Lattice chirality: a mission (im)possible?

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hep-lat/0605003 PRD74(2006)08528

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hep-lat/0701004 JHEP0710, 076
& work in progress

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arXiv:0706.1043 [hep-th] JHEP0708, 081
arXiv:0801.0587 [hep-lat] IJMPA23(2008)4545 - explanatory /reply to misguided criticism/
arXiv:0901.3402 [hep-lat] JHEP0903, 103

arXiv:1003.5896 [hep-lat] IJMPA(2010)2761- review article (1st half nontechnical)

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"Without a proper lattice formulation of a chiral gauge theory, it is unclear whether such models make any sense as fundamental field theories."

M. Creutz (2004)

"If a solution to putting chiral gauge theories on the lattice proves to be a complicated and not especially enlightening enterprise, then it probably is not worth the effort (unless the LHC finds evidence for a strongly coupled chiral gauge theory!)."

D.B. Kaplan (2009)

I simply find this a fun problem to spend some time thinking about

GOAL

NOT about LHC physics via strong chiral gauge dynamics
(hence, won't tell you which chiral gauge theory breaks EW symmetry and generates fermion masses with small S-parameter and no FCNCs...)

RATHER, I'd like to tell you where the lattice chiral gauge theory problem is at, and what are our attempts at improvement and progress

HOPING to convince you that it is a theoretically appealing problem, fun to think about ...and that doing this may even turn out to be useful, someday...

(many tools come together - both theoretical and "experimental")

OUTLINE:

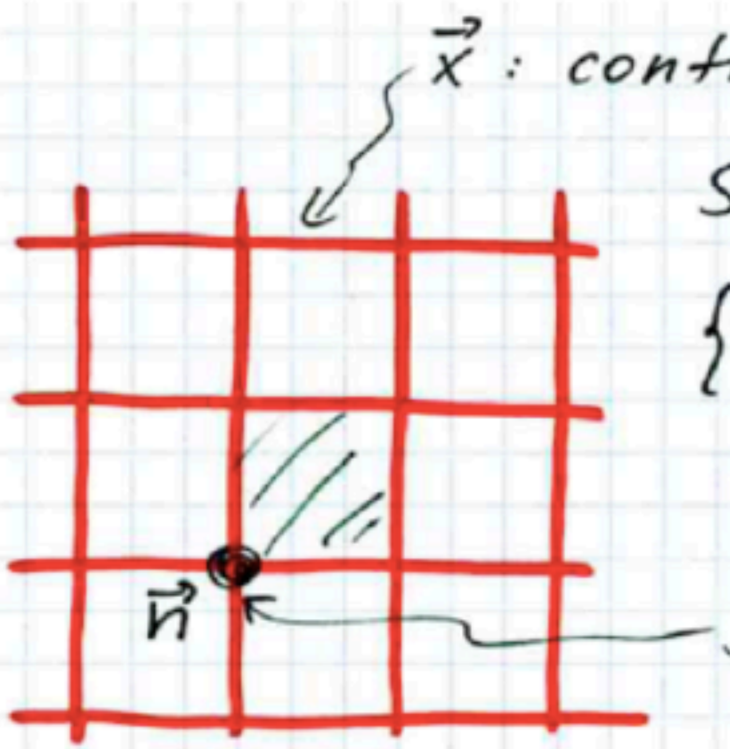
reminder/review or lightning intro/ of global chiral symmetry in vectorlike lattice gauge theories via Ginsparg-Wilson fermions

why do chiral gauge theories present a challenge?

what we are proposing, how is it different from the past, where does it stand, and where is it going?

Ginsparg-Wilson relation, its solution, and consequences:

[GW] $\{D, \gamma_5\} = a D \gamma_5 D$ *Ginsparg & Wilson, 1982: "A remnant of chiral symmetry on the lattice"*



$$S[\Psi] = \int d^d x \, \bar{\Psi} D \Psi(x)$$

$$\{D, \gamma_5\} = 0.$$

$$\chi_{\vec{n}} \equiv \frac{1}{a^d} \int_{\vec{n}} d^d x \, \Psi(x)$$

"block-spin" action for average lattice fermions is then

$$S[\chi] = \sum_{n,n'} \bar{\chi}_n D^{nn'} \chi_n$$

with lattice D obeying "GW relation"

$$\{D, \gamma_5\} = a D \gamma_5 D$$

Ginsparg-Wilson relation, its solution, and consequences:

[GW] $\{D, \gamma_5\} = a\, D \gamma_5 D$ *Ginsparg & Wilson, 1982: "A remnant of chiral symmetry on the lattice"*

Being only formally defined, GW was forgotten until ressurected in 1997 by *Neuberger* and by *Hasenfratz, Laliena, Niedermayer* after a fascinating development, worth a separate talk...

... Callan-Harvey, Kaplan, Narayanan-Neuberger, Neuberger, P.Hasenfratz-Laliena-Niedermayer, Neuberger, Luescher

For us, only imporant that an explicit form of a lattice D obeying GW exists!

Everything below - but explicit form of D - known to GW in1982 (this notation due to Luescher, 1998).

Given such D, define: $\hat{\gamma}_5 = (1 - aD)\gamma_5$

Then, GW is equivalent to: $\hat{\gamma}_5^2 = 1$ or $\hat{\gamma}_5 D = -D\gamma_5$

and the lattice action $S = \sum_{x\,y} \bar{\psi}_x D_{xy} \psi_y$ of an exactly massless Dirac fermion without doublers

has an exact chiral symmetry for any \`N\` and \`a\`

$$\psi \to e^{i\alpha\gamma_5}\psi \qquad \bar{\psi} \to \bar{\psi}\, e^{i\alpha\hat{\gamma}_5}$$

really, we have $\bar{\psi}_x \to \sum_{x'} \bar{\psi}_{x'} \left(e^{i\alpha\hat{\gamma}_5}\right)_{x'x}$ with $\hat{\gamma}_{5\,x'x} \sim e^{-\frac{|x'-x|}{a}}$

not the usual chiral symmetry, but mode-dependent, reducing to continuum at E <<1/a

deformed symmetry avoids Nielsen-Ninomiya theorem, giving massless fermions with chiral symmetry & no doublers

Ginsparg-Wilson relation, its solution, and consequences:

further,

$$\psi \to e^{i\alpha_\pm P_\pm}\,\psi$$

$$\bar\psi \to \bar\psi\, e^{-i\alpha_\pm \hat{P}_\mp}$$

global L and R symmetries of action $U(1)_+ \times U(1)_-$

field dependence of transformation leads to nontrivial Jacobian

$$1 \mp \frac{\alpha_\pm}{2}\,\mathrm{tr}(\gamma_5 D)$$

Jacobian vanishes for vector U(1), where both + and - done with same parameter

then properties of D are useful to (easily, really) to show "index theorem in QCD with finite cutoff"

$$-\frac{1}{2}\,\mathrm{tr}(\gamma_5 D) = n_+^0 - n_-^0$$

it will be important for us that exact chirality allows to introduce chiral components for each field, using appropriate projectors:

$$\psi_\pm \equiv P_\pm \psi$$

$$\bar\psi_\pm = \bar\psi\,\hat{P}_\mp$$

moral: exact lattice chiral symmetry (not usual one for all modes!), exact (including anomalous) Ward identities, axial charge violation, ...

In vectorlike theories - big success!

What is so different about chiral gauge theories?

Ginsparg-Wilson relation, its solution, and consequences:

We just learned that the Dirac fermion splits into L and R (+ or -) at finite 'a', so why not just integrate over those components in the path integral and be done? ...after all, in anomaly-free case one does not really expect a problem...

