# Lattice chirality and the decoupling of mirror fermions

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with Tanmoy Bhattacharya, Matthew Martin — Los Alamos, hep-lat/0605003

with Joel Giedt — (Minnessota) Rennselaer, hep-lat/0701004

with Yanwen Shang — Toronto, arXiv:0706.1043 [hep-th]

& work in progress with Joel Giedt, Yanwen Shang

## why should one care?

currently popular scenarios for LHC-scale physics:
weakly coupled models of electroweak symmetry breaking (S,T...)

keep in mind, however, that the kinds of strong-coupling gauge dynamics we understand - from experiment, theory, or numerics - are only a few

for example, while QCD-type technicolor models are out by EWPT, there may exist other kinds of dynamics that work just fine

in particular:

how well do we understand strong chiral gauge dynamics?

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- what tools do we have?

| tools one trusts | tools you don't really know whether to trust unless confirmed by other means - experiment, numerics, or the tools on the l.h.s. |
|---|---|
| 't Hooft anomaly matching<br><br>in SUSY:<br>"power of holomorphy" | "MAC"<br><br>truncated Schwinger-Dyson equations |

- there's not much there...

the recently popular AdS/CFT-QCD type duals
are not of great use in chiral gauge theories

large-N limit:

SU(5) with 5* and 10 ---- SU(N) with (N-4) N* and an N(N-1)/2

have different symmetries and symmetry realizations, as easily made evident, e.g., from the study of the supersymmetric case

more concretely, at large-N “quark” loops are not suppressed [e.g., N(N-1)/2 - representation], hence “mesons” are not free at infinite N; one doesn’t expect a nice classical “supergravity” description in a slice of AdS or a deformation thereof while I motivated the desire to study chiral dynamics via "beyond the Standard Model" physics, recall that the SM itself is a chiral gauge theory

albeit weakly coupled at energies < O(TeV)

Do we have a nonperturbative formulation of the SM?

( A PURIST'S QUESTION: "DOES THE SM EXIST?")

We should recall that the lattice is the best way to calculate many things in QCD - though not all! - notably spectra and various matrix elements (meson decay constants, for one), in addition to giving a nonpertubative definition of the theory

Can we apply similar methods to non-QCD-like theories?

can we apply similar methods to non-QCD-like theories?

**supersymmetric** - some recent developments;
notably good in lower dimensional, nonchiral, superrenormalizable cases
phenomenologically interesting N=1 + matter, 4d case ... still open...

see, e.g., Joel Giedt's review hep-lat/0602007

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**CHIRAL? - whether supersymmetric, or not - THIS TALK:**

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NOT about LHC physics via strong chiral gauge dynamics
(hence, will not discuss a potential theory of the world)

RATHER, I'd like to tell you where the lattice chiral gauge theory problem is at, and what attempts are being made at improvement and progress

HOPING to convince you that it is an interesting, theoretically appealing problem, fun to think about...

...and that doing this may even turn out to be useful!

many tools come together - some foreign to us before - both theoretical and "experimental" the approach I'd like to discuss today is a combination of "old" and "new"

will put in larger perspective shortly

*the logic basically goes like this:*

formulating vectorlike gauge theories (like QCD) on the lattice is not too much of a problem - there are doublers, of course, but we learned to deal with them

so, one can ask a natural question -

can one start with a vectorlike theory, for example:

$$\begin{array}{lcc} SU(5) \text{ with} & 5^* & 5 \\ & 10 & 10^* \\ \text{names:} & \text{"light"} & \text{"mirror"} \end{array} \quad \left.\begin{array}{c} \\ \\ \end{array}\right\} \text{all Weyl, L}$$

and then, deform the theory in such a way that

- mirrors decouple from the low-energy spectrum
- the gauge symmetry remains unbroken ?

before attempting to answer: WHY DO WE DO THIS?

a lightning review of current situation with chiral lattice gauge theories:

based on seminal works of Ginsparg, Wilson (1982); D.B. Kaplan (1992); Narayanan, Neuberger (1994); Neuberger (1997); P. Hasenfratz, Niedermaier (1997); Luescher (1998); Neuberger (1998),

Luescher has proven (1999-2000) that
**an exactly gauge invariant lattice action and measure exist**
for an anomaly free chiral gauge theory*

however, outside of perturbation theory, there is
**no explicit formulation** of the fermion path integral measure!

- fascinating theoretical achievement, but not good enough for practical use, e.g. numerical simulations

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* for the mathematically inclined:
for U(1) gauge groups in finite volume and SU(2)xU(1) in infinite volume **only**; other cases open to proof!

thus,

***attempts to construct a chiral lattice gauge theory via decoupling the mirrors from a vectorlike theory - where the measure is known explicitly - are still worthwhile and of possible practical importance****

in particular,
*Bhattacharya, Csaki, Martin, Shirman, and Terning (2005)*
proposed exactly such a construction combining Kaplan's domain wall fermions with ideas coming from higgsless models in slices of AdS

while very imaginative and new, the *BCMST* construction was

- never fully latticized (only 4d deconstructed version of 5d AdS slice)
- chiral symmetries were realized only in appropriate asymptotic limits

hence, while trying to understand a 2d version of the model, we (*Bhattacharya, Martin, EP, 2006*) started thinking of other possible ways to accomplish same goal...

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* another message for enthusiasts:
there may be other, not thought of yet, ways to do this!

back to our question -

can one start with a vectorlike theory, for example:

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- a "normal" continuum physicist would say: NO!

- however, the lattice affords possibilities a "normal" continuum physicist rarely thinks of!

