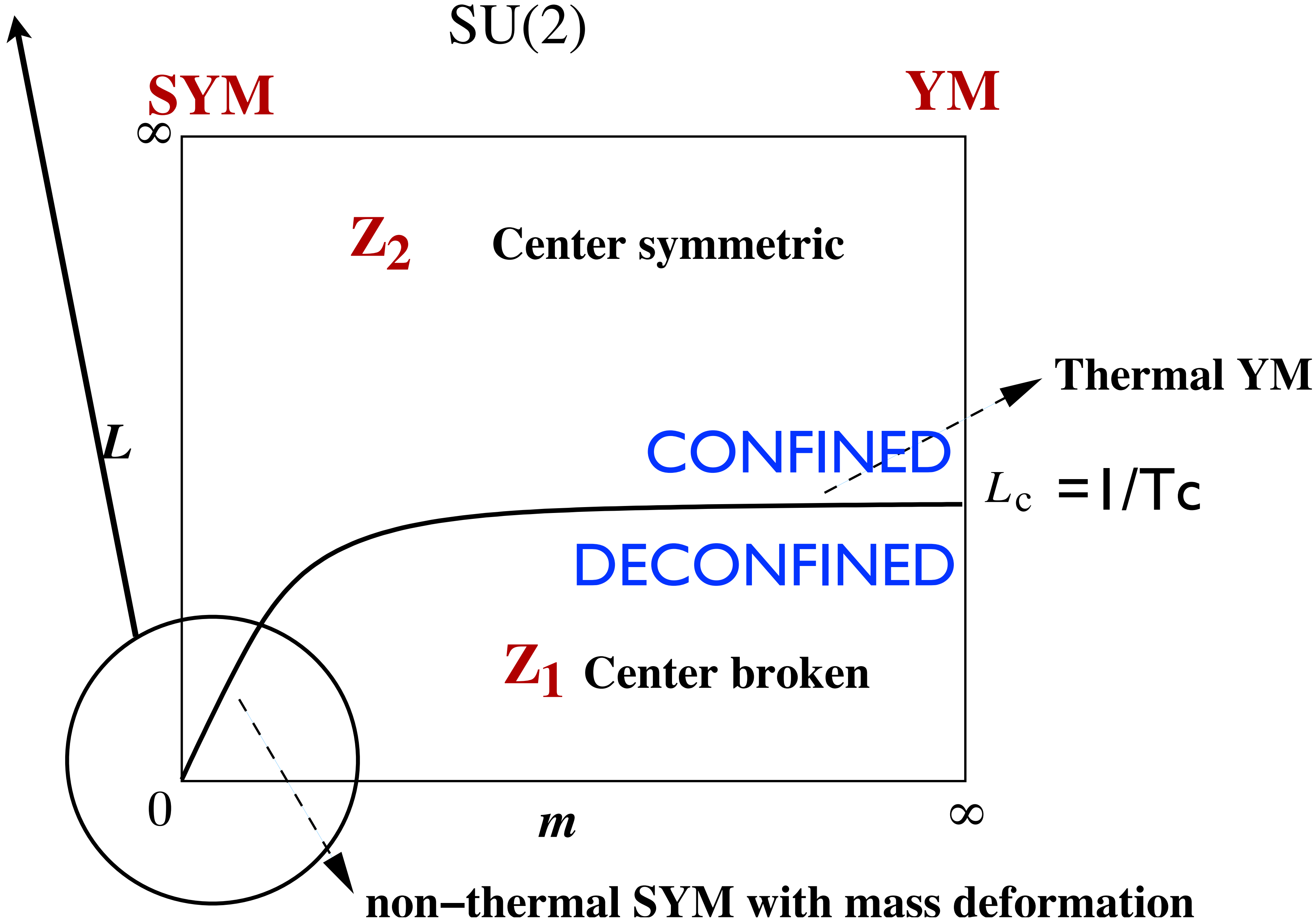


at small m, non-thermal compactification L (at m=0, partition function=Witten index)

at infinite m, thermal circle: L=1/T, thermal pure YM theory

quantum phase transition, Z_2 breaking

At small m,L, the transition can be studied in a theoretically controlled manner. A variety of novel topological excitations and perturbative contributions yield competing effects, resulting in a Z_2 breaking transition as $m\Lambda^{-3}L^{-2}$ **varies.**



quantum phase transition,
Z_2 breaking

thermal deconfinement transition,
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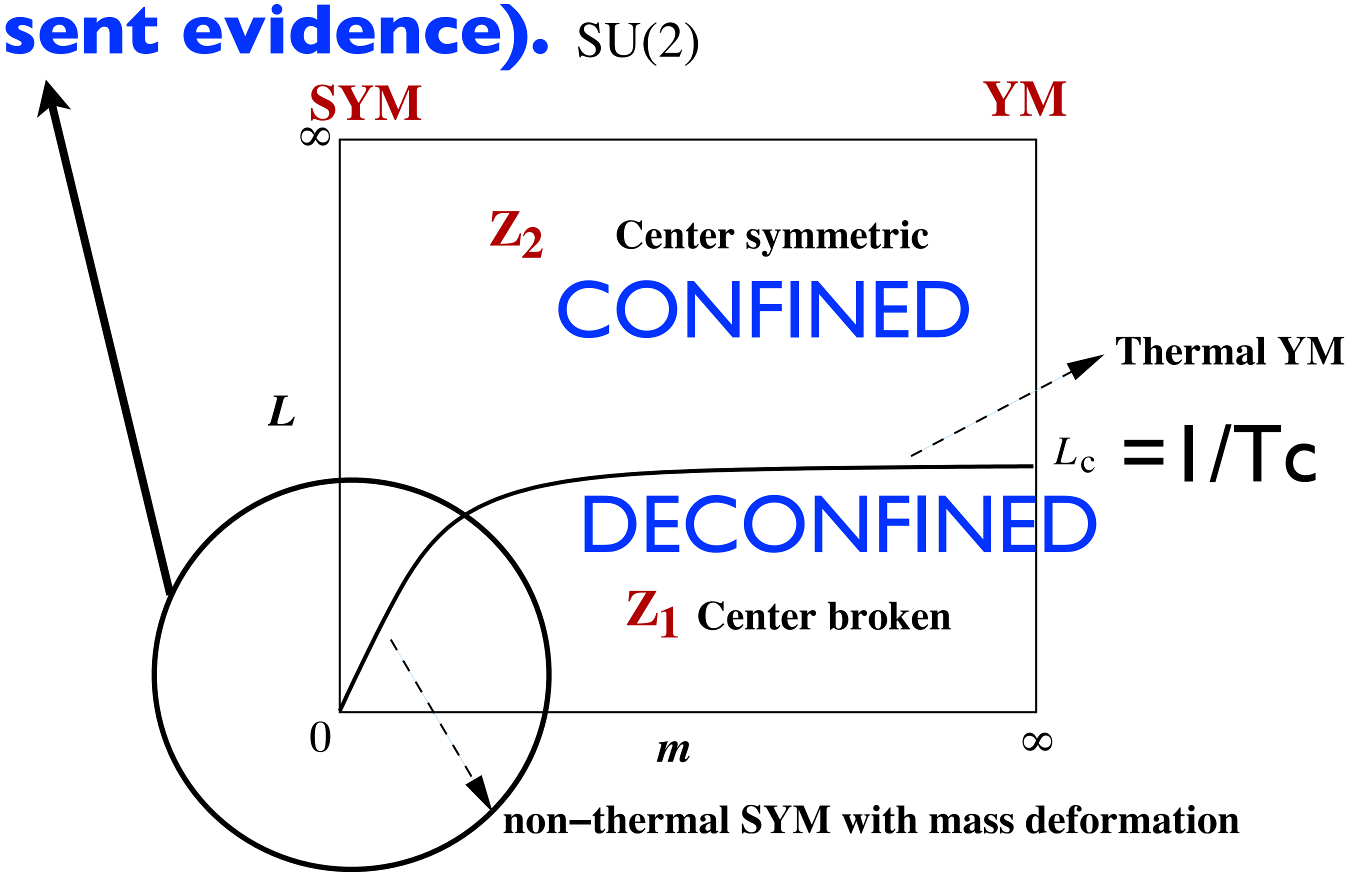
At small m,L, the transition can be studied in a theoretically controlled manner. A variety of novel topological excitations and perturbative contributions yield competing effects, resulting in a Z_2 breaking transition as $m\Lambda^{-3}L^{-2}$ **varies.**

We conjectured that continuously connected to deconfinement in pure YM (will present evidence).

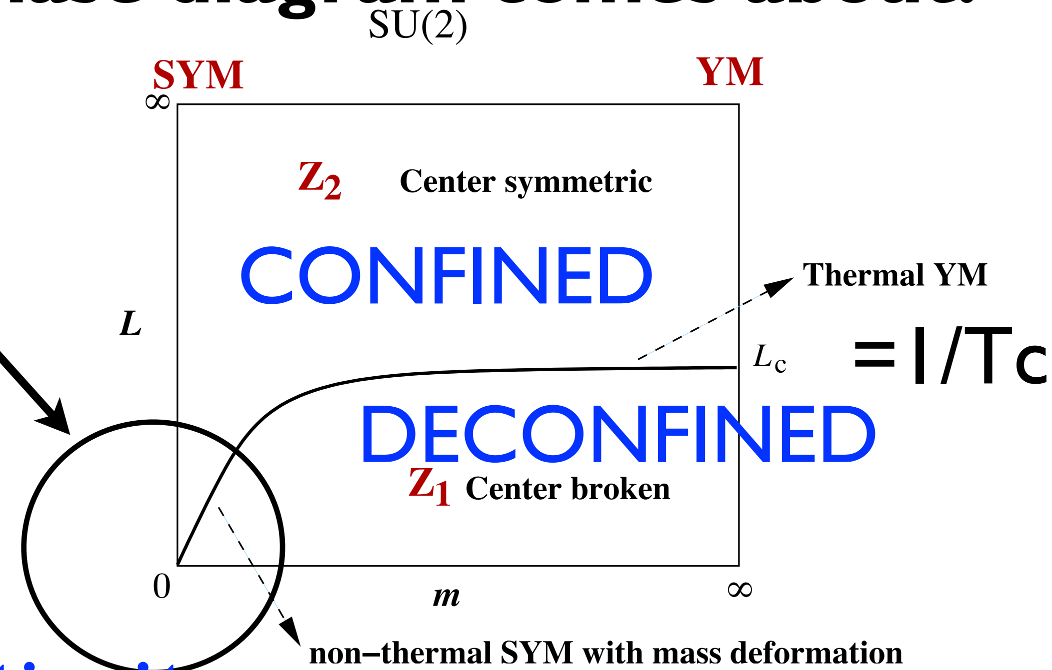


Mechanism behind semiclassical transition is universal, valid for all gauge groups *(that we have studied)*, **with or without center.**

Order of transition is same as in corresponding pure YM in all cases.

Some qualitative properties - theta-dependence of Tc and the strength of transition- first predicted at small-m,L have been verified in recent experiments *(i.e. lattice simulations of pure thermal YM theory)*.

I will tell you how this part of the phase diagram comes about.



First, the evidence in support of continuity:

- **same 'universality'** ('...': most 1st order) **class: Z_N breaking, for SU(N)**

- **same order of transition:**
  - **1st order at N>2, as seen on lattice**
  - **1st order for G_2, as seen on lattice**

    not associated with symmetry breaking

EP, Schaefer, Unsal, 2012

our jump at "Tc"

$$\langle \mathrm{tr}\Omega \rangle = -0.15\, \frac{g^2}{4\pi}$$

$$\langle \mathrm{tr}\Omega \rangle = 3.21\, \frac{g^2}{4\pi}$$

vs

lattice study of G_2

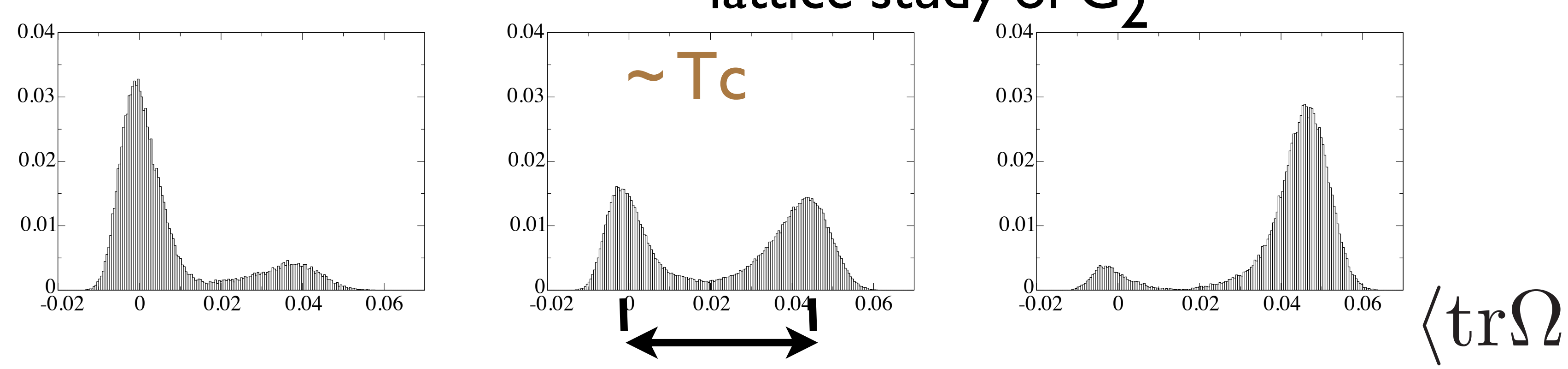


Figure 4: Polyakov loop probability distributions in the region of the deconfinement

[Pepe, Wiese 2006; Cossu et al. 2007]