In other words, why not define the formal continuum expression

$$Z = \int d\psi_- d\bar{\psi}_- e^{(\bar{\psi}\cdot D[A]\cdot\psi)}$$

for a chiral gauge theory partition function

recall these vectors are A-dependent

by choosing chirality eigenvectors, e.g.:

$$\gamma_5 v_i = v_i \quad , \qquad \gamma_5 t_i = -t_i$$
$$\hat{\gamma}_5 u_i = -u_i \quad , \qquad \hat{\gamma}_5 w_i = w_i$$

introducing the chiral components of the spinors: $\psi_- = \sum_i \alpha_-^i t_i \; , \quad \overline{\psi}_- = \sum_i \overline{\alpha}_-^i w_i^\dagger[A].$

and defining Z as an integral over $(\alpha_-^i, \overline{\alpha}_-^i)$,

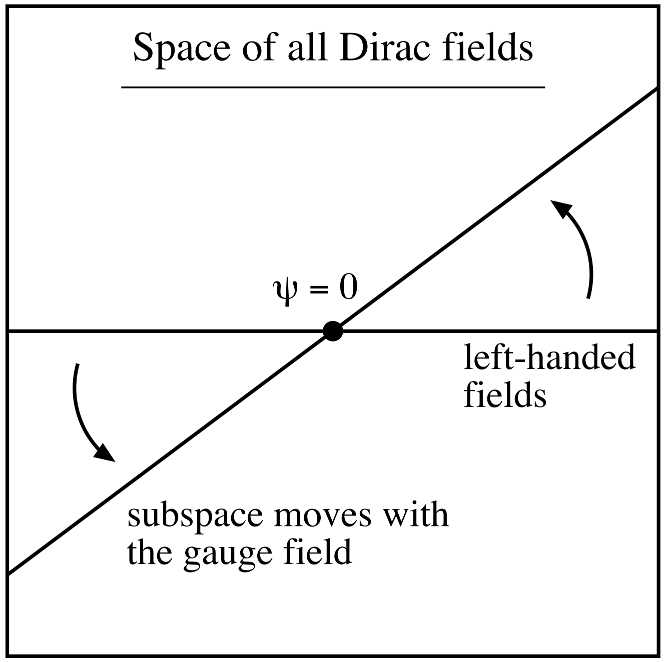
$$Z = \int d\alpha_- d\bar{\alpha}_- e^{\bar{\alpha}_-^i \alpha_-^j (w_i^\dagger[A]\cdot D[A]\cdot t_j)} = \det_{ij}\left(w_i^\dagger[A]\cdot D[A]\cdot t_j\right)$$

As I will now explain, the source of difficulty is the A-dependence of the chirality eigenvectors, which makes the chiral partition function eigenvector dependent. The phase ambiguity of choosing an eigenvector basis becomes A-dependent and, if not "treated," violates gauge invariance, even in anomaly-free models.

In contrast: the vectorlike Z is not eigenvector-dependent: one can define it simply as an integral over the x-values of the spinors, as no projection is needed.

Difficulties for chiral lattice gauge theories with GW fermions:

Let's try finding chiral Z[A] by starting with Z[some A_0, e.g.=0] and integrating small changes...



As A (gauge background) varies, the space of left-handed fields varies. Z changes with A: Z depends on eigenvectors AND operators (O=say D, or functions thereof):

$$\delta \log Z[A] = \sum_i (\delta w_i^\dagger[A] \cdot w_i[A]) + \langle \frac{\delta S}{\delta O} \delta O \rangle$$

factorization: "splitting theorem" for general chiral actions (Shang, EP 2007)
not only for bilinear (Neuberger, Luescher late 1990s)

We can find how w[A] change with A by solving:

$$\hat{\gamma}_5[A] w_i[A] = w_i[A]$$

perturbatively for small variations of "parameter" A, thinking of $\hat{\gamma}_5$ as Hamiltonian and w_i as eigenvectors.

Now, recall QM perturbation theory. Change of eigenvector in direction perpendicular to unperturbed eigenspace completely determined by solving perturbatively for change of eigenvector due to small changes of the Hamiltonian.

Change in parallel direction - a phase - is undetermined & usually ignored in QM, unless it can't be (e.g., Berry's phase...). Note that this undetermined phase precisely gives the change of Z due to eigenvector, as per "splitting theorem" above, which is a pure phase! This phase of Z can not be chosen arbitrary: the "Berry connection" = $\sum (\delta w_i^\dagger[A] \cdot w_i[A])$ has a "curvature", which is a known gauge invariant functional of A.

Difficulties for chiral lattice gauge theories with GW fermions:

Again, since $\delta \log Z[A] = \sum_i (\delta w_i^\dagger[A] \cdot w_i[A]) + \langle \frac{\delta S}{\delta O} \delta O \rangle$, aka "measure current"

arg(Z) has a contribution from the eigenvectors (a "Berry connection") whose Berry curvature is nonzero and depends on A. Hence this contribution to arg(Z) must be A-dependent. If not, partition function is not gauge invariant, even in anomaly free case.

In anomalous case -Neuberger, 1998- the Berry curvature integrates to a nonzero integer over some closed 2-cycles in field space, hence no nonsingular "Berry connection" can be found - and no nonsingular Z[A] exists then (e.g., simple examples in Shang, EP, 2007).

Luescher later (1999-00) proved a "reconstruction theorem:" ***for anomaly free U(1) theories no other obstructions exist and a "Berry connection" exists, which renders Z[A] smooth wrt A, gauge invariant, and (exponentially) local.***

No explicit form for "Berry connection" given -Luescher's is an existence proof- but an algorithm for constructing phase for any given A can be devised for U(1) and SU(2)xU(1) anomaly free models (Kadoh, Kikukawa 2008; has not been yet (2010) implemented, even in 2d).

Finally, on the "existence proof frontier," the proof has not been generalized to nonabelian case. Poses rather formidable mathematical problem...
(e.g., topological classification of nonabelian admissible lattice fields, exact form of nonabelian anomaly on the lattice at finite 'a', 'N' not known, but needed to find Berry connection nonperturbatively...)

- end of the intro/lightning review/ -

what we are proposing, how is it different from the past?

We have proposed a way to avoid this hard math AND provide an explicit definition of Z, with no need to find and fix a "Berry current".

Question:
since defining Z for a chiral theory is so hard, why not start with a vectorlike theory?

Ask if it is possible to begin with, e.g.:

SU(5) with 5^* 5
10 10^* all Weyl, L

names: "light" "mirror"

and then, "deform" the theory in such a way that
- mirrors decouple from the low-energy spectrum
- the gauge symmetry remains unbroken

and, at low energies, get the "quintessential" example of a 4d chiral gauge theory ?

- a normal continuum field theorist would say: no!
- a string theorist might say: may be
e.g., if one allows the liberty to think of orbifolding as decoupling of states

what we are proposing, how is it different from the past?

Imagine "deforming" the theory by adding four-Fermi interactions that act only on the mirror fields and taking them strong...

$$
SU(5) \quad \underbrace{\begin{array}{c} 5^* \\ 10 \\ 1 \end{array}}_{\text{"light"}} \quad \underbrace{\begin{array}{c} 5 \\ 10^* \\ 1 \end{array}}_{\text{"mirror"}}
$$

$$g_1 \quad 10^* - 5 - 5 - 1$$

$$g_2 \quad 10^* - 10^* - 10^* - 5$$

desired outcome (fantasy picture):

strong interactions bind, or "confine", mirrors into vectorlike or singlet composites; these can gain mass without breaking SU(5) gauge symmetry

What makes one think this is even remotely plausible?

strong-coupling symmetric phases exist in lattice 4-Fermi or Yukawa models

many old refs, I learned from Eichten and Preskill, 1986 [E-P]

Let's study a toy example (invented to fit on transparency):

what we are proposing, how is it different from the past?