- for example, everybody knows that four-fermi interactions, if taken strong enough, break chiral symmetries

$$\frac{g}{\Lambda^2}(\bar\Psi\Psi)(\bar\Psi\Psi)\ , \quad gN > 8\pi^2$$

as per the NJL "gap equation" *made "believable" via large-N, gN=const (aka "mean field")*

- few "continuum people" know, however, that if one takes coupling even stronger, the theory enters a "**strong-coupling symmetric phase**," with only massive excitations and unbroken chiral symmetry

- why haven't most people heard about these phases?

because these phases are a "lattice artifact" - the physics is that of "lattice particles" with small hopping probability

thus, these "lattice particles" are "heavier than the UV cutoff" (think of an almost-insulator)

I’m not sure who discovered them first

Eichten, Preskill (1986; “Chiral gauge theories on the lattice”)
- 4-fermi interactions ... [E-P]

A. Hasenfratz, Neuhaus (1988), strong Yukawa case - similar!

E-P story “retold”

$$SU(5) \qquad \underbrace{\begin{matrix}5^* \\ 10 \\ 1\end{matrix}}_{\text{“light”}} \qquad \underbrace{\begin{matrix}5 \\ 10^* \\ 1\end{matrix}}_{\text{“mirror”}}$$

speaking in a continuum language

one could include interactions for mirrors only:

$$g_1 \quad 10^* - 5 - 5 - 1$$

$$g_2 \quad 10^* - 10^* - 10^* - 5$$

continuum language description - **strong** 4-fermi causes

a.) 10*-5-5 bound state fermion, gets Dirac mass with singlet
b.) $(10^*)^3$ bound state fermion, gets Dirac mass with 5

none of these involves breaking SU(5) - here a mere spectator to strong 4-fermi

---- "strong-coupling symmetric" phase

I called this phase a "lattice artifact" - E-P established its existence using, at large 4-fermi, a "hopping expansion," i.e. treating kinetic terms as perturbation and 4-fermi as leading piece in action - to get a flavor of how it works, an easily manageable toy example:

an easily manageable toy example SU(4) "chiral" symmetry

$$H_{4\psi} = \sum_x g(\psi_a \psi_b \psi_c \psi_d \epsilon^{abcd} + hc)$$

space lattice only, canonical anticommutation relations:

$$\{\psi_a(x), \psi_b^+(y)\} = \delta_{ab}\delta_{xy}$$

at g>>1 in lattice units, hopping is negligible:

$$H = \underbrace{\sum_x H_{0\,x}}_{\text{4-fermi}} + \underbrace{H_1}_{\text{hopping} \sim \psi^+ \vec{\alpha}\cdot\vec{\nabla}\psi}$$

to leading order, at every site the same simple 4-fermion QM problem, rename: $\psi_{a\,x} \to a_a$ $\psi_{b\,x} \to a_b^+$

$$H_0 = g(a_a a_b a_c a_d + a_a^+ a_b^+ a_c^+ a_d^+)\epsilon^{abcd}$$

H conserves F(mod 4); 16 states = 1 + 1' + 4 + 4* + 6 under SU(4)

H connects only 1 (= all fermions empty) and 1' (= all fermions occupied)

so: (1-1') has energy -g; (1+1') has energy +g, 4,4*,6 have energy 0.

so, in the infinite-g limit, the lattice theory ground state is

- unique
- an SU(4) - chiral symmetry - singlet

at first order in 1/g, hopping will turn on and (-g) site-localized states form a band,

$$H = \begin{pmatrix} -g & & & & \\ & -g & & & \\ & & -g & & \\ & & & \ddots & \\ & & & & -g \end{pmatrix} + \begin{pmatrix} 0 & 1 & & & 1 \\ 1 & 0 & \ddots & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & & 1 \\ 1 & & & 1 & 0 \end{pmatrix}$$

causing the SU(4)-singlet states to propagate...

states are heavy, mass ~ g/a >> 1/a, the UV-cutoff

of course, there are non-singlet heavy states propagating, too, but no massless states

as usual, the 1/g (strong-coupling) expansion has finite radius of convergence, hence above story should represent true ground state of theory for sufficiently large g

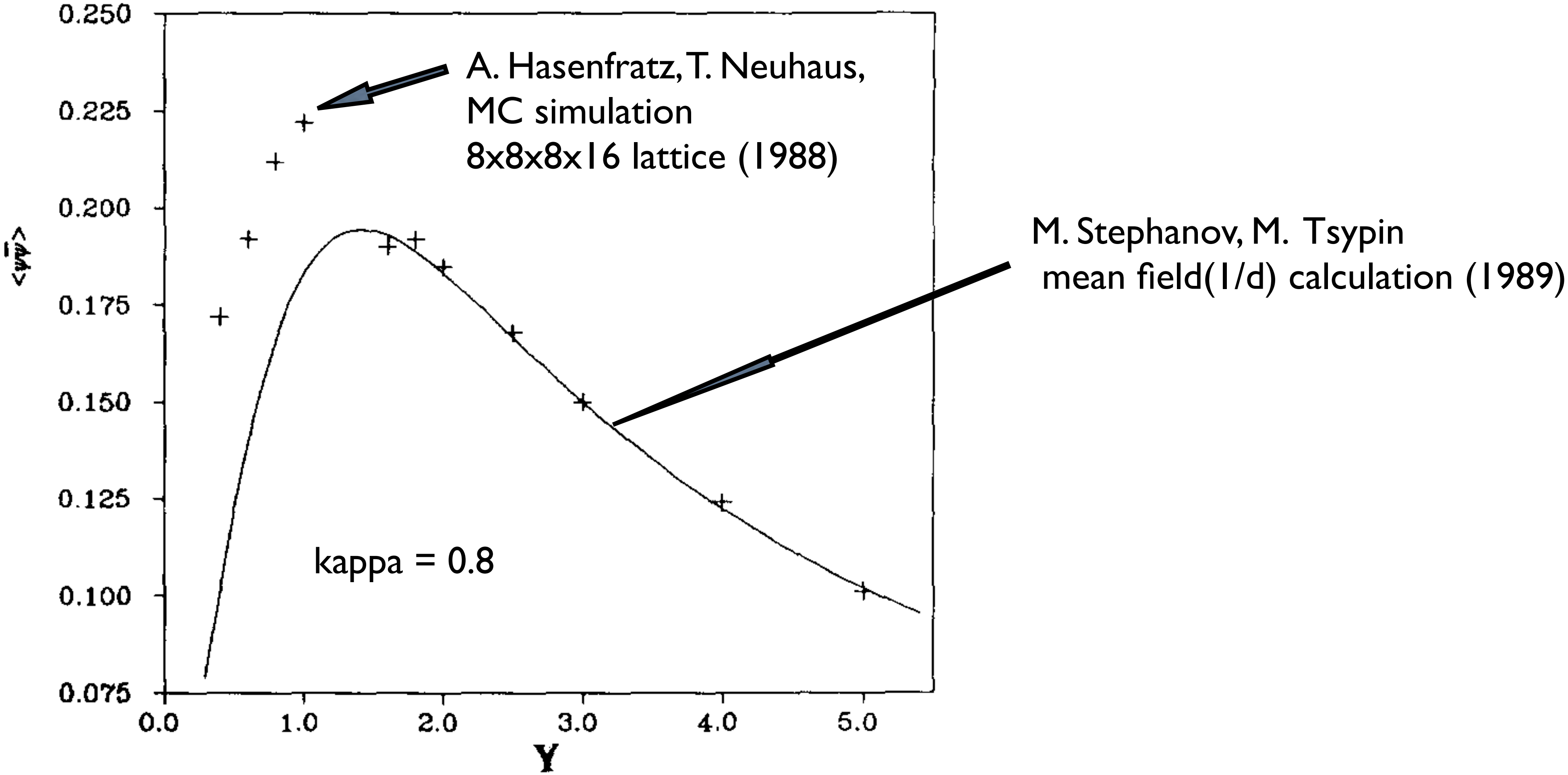
very much like "static limit" of lattice QCD, but infinite mass limit replaced by infinite four fermi