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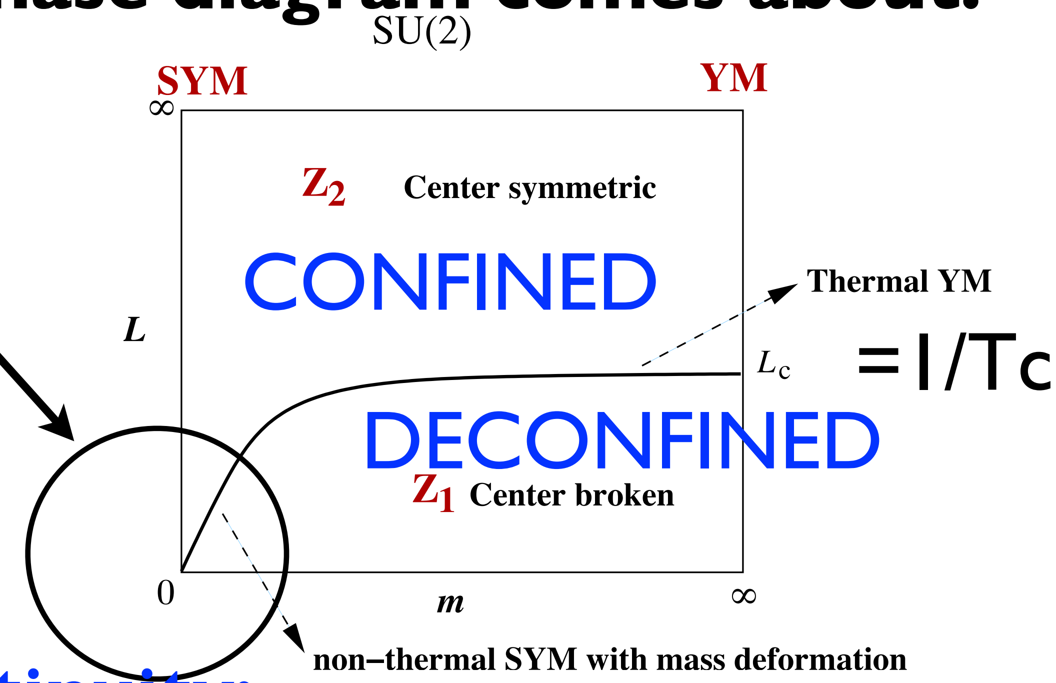

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  EP, Schaefer, Unsal, 2012

  EP, Sulejmanpasic, 2013

  - **with massive fundamental quarks transition becomes crossover as seen on lattice**

- **theta-angle dependence of transition**

  these were actually predicted - Unsal 2012; EP, Schaefer, Unsal, 2012; Anber 2013

  - **Tc decreases with increasing theta; seen on lattice** [D' Elia, Negro 2012]
  - **disc of Polyakov loop at Tc, for Nc>2, increases with increasing theta** [predicted Mohamed Anber 2013] **and seen on lattice** [D' Elia, Negro 2013]

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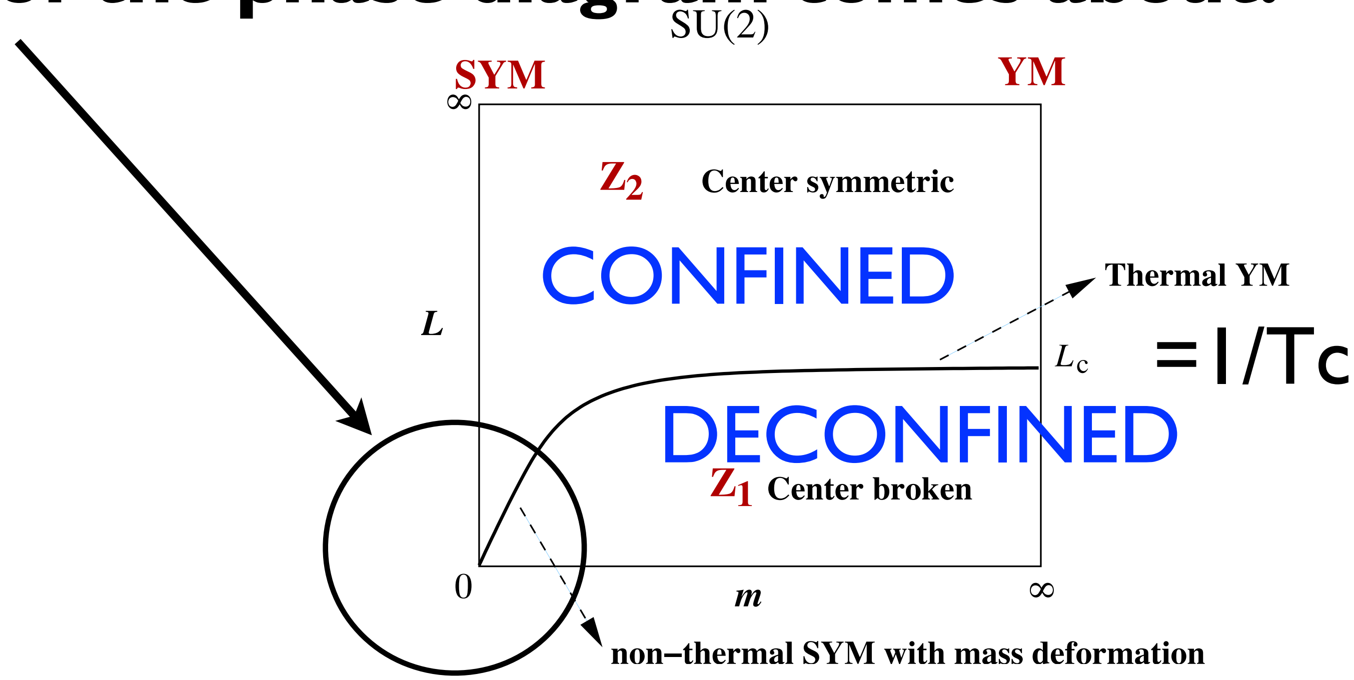


What is the role of SUSY?

theory is weakly coupled at small L - abelian!, not just asymptotic freedom

thus

allows us to have calculable non-perturbative effects $\quad roughly \sim e^{-\frac{\mathcal{O}(1)}{g^2}}$

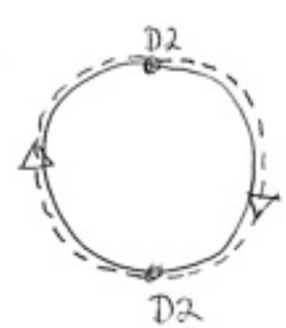
and

calculable perturbative effects - which are suppressed by m - $\quad roughly \sim g^2 m$

so the two can compete and result in a calculable transition

major players: monopole-instanton "BPS" and twisted "KK" [Piljin Yi, Kimeyong Lee, 1997]

and various "topological molecules made thereof"

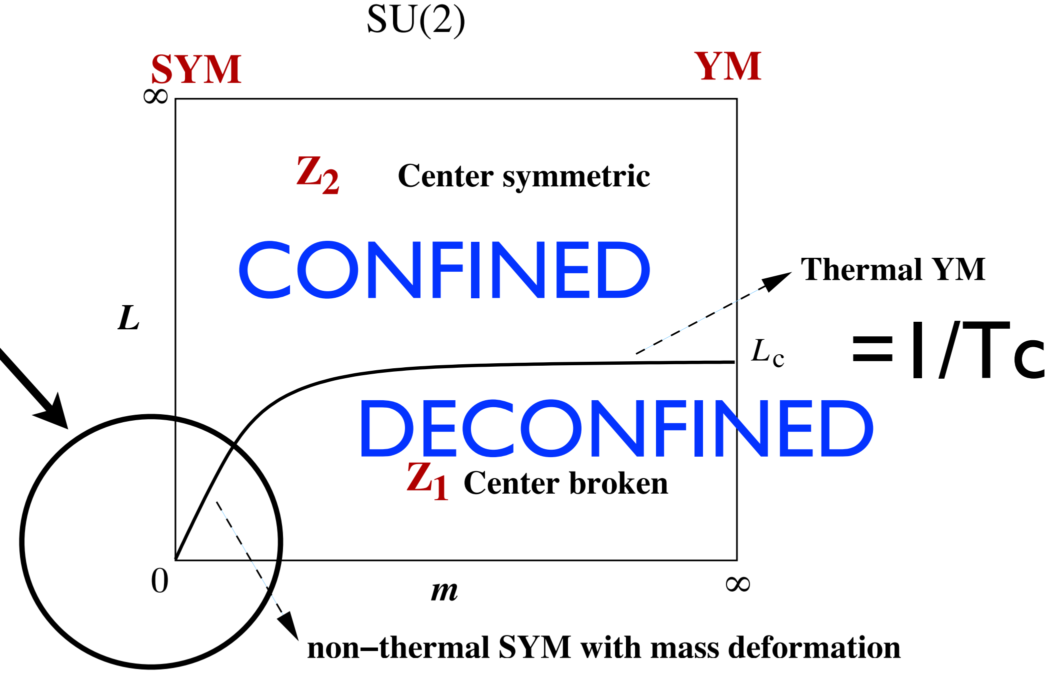


[Unsal 2007, Unsal EP 2011]

2. topological

... how this part of the phase diagram comes about ...

- small-L theory is abelian SU(2) breaks to U(1)
- no light charged states (remember this is T=0 quantum transition!)



relevant bosonic fields: A_4 - gauge field in compact direction - and A_i - 3d gauge field - in the unbroken U(1) of SU(2), equivalent to:

σ - 3d dual to A_i = "dual photon" (potential for magnetic charge)

ϕ - deviation of A_4 from center symmetric value $\mathrm{Tr}\,\Omega = 0$

...without taking into account nonpertubative physics, these are FREE...

2. *topological*

- small-L theory is abelian
  SU(2) breaks to U(1)

- no light charged states
  (remember this is T=0 quantum
  transition!)

all (almost) dynamics is due to nonperturbative objects: vacuum of the theory is a dilute 3d "gas" of "molecules" interacting via long-range forces due to (dual) photon, scalar modulus, and fermion zero-mode hopping

???

relevant bosonic fields: A_4 - gauge field in compact direction - and A_i - 3d gauge field - in the unbroken U(1) of SU(2), equivalent to:

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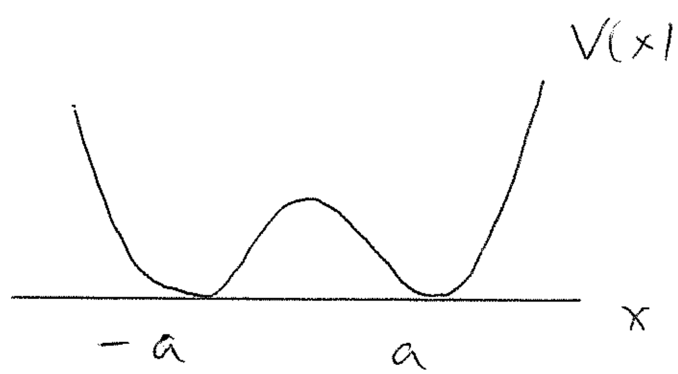
  (remember this is T=0 quantum
  transition!)

all (almost) dynamics is due to nonperturbative objects: vacuum of the theory is a dilute 3d "gas" of "molecules" interacting via long-range forces due to (dual) photon, scalar modulus, and fermion zero-mode hopping

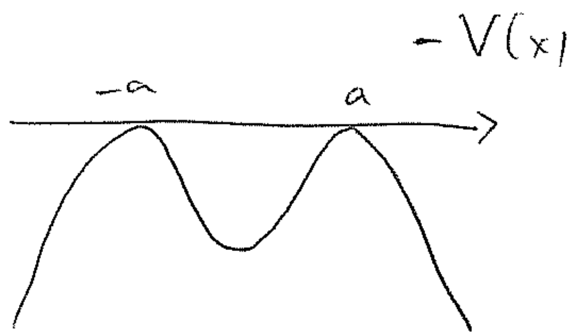
QM:

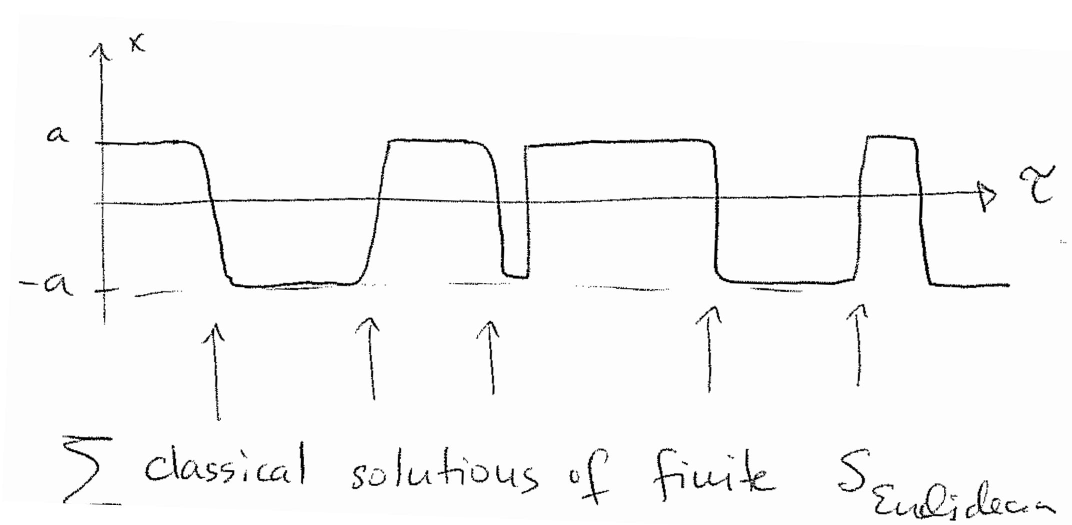
$$\langle a| e^{-T\hat{H}} |a\rangle \underset{``\hbar \to 0"}{\sim} \sum_{\substack{\text{classical} \\ \text{trajectories} \\ a \to a \\ \text{Euclidean!}}} e^{-S_{class}[\ldots]}$$

$$\frac{S_{cl.}}{\hbar} \gg 1$$



t => i t

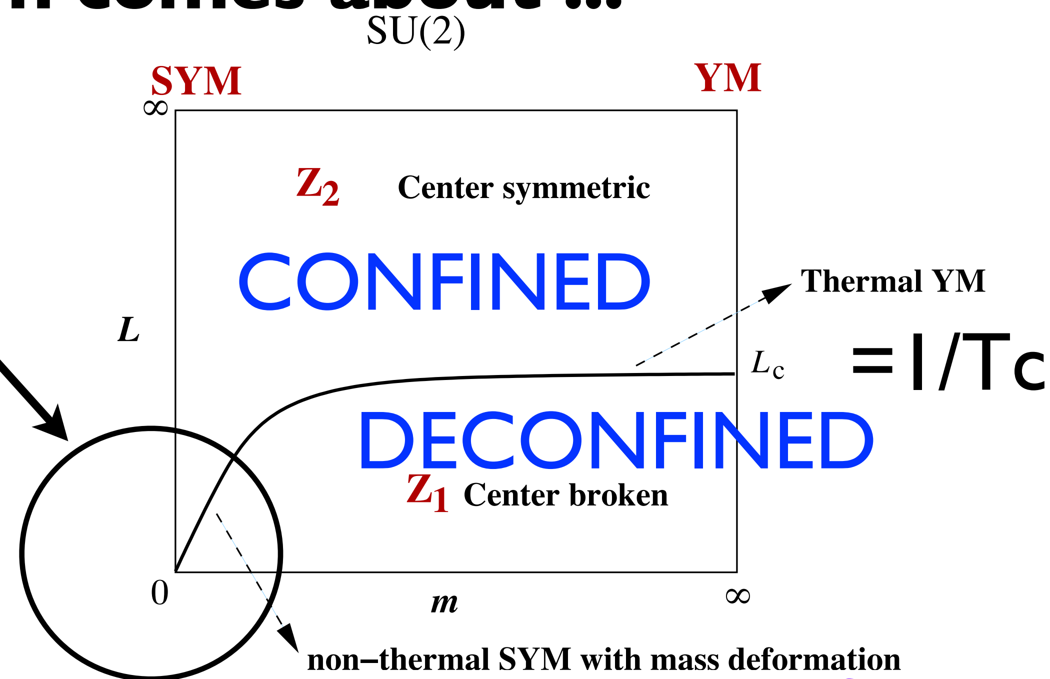




ground state ~
a dilute 1d "gas" of
"tunneling events"
or instantons

2. *topological*

... how this part of the phase diagram comes about ...



