Magnetic bions, multiple adjoints, and Seiberg-Witten theory

Erich Poppitz   U of Toronto

work with   Mithat Ünsal   SLAC/Stanford   (in progress)

(will mention also some recent work with Mohamed Anber, Toronto, 1105.0940)

ABSTRACT: (as submitted to CAQCD organizers)

We first study properties of magnetic bions in multi-adjoint-fermion theories on R^3xS^1, notably the flavor and radius dependence of the mass gap.

[Anber, EP, 2011]

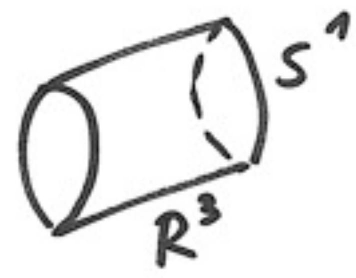
We will also argue that ``Polyakov-like" magnetic bion confinement can be continuously related to the four-dimensional confinement mechanism in Seiberg-Witten theory by appropriately resumming the contributions of the 3d monopole-instanton tower.

[Unsal, EP, 2011]

But, first I really need to tell (remind) you what it's all about.

The theme of my talk is about inferring properties of infinite-volume theory by studying (arbitrarily) small-volume dynamics.

The small volume may be



most of this talk

or

of characteristic size “L”

“Eguchi-Kawai” ... “large-N volume independence”...

long history of stumbling (1980-2008) that I won’t review

some recent (2008+) excitement:

remedy by Unsal, Yaffe 2008

EK reduction valid to arbitrarily small L (single-site) if either:

periodic adjoint fermions
(more than one Weyl) - no center breaking, so EK reduction holds at all L

double-trace deformations
deform measure to prevent center breaking
at infinite-N, deformation does not affect (connected correlators of "untwisted") ***observables***

used for current **lattice studies** of
"minimal walking technicolor"

is 4 ...3,5... Weyl adjoint theory conformal or not?

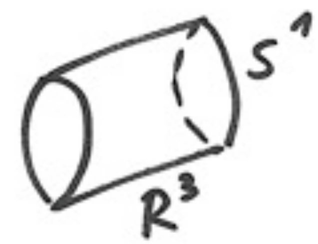
small-L(=1) large-N (~20 or more...) simulations (2009-)
Hietanen-Narayanan; Bringoltz-Sharpe; Catterall et al

small-N large-L simulations (2007-)
Catterall et al; del Debbio et al; Hietanen et al...

THIS TALK:

theoretical studies

Unsal;
Unsal-Yaffe;
Shifman-Unsal;
Unsal-EP 2007-



fix-N, take L-small: semiclassical studies of **confinement** due to novel strange "oddball" (nonselfdual) topological excitations, whose nature depends on fermion content

- for vectorlike or chiral theories, with or without supersymmetry

- **a complementary regime to that of volume independence, which requires infinite N - a (calculable!) shadow of the 4d "real thing".**

For this talk only consider 4d SU(2) theories

with N_W = multiple adjoints ***Weyl fermions***

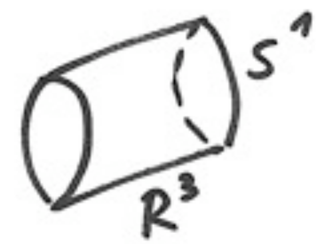
"applications":

N_W=1 is N=1 SUSY YM ~ Seiberg-Witten theory with soft-breaking mass

N_W=4
- "minimal walking technicolor"
- happens to be N=4 SYM without the scalars

N_W=5.5 asymptotic freedom lost

theoretical studies



Unsal;
Unsal-Yaffe;
Shifman-Unsal;
Unsal-EP 2007-

fix-N, take L-small: **semiclassical studies of confinement** due to novel strange "oddball" (nonselfdual) topological excitations, whose nature depends on fermion content

In 4d theories with periodic adjoint fermions, for small-L, dynamics is semiclassically calculable (including confinement).

Polyakov's 3d mechanism of confinement by "Debye screening" in the monopole-anti-monopole plasma extends to (locally) 4d theories.

However, the "Debye screening" is now due to composite objects, the "magnetic bions" ***of the title.***

“Magnetic bions, multiple-adjoints, and Seiberg-Witten theory”

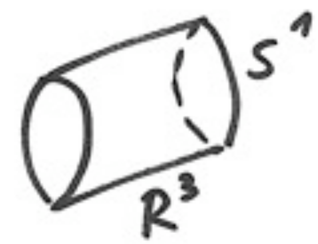
For this talk only consider 4d SU(2) theories

with N_W = $\boxed{\text{multiple adjoints}}$ ***Weyl fermions***

“applications”:

N_W=1 is N=1 SUSY YM = $\boxed{\text{Seiberg-Witten theory}}$ with soft-breaking mass

theoretical studies



Unsal;
Unsal-Yaffe;
Shifman-Unsal;
Unsal-EP 2007-

N_W=4
- “minimal walking technicolor”
- happens to be N=4 SYM without the scalars

N_W=5.5 asymptotic freedom lost

fix-N, take L-small: **semiclassical studies of confinement** due to novel strange “oddball” (nonselfdual) topological excitations, whose nature depends on fermion content

In 4d theories with periodic adjoint fermions, for small-L, dynamics is semiclassically calculable (including confinement).

Polyakov’s 3d mechanism of confinement by “Debye screening” in the monopole-anti-monopole plasma extends to (locally) 4d theories.

However, the “Debye screening” is now due to composite objects, the $\boxed{\text{“magnetic bions”}}$ ***of the title.***

In 4d theories with periodic adjoint fermions, for small-L, confining dynamics is semiclassically calculable.

$$S^1: \quad x^4 \sim x^4 + L$$

A_4 is now an adjoint 3d scalar Higgs field $\quad \partial_4 + A_4 \longrightarrow \frac{2\pi n}{L} + A_4$

but it is a bit unusual -
a compact Higgs field: $\quad \langle A_4 \rangle \sim \langle A_4 \rangle + \frac{2\pi}{L}$

such shifts of A_4 vev absorbed into shift of KK number "n" $\quad A_4 \to A_4 + \partial_4\left(\frac{2\pi x_4}{L}\right)$

"large" gauge transform

thus, natural
scale of "Higgs vev" is $\quad \langle A_4 \rangle \sim \frac{\pi}{L} \quad$ leading to

$$SU(2) \xrightarrow{\frac{1}{L}} U(1)$$

hence, semiclassical if L << inverse strong scale

exactly this happens in theories with more than one periodic Weyl adjoints

follows from two things, without calculation:

1.) existence of deconfinement transition in pure YM and 2.) supersymmetry

in pure YM, at small L (high-T), Veff min at A_4=0 & max at pi/L (Gross,Pisarsky,Yaffe 1980s)

in SUSY Veff=0, so one Weyl fermion contributes the negative of gauge boson Veff

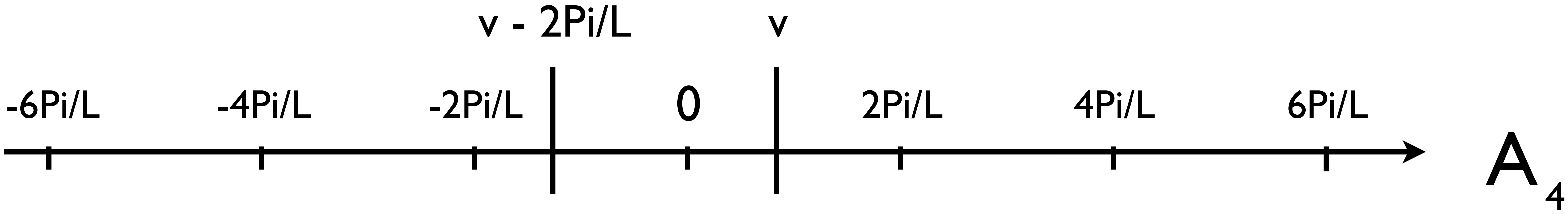
Q.E.D.