# for the skeptics: a 4d Z2-chiral invariant Yukawa-Higgs lattice theory, naive fermions

$$S_{\mathrm{B}} = -2k \sum_{x,\mu} \phi_x \phi_{x+\mu} \, ,$$

$$S_{\mathrm{F}} = \tfrac{1}{2} \sum_{x,\mu} \bar{\psi}_x \gamma_\mu (\psi_{x+\mu} - \psi_{x-\mu}) \equiv \sum_{x,\mu} \bar{\psi}_x K_{xy} \psi_y$$

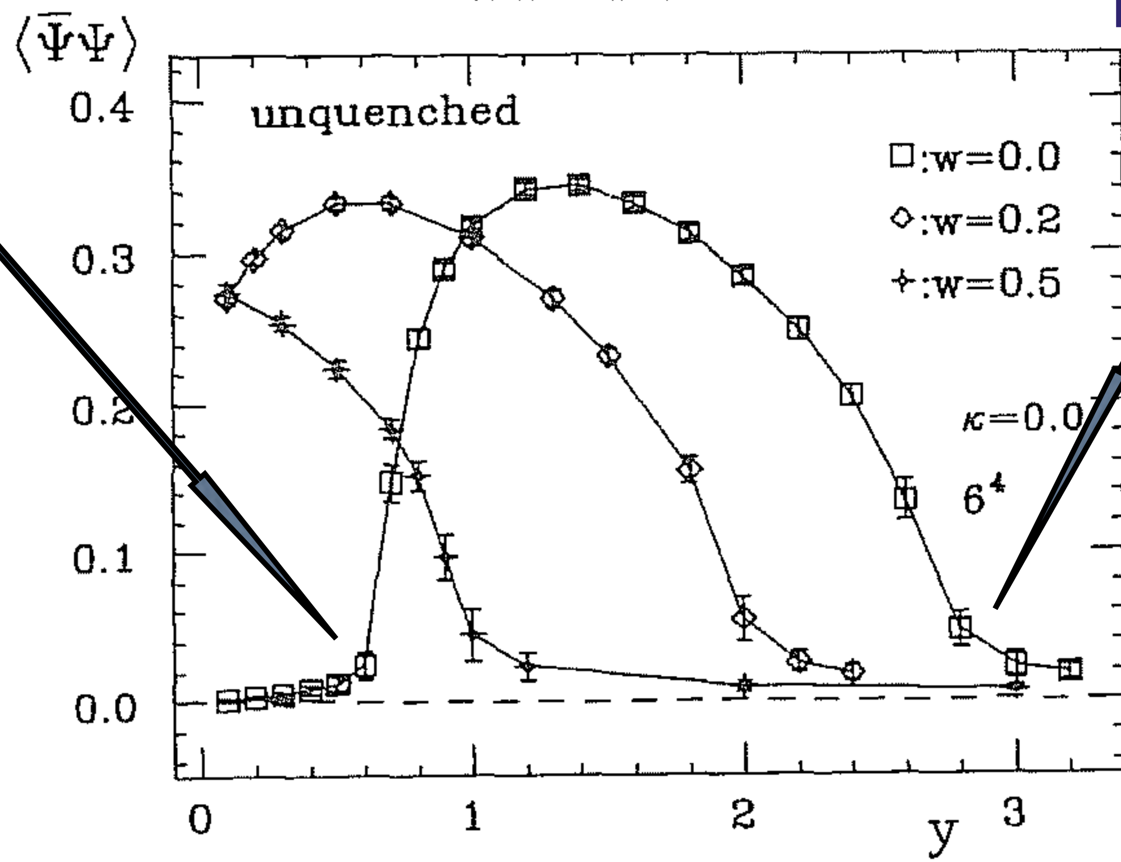
$$S_{\mathrm{Y}} = \sum_{x} Y \phi_x \bar{\psi}_x \psi_x \, , \quad \mu = 1, 2, \ldots, d, \quad \phi_x = \pm 1 \; .$$



for the skeptics:

W. Bock, A. De, K. Jansen, J. Jersak, T. Neuhaus, J. Smit (1990)

6x6x6x6 lattice




NJL transition

transition to "strong coupling symmetric phase"

a 4d SU(2)_L x SU(2)_R model with naive fermions
(or Wilson, w>0, - explicitly broken chiral symmetry)

simple SU(4) exersize, with a bit more group theory, can be repeated for SU(5) of E-P

(btw, singlet needed by E-P to have sensible "static limit" of Euclidean fermion path integral)

$$g_1 \quad 10^* - 5 - 5 - 1$$

$$g_2 \quad 10^* - 10^* - 10^* - 5$$

showing that at infinite g SU(5) ground state unique and singlet

- the "E-P dream" was, essentially, to use this* phase to decouple the mirrors

*similar, can't go there now; ask me later

- no continuum limit of this mirror theory - "everything mirror" is cutoff scale and heavier and decoupled from IR physics... ideally

- gauge field appears only in hopping terms and so contributions of mirror sector to gauge field action should be ~ 1/g