SU(4) "chiral" symmetry (the one to be gauged)

$$H_{4\psi} = \sum_x g\left(\psi_a \psi_b \psi_c \psi_d \, \epsilon^{abcd} + hc\right)$$

space lattice only (any dimension); canonical anticommutation relations:

$$\{\psi_a(x), \psi_b^+(y)\} = \delta_{ab}\,\delta_{xy}$$

at g>>1 in lattice units, hopping is negligible:

$$H = \underbrace{\sum_x H_{0\,x}}_{\text{4-fermi}} + \underbrace{H_1}_{\text{hopping} \sim \psi^+ \vec{\alpha}\cdot\vec{\nabla}\,\psi}$$

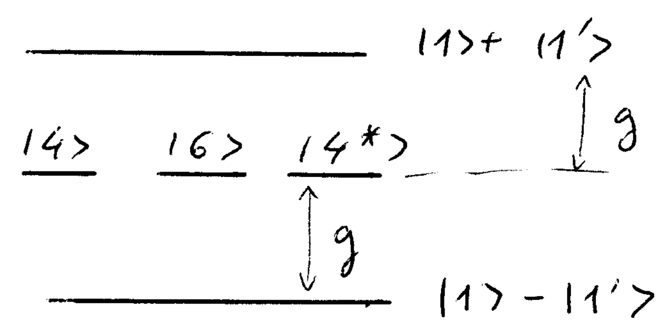
to leading order, at every site the same simple 4-fermion QM problem, rename: $\psi_{a\,x} \to a_a$, $\psi^+_{b\,x} \to a^+_b$

$$H_0 = g\left(a_a a_b a_c a_d + a_a^+ a_b^+ a_c^+ a_d^+\right)\epsilon^{abcd}$$

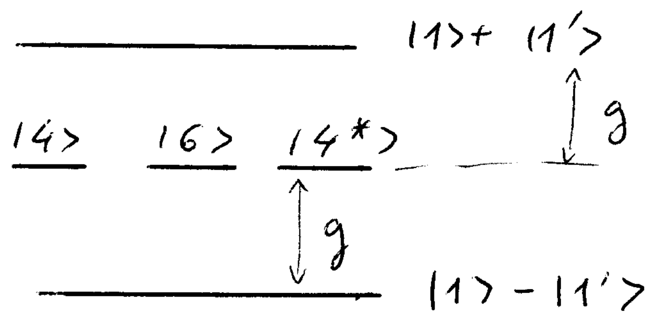
2^4 states	SU(4)	F
$\vert 0\rangle$	$\vert 1\rangle$	0
$a_a^+\vert 0\rangle$	$\vert 4\rangle$	1
$a_a^+ a_b^+\vert 0\rangle$	$\vert 6\rangle$	2
$a_a^+ a_b^+ a_c^+\vert 0\rangle$	$\vert 4^*\rangle$	3
$a_1^+ a_2^+ a_3^+ a_4^+\vert 0\rangle$	$\vert 1'\rangle$	4

$\langle 1'|H_0|1\rangle \sim g$ ONLY NONZERO

$\langle 4|H_0|1\rangle = \langle 4|H_0|6\rangle = \ldots = 0$



what we are proposing, how is it different from the past?



in the infinite-g limit,
the lattice theory ground
state is unique and an SU(4) - "chiral" - singlet
with a mass gap = g in lattice units

infinite-g ground state unique and SU(4) "chiral" singlet

hence the name: "strong-coupling symmetric phase"

at first order in 1/g, hopping turns on, site-localized states form bands and propagate; they are heavier than the lattice cutoff, mass ~ 1/(g a)

the 1/g expansion has finite radius of convergence, hence this story represents the true ground state of theory, for g sufficiently large

what we are proposing, how is it different from the past?

homework: repeat toy model study for SU(5) (E-P, 1986, reformulated for our purposes)

For the continuum folks among us,
A FEW GENERAL WORDS ABOUT SUCH PHASES, TRUE WHENEVER THEY EXIST:

- very much like "static limit" of lattice QCD, but infinite mass limit replaced by infinite four-fermi
- large-g phase same in any dimension provided static limit exists (= unique ground state at every site); major differences between dimensions occur at small-g
- like high-T statistical mechanics where disorder always wins: neglect of kinetic terms = uncorrelated fluctuations at neigboring sites, maximum "entropy"

preemptive answers to FAQs:

- we are not interested in a continuum limit of the "mirror" theory - everything "mirror" is cutoff scale (and heavier) and decoupled from IR physics
- gauge field appears only in hopping terms and so contributions of heavy "mirror" sector to gauge field action should be ~ 1/g (recall g is the strong 4-fermi...)

(see fun 2d study of a similar story in gauged model with scalars Shang, EP (2008), designed to answer some misguided criticism...)

Homework: repeat toy model study for SU(5) (E-P, 1986)

what we are proposing, how is it different from the past?

SU(5)		
	5^*	5
	10	10^*
	1	1
	"light"	"mirror"

deformed by

g_1 $\quad 10^* - 5 - 5 - 1$

g_2 $\quad 10^* - 10^* - 10^* - 5$

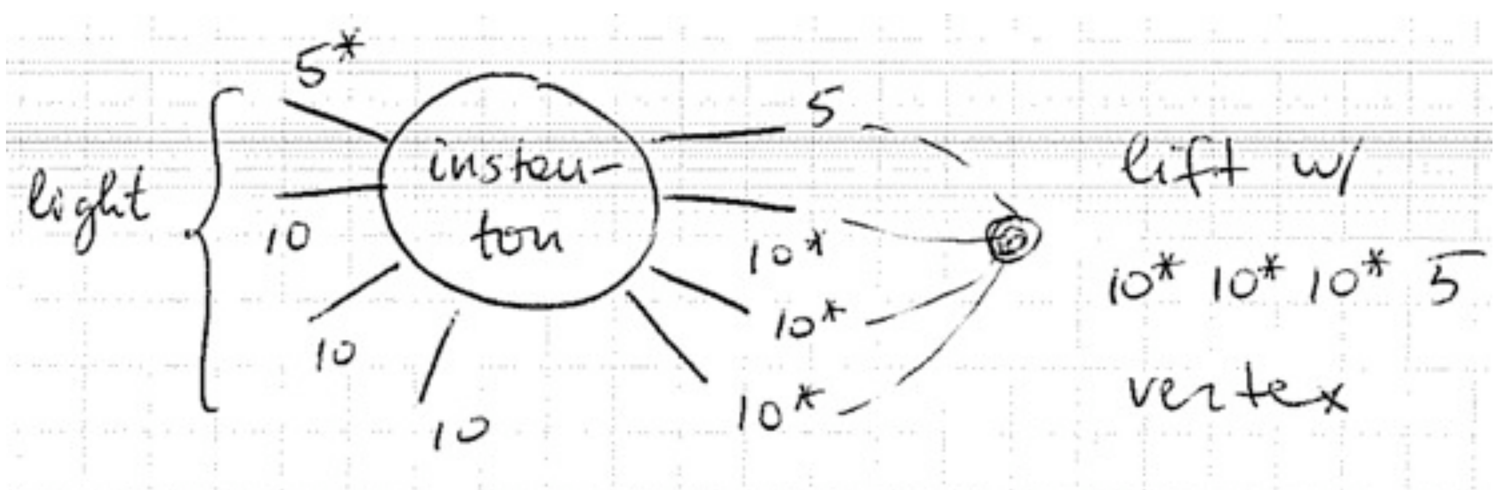
why singlet?

needed to have sensible "static limit" of Euclidean fermion path integral; E-P used Euclidean, not Hamiltonian, strong-couping expansion showing that at infinite-g SU(5) ground state unique and singlet; hence a "strong-coupling symmetric phase"

why two 4-fermi terms?

I. so that static limit exists

II. mirror global symmetries, including anomalous ones, must be broken, or else get extra zero modes in instanton - wrong 't Hooft vertex



III. unbroken mirror global symmetries have 't Hooft anomalies, 't Hooft anomaly matching, valid also here when GW fermions used (Shang, EP 2009) then implies extra light states

zero modes and lifting by g_2-coupling analogy: t-quark decoupling from QCD 't Hooft vertex vs non-decoupling from SU(2) 't Hooft vertex, no matter how heavy, $\Delta B = 3$

the "E-P dream" was, essentially, to use this* phase to decouple the doublers

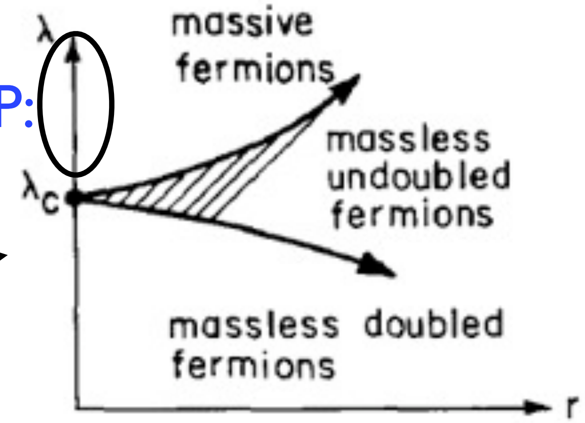
*I am simplifying E-P story - here's their dream phase diagram:

what we are proposing, how is it different from the past?