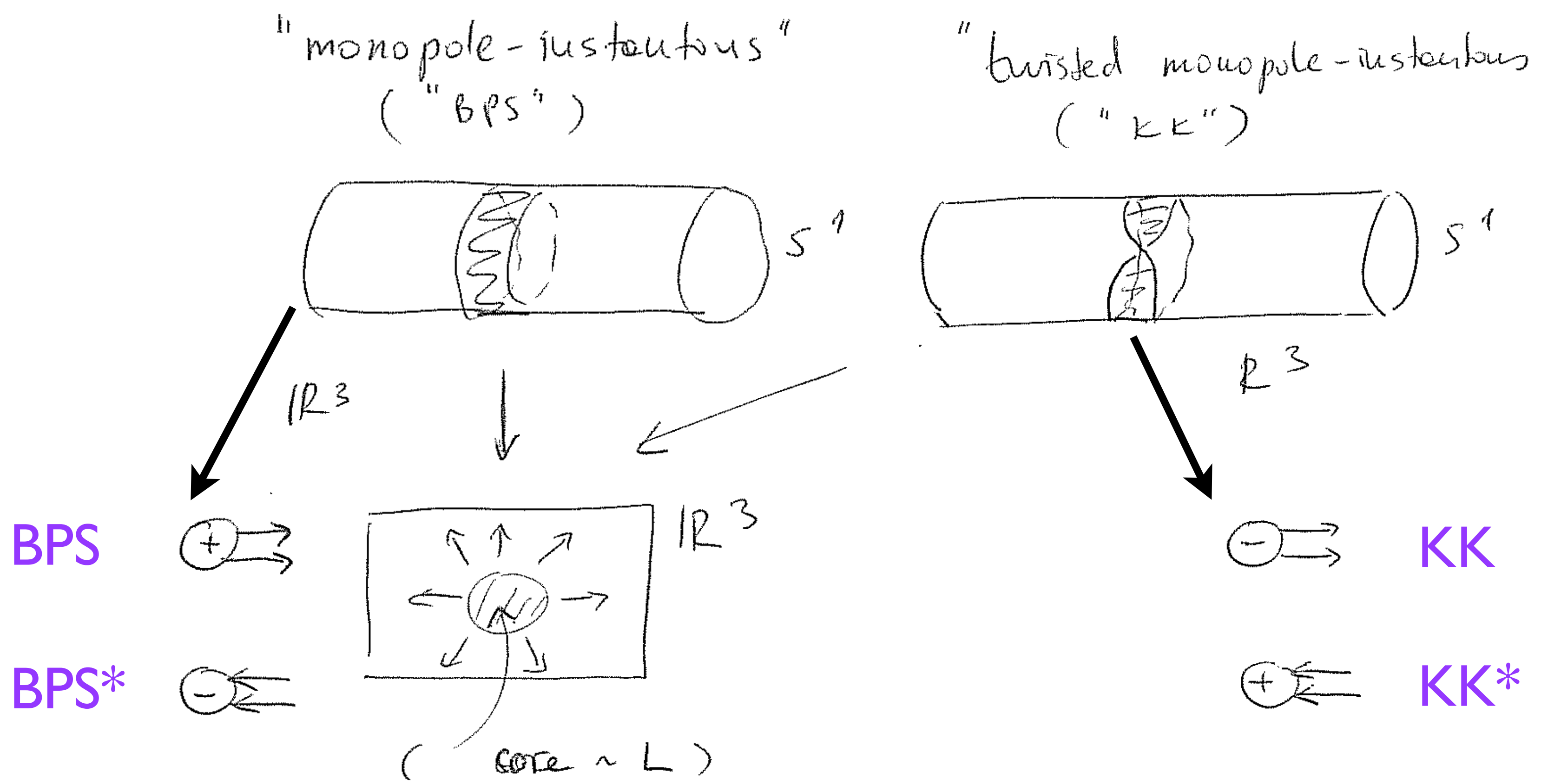
- small-L theory is abelian SU(2) breaks to U(1)
- no light charged states (remember this is T=0 quantum transition!)

all (almost) dynamics is due to nonperturbative objects: vacuum of the theory is a dilute 3d "gas" of "molecules" interacting via long-range forces due to (dual) photon, scalar modulus, and fermion zero-mode hopping

QFT: $\frac{S_{cl.}}{\hbar} \gg 1$

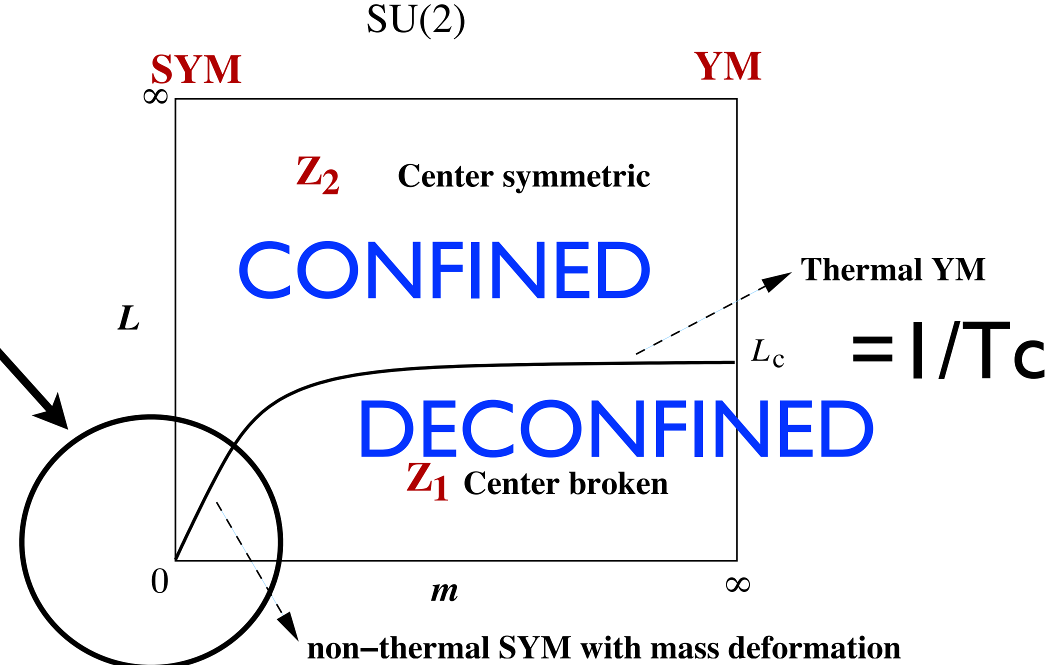
$$\langle 0 | e^{-T\hat{H}} | 0 \rangle \overset{\text{"}\hbar \to 0\text{"}}{\sim}$$

$$\sim \sum \left(\begin{array}{l} \text{finite action} \\ \text{saddle points} \\ \text{of path integral} \end{array} \right)$$



2. *topological*

... how this part of the phase diagram comes about ...



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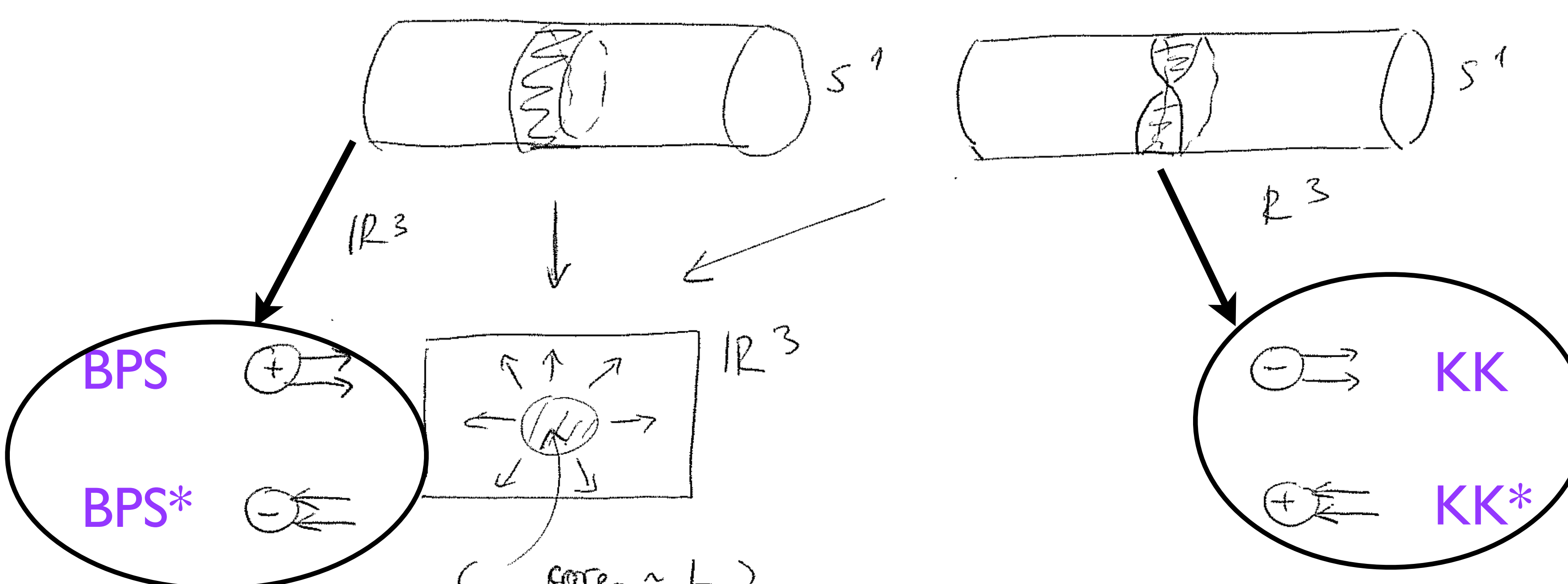
QFT: $\frac{S_{cl.}}{\hbar} \gg 1$

$\langle 0 | e^{-T\hat{H}} | 0 \rangle \overset{\text{"}\hbar \to 0\text{"}}{\sim} \sim \sum \left(\begin{array}{l} \text{finite action} \\ \text{saddle points} \\ \text{of path integral} \end{array} \right)$

"monopole-instantons" ("BPS")

"twisted monopole-instantons" ("KK")

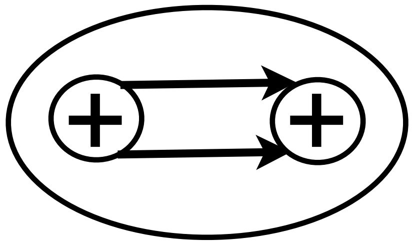


these main "players", as they interact, can form "molecules" - "correlated tunneling events"

2. topological

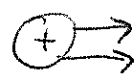
… how this part of the phase diagram comes about …

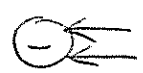
B:



BPS KK*

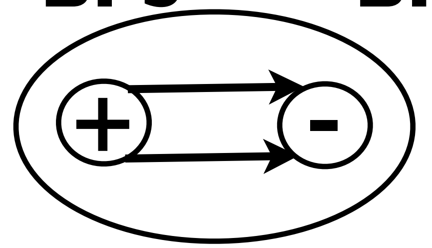
$$e^{-2S_0} e^{+i2\sigma}$$

BPS 

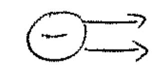
BPS* 

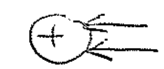
BPS BPS*

N:



$$e^{-2S_0} e^{-2\phi}$$

 KK

 KK*

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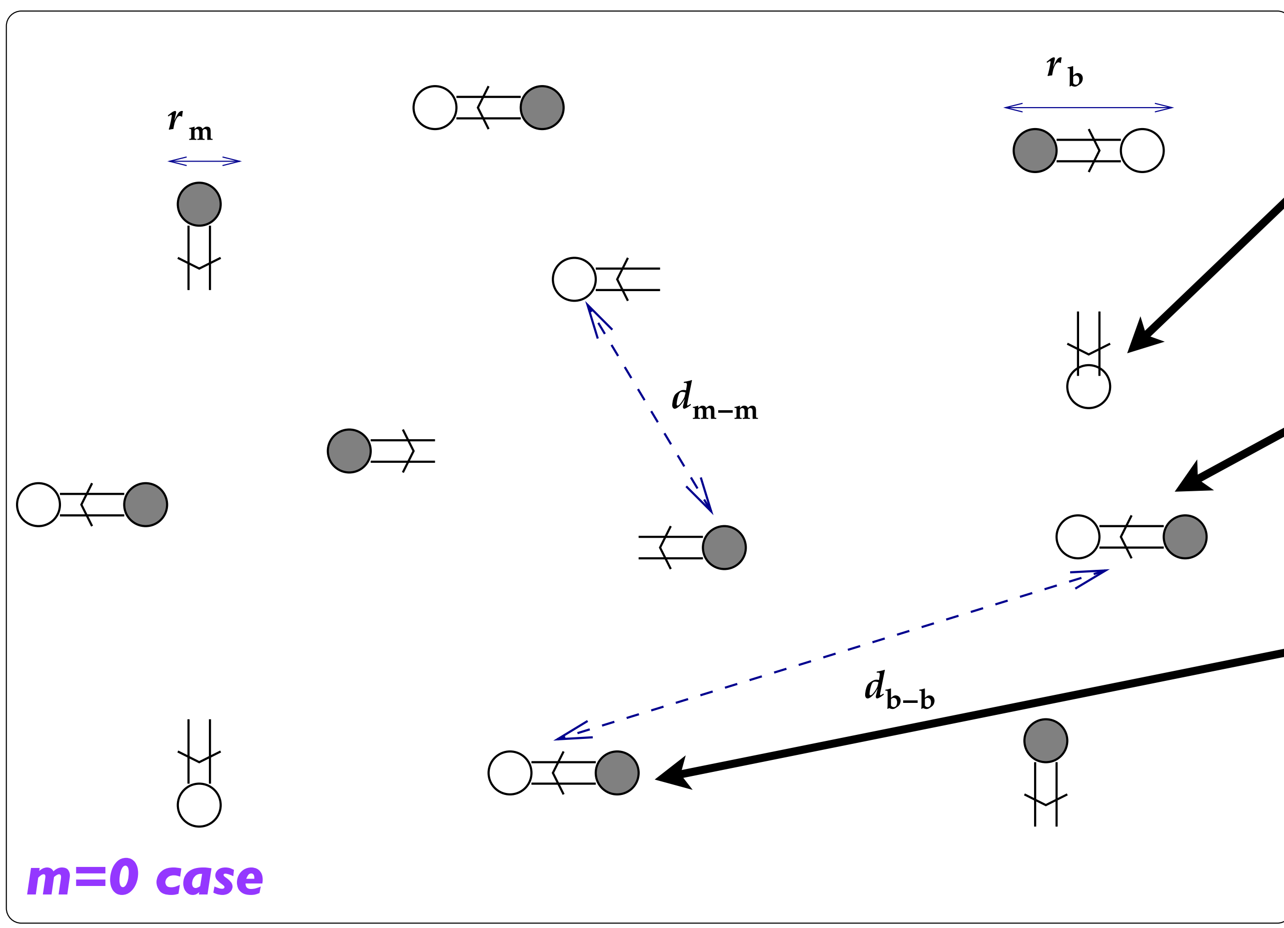


monopole-instantons (M,KK+*)

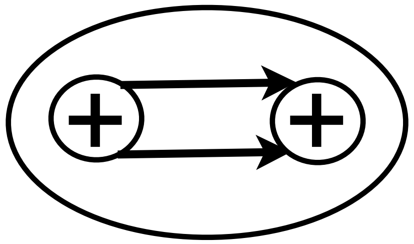
magnetic bion "molecules"

neutral bion "molecules"

2. topological

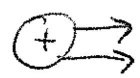
... how this part of the phase diagram comes about ...

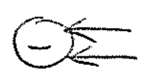
B:



BPS KK*

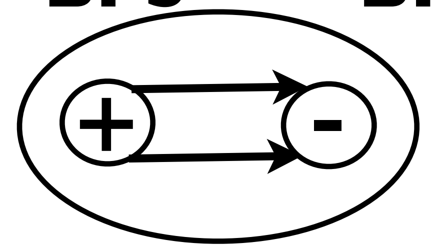
$$e^{-2S_0}e^{+i2\sigma}$$

BPS 

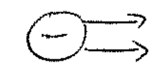
BPS* 

BPS BPS*

N:

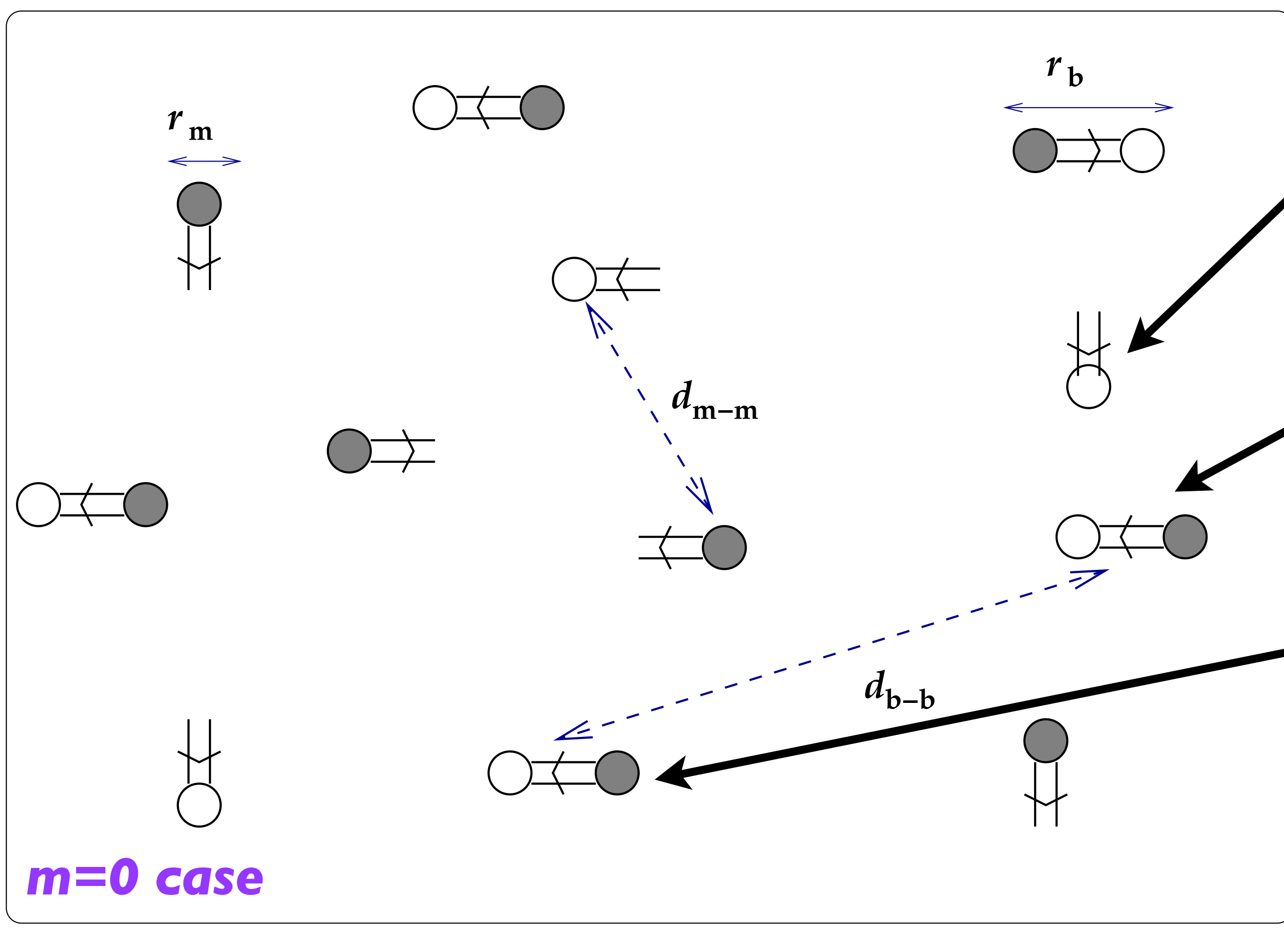


$$e^{-2S_0}e^{-2\phi}$$

 KK

 KK*

all (almost) dynamics is due to nonperturbative objects: vacuum of the theory is a dilute 3d "gas" of "molecules" interacting via long-range forces due to (dual) photon, scalar modulus, and fermion zero-mode hopping



monopole-instantons (M,KK+*)

the ones with arrows: fermion zero modes carry magnetic charge 1

magnetic bion "molecules"

carry magnetic charge 2

[mass gap; breaking discrete chiral symmetry]

neutral bion "molecules"

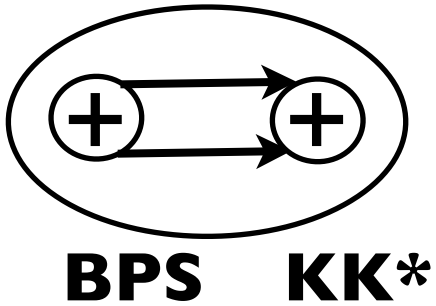
carry scalar (modulus) charge 2

[Z2 center symmetry stabilization]

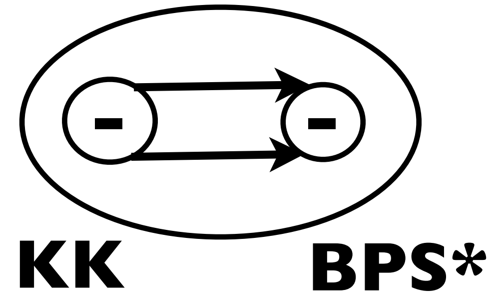
[aside: BB*~renormalons? ..."resurgence"]

(BPS-KK* “molecules”) “magnetic bions” - confinement!

B:



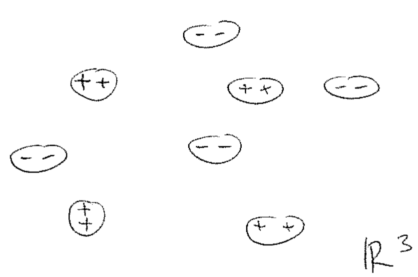
B*:



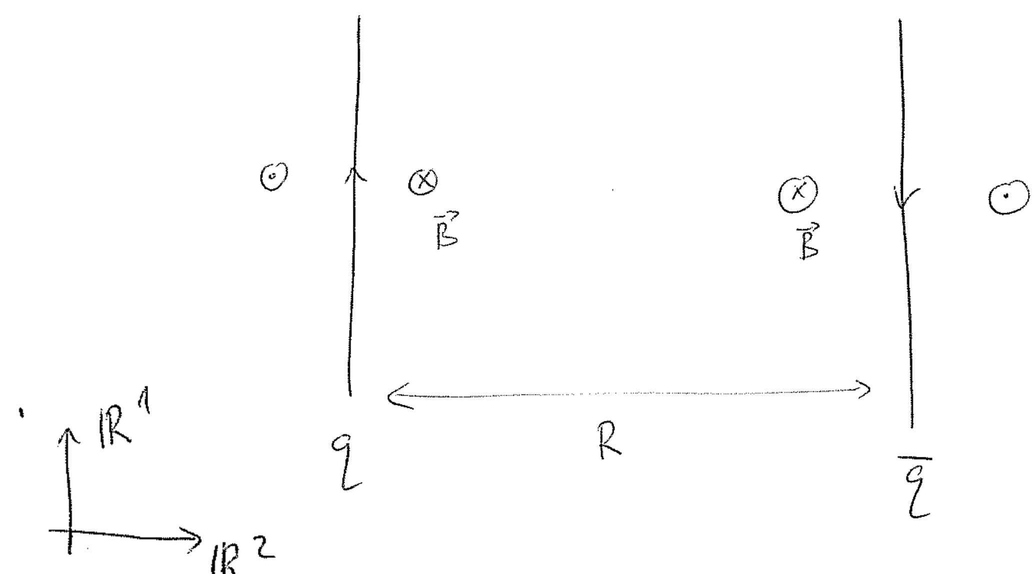
$$e^{-2S_0} e^{+i2\sigma} \qquad\qquad e^{-2S_0} e^{-i2\sigma}$$

m=0 case - physics is that of 3d Debye screening - mass gap and confinement:

magnetic bion gas: classical 3d Coulomb plasma



if nonperturbative saddle points are not summed over...

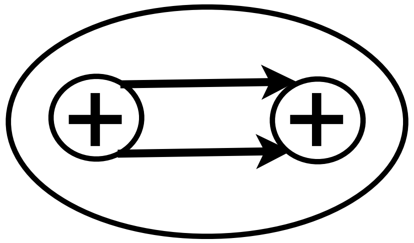


$(\mathbb{R}^{1,2})$

$V_{q\bar{q}} \sim g_3^2 \log R$ ***- 2d Coulomb potential***

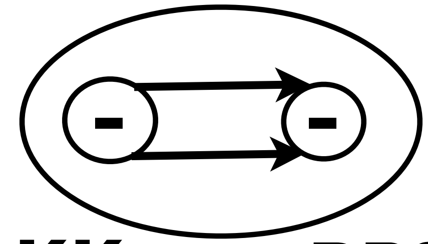
(BPS-KK* “molecules”) “magnetic bions” - confinement!

B:



BPS **KK***

B*:



KK **BPS***

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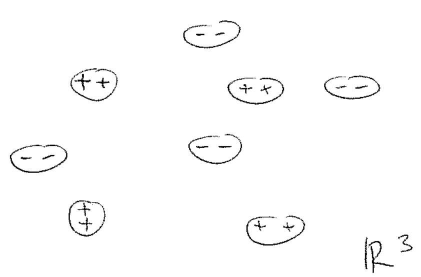
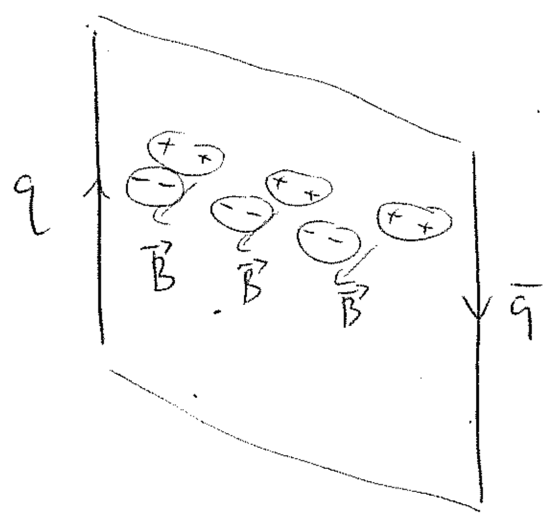
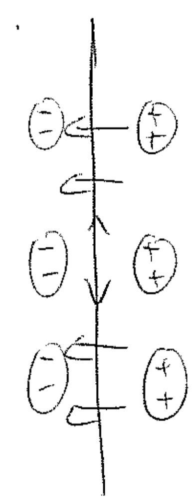


... in reality, B-B* plasma screens magnetic field of external probes





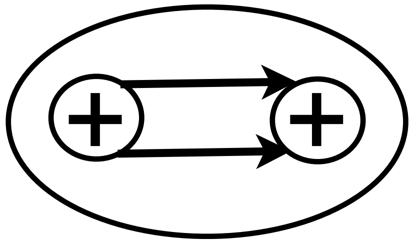
“string worldsheet”: B-B* dipole layer

[Polyakov 1977]

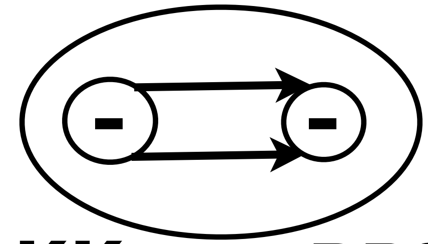
“monopole condensation” is due to composite “molecular” objects - this theory does not confine in 3d limit [Unsal 2007]

$$V_{q\bar{q}} \sim g_3^2 \log R \Longrightarrow \sigma R$$

(BPS-KK* "molecules") "magnetic bions" - confinement!