**- did the "E-P dream" work?**

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no - the reason was, in essence, that, in 1986, there was no way to define chiral components of a spinor field on the lattice - even a two-component Weyl field on the lattice (that they used) has opposite chirality massless excitations in it, because of the fermion doubling

because of the lack of L/R separation on lattice - notice that L/R separation requires the notion of chiral symmetry - the strong 4-fermi was "felt" by both "mirror" and "light" fermions

hence, both "mirror" and "light" fermions became heavy at strong-4 fermi, while at weak 4-fermi, both "mirror" and "light" were massless, i.e. the theory was vectorlike

- study of E-P model by Golterman, Petcher, Rivas (1993)

**- so what has changed?**

## - so what has changed?

after a series of seminal papers in the 90's (Kaplan, Narayanan/Neuberger, Neuberger, Hasenfratz/Niedermayer, Luescher, Neuberger) it was realized that there is an exact definition of chirality at any nonzero lattice spacing

...rediscovering, in 1997, Ginsparg/Wilson of 1982!

definition of L and R components of Dirac fermions is possible

- somewhat complicated, but exact at any (a, N)!
- exact chirality transforms, anomaly, Ward identity, index theorem...

so, naturally, one can ask, can the "E-P dream" be resurrected as well?

Bhattacharya, Martin, EP, 2006

one note: Creutz, Rebbi, Tytgat, Xue, 1996, similar proposal using E-P + domain wall - before GW operator and exact chirality - symmetries become exact only as size becomes infinite, so less pretty, and **much more difficult to study theoretically,** so, there was no follow-up work whatsoever...

to explain our proposal and later/current work in more detail, need a quick review of

## Ginsparg-Wilson relation, its solution and consequences:

## Ginsparg-Wilson relation, its solution and consequences:

$$\{D_q, \gamma_5\} = D_q \gamma_5 D_q$$ *Ginsparg-Wilson [GW], 1982*

but what is D? - ressurection by *Neuberger, 1997, D is "local", but with an exponential tail*

given D, define:

$$\hat{\gamma}_5 \equiv (1 - D)\gamma_5$$

GW implies:

$$\hat{\gamma}_5^2 = 1 \qquad \hat{\gamma}_5 D_q = -D_q \gamma_5$$

two sets of chiral projectors:

$$P_\pm = (1 \pm \gamma_5)/2$$

$$\hat{P}_\pm = (1 \pm \hat{\gamma}_5)/2$$

then, there is an exact chiral symmetry (GW, 1982; formulation of Luscher, 1999)

$$\Psi \to e^{i\alpha\gamma_5}\Psi \qquad \bar{\Psi} \to \bar{\Psi} e^{i\alpha\hat{\gamma}_5}$$

of lattice action

$$S_{kin} = \sum_{x,y} \bar{\Psi}_x D_{qxy} \Psi_y$$

note that, really, we have $\bar{\Psi}_x \to \sum_{x'} \bar{\Psi}_{x'} \left(e^{i\alpha\hat{\gamma}_5}\right)_{x'x}$

$$\Psi_q \rightarrow e^{i\alpha_{q,\pm}P_\pm}\Psi_q$$
$$\bar{\Psi}_q \rightarrow \bar{\Psi}_q e^{-i\alpha_{q,\pm}\hat{P}_\mp}$$

global L and R symmetries of action $U(1)_{q,-} \times U(1)_{q,+}$

field dependence of transformation leads to Jacobian (vanishes for vector)

$$\left[1 \pm i\alpha_{q,\pm}\mathrm{Tr}\left(\gamma_5 - \frac{1}{2}D_q\gamma_5\right)\right]$$

then properties of D are useful to (easily, really!) to show that:

$$\mathrm{Tr}(\gamma_5 - \tfrac{1}{2}D_q\gamma_5) = n_+^0 - n_-^0$$

*moral:*

exact lattice chiral symmetry (not usual one for all modes!),
exact (anomalous) Ward identities, axial charge violation, ...
in vectorlike theories - **big success!**

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**biggest drawback – no Hamiltonian formulation!** only evidence for unitarity

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# finally, can explain our formulation [Bhattacharya, Martin, EP, hep-lat/0605003]

- because simulations are cheapest in 2d, consider a 2d chiral U(1) theory, the "345" model (easily formulated for 4d, 5*-10 theory, but no evidence, so far, except words...)

"345" theory fields: 3- 4- 5+ 0+ and their mirrors: 3+ 4+ 5- 0-

$$S_{kin} = \sum_{q=0,3,4,5} \sum_{x,y} \bar{\Psi}_q(x) D_q(x,y) \Psi_q(y)$$

8 global chiral U(1)s are symmetries of $S_{kin}$:

$$\prod_{q=0,3,4,5} U(1)_{q,-} \times U(1)_{q,+}$$

while target 3-4-5 theory has following exact classical

$$U(1)_{3,-} \times U(1)_{4,-} \times U(1)_{5,+} \times U(1)_{0,+}$$

introduce chiral components for each field, using appropriate projectors:

$$\Psi_\pm = P_\pm \Psi$$

$$\bar{\Psi}_\pm = \bar{\Psi} \hat{P}_\mp$$

include Yukawa couplings involving mirrors that violate all unwanted U(1)s

(gauge invariant, with unitary Higgs field with no kinetic term ~ multifermion)

e.g.: $\bar{\Psi}_{0,-}(\phi^*)^3\Psi_{3,+}$ (Dirac) and $\Psi^T_{3,+}\gamma_2(\phi^*)^8\Psi_{5,-}$ (Majorana)

from the remaining classical symmetries of the action

$$U(1)_{3,-} \times U(1)_{4,-} \times U(1)_{5,+} \times U(1)_{0,+}$$

three, 345, 133, $U(1)_0$, are exact and one is anomalous, 111, obeying an exact anomalous Ward identity (exactly as in continuum!):

$$\langle \delta_{\alpha_{111}} \mathcal{O} \rangle = i\,\frac{\alpha}{2}\,\langle \mathcal{O}\ \mathrm{Tr}\left[\gamma_5(D_3 + D_4 - D_5)\right]\rangle$$

- this completes the definition of the model.