Polyakov's 3d mechanism of confinement by "Debye screening"
in the monopole-anti-monopole plasma extends to (locally) 4d theories.
However, the "Debye screening" is now due to composite
objects, the "magnetic bions" ***of the title.***

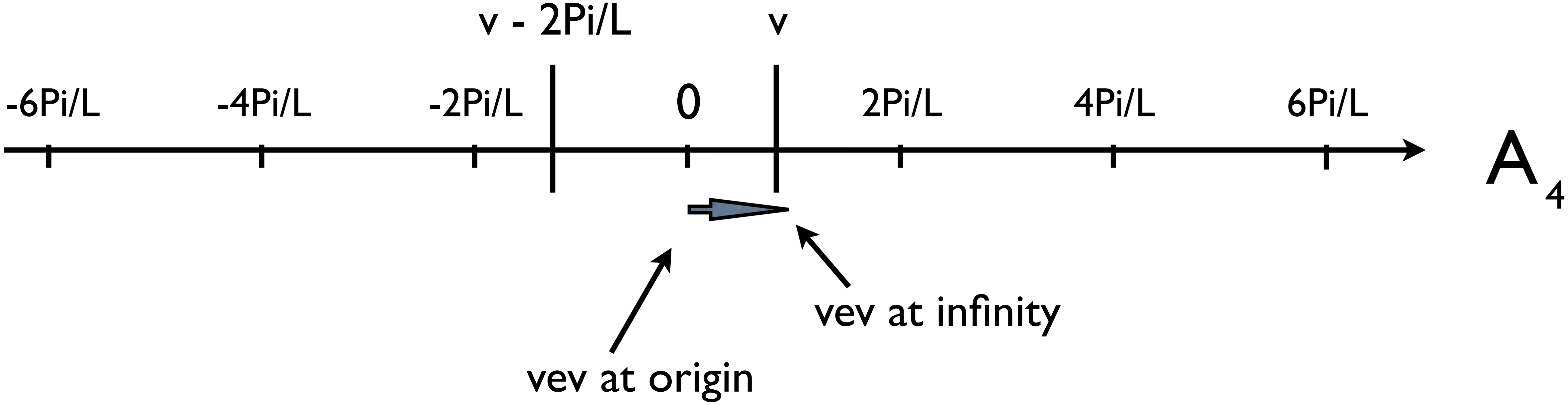
since SU(2) broken to U(1) at scale 1/L

there are monopole-instanton solutions of finite Euclidean action, constructed
as follows:

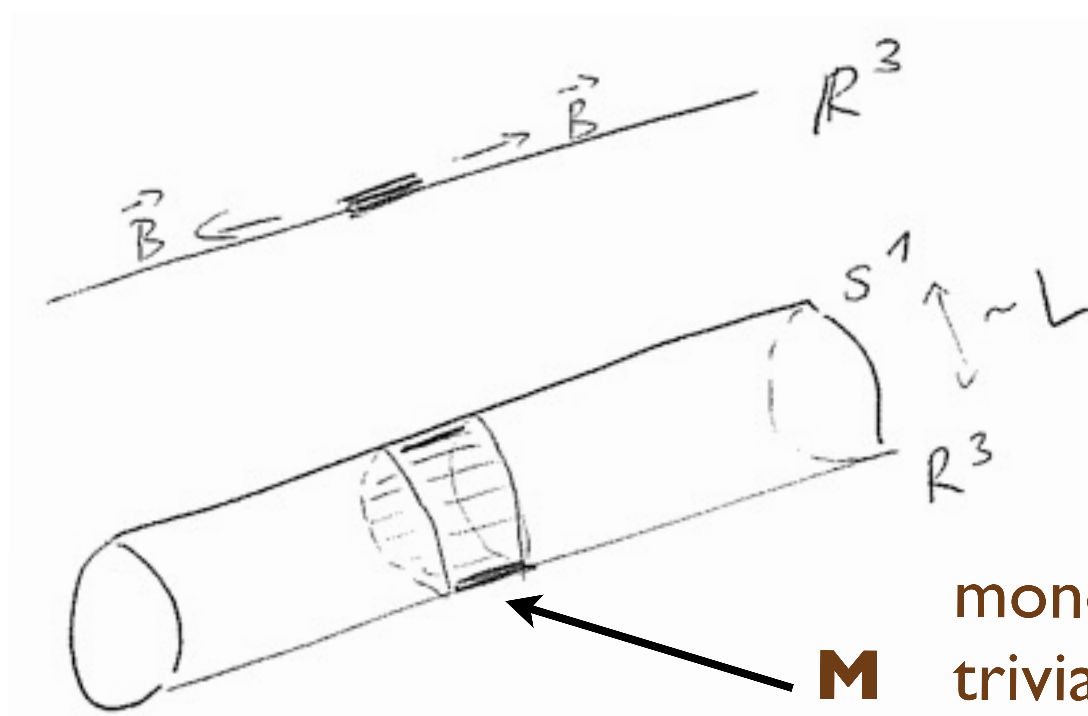
gauge equivalent vevs



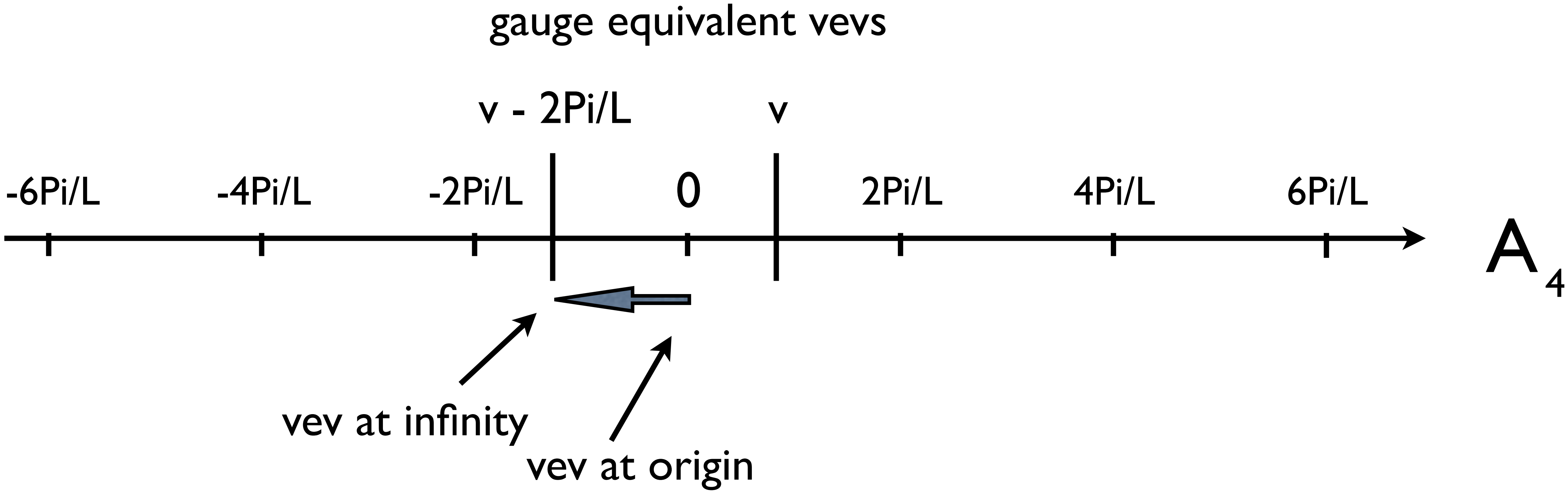
gauge equivalent vevs



monopole-instanton of action ~ v/g_3^2



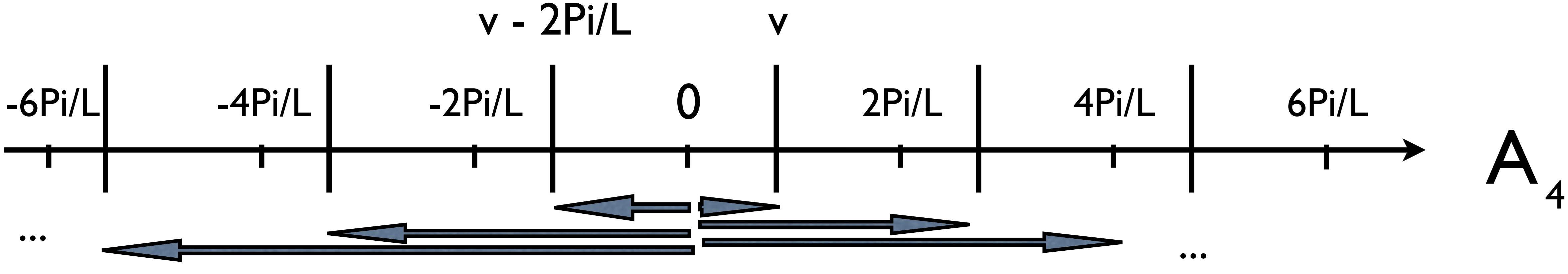
M monopole trivially embedded in 4d



monopole-instanton of action ~ $|2\,\mathrm{Pi}/L - v|/g_3^2$

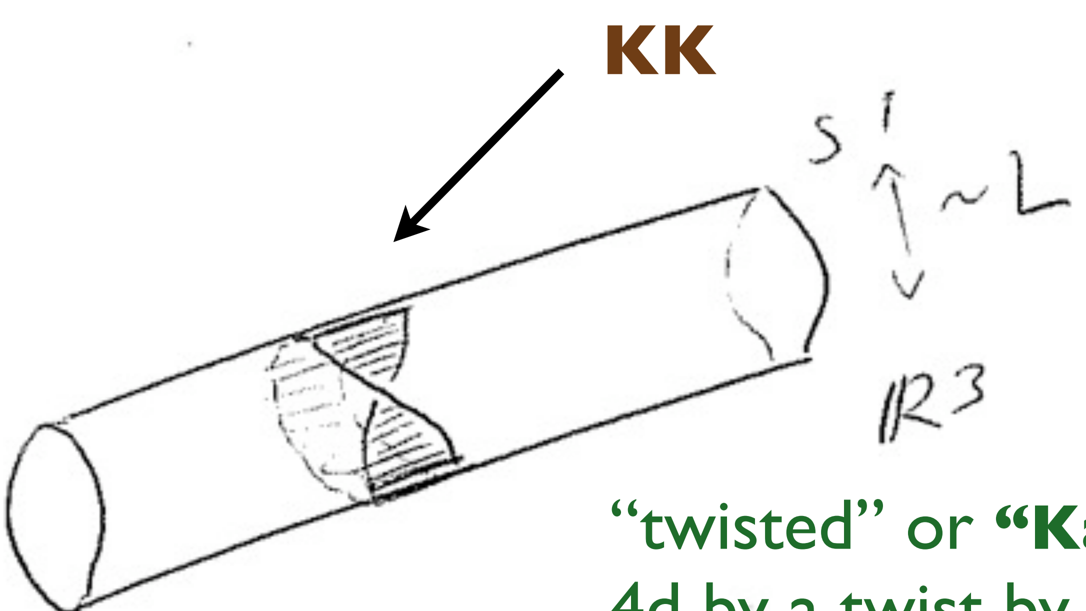
- use a large gauge transformation to make vev at infinity = v
- action does not change
- x_4-dependence is induced, hence called “twisted”