B:  $e^{-2S_0} e^{+i2\sigma}$

BPS **KK***

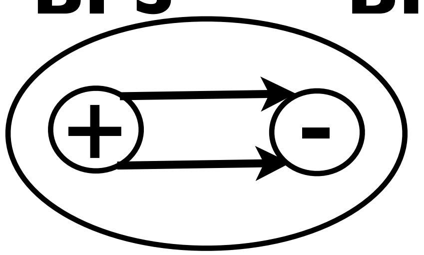
B*:  $e^{-2S_0} e^{-i2\sigma}$

KK **BPS***

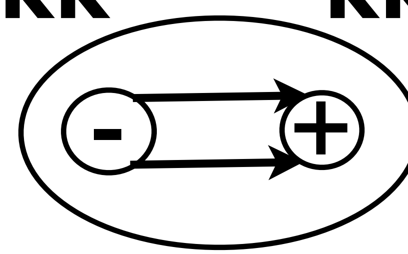
(BPS-BPS*,KK-KK* "molecules") "neutral bions"

in pure-SYM: center-stabilizing

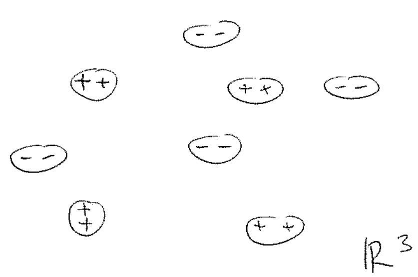
BPS **BPS***

N:  $e^{-2S_0} e^{-2\phi}$

KK **KK***

N*:  $e^{-2S_0} e^{+2\phi}$

magnetic bion gas: classical 3d Coulomb plasma



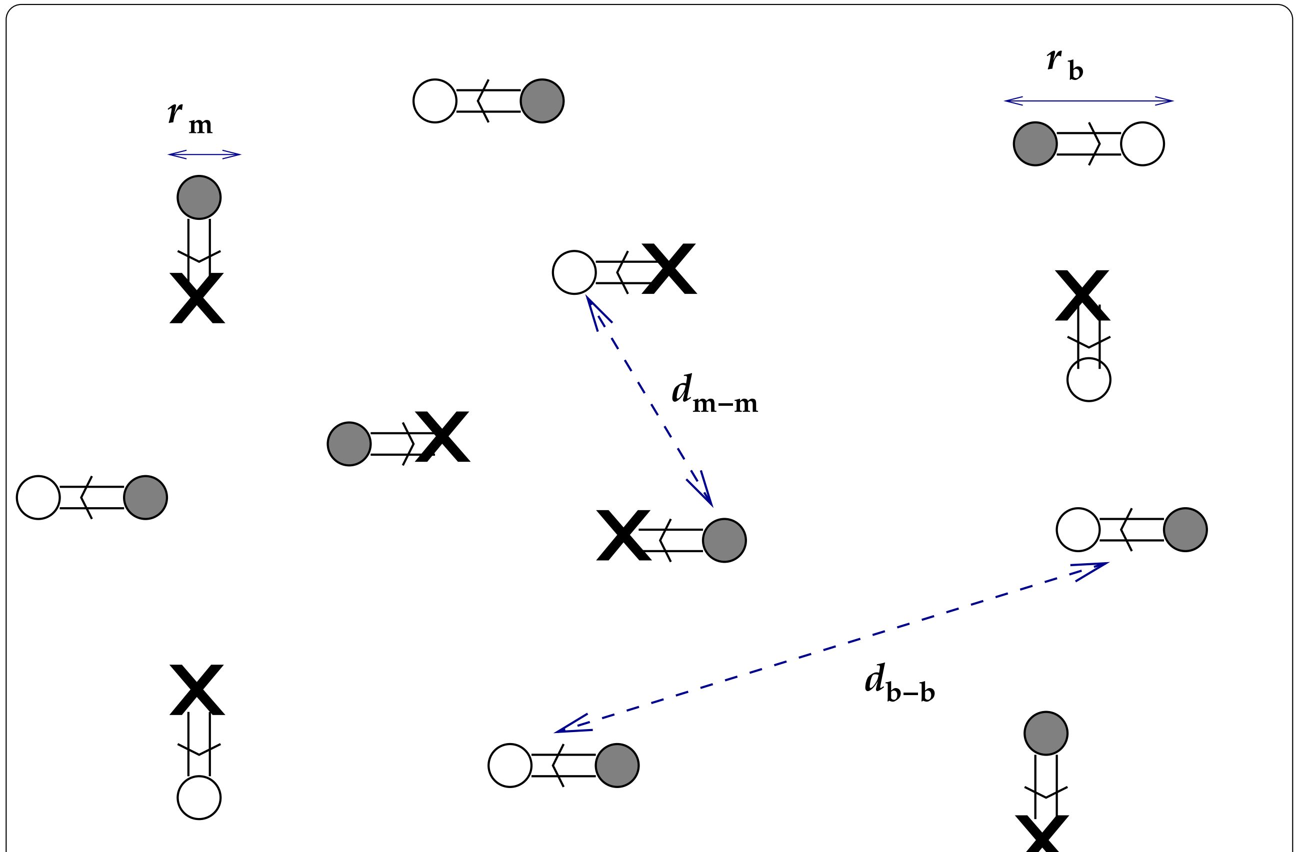
magnetic bions: break chiral Z_2, mass gap for dual photon

neutral bions: stabilize center Z_2, mass gap for modulus (phi=0 - center stable)

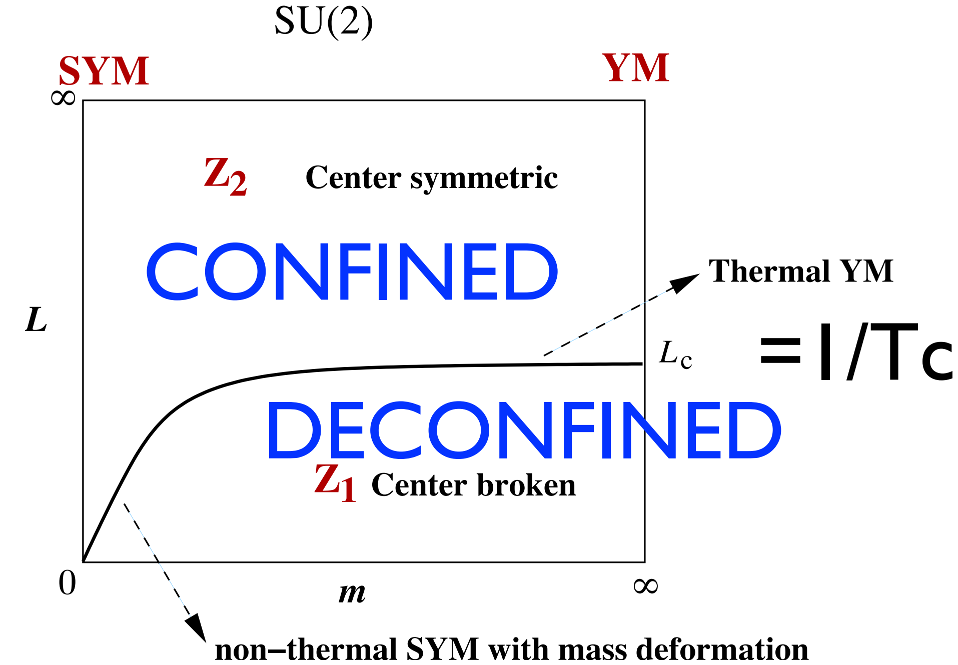
Our interest is in the center Z_2 (as chiral Z_2 broken at m>0)

Recall it is the center Z_2 which becomes the thermal center symmetry of pure YM when m goes to infinity.

... how this part of the phase diagram comes about ...



m>0 case: breaks chiral symmetry, yields:



= 1/Tc

monopole-instantons (BPS,KK+*)

magnetic bion "molecules"

[breaking of discrete chiral symmetry]

neutral bion "molecules"

[stability of Z2 center symmetry [non-thermal]]

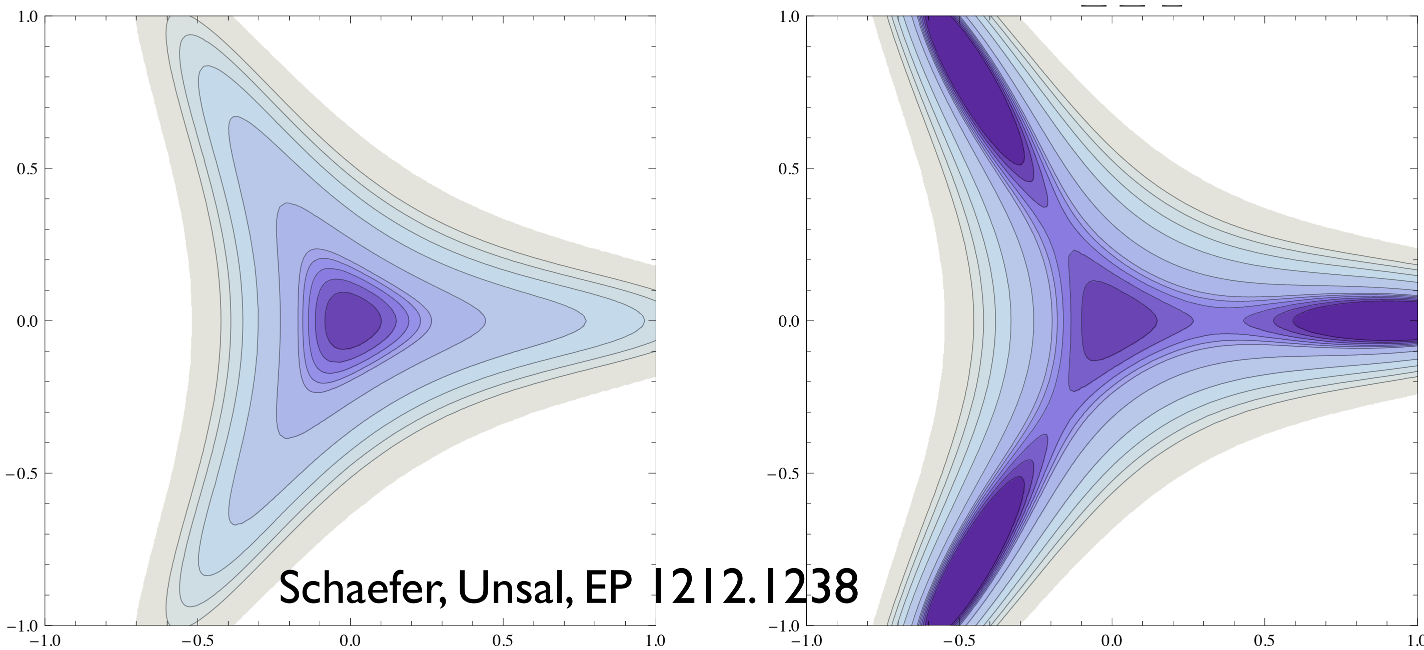
1. extra nonperturbative contributions from monopole-instantons (no fermion zero modes)

2. extra perturbative Gross-Pisarski-Yaffe-like contribution (small since m is small)

small SUSY breaking "m" allows us to have perturbative and nonperturbative contributions compete while under theoretical control, resulting in a center-breaking transition as $\frac{m}{L^2\Lambda^3}$ becomes O(1) (2nd order for SU(2); 1st for SU(N)...)

=8, so if at m>5Λ decoupled, as quarks in QCD, $1/L_c = \Lambda\sqrt{8\Lambda/m}$ $\rightarrow$ $T_c \simeq \Lambda$

instead of formulae, plot of potential due to "neutral bions" for SU(3): Z3-symmetric vs Z3-breaking as $\frac{m}{L^2\Lambda^3}$ ***increases*** (deviation of Ω EVs from Z3)



Same objects that were identified in SYM also exist in pure thermal YM. What is lost is the theoretical control...