since for GW fermions - as a consequence of exact lattice chiral symmetry, exactly as in continuum:

$$\bar{\Psi}_q D_q \Psi_q = \bar{\Psi}_{q,+} D_q \Psi_{q,+} + \bar{\Psi}_{q,-} D_q \Psi_{q,-}$$

and since the Yukawa (multi-fermi) are written only in terms of the mirror fields, the action is completely "light"/"mirror" split

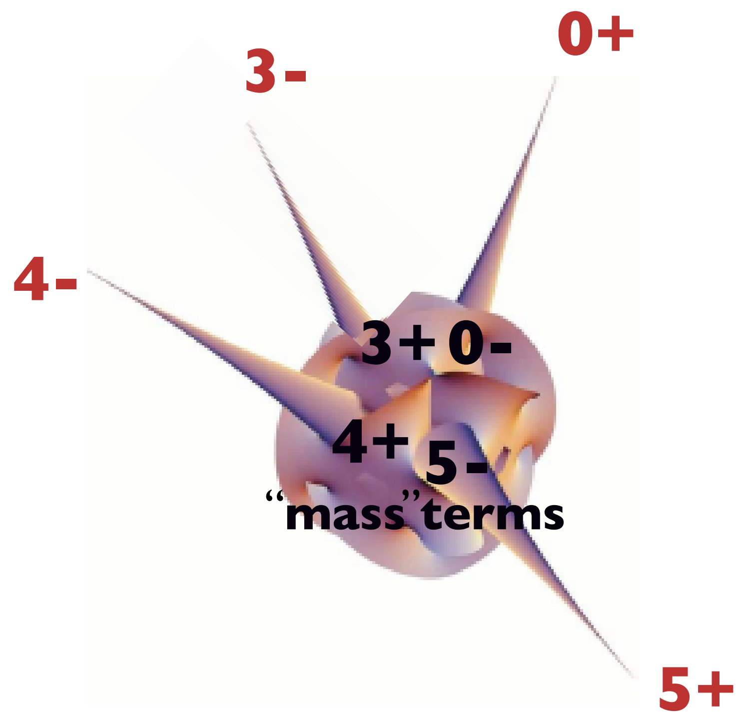
- "light" fields do not feel the strong mirror interactions!

recall - cause of problem for E-P "dream"

**so, we have proposed a lattice formulation of chiral gauge theories, which is, unlike any other:**

- **exactly gauge invariant**
- **global symmetries, incl. anomalous ones, are as in desired target continuum theory**
- **the fermion measure is that of a vectorlike theory, so it is explicitly defined**

in other words, it looks like we have realized the E-P dream:




**why aren't we opening the champaign, then?**

note that, really, we have

$$\bar{\Psi}_x \to \sum_{x'} \bar{\Psi}_{x'} \left(e^{i\alpha\hat{\gamma}_5}\right)_{x'x}$$

so L/R components defined with

$\hat{\gamma}_5 \equiv (1-D)\gamma_5$ are slightly nonlocal...

## a "few" questions remain:

1 with these slightly nonlocal Yukawa/4-fermi mirror interactions, is it still true that a "strong coupling symmetric phase" exists?
are the mirrors heavy?

2 in typical models, there is more than one strong Yukawa/4-fermi interaction - needed to break all classical mirror global symmetries, less there will be extra unlifted instanton mirror zero modes - and there can be a nontrivial phase structure as their ratios change

1 and 2 can be addressed without gauge fields, but NEED TO USE NUMERICS; no simple analytic strong-coupling expansion as in original models with non-exactly chiral fermions - beauty has a price! - then, adding gauge fields brings in a new set of questions:

3 what happens if one tries to decouple an anomalous mirror representation?

4 with gauge fields included, is the long-distance theory unitary?
note the different treatment of conjugate mirror fermion variables with respect to interactions with gauge field through the 4-fermi/Yukawa chiral projectors; hopes that this is irrelevant...

**and we don't know all the answers yet...**

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Joel Giedt, EP, hep-lat/0701004
P. Gerhold, K. Jansen, arXiv:0707.3849[hep-lat] + ...

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in progress

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# is it still true that a "strong coupling symmetric phase" exists?

Joel Giedt, EP (2007)

toy 2d Yukawa-unitary Higgs model with Ginsparg-Wilson fermions

exact chiral symmetry, zero gauge fields in simulation:

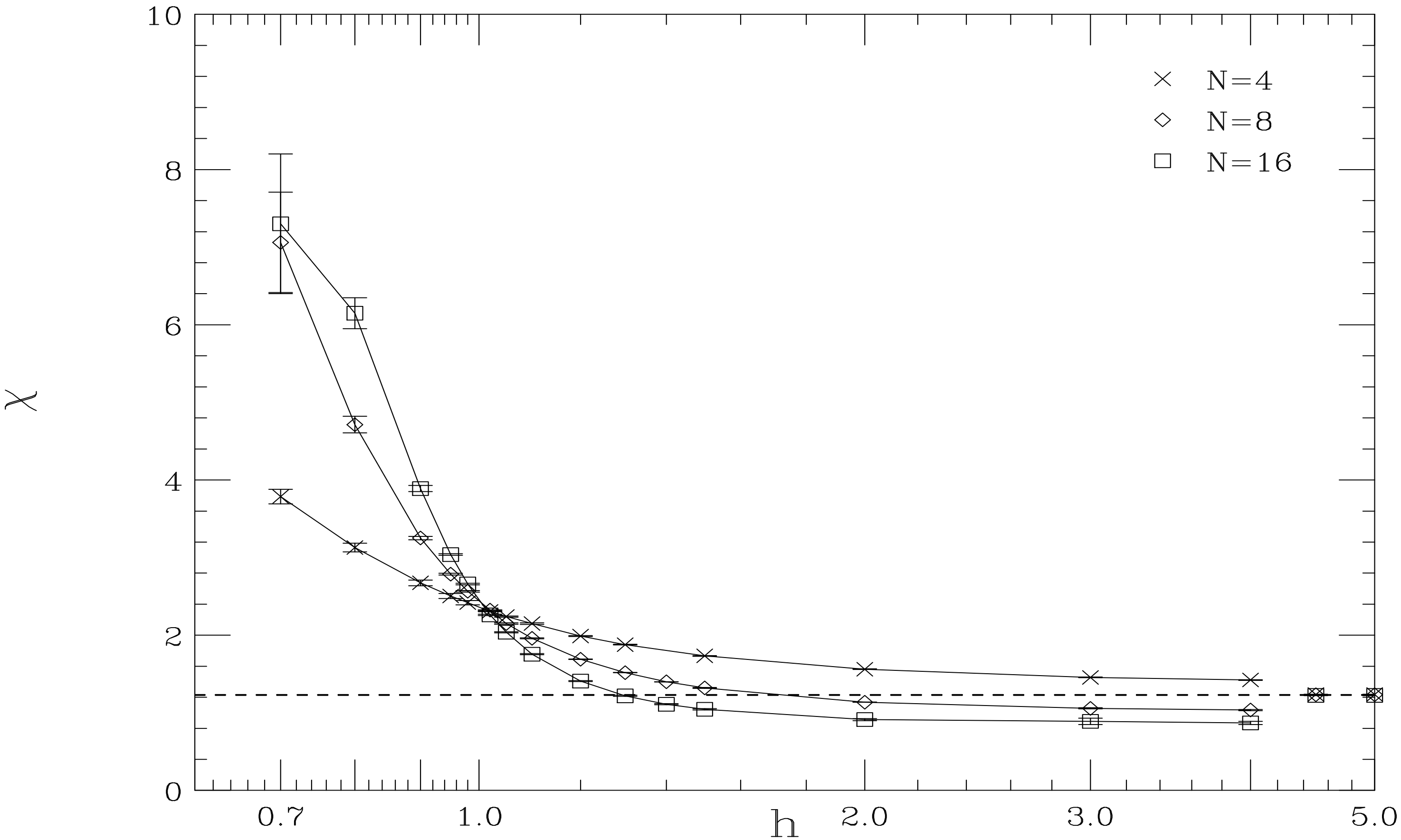
$$\begin{aligned} S &= S_{light} + S_{mirror} \\ S_{light} &= \left(\bar{\psi}_+, D_1\psi_+\right) + \left(\bar{\chi}_-, D_0\chi_-\right) \\ S_{mirror} &= \left(\bar{\psi}_-, D_1\psi_-\right) + \left(\bar{\chi}_+, D_0\chi_+\right) \\ &+ y\left\{\left(\bar{\psi}_-, \phi^*\chi_+\right) + \left(\bar{\chi}_+, \phi\psi_-\right) + h\left[\left(\psi_-^T, \phi\gamma_2\chi_+\right) - \left(\bar{\chi}_+, \gamma_2\phi^*\bar{\psi}_-^T\right)\right]\right\} \end{aligned}$$

$$S_\kappa = \frac{\kappa}{2}\sum_x\sum_{\hat{\mu}}\left[2 - \left(\phi_x^*\, U_{x,x+\hat{\mu}}\, \phi_{x+\hat{\mu}} + \mathrm{h.c.}\right)\right]$$

all simulations done at infinite y (economics reasons!) i.e. dropping mirror kinetic terms

2d Yukawa-unitary Higgs model with Ginsparg-Wilson fermions,

show one plot only: scalar susceptibility at infinite y, as function of h, kappa=0.1
(~ inverse “mass squared” of scalar in lattice units)




Joel Giedt, EP (2007)

- also measured other order parameters:
Binder cumulant, fermion composite -”Dirac”and “Majorana”- susceptibilities, and vortex density - all show similar behavior as a function of h, no indication of long-range correlations for h > 1

***- strong coupling symmetric phase exists***

strong coupling symmetric phase exists also in at least one 4d model with exactly chiral fermions:

# SU(2)_L x SU(2)_R chirally invariant Yukawa-Higgs model with GW fermions

P. Gerhold, K. Jansen (2007)

different motivation..., single Yukawa coupling- only Dirac, no Majorana

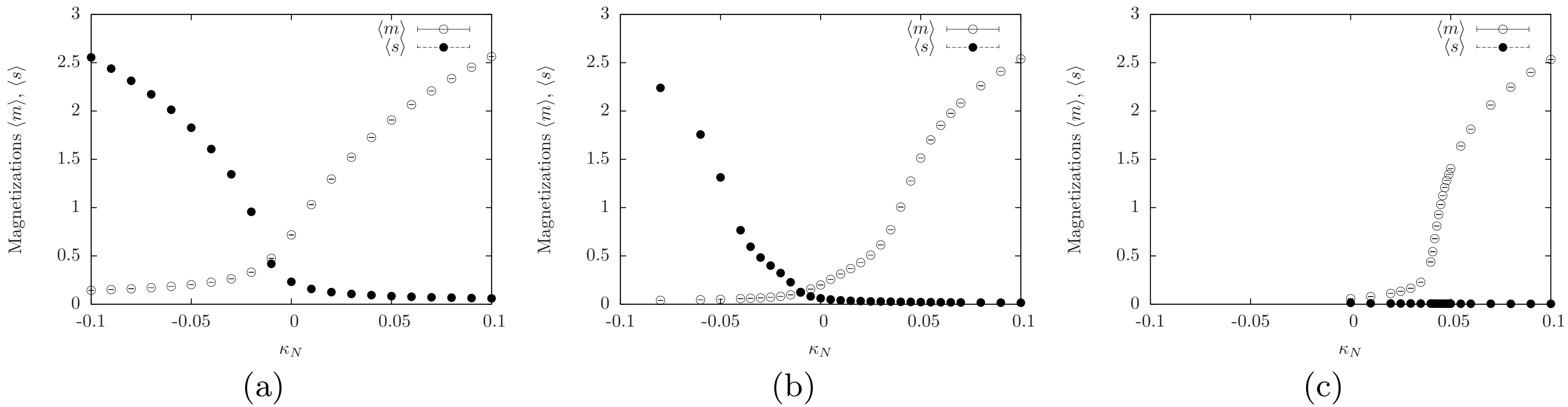



FIG. 8: The behaviour of the average magnetization $\langle m \rangle$ and staggered magnetization $\langle s \rangle$ as a function of $\kappa_N$ on a $4^4$- (a), $8^4$- (b) and $16^4$-lattice (c). In the plots we have chosen $\tilde{y}_N = 30$, $\tilde{\lambda}_N = 0.1$ and $N_f = 2$.