gauge equivalent vevs

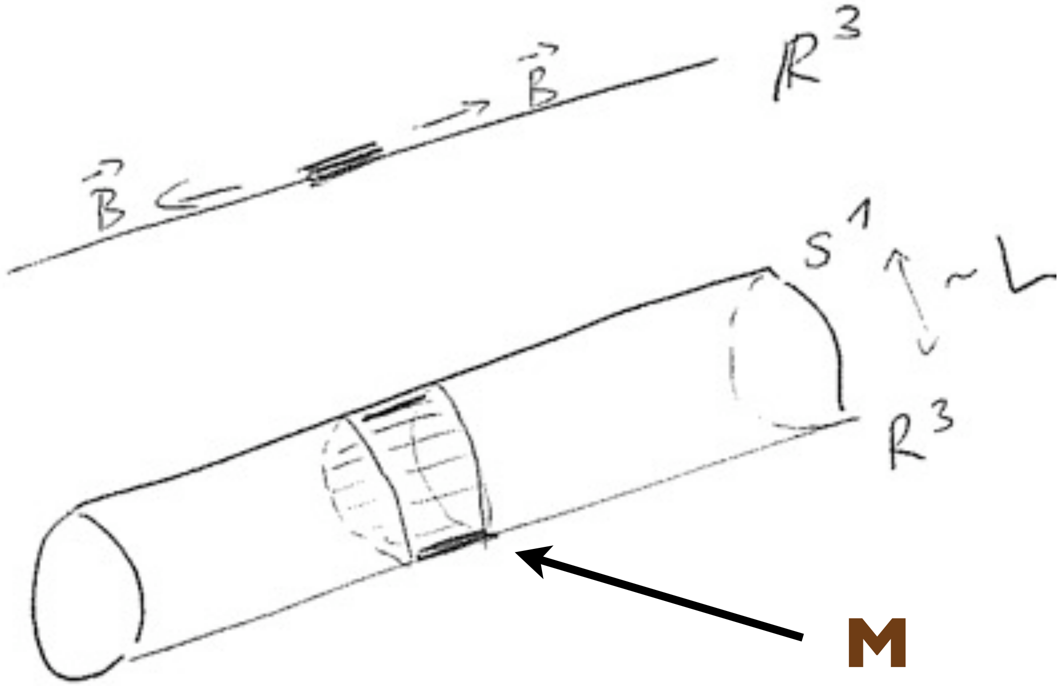


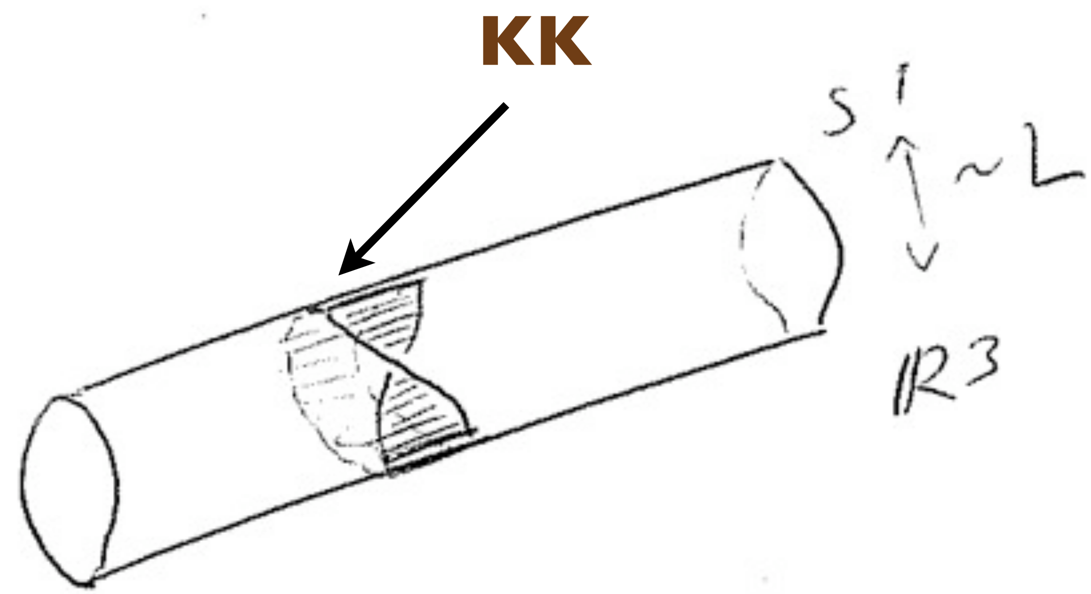
monopole-instanton tower; action ~ $|2\, k\, \mathrm{Pi}/L - v|/g_3^2$

the lowest action member of the tower can be pictured like this (as opposed to the no-twist):



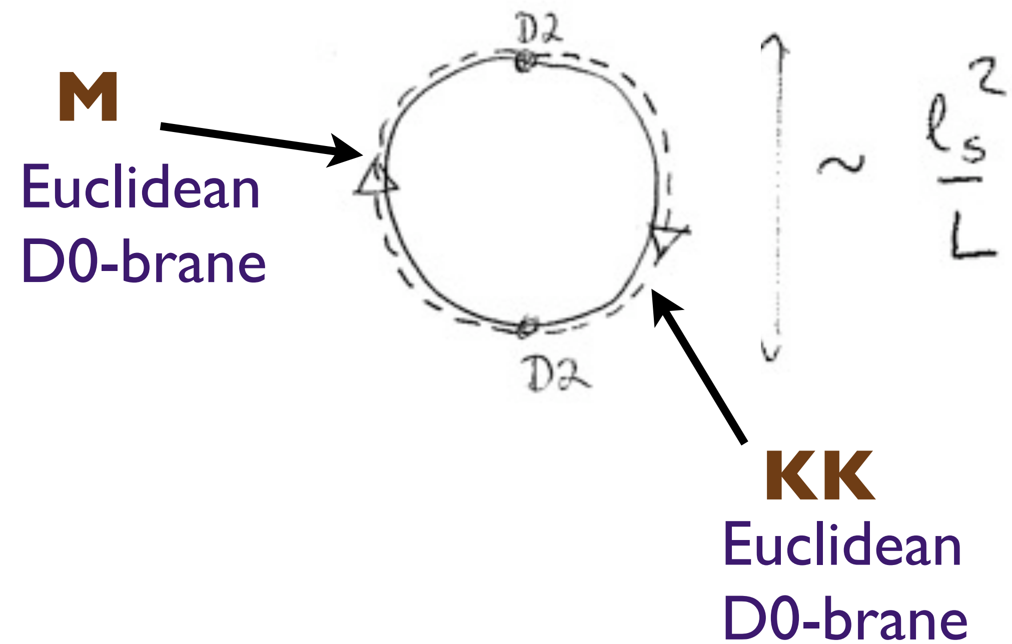
"twisted" or **"Kaluza-Klein":** monopole embedded in 4d by a twist by a "gauge transformation" periodic up to center - in 3d limit not there! (infinite action)





K. Lee, P. Yi, 1997

	magnetic	topological	suppression
M	+1	1/2	e^{-S_0}
KK	−1	1/2	e^{-S_0}
BPST	0	1	e^{-2S_0}



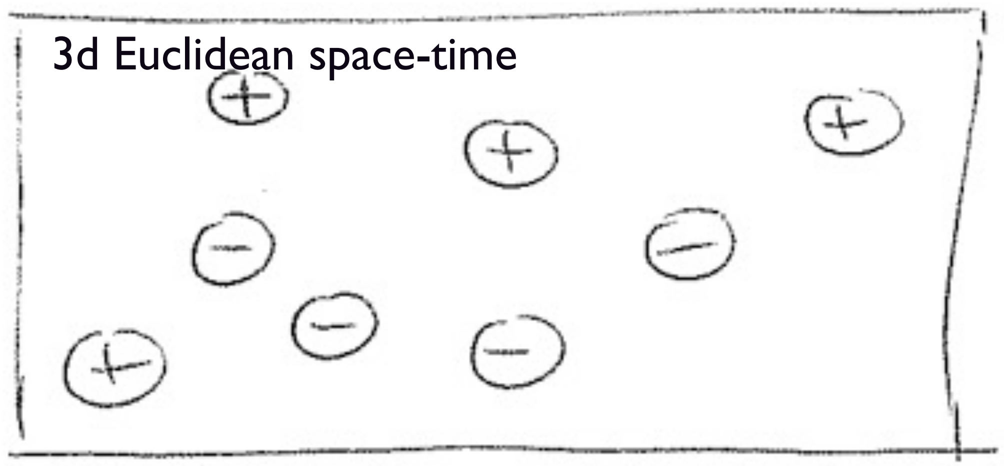
M & KK have 't Hooft suppression given by:

$$e^{-S_0} = e^{-\frac{4\pi v}{g_3^2}} = e^{-\frac{4\pi^2}{L g_3^2}} = e^{-\frac{4\pi^2}{g_4^2(L)}}$$

center-symmetric vev coupling matching

in SU(N), 1/N-th of the 't Hooft suppression factor

in a purely bosonic theory, vacuum would be a dilute M-M* plasma - but interacting, unlike instanton gas in 4d (in say, electroweak theory)



physics is that of Debye screening

analogy:

electric fields are screened in a charged plasma ("Debye mass for photon")
in the monopole-antimonopole plasma, the dual photon (3d photon ~ scalar) obtains mass from screening of magnetic field:

$$\mathcal{L}_{eff} = g_3^2 (\partial\sigma)^2 + (\#)\, v^3 e^{-S_0}\left(e^{i\sigma} + e^{-i\sigma}\right) + \ldots$$