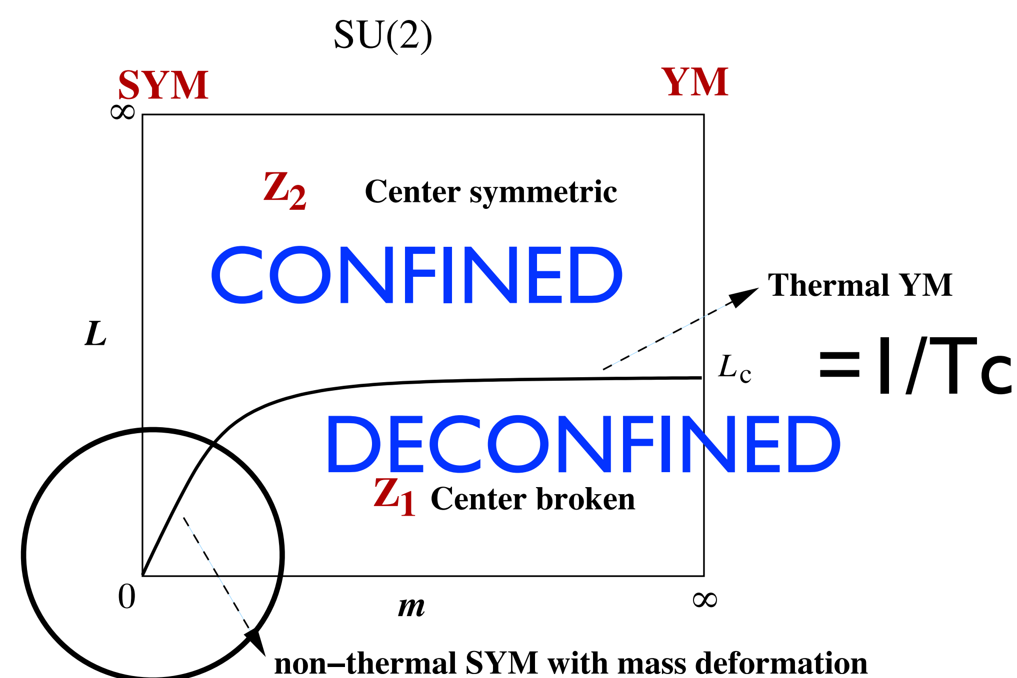
Instanton-liquid type models of the deconfinement transition can be considered, incorporating "molecular" contributions...
[Shuryak, Sulejmanpasic...'13]

- one can build models and/or compare small-L calculations with lattice ... eventually entire m/L

So far I told you about

1. a quantum center-breaking transition continuously connected (? ... gave evidence) to thermal deconfinement

2. driven by topological molecules, incl. some rather strange ones -
appear related to renormalons and needed to make sense of the divergent perturbation series... and even define the theory?
[Argyres, Dunne, Unsal ... 2012-]



All of this was non-thermal -but quantum connected to thermal (electric charges were not directly present).

Can one have a controllable thermal deconfinement transition? - YES

3. $R^2 x S^1 x S^1$ compactifications

non-thermal

thermal

[Simic, Unsal 2010
Unsal 2012

Anber, EP, Unsal 2011
Anber, Collier, EP 2012
Anber, Collier, Strimas-Mackey, Teeple, EP 2013]

"deformed" pure-YM

"QCD(adj)" = YM with many massless adjoint Weyl fermion

In the process of unraveling the above map, SUSY played a crucial role...

- notice the nf=1 adjoint theory is N=1 SYM

(already mentioned relation of QCD(adj) to a large-N limit of QCD(fund.) via various equivalences)

"QCD(adj)" on $R^3 x S^1$ with fermions periodic around the circle, retains many features of SYM.

Consider first theory on $R^3 x S^1$ with fermions periodic around the circle and then study nonzero-T of this theory (i.e. add a second "thermal circle").

Go back to my SYM slide... and proceed by applying

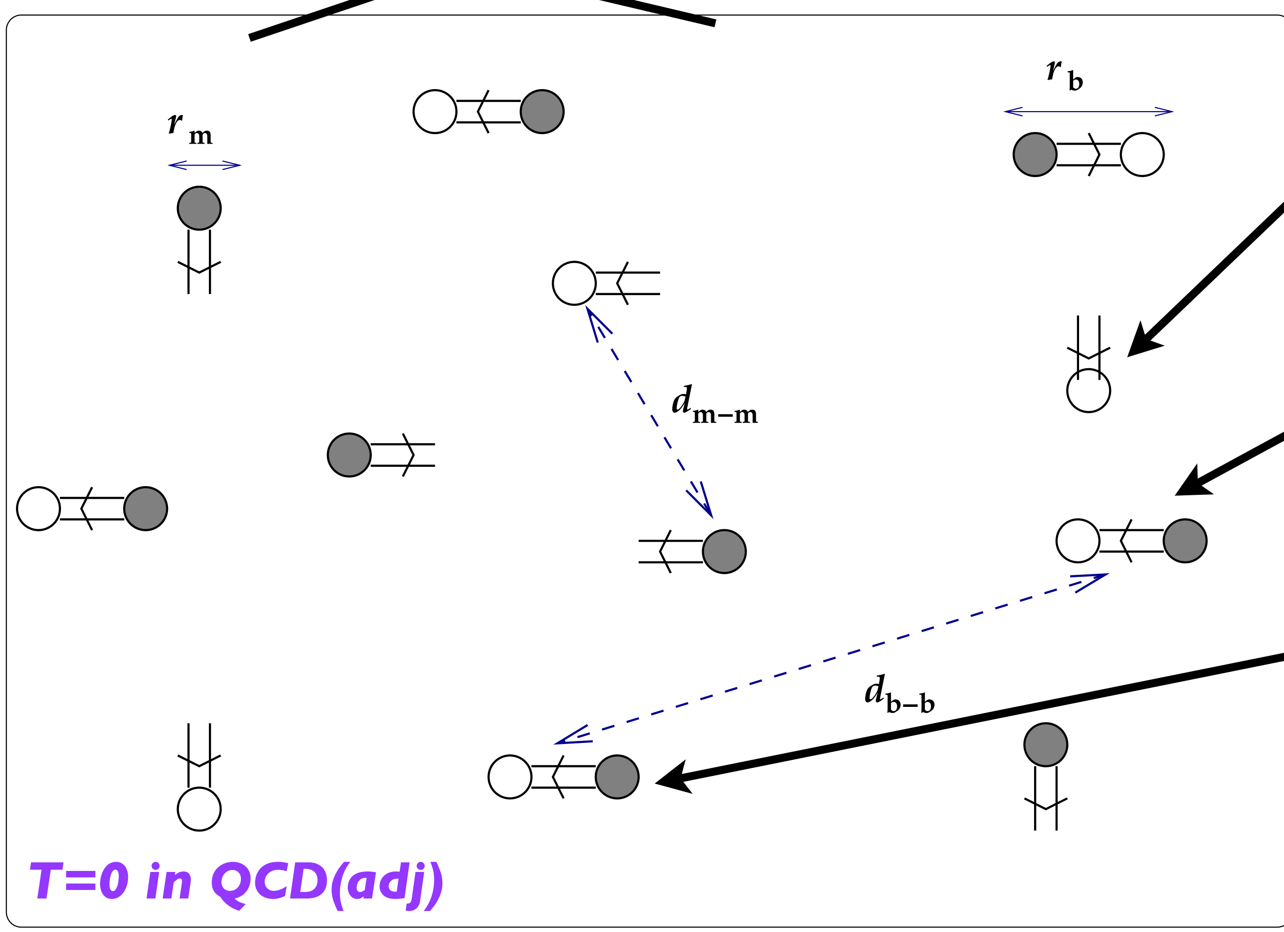
QCD(adj) on $R^3 \times S^1$ (spatial)

- small-L theory is abelian
  SU(2) breaks to U(1)

- no light charged states
  (remember this is T=0 quantum
  transition!)

(same features as SYM before)

all (almost) dynamics is due to nonperturbative objects: vacuum of the theory is a dilute 3d “gas” of “molecules” interacting via long-range forces due to (dual) photon, ~~scalar modulus~~, and fermion zero-mode hopping



monopole-instantons (M,KK+*)

magnetic bion “molecules”

~~neutral bion “molecules”~~

more precisely:
no role in thermal center; spatial-circle center stabilized perturbatively

QCD(adj) on $R^3 \times S^1$ (spatial)

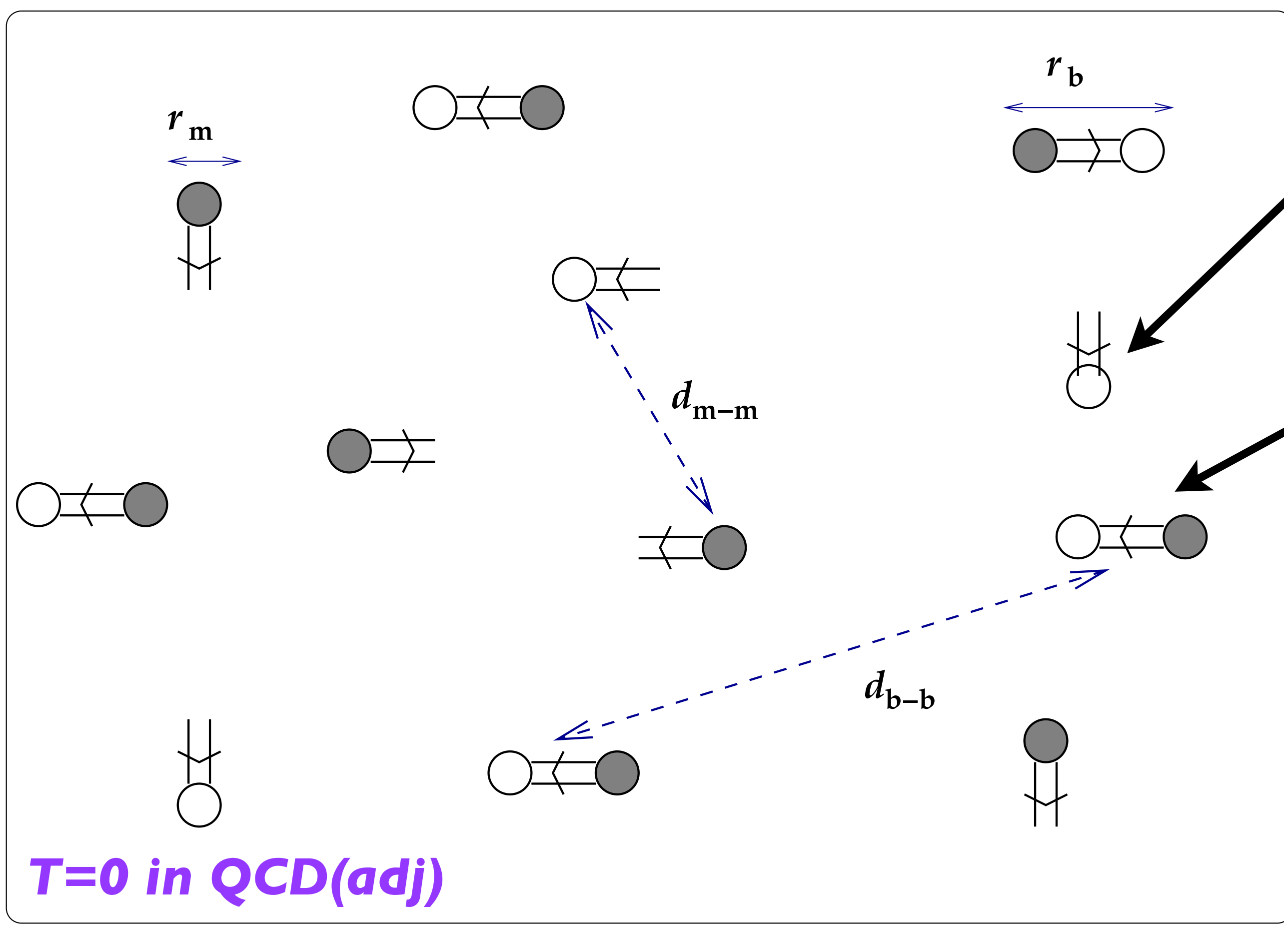
T=0 vacuum:

- a Coulomb plasma of magnetic bions (charge 2)
- Debye screening in the plasma of magnetic charges = mass gap for dual photon
  confinement of electric charges ~ confining string tension [Polyakov 1977];

But notice "monopole condensation" is due to composite "molecular" objects - this theory does not confine in 3d limit! [Unsal 2007]

THUS, FOR WHAT I DESCRIBE, FINITE SIZE OF L-CIRCLE IS CRUCIAL.

What about the T>0 dynamics? **QCD(adj) on $R^2 \times S^1$ (spatial)$\times S^1$ (thermal)**



monopole-instantons (M,KK+*)

magnetic bion "molecules"

QCD(adj) on $R^2 \times S^1$ (spatial) $\times S^1$ (thermal)

Near Tc for deconfinement, the theory is approximately two-dimensional
- a thermal, not a quantum transition

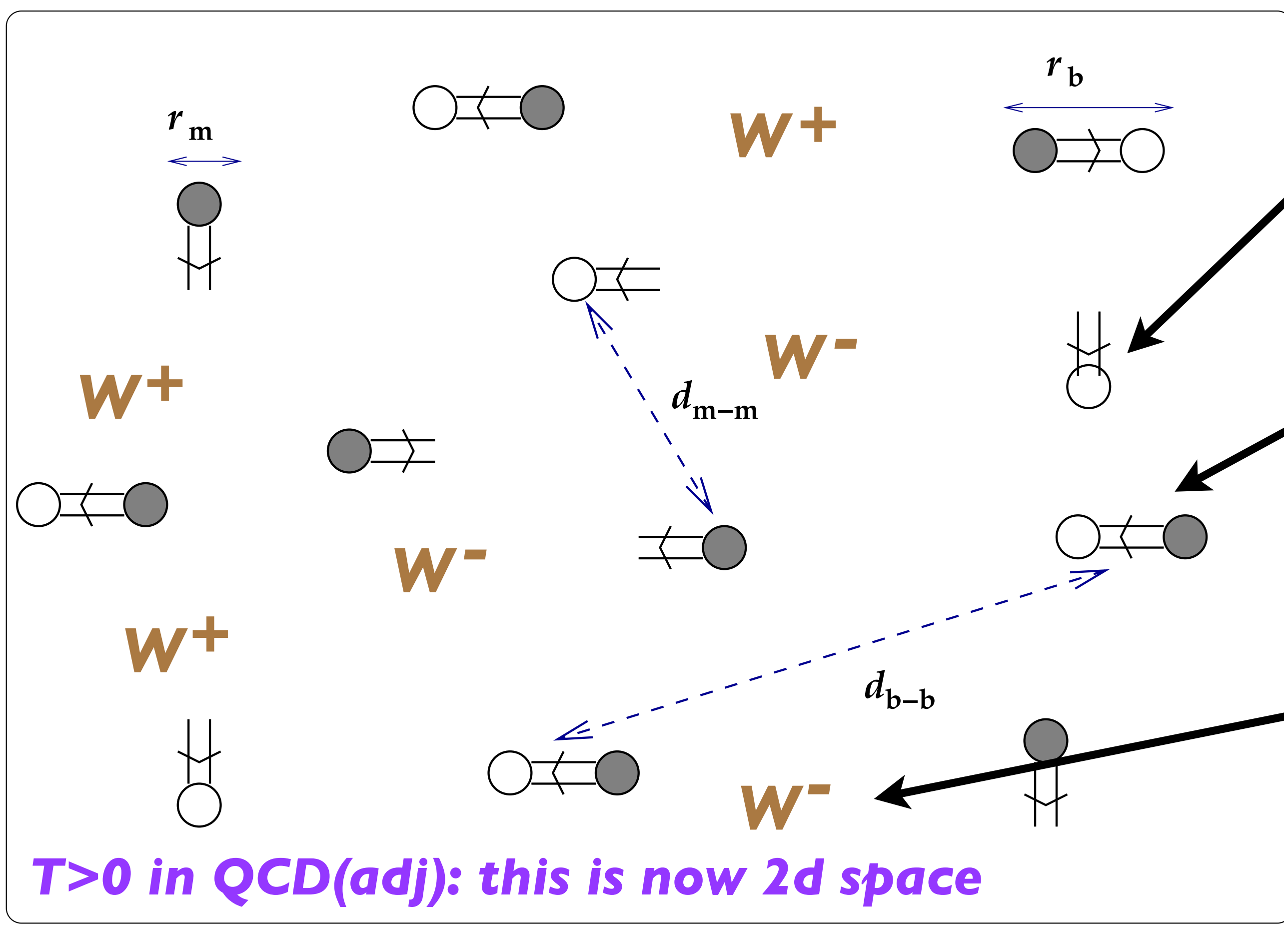
W-bosons can be excited thermally = a 2d gas of heavy electrically charged states - from SU(2) to U(1) breaking (1/L)

these are genuine "particles"

magnetic bions = a 2d gas of magnetic charges

these are "pseudoparticles" (instantons, localized in time)

both appear as "particles" in the 2d plasma



monopole-instantons (M,KK+*)

magnetic bion "molecules"