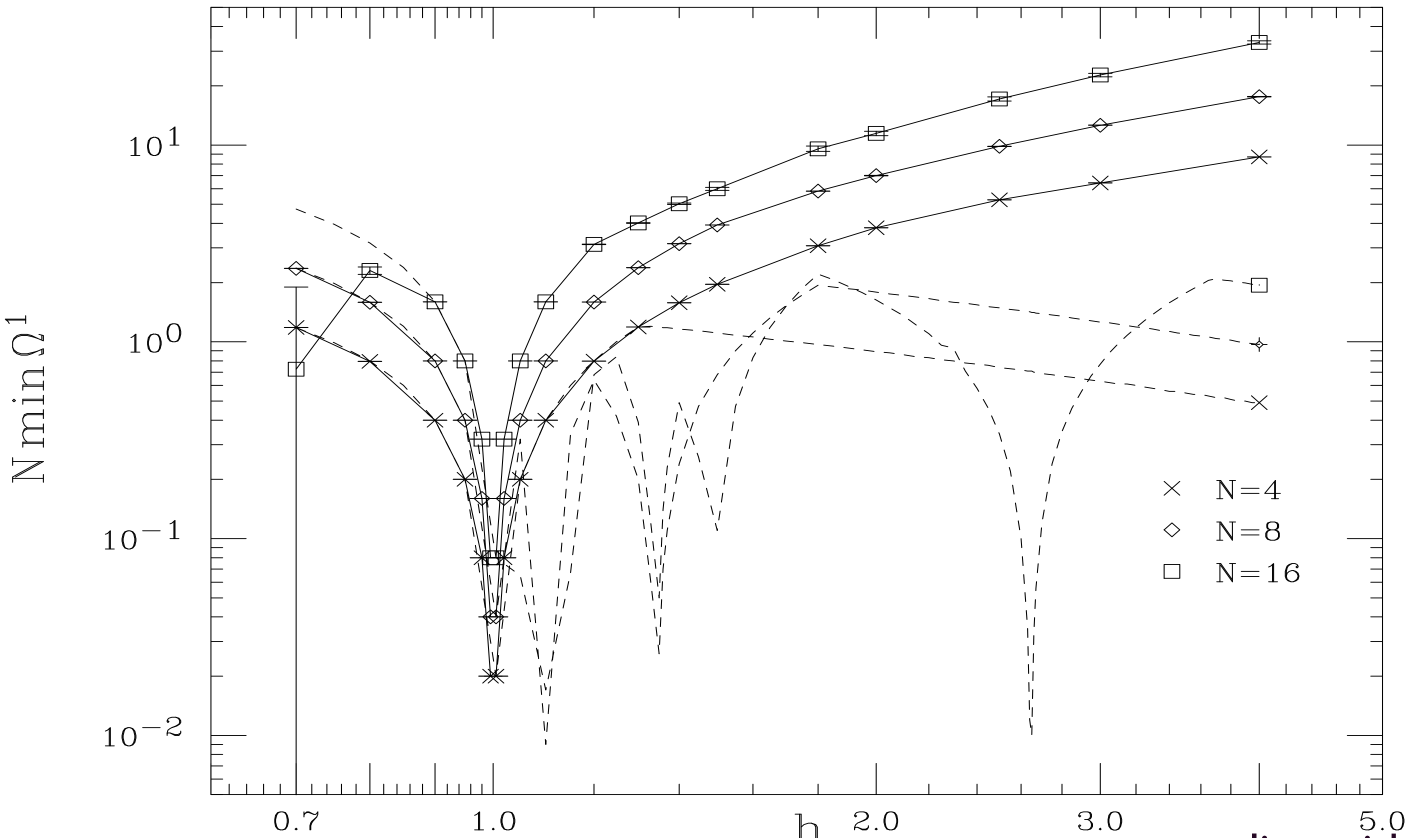
# are mirrors heavy?

Joel Giedt, EP (2007)

2d Yukawa-unitary Higgs model with Ginsparg-Wilson fermions

lowest, as function of momentum, inverse eigenvalue of the L-R components of mirror fermion Green's function (zero means massless pole)

dotted lines: "broken" (spin-wave) phase values, where perturbation theory good, check that agrees with MC




in units of inverse size of system L

scaling with y, N, L: $\sim y\, N^x\ L^{-1}$

(values of exponent x depend weakly on kappa, h, but x is usually about 1)

***- mirrors look heavy -***

***but more to come -*** have we decoupled an anomalous representation without a trace?

# what happens if one tries to decouple an anomalous mirror representation?

Yanwen Shang, EP, arXiv:0706.1043[hep-th] + in progress

I discussed split of action into "light" and "mirror" components,
using GW to split kinetic term and defining Yukawa/4-fermi only in "mirror" terms:

$$\bar{\Psi}_q D_q \Psi_q = \bar{\Psi}_{q,+} D_q \Psi_{q,+} + \bar{\Psi}_{q,-} D_q \Psi_{q,-}$$

but does measure split similarly?

$$\Psi_x = \sum_i c_i^+ w_i(x) + c_i^- u_i(x) \longleftarrow \gamma_5$$

eigenvectors of

$$\bar{\Psi}_x = \sum_i \bar{c}_i^+ t_i^\dagger(x) + \bar{c}_i^- v_i^\dagger(x) \longleftarrow \hat{\gamma}_5$$

complete set of eigenvectors of modified $\hat{\gamma}_5$ - t, v depend on gauge field

split measure as:

$$\prod_x \mathrm{d}\Psi_x \mathrm{d}\bar{\Psi}_x = \frac{1}{J} \prod_i \mathrm{d}c_i^+ \mathrm{d}c_i^- \mathrm{d}\bar{c}_i^+ \mathrm{d}\bar{c}_i^-$$

$$J = \det||w_i(x) u_j(x)|| \, \det||v_i^\dagger(x) t_j^\dagger(x)||$$

both mirror and light partition functions now depend on gauge field through:

- operators depending on gauge field

- eigenvectors t,v depend on gauge field (and action depends on t,v now)

remarkably, the change of the basis vectors factorizes in the change of Z,
no matter what the action -