E. Eichten, J. Preskill / Chiral gauge theories 191

lambda-axis of E-P = my g_1, g_2

region studied by E-P:



"NJL" phase missing, for some reason...

Fig. 3. Phase diagram in the $\lambda - r$ plane, assuming $\lambda_c \neq 0$. Composite fermion states go to threshold along the curves shown. In the shaded region, there is a massless undoubled fermion mode.

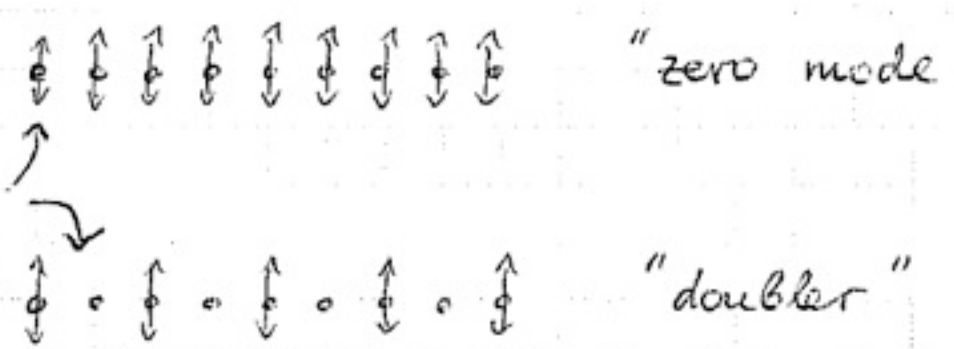
even a two-component Weyl field on the lattice, as E-P used, has opposite chirality massless excitations in it (fermion doubling)

a 1+1dim reminder: spatial lattice hermitean Hamiltonian of a 1-component Weyl fermion

remember (de)construction

$$i \dot{\psi}_i = i(\psi_{i+1} - \psi_{i-1})$$

lattice sites



(symmetry)

to deal with doublers of Weyl lattice fermions, E-P introduced "r"-axis:

- more 4-fermi terms, this time with derivatives in them, must break symmetry between "doubler" and "light" modes

- hope to tune "r" to make light massless, while doublers heavy (were able to only study one region)

what we are proposing, how is it different from the past?

Clearly, this dream was not likely to be easily fulfilled...

Only one further study, by Golterman, Petcher, Rivas, 1993: no proof, by all means, **but in all regions that they could study using 1/N, strong- and weak-coupling expansions in r, lambda both "mirror" and "light" fermions became heavy at strong-4 fermi /"r"/, while at weak 4-fermi, both "mirror" and "light" were massless, i.e. the theory was always vectorlike...**

In hindsight, one expects E-P story to fail in the absence of symmetries: E-P is complicated by the fact that the strong 4-fermi interactions are felt by both "mirror" (here - doubler) and "light" fermions (no separation was known in 1986).

There was no symmetry distinguishing light from heavy modes, needed to protect the light modes and allow the heavy ones to become massive. It was expected to "emerge" at some value of "r" ...which was never found.

Things would be a lot cleaner if the strong interactions only acted on "mirror" modes and if one could separate mirror and light already at finite (a,V)

- have to only deal with lambda-axis of E-P = my g_1, g_2 and avoid tuning r

- *if chiral symmetries could be clearly and unambiguously defined, expect that unbroken exact chiral symmetry, if such a thing existed at finite (a,V), would protect the light fermions*

what we are proposing, how is it different from the past?

Since we now know how define of L and R components of Dirac - not Weyl, like E-P - fermions then, one can ask whether the "E-P dream" be resurrected as well?

We need to only replace "doubler" by "mirror"... Bhattacharya, Martin, EP, 2006

(Turns out, Creutz, Rebbi, Tytgat, Xue, 1996, had made similar proposal using E-P + domain wall - before GW operator and exact chirality. But symmetries become exact only as size becomes infinite, so less "pretty," hence more difficult to study - there has been no follow-up work whatsoever.)

For any lattice vectorlike theory, we know how to define L and R fermions, and can write 4-fermi or Yukawa interactions that only act on the mirror fields.

- measure of Z_vector is explicitly defined by the usual vector theory measure, no phase ambiguity: could simulate "right now"...if infinite power available... but it's probably a good idea to first examine how the parts perform!

to this end, note Z[A] separates into "light" and "mirror" explicitly in any A (separation useful for mirror dynamics studies, as will become clear)

what we are proposing, how is it different from the past?

4-fermi breaking all mirror global symmetries except the one to be gauged

$$Z_{vector}[A] = Z_{light}[A] \times Z_{mirror}[A]$$

$$Z_{light}[A] = \int \Pi \, d\bar{c} \, dc \; e^{\,\bar{c}^k D_{kp}[A] c^p}$$

$$Z_{mirror}[A] = \int \Pi \, d\bar{b} \, db \; e^{\,\bar{b}^k D_{kp}[A] b^p + S_{4fermi}[\bar{b}, b, A]}$$

most importantly, the global symmetries, including anomalous ones, are exactly the ones of the target continuum theory

- there are exact chiral symmetries protecting light modes
- all chiral symmetries acting on mirrors are explicitly broken (none of previous "mirror decoupling" approaches did this)

This gives an elegant gauge invariant formulation of "E-P like" ideas that has all the symmetries of the desired target theory...

...should we be opening the champagne, then?

should we be opening the champagne, then?

not yet - a “few” questions remain to be answered first:

1 with the exp.-local Yukawa/4-fermi mirror interactions, is it still true that a “strong coupling symmetric phase” exists?
are the mirrors heavy?

2 in typical models, there is more than one strong Yukawa/4-fermi interaction - needed to break all classical mirror global symmetries - and there can be a nontrivial phase structure as their ratios change
not necessarily a problem, but an issue to understand

3 what happens if one tries to decouple an anomalous mirror representation?

- 1,2, and 3 can be addressed with background nondynamical gauge fields only
- NEED TO USE simulations: no simple analytic strong-coupling expansion as in original models with non-exactly chiral fermions - beauty has a price...
- however, in this matter of principle, we can stay in 2d at first
- adding dynamical gauge fields brings in a new set of questions, for example:

4 with dynamical gauge fields included, is the long-distance theory unitary?
we have defined a complex Euclidean partition function: different treatment of conjugate mirror fermion variables through the different chiral projectors

5 suppose all checks above are fine - apart from gaining intellectual satisfaction, what can we now learn about strong chiral gauge dynamics?
can we calculate with $T_{simulation} < O(Gy)$?

but we are (slowly) learning:

- with the exp.-local Yukawa/4-fermi mirror interactions, is it still true that a "strong coupling symmetric phase" exists?

yes, in the 2d models studied
Joel Giedt, EP, hep-lat/0701004

yes, in the 4d model studied
P. Gerhold, K. Jansen, arXiv:0707.3849[hep-lat]
(not all symmetries broken, due to different motivation;
unlifted "mirror" zero modes quite easy to predict and spot)

are the mirrors heavy?

- it depends... this talk

but we are (slowly) learning:

2 in typical models, there is more than one strong Yukawa/4-fermi interaction - needed to break all classical mirror global symmetries - and there can be a nontrivial phase structure as their ratios change

not necessarily a problem, but an issue to understand

there is a nontrivial phase structure in the 2d model studied

reaching symmetric phase at strong coupling does not require fine-tuning (a large region in coupling space)

Joel Giedt, EP, hep-lat/0701004

but we are (slowly) learning:

3 what happens if one tries to decouple an anomalous mirror representation?

important to differentiate between options
- massless mirror fermion, Green-Schwarz field, nonunitarity ???

Yanwen Shang, EP, arXiv:0706.1043[hep-th]; 0901.3402[hep-lat]

4 with gauge fields included, is the long-distance theory unitary?

this talk...

we have defined a complex Euclidean partition function: different treatment of conjugate mirror fermion variables through the different chiral projectors

not obvious, but some indications Yanwen Shang, EP, arXiv:0901.3402[hep-lat]

5 suppose all checks above are fine - apart from gaining intellectual satisfaction, what can we now learn about strong chiral gauge dynamics?
can we calculate with $T_{\text{simulation}} < O(Gy)$?

...left for future work

where does it stand?