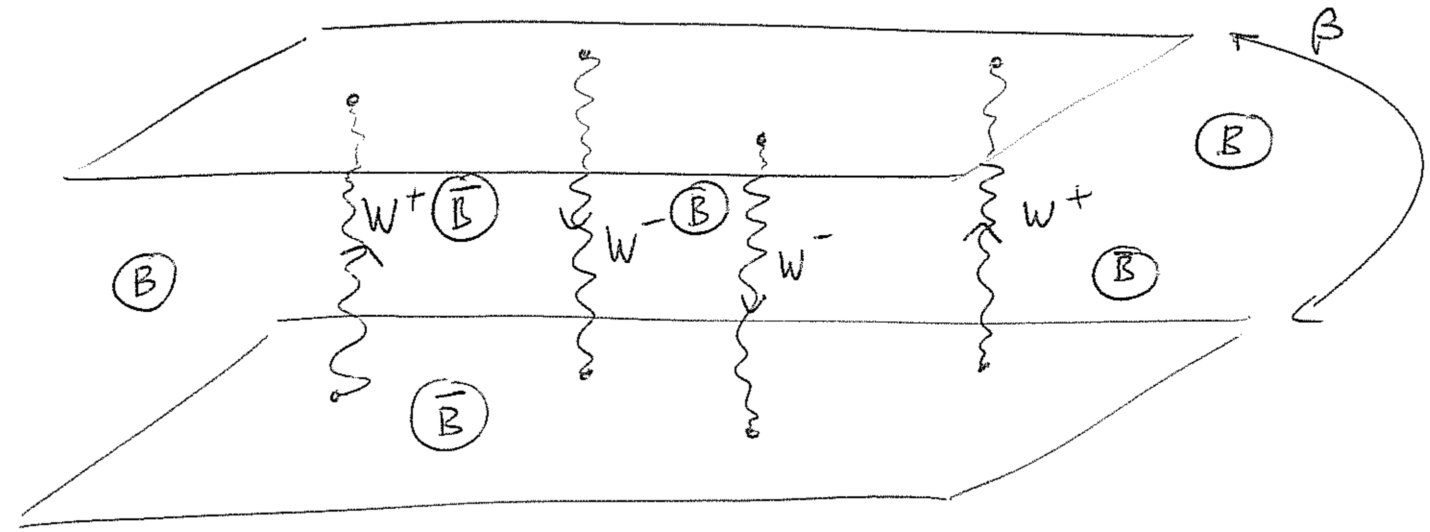
"W-bosons" = electric charges

3. thermal gases of electric and magnetic charges

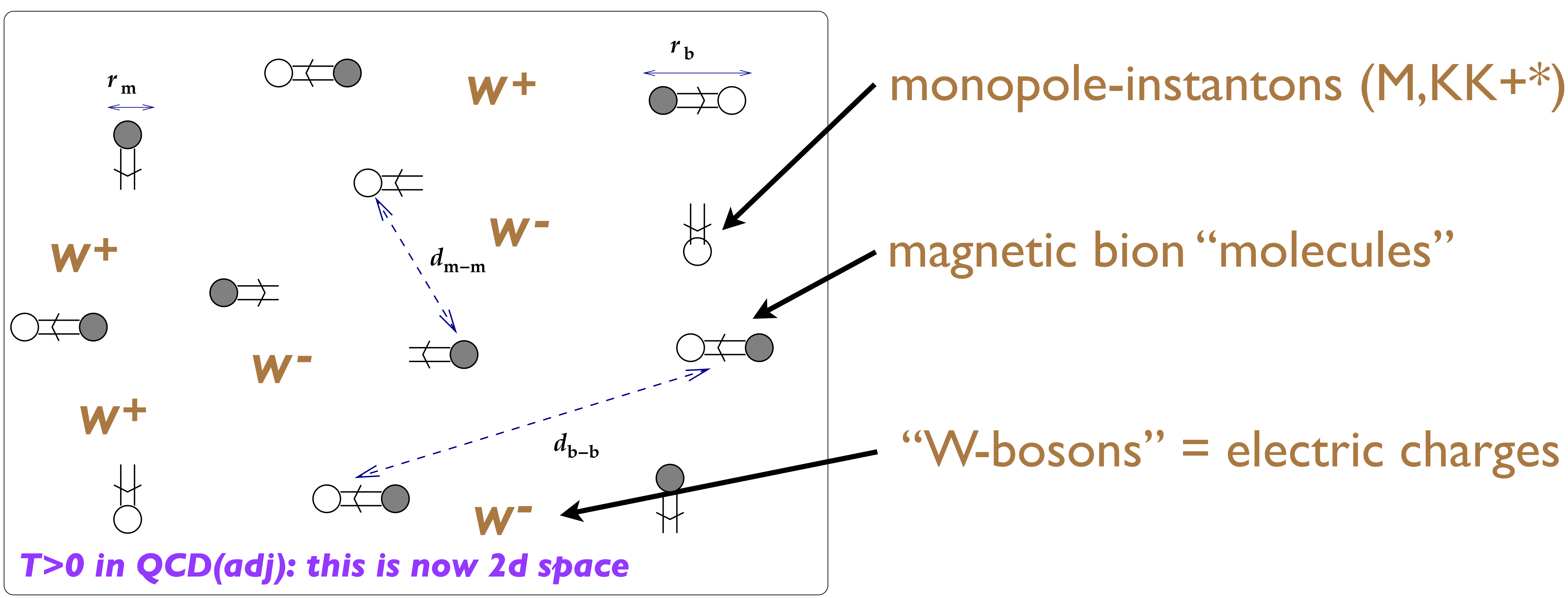
QCD(adj) on $R^2 \times S^1$ (spatial) $\times S^1$ (thermal)

At T near Tc for deconfinement, the theory is approximately two-dimensional
- a thermal, not a quantum transition.

The partition function of the theory is that of a classical 2d gas of electric and magnetically charged particles.



Not just words: due to weak coupling at small-L, reduction of Z to the gas can be justified and corrections estimated and computed.



3. thermal gases of electric and magnetic charges

QCD(adj) on R^2x S^1 (spatial)x S^1 (thermal)

for potential use in nonequilibrium keep 3d
(...particles in instanton monopole fields...)

The partition function of the theory is that of a classical 2d gas of electric and magnetically charged particles... depends on fugacities, charges and coupling strength: all mapped to 4d theory parameters:

For SU(N_c)

$$Z = \sum_{(N^i_{e\pm}\geq 0,\, i\geq 0,\, q_a=\pm 1)} \sum_{(N^i_{m\pm}\geq 0,\, j\geq 0,\, q_A=\pm 1)} \frac{\left(\frac{y_m}{a^2}\right)^{\sum_i (N^i_{m+}+N^i_{m-})} \left(\frac{y_e}{a^2}\right)^{\sum_i (N^i_{e+}+N^i_{e-})}}{\prod_i N^i_{m+}!N^i_{m-}!N^i_{e+}!N^i_{m-}!}$$

$$\times \int \prod_{a,i} d^2 R^i_a \int \prod_{A,i} d^2 R^j_A$$

$$\times \exp\left[\kappa_e \sum_{i\geq j}^{N_c} \sum_{A>B}^{N_e} q_A q_B \vec{\alpha}_i \cdot \vec{\alpha}_j \ln \frac{|\vec{R}^i_A - \vec{R}^j_B|}{a} + \frac{4}{\kappa_m} \sum_{i\geq j}^{N_c} \sum_{a>b}^{N_m} q_a q_b \vec{Q}_i \cdot \vec{Q}_j \ln \frac{|\vec{R}^i_a - \vec{R}^j_b|}{a}\right.$$

$$\left. + 2i \sum_{i,j}^{N_c} \sum_{a,B}^{N_m, N_e} q_a q_B \vec{\alpha}_j \cdot \vec{Q}_i \,\Theta(|\vec{R}^i_a - \vec{R}^j_B|)\right]$$

$\vec{Q}_i = \vec{\alpha}_i - \vec{\alpha}_{i-1}$ magnetic bion charges

$\vec{\alpha}_i$ affine roots = W charges

3. thermal gases of electric and magnetic charges

QCD(adj) on R^2x S^1 (spatial)x S^1 (thermal)

$$\kappa_e(a) = \kappa_m(a) \leftrightarrow \frac{g^2}{2\pi LT}$$

strength of W-W Coulomb interaction

For SU(N_c)

magnetic bion fugacity

electric (W) fugacity

$$Z = \sum_{(N^i_{e\pm}\geq 0,\, i\geq 0,\, q_a=\pm 1)} \sum_{(N^i_{m\pm}\geq 0,\, j\geq 0,\, q_A=\pm 1)} \frac{\left(\frac{y_m}{a^2}\right)^{\sum_i (N^i_{m+}+N^i_{m-})} \left(\frac{y_e}{a^2}\right)^{\sum_i (N^i_{e+}+N^i_{e-})}}{\prod_i N^i_{m+}!N^i_{m-}!N^i_{e+}!N^i_{m-}!}$$

$$\times \int \prod_{a,i} d^2 R^i_a \int \prod_{A,i} d^2 R^j_A$$

sum/integral over all coordinates/charges

$$\times \exp\left[\kappa_e \sum_{i\geq j}^{N_c} \sum_{A>B}^{N_e} q_A q_B \vec{\alpha}_i \cdot \vec{\alpha}_j \ln\frac{|\vec{R}^i_A - \vec{R}^j_B|}{a} + \frac{4}{\kappa_m} \sum_{i\geq j}^{N_c} \sum_{a>b}^{N_m} q_a q_b \vec{Q}_i \cdot \vec{Q}_j \ln\frac{|\vec{R}^i_a - \vec{R}^j_b|}{a}\right.$$

W-charges, electric Coulomb interaction

magnetic bion charges, magnetic Coulomb interaction

$$\left. + 2i \sum_{i,j}^{N_c} \sum_{a,B}^{N_m, N_e} q_a q_B \vec{\alpha}_j \cdot \vec{Q}_i \, \Theta(|\vec{R}^i_a - \vec{R}^j_B|)\right]$$

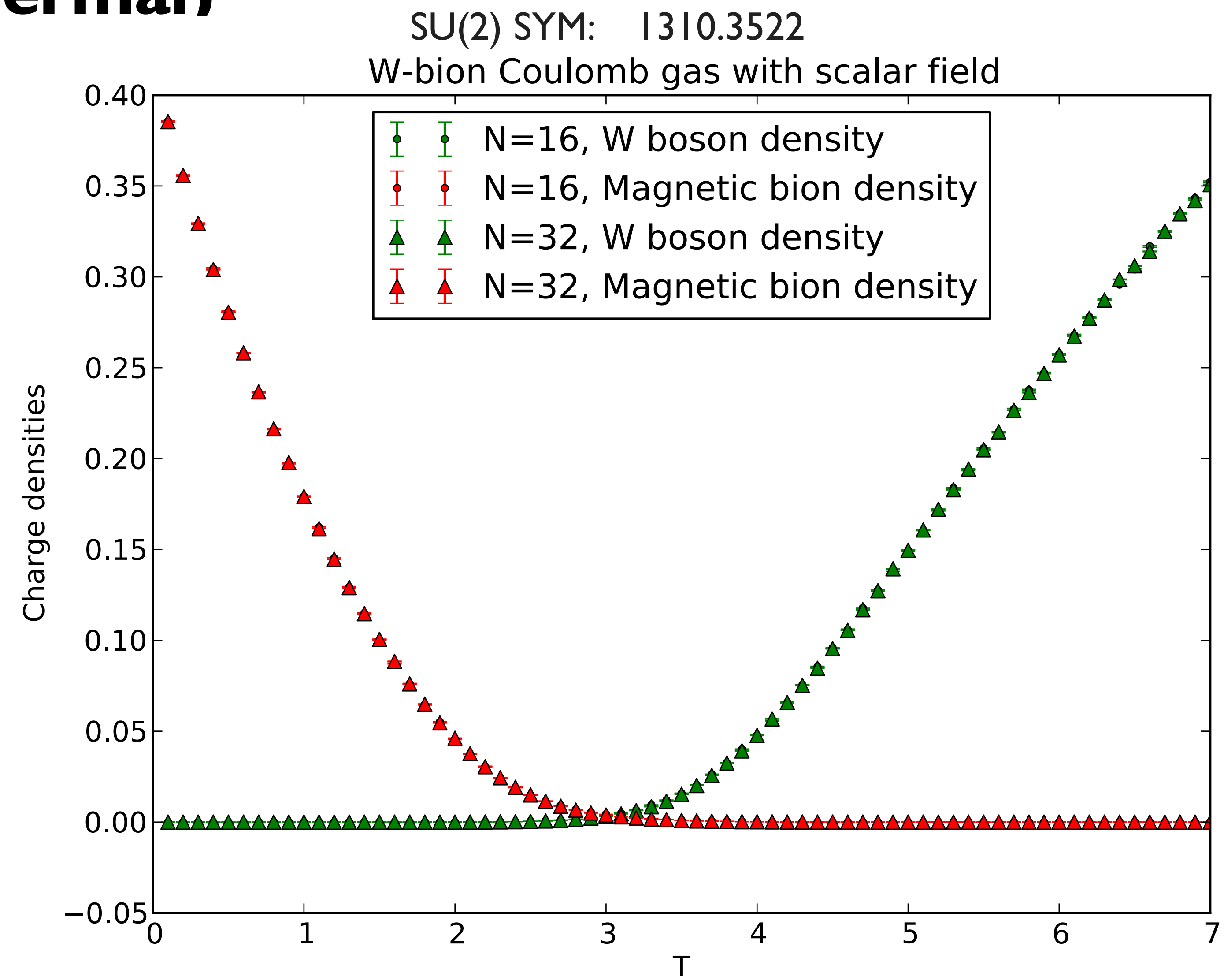
$\vec{Q}_i = \vec{\alpha}_i - \vec{\alpha}_{i-1}$ magnetic bion charges