Shang, EP, 2007: proved "splitting theorem" for a ***general variation***
of a ***general chiral lattice action*** - e.g. our "mirror" w/ Yukawa:

$$\delta \log Z[U] = \sum_i (\delta t_i^\dagger \cdot t_i) + \left\langle \frac{\delta S}{\delta O} \, \delta O \right\rangle$$

---

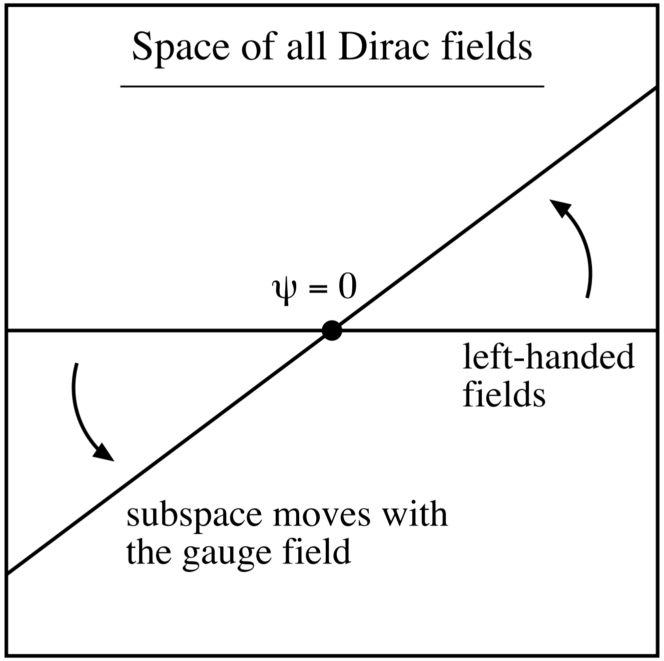
why we care?

because most of the "action" regarding anomalies happens in the $\hat{\gamma}_5$ -eigenvectors and in their A-dependence!

- change in direction perpendicular to eigenspace completely determined by solving perturbatively for change of eigenvector due to small changes of "parameter" A:

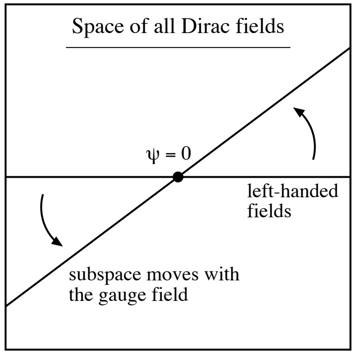
$$\hat{\gamma}_5 \, t_i = -t_i$$

- change in parallel direction - as usual in perturbation theory - not determined (phase of Z); but not always completely arbitrary (Berry phase!)




since Z(chiral) defined via t,v - to what extent is Z(chiral) arbitrary? where/when does this show up?

## Neuberger, 1998; Luescher, 1999:




in anomalous case, sum of "Berry curvatures" for the "eigenstates" $t_i$ of the "Hamiltonian" $\hat{\gamma}_5$ (as a function of gauge bckgd) integrates to an integer over some closed 2-manifolds in gauge field space

hence, no smooth, wrt gauge field, choice of $t_i$ exists (more precisely, of the corresponding Berry connection) - so, partition function is not smooth wrt A for an anomalous representation

similar to continuum topological classification - Alvarez-Gaume, Ginsparg, 1985:
consider gauge loop in gauge connection space

$$A^\theta = g^{-1}(\theta) A g(\theta) + g^{-1}(\theta)\, \mathrm{d}_x g(\theta) \qquad g(\theta, x): \quad S^1 \times S^{2n} \to G \quad (\pi_5(G))$$

change of phase w of chiral determinant along loop = winding number $n = \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\theta \, \frac{\partial w}{\partial \theta}(\theta, A)$

such loops with nonzero n exist iff a non-contractible two-sphere in gauge orbit space

$$\pi_5(G) = \pi_2(\mathcal{A}/\mathcal{G}) \quad \text{if } \pi_1(G) = 0$$

---

$$\delta \log Z[U] = \sum_i (\delta t_i^\dagger \cdot t_i) + \left\langle \frac{\delta S}{\delta O}\, \delta O \right\rangle$$

***since change of ImLog Z due to change of A largely controlled by eigenvectors, our "splitting theorem" encodes, on the lattice, the fact that anomalies do not depend on the action***

furthermore, using our splitting theorem, combined with Neuberger/Luescher results, we showed that:

- in anomaly-free mirror case, mirror partition function is a smooth function of A (in finite volume only; use "splitting theorem" recursively to show smoothness to all orders)

- in anomalous mirror case, mirror partition function has singularities (a trivial consequence of Neuberger/Luescher)

now, our simulations used precisely this singular mirror partition function, defined via eigenvectors, which are discontinuous at A=0; expect different (from A=0) spectrum once path integral over A done

in regard to this "would-be-anomalous" model, current work in two directions:

- simulations with gauge fields - Joel Giedt, in progress...

- one can still learn a lot (e.g. unitarity!) by studying polarization operator in mirror, as well as higher correlation functions, around A=0, in progress...

anomaly-free models a lot more interesting (but expensive!)
singular "light"-"mirror" splits do not afflict them

but it is still good (and cheaper!) to understand precisely what happens in anomalous case, at least for purely theoretical reasons
(and how/if problems are resolved in anomaly-free case)

this talk was

NOT about LHC physics via strong chiral gauge dynamics

I did not discuss a potential theory of the world

RATHER, I told you where the lattice chiral gauge theory problem is at, and what attempts are being made at improvement and progress

I HOPE to have convinced you that it is an interesting, theoretically appealing problem, fun to think about...

...and that doing this may even turn out to be useful!

many tools come together - some foreign to us before - both theoretical and "experimental"

IN SHORT:

the "**decoupling of mirror fermions via strong-coupling symmetric phases**" idea, combined with "**exact lattice chirality**" leads to a proposed formulation of chiral lattice gauge theories which is:

a.) exactly gauge invariant

b.) has explicit definition of path integral action and measure

c.) has the correct - anomalous or not - Ward identities of the continuum target theory

*but*

d.) requires (more*) numerical work to study

*and*

e.) ***we have not seen reasons to give up, yet!***

*a lot more...