"(anti-)monopole operators"

aka **"disorder operators"** - not locally expressed in terms of original gauge fields (Kadanoff-Ceva; 't Hooft - 1970s)

also by analogy with Debye mass:

dual photon mass^2 ~ M-M* plasma density

$$m_\sigma \sim v\, e^{-S_0/2} = v\, e^{-\frac{4\pi v}{2 g_3^2}}$$

(for us, v = pi/L)

Polyakov, 1977: ***dual photon mass ~ confining string tension***

"Polyakov model" = 3d Georgi-Glashow model or compact U(1) (lattice)

but our theory has fermions and M and KK have zero modes

each have $2N_W$ zero modes

index theorem
Nye-Singer 2000,

for physicists:
Unsal, EP 0812.2085

disorder operators:

M: $e^{-S_0} e^{i\sigma} (\lambda\lambda)^{N_w}$

KK: $e^{-S_0} e^{-i\sigma} (\lambda\lambda)^{N_w}$

M*: $e^{-S_0} e^{-i\sigma} (\bar\lambda\bar\lambda)^{N_w}$

KK*: $e^{-S_0} e^{i\sigma} (\bar\lambda\bar\lambda)^{N_w}$

chiral symmetry $SU(N_w) \times U(1)$

U(1) anomalous, but $\mathbb{Z}_{4N_w}: \lambda \to e^{i\frac{2\pi}{4N_w}} \lambda \qquad \sigma \to \sigma + \pi$ is not

topological shift symmetry is intertwined with exact chiral symmetry

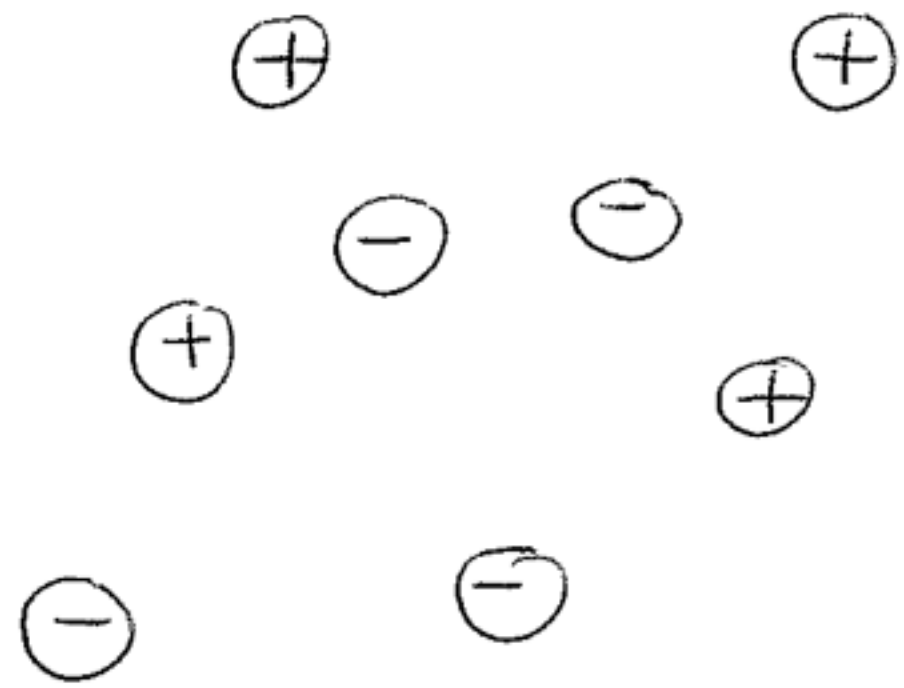
$\cancel{\cos\sigma} \qquad \cos(2\sigma)$ ✓ ...

potential (and dual photon mass) allowed, but what is it due to?

Unsal 2007: **dual photon mass is induced by magnetic "bions" - the leading cause of confinement in SU(N) with adjoints at small L** (including SYM)

3d pure gauge theory vacuum monopole plasma

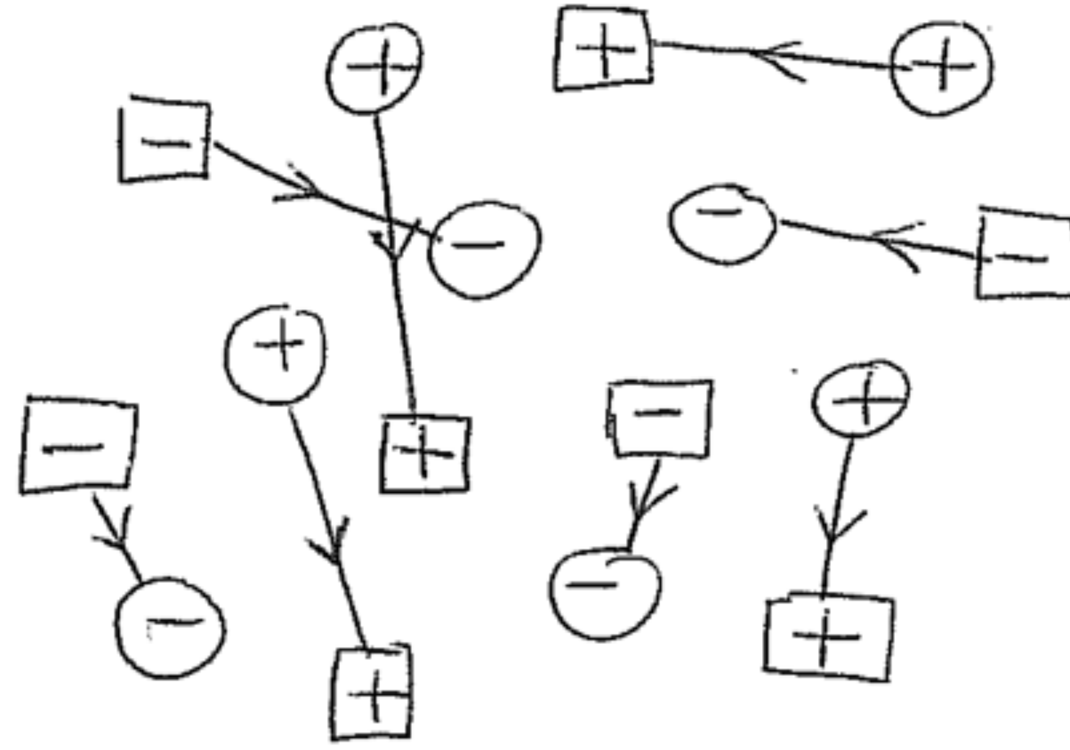
Polyakov 1977



circles = M(+)/M*(-)

4d QCD(adj) fermion attraction M-KK* at small-L

Unsal 2007, ...