"simple" case:

massless Schwinger model + singlet massless fermion + strong mirror interaction

will also call it "1-0 model"

$$\begin{aligned} S &= S_{light} + S_{mirror} \\ S_{light} &= \left(\bar{\psi}_+, D_1 \psi_+\right) + \left(\bar{\chi}_-, D_0 \chi_-\right) \\ S_{mirror} &= \left(\bar{\psi}_-, D_1 \psi_-\right) + \left(\bar{\chi}_+, D_0 \chi_+\right) \\ &+ \; y \left\{ \left(\bar{\psi}_-, \phi^* \chi_+\right) + \left(\bar{\chi}_+, \phi \psi_-\right) + h \left[\left(\psi_-^T, \phi \gamma_2 \chi_+\right) - \left(\bar{\chi}_+, \gamma_2 \phi^* \bar{\psi}_-^T\right) \right] \right\} \end{aligned}$$

$$S_\kappa = \frac{\kappa}{2} \sum_x \sum_{\hat{\mu}} \left[2 - \left(\phi_x^* \, U_{x,x+\hat{\mu}} \, \phi_{x+\hat{\mu}} + \text{h.c.} \right)\right] \qquad \phi(x) = e^{i\alpha(x)}$$

mirror theory action

Joel Giedt, EP (2007), studied mirror partition function at A=0;
clearly "light" theory is anomalous so don't expect decoupling
but cheap to study while still has nontrivial dynamics

where does it stand?

$$S_{mirror} = \cancel{-(\overline{\psi}_- \cdot D_1 \cdot \psi_-) - (\overline{\chi}_+ \cdot D_0 \cdot \chi_+)}$$

y=infinity - drop kinetic term

$$+\, y \left\{ (\overline{\psi}_- \cdot \phi^* \cdot \chi_+) + (\overline{\chi}_+ \cdot \phi \cdot \psi_-) + h \left[(\psi_-^T \cdot \phi \gamma_2 \cdot \chi_+) - \left(\overline{\chi}_+ \cdot \gamma_2 \cdot \phi^* \cdot \overline{\psi}_-^T \right) \right] \right\}$$

$$S_\kappa = \frac{\kappa}{2} \sum_x \sum_{\hat\mu} [2 - (\, \phi_x^* \, U_{x,x+\hat\mu} \, \phi_{x+\hat\mu} + \text{h.c.}\,)]$$

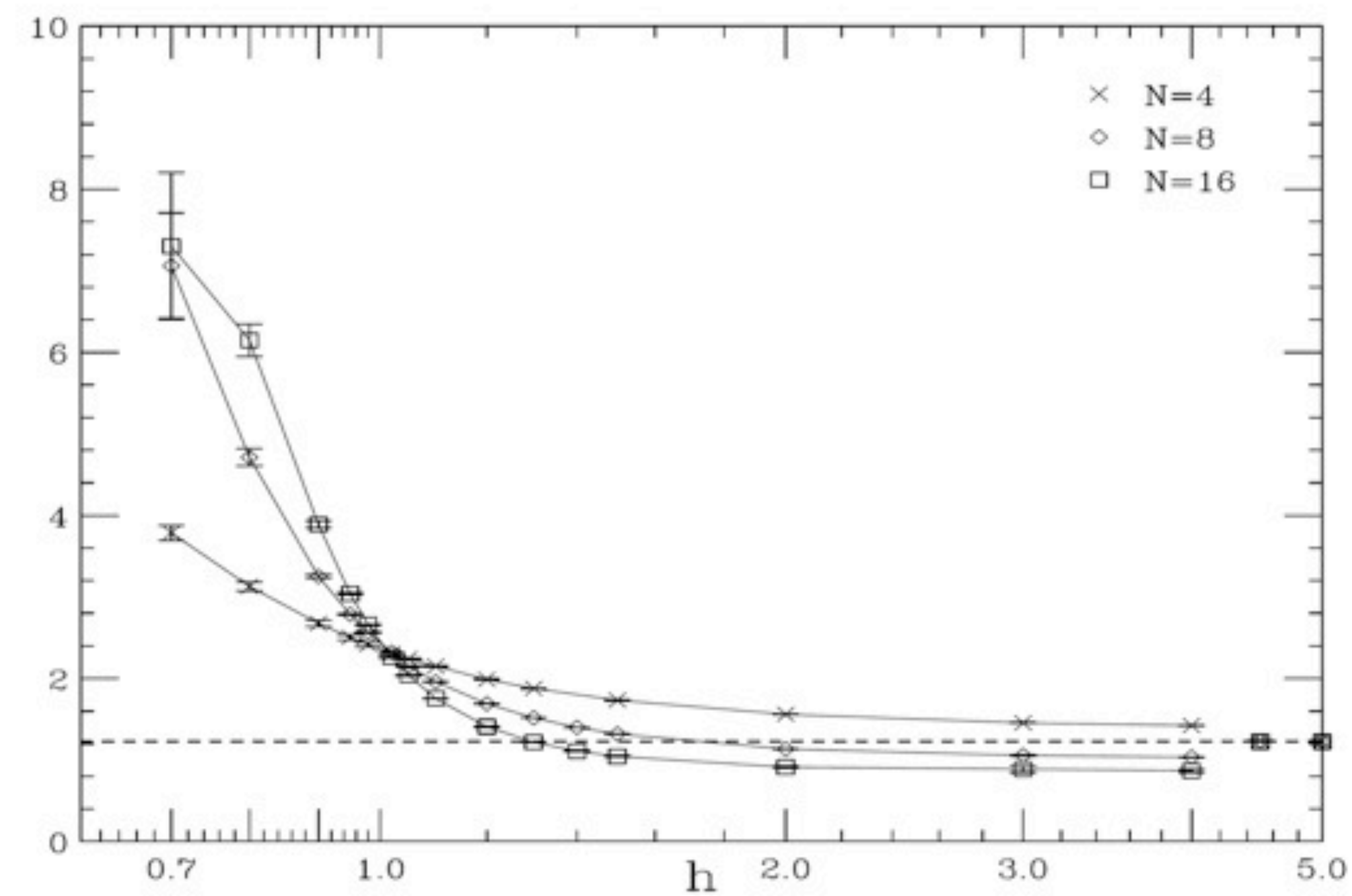
$$\varkappa < \varkappa_c \sim 1$$

Giedt, EP, 2007

$$\phi(x) = e^{i\alpha(x)}$$

$$\chi = \int d^2x \, \langle \phi^*(x) \phi(0) \rangle \sim \frac{1}{m_\phi^2}$$

$$\begin{pmatrix} \text{long} \\ \text{range} \\ \text{order} \end{pmatrix} \sim \int d^2x \, \frac{1}{|x|^{2-\eta}} \sim L^\eta$$



- strong coupling symmetric phase with GW fermions exists, for a range of h

(similar findings of 4d studies of Gerhold and Jansen, different motivation)

where does it stand?

Next, probe charged spectrum by studying polarization operator :

$$\Pi_{\mu\nu}(x,y) \equiv \left. \frac{\delta^2 \ln Z[A]}{\delta A_\mu(x) \delta A_\nu(y)} \right|_{A=0}$$, since Z[A] of 1-0 model gauge invariant: $\nabla^*_\mu \Pi_{\mu\nu} = 0$

we know exactly how lattice Z[A] splits into light + mirror, for arbitrary A:

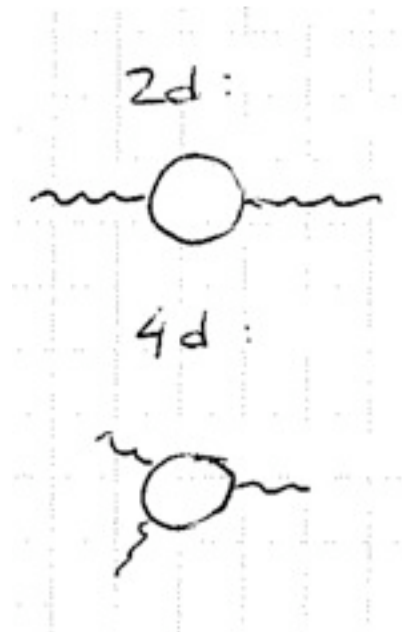
$$\ln Z[A] = \ln Z_+[A] + \ln Z_-[A]$$

and since light + mirror split of Z is locally smooth in A, polarization operator splits, too:

$$\Pi_{\mu\nu}(x,y) = \Pi^+_{\mu\nu}(x,y) + \Pi^-_{\mu\nu}(x,y)$$

but the light theory has a chiral charged fermion, hence its polarization operator is not transverse but gives anomaly of chiral light GW fermion:

$$\nabla^*_\mu \Pi^{+/J}_{\mu\nu} \sim \frac{\delta}{\delta A_\nu}\left(\epsilon_{\alpha\beta} F^{\alpha\beta}\right)$$



but total polarization operator is transverse, so mirror must also be non-transverse, i.e. have opposite anomaly:

$$\nabla^*_\mu \Pi^-_{\mu\nu} = - \nabla^*_\mu \Pi^{+/J}_{\mu\nu}$$

These are very usual considerations in continuum, leading to 't Hooft anomaly matching... It is a remarkable consequence of exact lattice chirality that they can be precisely transcribed, with all i's and pi's (and some extras like N,a + ...) to a lattice. (Could easily do in 4d, but need 3pt function.)