$\vec{\alpha}_i$ affine roots = W charges

Aharonov-Bohm interaction of magnetic bions and W-bosons

3. thermal gases of electric and magnetic charges

QCD(adj) on R^2x S^1 (spatial)x S^1 (thermal)



$\kappa_e(a) = \kappa_m(a)$

For SU(N_c)

$$Z = \sum_{(N^i_{e\pm}\geq 0,\, i\geq 0,\, q_a=\pm 1)} \; \sum_{(N^i_{m\pm}\geq 0,\, j\geq 0,\, q_A=\pm 1)} \frac{\left(\frac{y_m}{a^2}\right)^{\sum_i (N}}{\prod_i N}$$

$$\times \int \prod_{a,i} d^2R^i_a \int \prod_{A,i} d^2 R^j_A$$

$$\times \exp\left[\kappa_e \sum_{i\geq j}^{N_c} \sum_{A>B}^{N_e} q_A q_B \vec{\alpha}_i \cdot \vec{\alpha}_j \ln \frac{|\vec{R}^i_A - \vec{R}^j_B|}{a} + \frac{4}{\kappa_m} \sum_{i\geq j}^{N_c} \sum_{a>b}^{N_m} q_a q_b \vec{Q}_i \cdot \vec{Q}_j \ln \frac{|R^i_a - R^j_b|}{a} \right.$$

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For SU(2) and SU(3): Kramers-Wanier duality (low-T/high-T); self dual point: Tc

3. thermal gases of electric and magnetic charges

QCD(adj) on R^2x S^1 (spatial)x S^1 (thermal)

How do we study the phase transition?

- SU(2): el.-m. Coulomb gas RGEs have a fixed line extending to weak coupling (fugacities); transition is second order; can calculate (some) critical exponents

- SU(N>2): small fugacity RGEs break down → map to XY "affine" spin model study via Monte Carlo; → Monte Carlo of Coulomb gas

For SU(N_c)

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$$\times \int \prod_{a,i} d^2R^i_a \int \prod_{A,i} d^2 R^j_A$$

$$\times \exp\left[\kappa_e \sum_{i\geq j}^{N_c} \sum_{A>B}^{N_e} q_A q_B \vec{\alpha}_i\cdot\vec{\alpha}_j \ln\frac{|\vec{R}^i_A-\vec{R}^j_B|}{a} + \frac{4}{\kappa_m}\sum_{i\geq j}^{N_c}\sum_{a>b}^{N_m} q_a q_b \vec{Q}_i\cdot\vec{Q}_j \ln\frac{|\vec{R}^i_a-\vec{R}^j_b|}{a}\right.$$

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  - → map to XY "affine" spin model study via Monte Carlo
  - → Monte Carlo of Coulomb gas

order of transition?
comparison with L=infinity?

Anber, EP, Unsal 2011
Anber, Collier, Strimas-Mackey, EP, Teeple 2013

SU(2): continuous, as in 4d
SU(3), SU(4): first order, as in 4d

physics of transition?
(critical exponents?)

Anber, Collier, EP 2012
TBA, 2013/4

Most answers so far were obtained in the spin-model picture (no sign problem). These are new "affine" XY-models (for SU(3), ignoring W's = dislocation theory of 2d triangular-lattice crystal melting of D. Nelson 1970's)

3. thermal gases of electric and magnetic charges

QCD(adj) on R^2x S^1 (spatial)x S^1 (thermal)

W bosons
=vortices of spin model

magnetic bions
=”external field” in spin model

$$-\beta H = \sum_{x;\hat{\mu}=1,2} \sum_{i=1}^{N_c} \frac{\kappa}{4\pi} \cos 2\vec{\nu}_i \cdot (\vec{\theta}_{x+\hat{\mu}} - \vec{\theta}_x) + \sum_x \sum_{i=1}^{N_c} \tilde{y} \cos 2(\vec{\alpha}_i - \vec{\alpha}_{i-1}) \cdot \vec{\theta}_x .$$

XY-spins - rank(G) of them: dual photons

weights of fundamental

physics at low-T:

- vortices (electric charges) are confined, almost none present, bound in neutral pairs
- external charges in spin model (magnetic bions) proliferate, breaking of discrete chiral symmetry

physics at high-T:

- vortices (electric charges) proliferate, breaking center symmetry
- external charges in spin model (magnetic bions) confined, only appear in pairs

3. thermal gases of electric and magnetic charges

QCD(adj) on R^2x S^1 (spatial)x S^1 (thermal)

physics of transition...

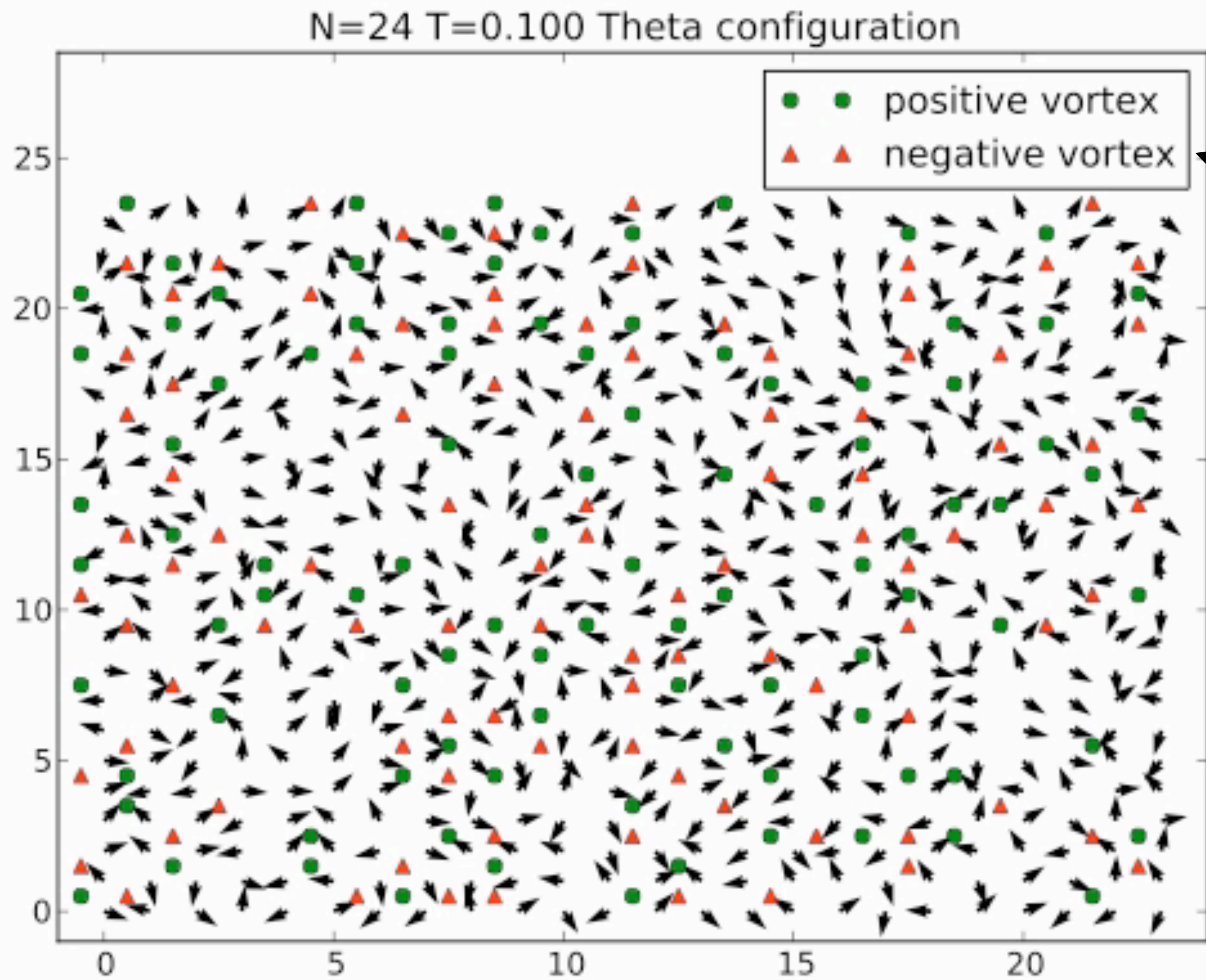
but in a different "duality frame" ... SU(2)

arrows = XY spins = photon (U(1) in SU(2))

(W-bosons are, now, represented by external field potential)

randomly fluctuating arrows =
small correlation length,
mass gap

ordered arrows =
center symmetry breaking
in spin model can probe confinement
with unphysical "half-electron" operator, so Z_4



vortices = magnetic bions

movie courtesy Seth Strimas-Mackey, Nov. 2013

3. thermal gases of electric and magnetic charges

QCD(adj) on R^2x S^1 (spatial)x S^1 (thermal)

one final slide for theorists... also showing detailed map to 4d parameters

same physics can be described by the quantum phase transition
of a 1-dimensional system with Hamiltonian: "dual sine-Gordon"

actually, any Nc

4d gauge coupling at 1/L

$$\mathcal{H} = \frac{1}{2}(\partial_x \vec{\Phi})^2 + \frac{1}{2}(\partial_x \vec{\Theta})^2 - \sum_{i=1}^{N_c=3} \left[\tilde{y} \cos\left[\frac{4\pi\sqrt{LT}}{g} (\vec{\alpha}_i - \vec{\alpha}_{i-1})\vec{\Phi} \right] + y \cos\left[\frac{g}{\sqrt{LT}} \vec{\alpha}_i \vec{\Theta} \right] \right]$$

$$[\Theta^i(x), \Phi^j(y)] = -i\delta^{ij}\theta(x-y)$$

L,T - (inverse) sizes of two circles

magnetic bion fugacity $\tilde{y} \sim \frac{1}{T}\frac{1}{L^3 g^{14-8n_f}}\, e^{-S_0}$ $\qquad S_0 = \frac{8\pi^2}{g^2}$

W-boson fugacity y $= (2n_f + 2)\frac{m_W T}{2\pi}\, e^{-\frac{m_W}{T}}$ $\qquad m_W = \frac{\pi}{L}$

For Nc=2,3 this is self dual. For Nc=2 at self-dual point c=1 (free field).

Can this representation of the problem be used to understand why SU(2) transition is continuous but SU(3,4...) is not?

BRIEF SUMMARY AND A FEW MORE QUESTIONS:

I told you about how SUSY can - directly or otherwise - help in finding calculable realizations of deconfinement - generally, a complicated strongly-coupled (non-BPS, non-protected, non-holomorphic) problem.

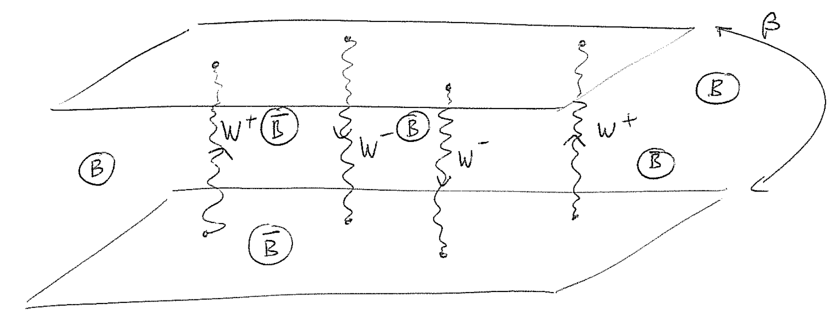
SYM with gaugino mass on $R^3\mathrm{x}S^1$

where a quantum phase transition appears continuously related to thermal deconfinement

QCD(adj) on $R^2\mathrm{x}S^1\mathrm{x}S^1$

where deconfinement maps to the transition in a "simple" electric magnetic "Coulomb gas"

(potential use in nonequilibrium?)



In both cases, various properties of the transition agree with known 4d lattice results.

We pointed out many erroneous assumptions/statements in existing models of deconfinement via topology.

Some new effort in "model building" ("instanton-monopole liquid"?)

Lattice work - in pure YM; in studying the phases of QCD(adj) on S^1; also incl. SYM.

BRIEF SUMMARY AND A FEW MORE QUESTIONS:

- A natural step is to extend these studies to all gauge groups.

… does it always "work"? why ?

- *is continuity of SYM*/thermal YM related to the item below?*

- We found novel topological excitations stabilizing center symmetry. Appear related to "renormalons" - a "semiclassical shadow" thereof - required to cancel ambiguities of perturbative series... Argyres, Unsal; Dunne,Unsal ...2012-

… how general ? how far can the "resurgence" idea be pushed in QFT ?

Stimulated by QM [Zinn-Justin Jentchura...]. Semiclassical series - all exp. small, power-law, log terms, e.g.,

$$E^{(N)}(g) = \sum_{\pm} \sum_{k=0}^{\infty} \sum_{l=1}^{k-1} \sum_{p=0}^{\infty} c^{\pm}_{k,l,p} \, \frac{e^{-k\frac{S}{g}}}{g^{k(N+\frac{1}{2})}} \left(\ln \left[\mp \frac{2}{g} \right] \right)^{l} g^{p}$$

is a "resurgent transseries"... can be "Borel-Ecalle summed",

hence obtained from an exact semiclassical result

- does it continue to strong coupling? - answer appears "yes" in QM [Unsal...'13/14]

- in cases where continuous connection exists (SYM*/thYM?) is a similar continuation to strong coupling the deep reason behind agreement we see?

a (very) long-term story, obviously...