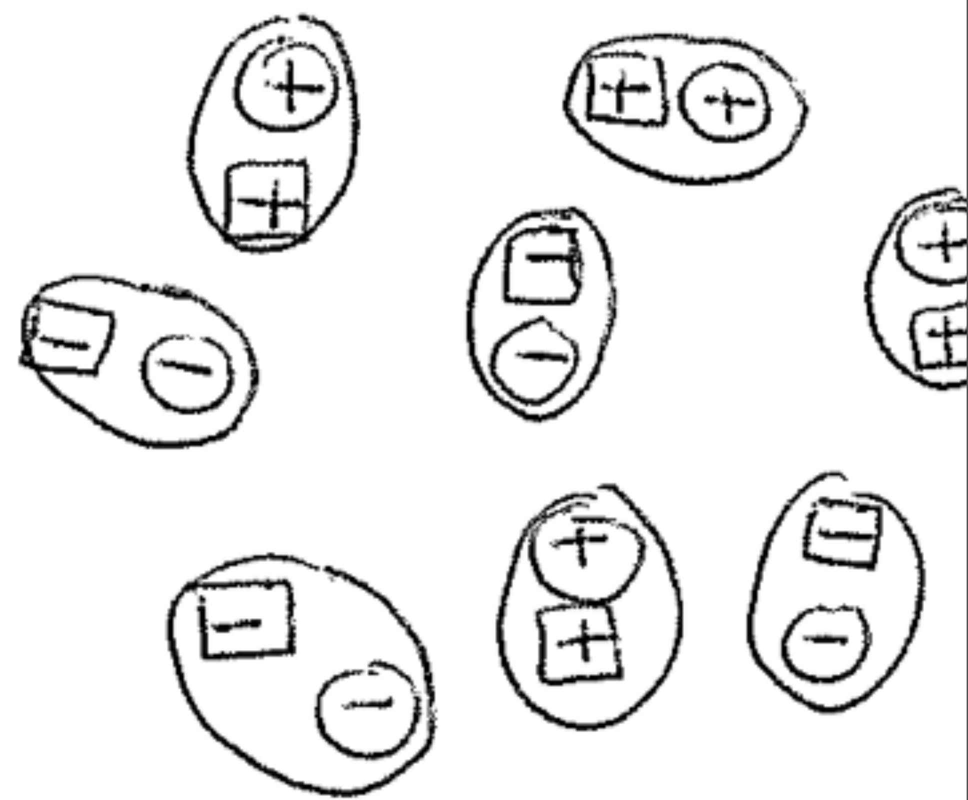


circles = M(+)/M*(-)

squares = KK(-)/KK*(+)

4d QCD(adj) bion plasma at small-L

Unsal 2007, ...

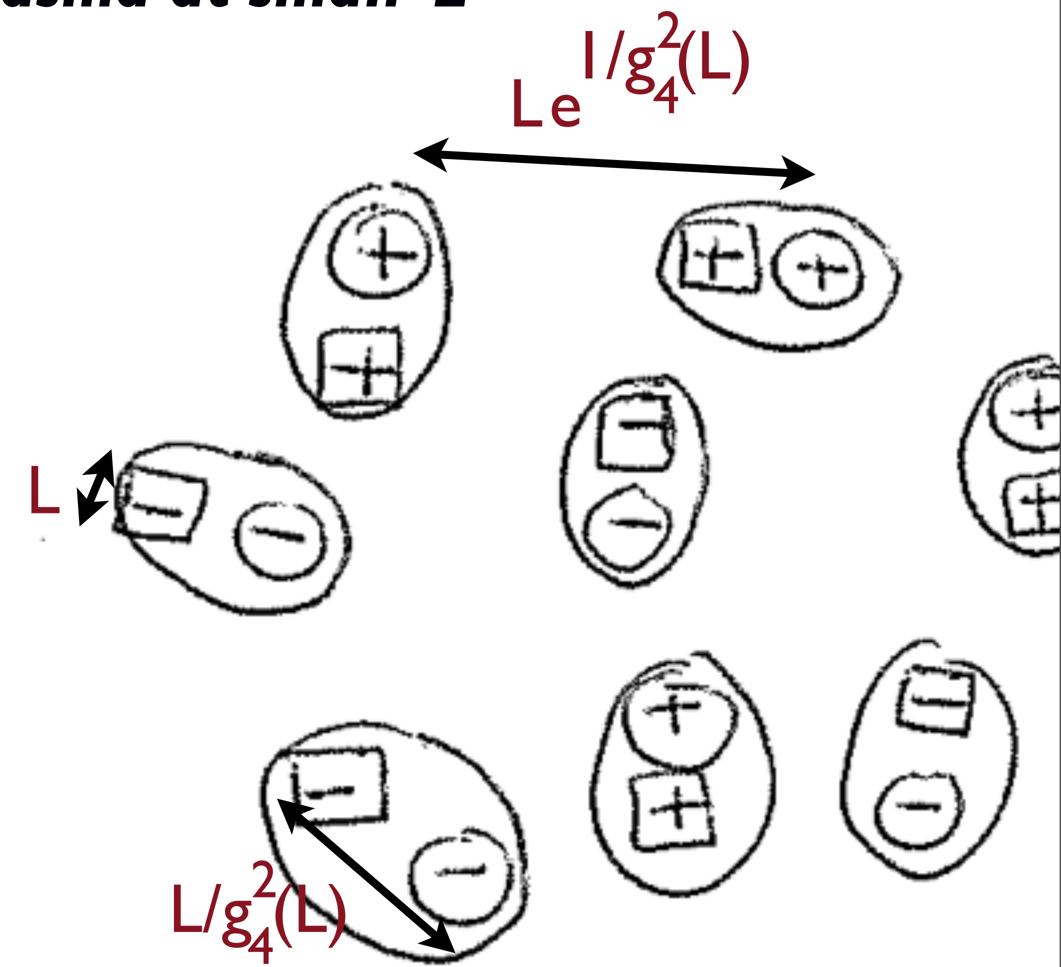


circles = M(+)/M*(-)

squares = KK(-)/KK*(+)

blobs = Bions(++)/Bions*(--)

4d QCD(adj) bion plasma at small-L
Unsal 2007, ...



M + KK* = B - magnetic "bions" -

-carry 2 units of magnetic charge
-no topological charge (non self-dual)
(locally 4d nature crucial: no KK in 4d)

bion stability is due to fermion attraction balancing Coulomb repulsion - results in scales as indicated

- bion/antibion plasma screening generates mass for dual photon

"magnetic bion confinement" operates at small-L in any theory with massless Weyl adjoints, including N=1 SYM (& N=1 from Seiberg-Witten theory)

it is "automatic": no need to "deform" theory other than small-L

first time confinement analytically shown in a non-SUSY, continuum, locally 4d theory

can calculate mass gap, string tension...

Unsal, EP 2009, Anber, EP 2011

$$\frac{\mathcal{M}}{\Lambda} \sim (\Lambda L)^{\frac{8-2n_{w}}{3}} e^{-2\pi\tilde{c}\left(\log\frac{1}{\Lambda L}\right)^{1/2}} \times (\text{less relevant contributions})$$

strong scale

O(1), positive

... how **dare** you study non-protected quantities?