Details are fun and of great interest (to me) but I'll spare you: Shang, EP, 2007,2009 ("splitting theorem"...).

where does it stand?

The moral is, in low-momentum limit, Fourier transform of the imaginary part of the mirror polarization operator obeys - we're in Euclidean, anomaly is in ImLogZ:

$$i q^{\mu} \tilde{\Pi}^{-}_{\mu\nu}(q) = \frac{1}{2\pi} \epsilon_{\nu\lambda} q^{\lambda} + \mathcal{O}(q^2)$$

and, so, Im-part of polarization operator should have some nonlocal contribution

- in a unitary, Lorentz invariant theory, means also real part should be nonlocal

- poles in real part of polarization operator mean massless charged particles, so mirror should have light states

This condition on the mirror dynamics is **exact:**

- independent on the strength of the mirror 4-fermi, Yukawa, etc. couplings - this argument never used explicit form of mirror action, only gauge invariance of full Z!

- true for any volume, lattice spacing ('N', 'a')

- ***analogous to 't Hooft anomaly matching in theories with strong IR dynamics***

  - strong non-gauge mirror dynamics has to comply with it

  - as usual, anomaly matching does not tell us what the pole in the real part is from - a Goldstone boson or massless fermion, and one needs to study the dynamics

where does it stand?

Must better understand how our attempt to decouple one chirality of the Schwinger model in a mirror strong-coupling symmetric phase fails:

- a check on unitarity
  could've imagined (as some did!) a nonlocal Im-part and a local Re-part - recall GW
  not explicitly lattice Hamiltonian

- relatively cheap exercise, if a bit long to set up

- tools developed to express mirror polarization operator in terms of mirror correlators are useful in current work (Giedt et al. studying anomaly-free case)

- hope to learn something about strong mirror dynamics, a la E-P, with GW fermions

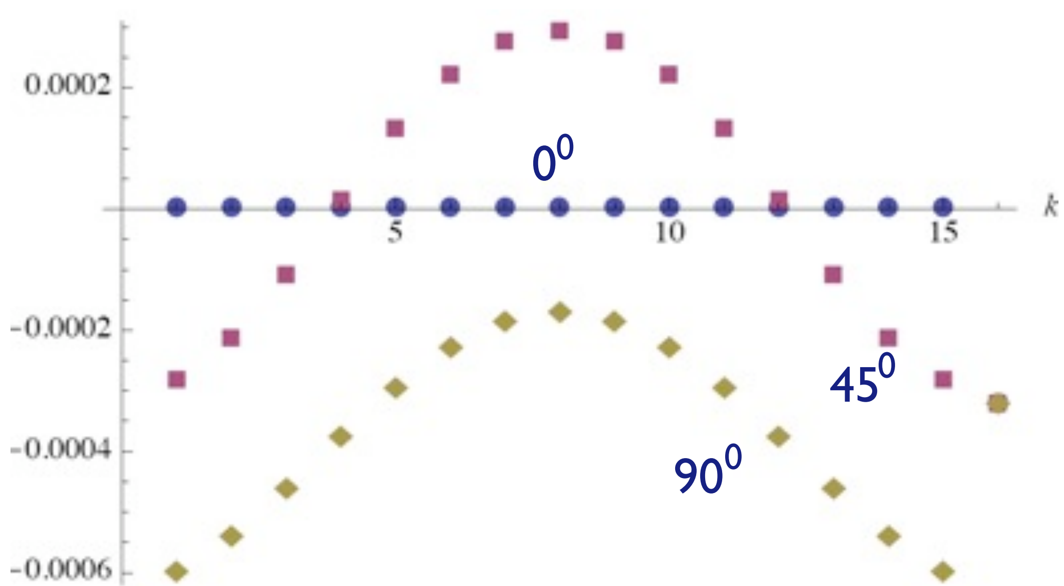
where does it stand?

remember from two slides ago, interested in Re(polarization operator) - tells us about spectrum:

and, so, Im-part of polarization operator should have some nonlocal contribution

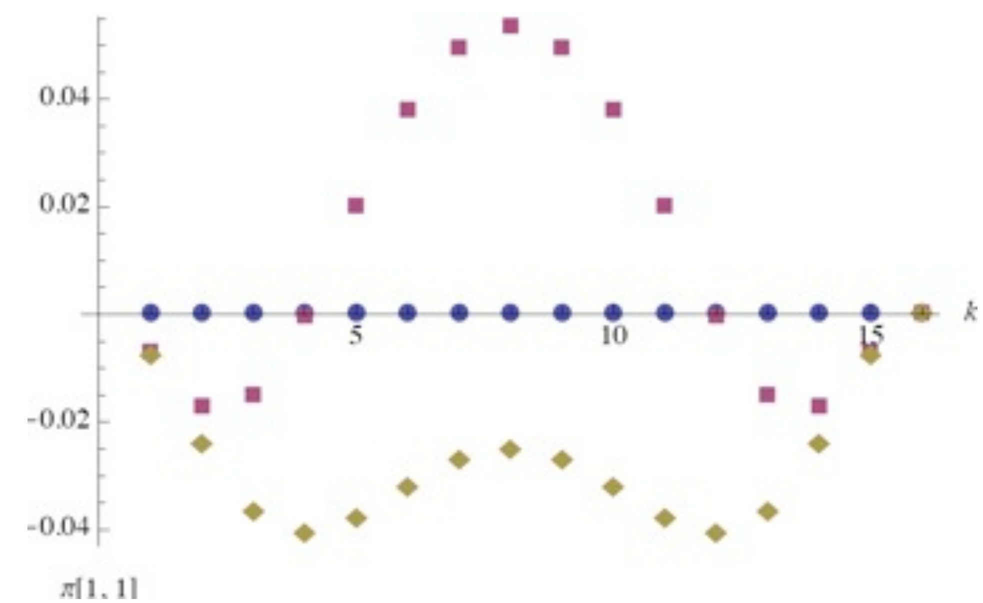
- **in a unitary, Lorentz invariant theory, means also real part should be nonlocal**
- **poles in real part of polarization operator mean massless charged particles, so mirror should have light states**

real part of polarization operator of free chiral GW fermion (= 1/2 vector) $\frac{1}{2}\,\delta_\nu \mathrm{Tr} D^{-1} \delta_\mu D$



in continuum, loop of massless particle in 2d:

$$\Pi_{11} \sim 1 - \frac{k_1^2}{k_1^2 + k_2^2}$$



to compare - loop of cutoff-scale mass particle shows no small-k discontinuity

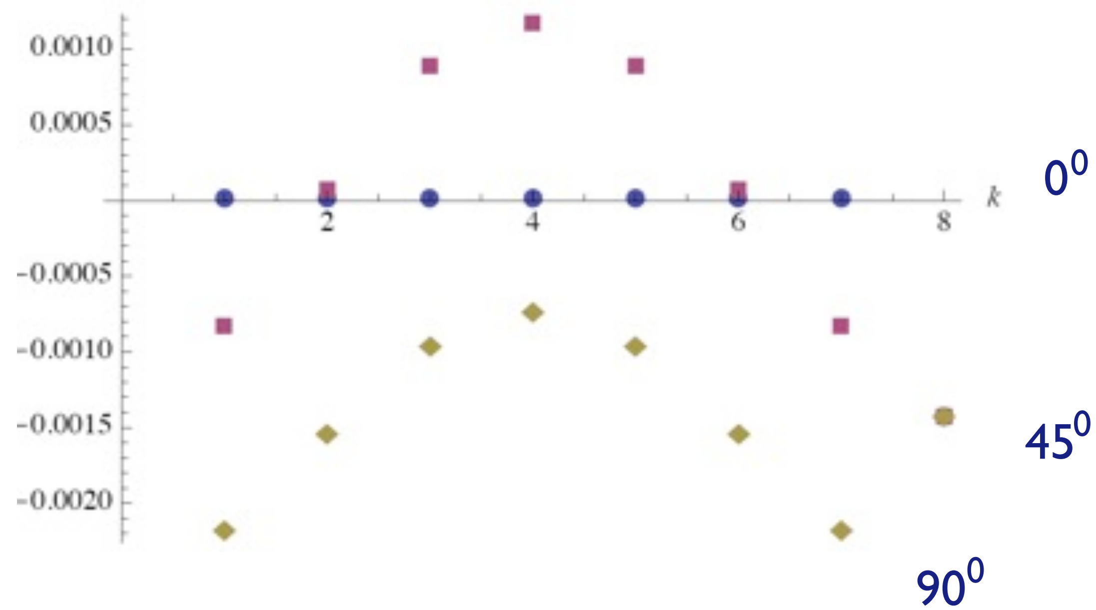
this is not Monte-Carlo but exact sum over loop momenta with Mathematica on 16x16 lattice

where does it stand?

This gives the "light" polarization operator
to compute mirror polarization operator:
must use Monte-Carlo as mirror theory is a strongly-coupled nonlinear system

small-k discontinuity seen even on 8x8 lattice:
(this plot, as well as one from previous page, has wrong absolute normalization)

$$\frac{1}{2}\,\delta_\nu \mathrm{Tr} D^{-1} \delta_\mu D$$



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where does it stand?

to compute mirror polarization operator: use Monte-Carlo, as mirror theory is a strongly-coupled nonlinear system

two steps involved:

1. find expression of polarization operator in terms of mirror correlation functions expressed in terms of variables of integration...
   long and tedious, but now we know how to do it, reasonably fast, for any theory- Shang, EP, 2009

2. use MC to calculate polarization operator...

- use expansions in terms of chiral eigenvectors to define Z(mirror)

- use of "splitting theorem" (crucial!) to find second derivative of LogZ(mirror) wrt gauge field (i.e., polarization operator)

where does it stand?

what comes out?

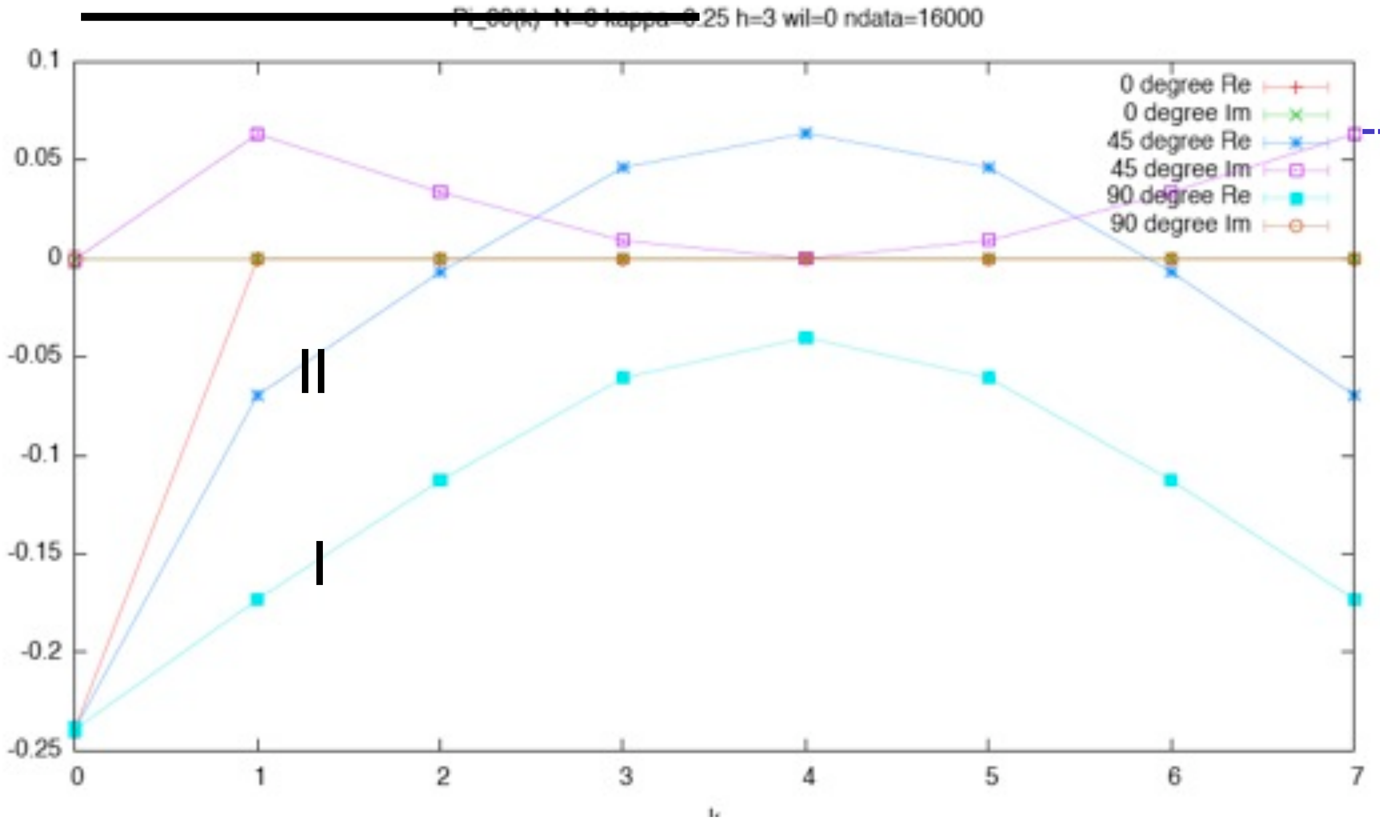
after numerous checks and balances, e.g.:

- check that the divergence of Im-part exactly cancels that of light fermion
- checks of other exact (i.e. coupling-independent) properties of mirror polarization operator that we had derived

lots of plots + long expressions - see 0901.3402 - will only show one

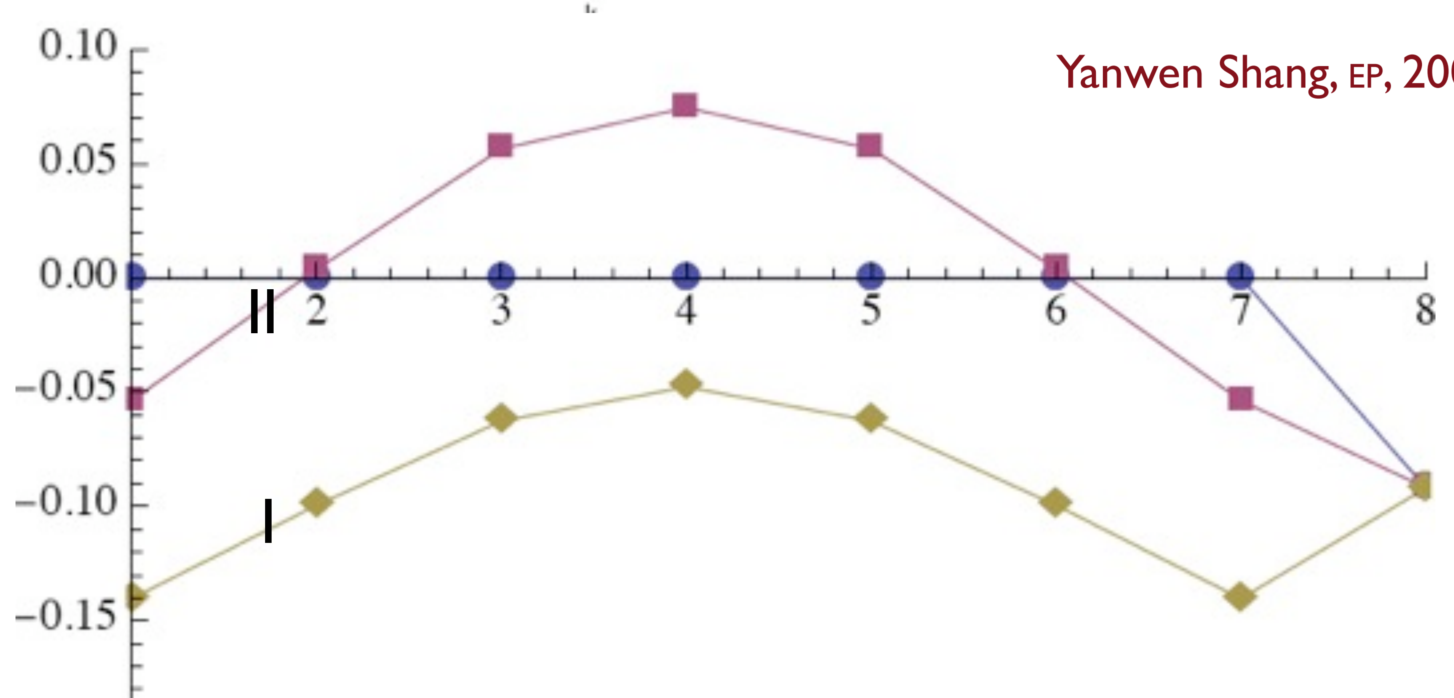
for h>1: real part of mirror polarization operator - probing number of massless charged modes - is like that of one charged massless fermion (this result is independent on h, so long as h>1)

where does it stand?



----never mind this curve

this it the Monte-Carlo calculation of the mirror polarization operator at **y=infinity, h=3, 8x8 lattice, disordered phase, 16000 field configurations (error bars are almost invisible)**



Yanwen Shang, EP, 2009

looking very much like the one from Mathematica

Monte-Carlo "proof" of 't Hooft anomaly matching at strong mirror coupling

I have not fudged anything!
different value at k=0 has to do with Wilson line needed to avoid singularity when computing loop, not with error bars of MC

where does it stand?

what comes out?

- for h>1: real part of mirror polarization operator - probing number of massless charged modes - is that of **one charged massless fermion** (this result is independent on h, so long as h>1)

- for h = 0: real part of mirror polarization operator like that of **three charged massless fermions** (since anomaly same for all h, must be one - chirality and a +/- chirality pair)

- can not interpolate through h=1, huge sign problem for h~0.7 *about where a KT-like transition lies*

- all this is in the "strong-coupling symmetric" phase, small \kappa;
  at large-\kappa - "broken phase" scalar Green-Schwarz field, massive gauge boson - understood perturbatively as well as via simulation (check)

- so, 't Hooft anomaly matching is obeyed, by having, for h>1, the minimal number of massless charged mirror fermions required by anomaly

- spectrum of mirror at strong coupling is consistent with long-distance unitarity
  - some thought it wouldn't be!

where does it stand?

what lessons are we learning?

- anomaly matching works; *mirror dynamics is "smart" - and appears unitary - both Re- and Im- nonlocal*

- Majorana couplings crucial - recall initially motivated by breaking all ungauged mirror global symmetries

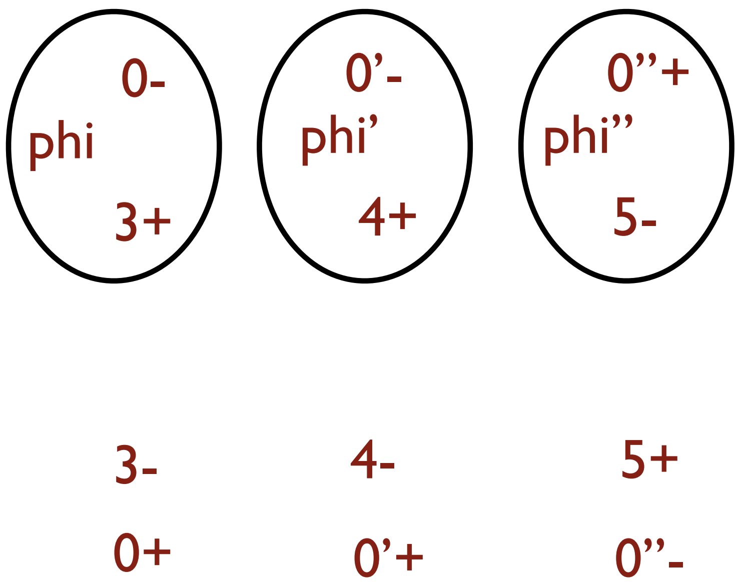
  without them massless spectrum at y=infinity always 3 doubler modes

where is it going?

finally, the question on everybody's mind: but what about anomaly free models?

best I can say for now is it will depend on how mirror is implemented (symmetries again)
in principle, 't Hooft implies conditions even in that case

- consider "345 model," an anomaly-free 2d theory with 3- 4- 5+ charged fermions
  3 is charge; - is chirality, etc.



this implementation is bound to have massless mirrors:

three disjoint copies of our 1-0 model (3-0, 4-0 and 5-0 models)

three exact global symmetries appear when g=0
couple light/mirror fermions
imagine gauging each one & argue by 't Hooft

where is it going?

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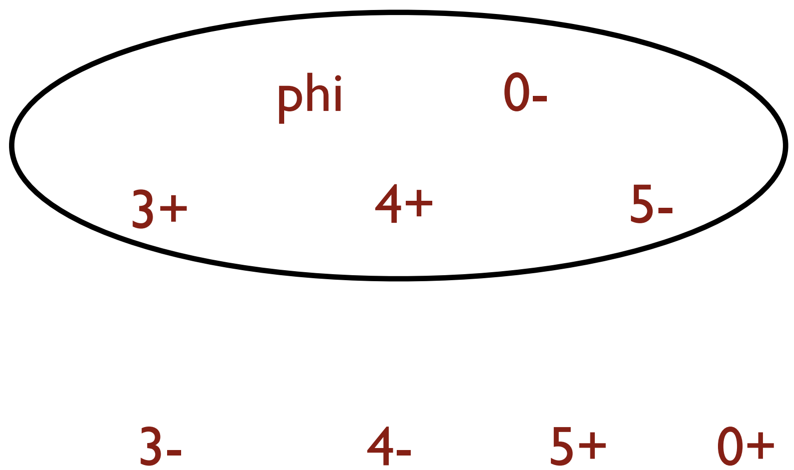

so change mirror implementation

break all global symmetries that involve mirror fields by allowing cross couplings between 3,4,5 mirrors (0- needed for static limit)

now at g=0 only global symmetries are the three light chiral U(1)s

- no reason to think that there will be massless mirror states for all values of the couplings, now that there's no symmetry reason for this **but if there are no reasons, beyond anomaly matching, then why should there always be massless mirror modes?**

- unless we, or anybody else willing to think about this, comes up with a general argument why there always should be massless states, only future "experiment" will tell for sure

- "experiment" is quite doable, as *no gauge fields are involved, and most of the groundwork is done*
  345 (or 11112) chiral models in 2d are an obvious first try...

there are people who can and will do it - Chen, Giedt (et al. ...), in progress.

SUMMARY

the
“**decoupling of mirror fermions via strong-coupling symmetric phases**” idea, combined with “**exact lattice chirality**” leads to a proposed formulation of chiral lattice gauge theories, which is:

a.) exactly gauge invariant

b.) has explicit definition of path integral action and measure
so one can study it numerically

c.) has the correct - anomalous or not - Ward identities
of the continuum target theory

but

d.) requires more - but feasible - numerical + analytic work to study *most importantly,*

e.) ***we have not seen reasons to give up -***

results on anomaly matching go in the right direction
 - min number of fermions needed to match anomalies remains massless in the strong-coupling symmetric phase -

hence, conjecture: **at strong coupling, if no anomalies to match - no massless fermions?**

At the same time, we don't know if we have succeeded or "not failed", yet!

I have not failed. I've just found 10,000 ways that won't work.

Thomas A. Edison