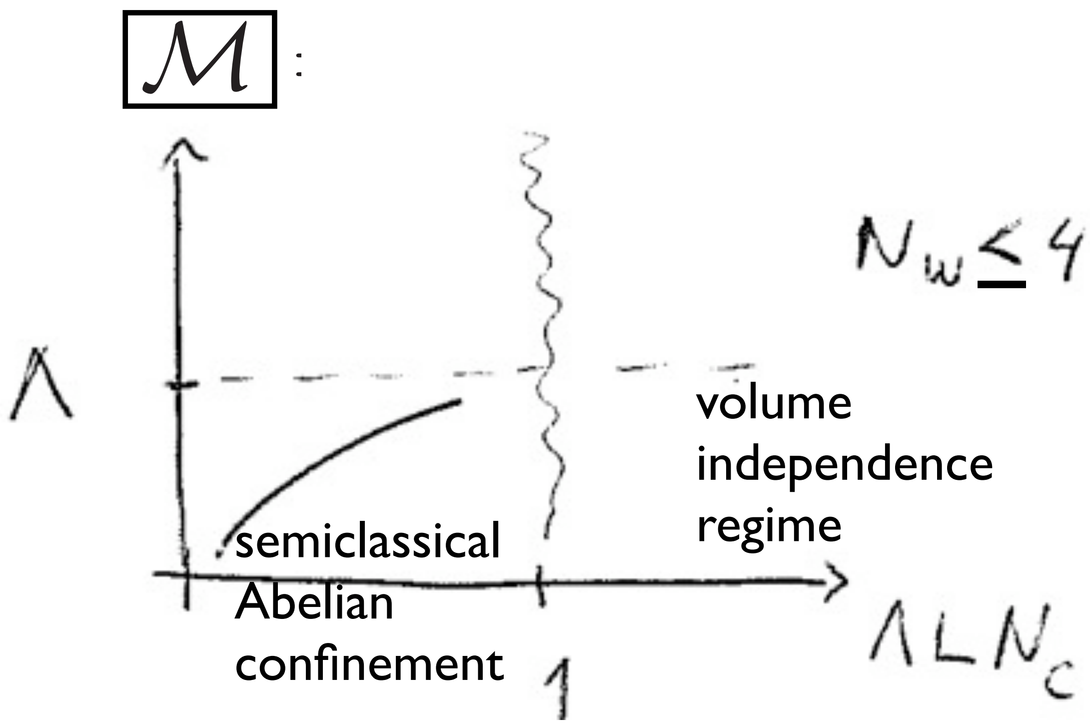
can calculate mass gap, string tension...

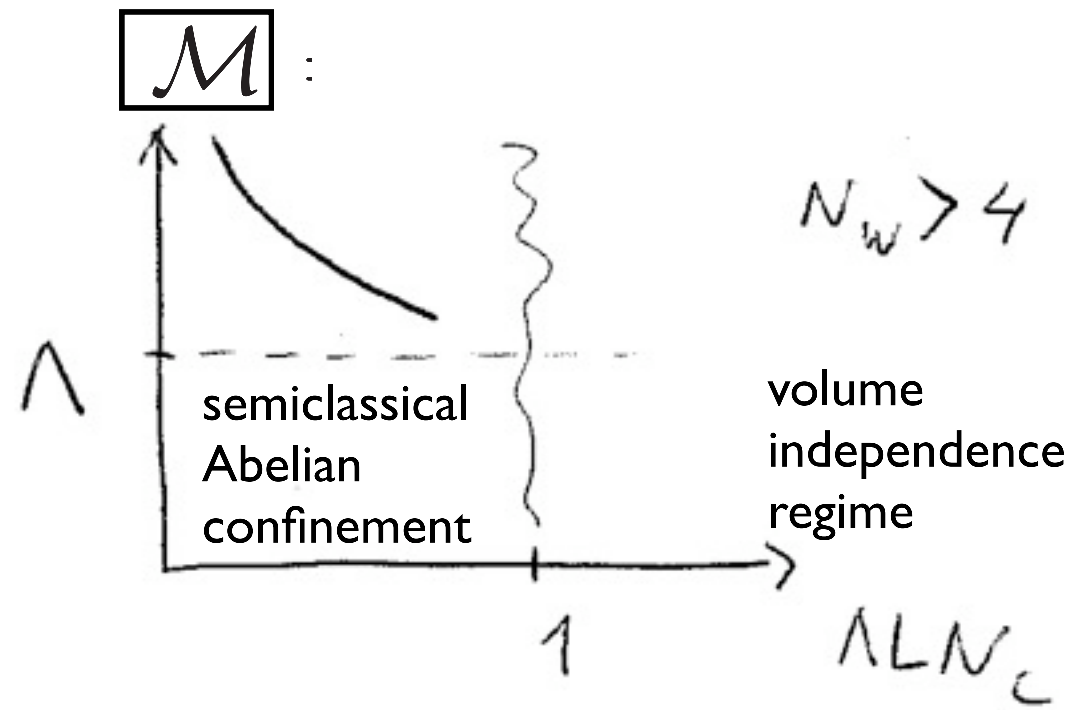
Unsal, EP 2009, Anber, EP 2011

$$\frac{\mathcal{M}}{\Lambda} \sim (\Lambda L)^{\frac{8-2n_W}{3}} e^{-2\pi\tilde{c}\left(\log\frac{1}{\Lambda L}\right)^{1/2}} \times \text{(less relevant contributions)}$$

strong scale

O(1), positive





Discussion on approach to R^4 in refs. - here only note for 4 and 5 massless Weyl adjoints appears that weak coupling IR fixed point at any L, hence Abelian confinement with exponentially small mass gap and string tension

$$\sim \frac{1}{L} e^{-\frac{\mathcal{O}(1)}{g_*^2}}$$

The question about the approach to infinite 4d in the non-SUSY case is very interesting...

... but let's turn to SUSY first:

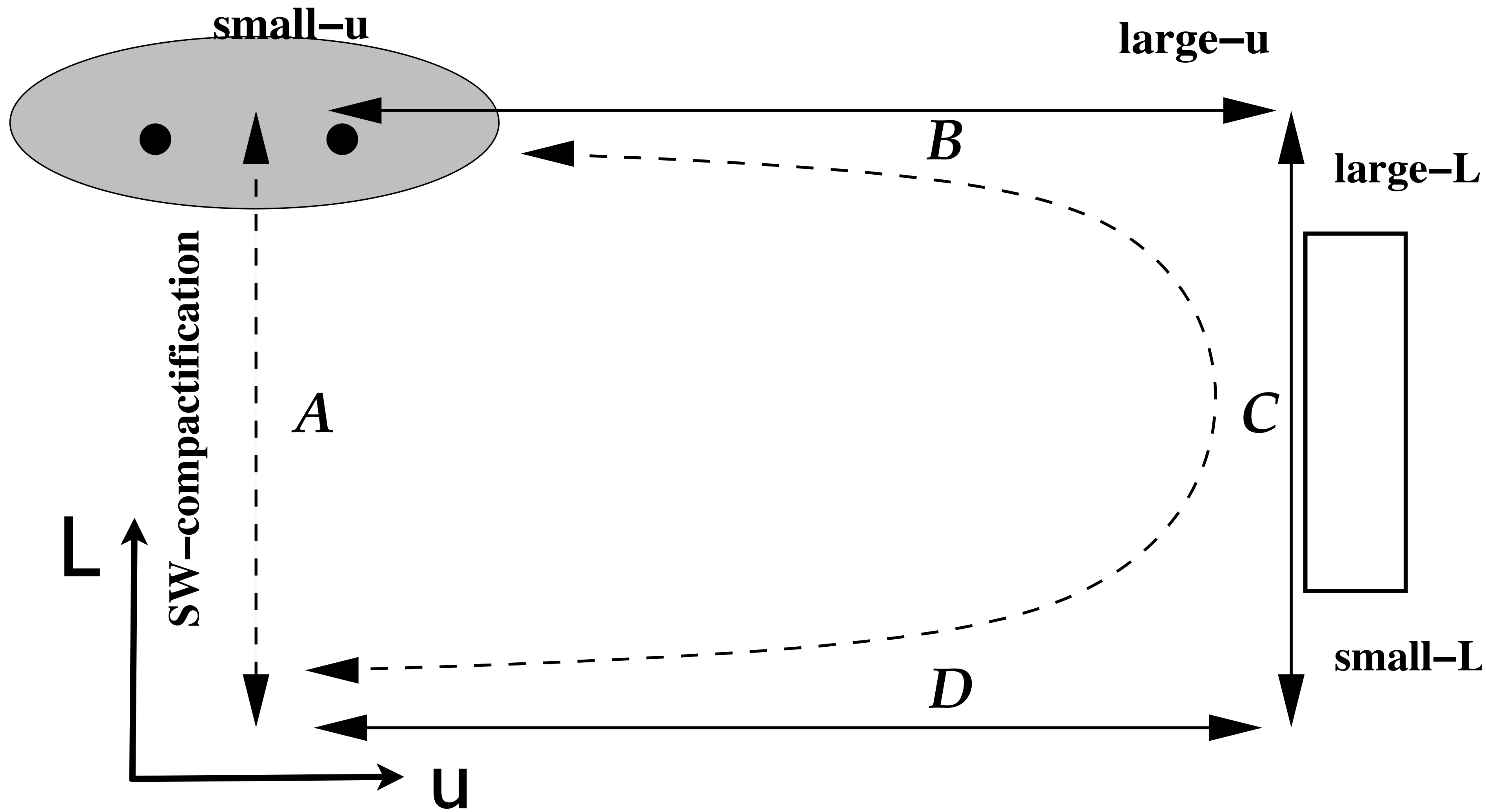
We argued that "magnetic bions" are responsible for confinement in N=1 SYM at small L - a particular case of our Weyl adjoint theory - a "Polyakov like" confinement.

This remains true if N=1 obtained from N=2 by soft breaking.

On the other hand, we know monopole and dyon condensation is responsible for confinement in N=2 softly broken to N=1 at large L (Seiberg, Witten `94)

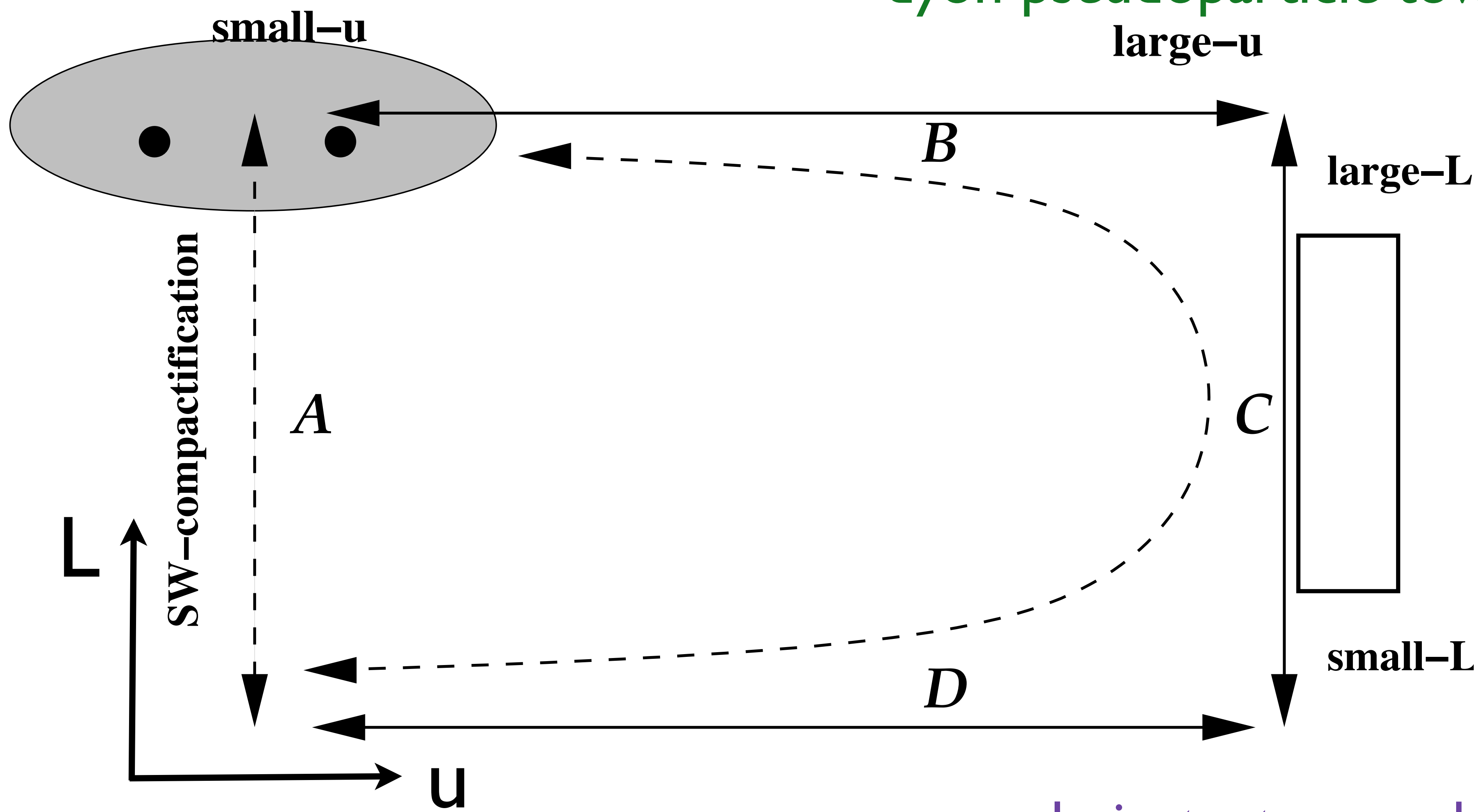
So, in different regimes we have different pictures of confinement in softly broken N=2 SYM.
(Both regimes are Abelian and quantitatively understood.)

Do they connect in an interesting way? Unsal, EP in progress path A - difficult: two mutually nonlocal descriptions at large L to merge into one at small L



path BCD - easier: C, D can be arranged always semiclassical dyon tower (sum over electric charges) of particles with Euclidean worldlines around S^1

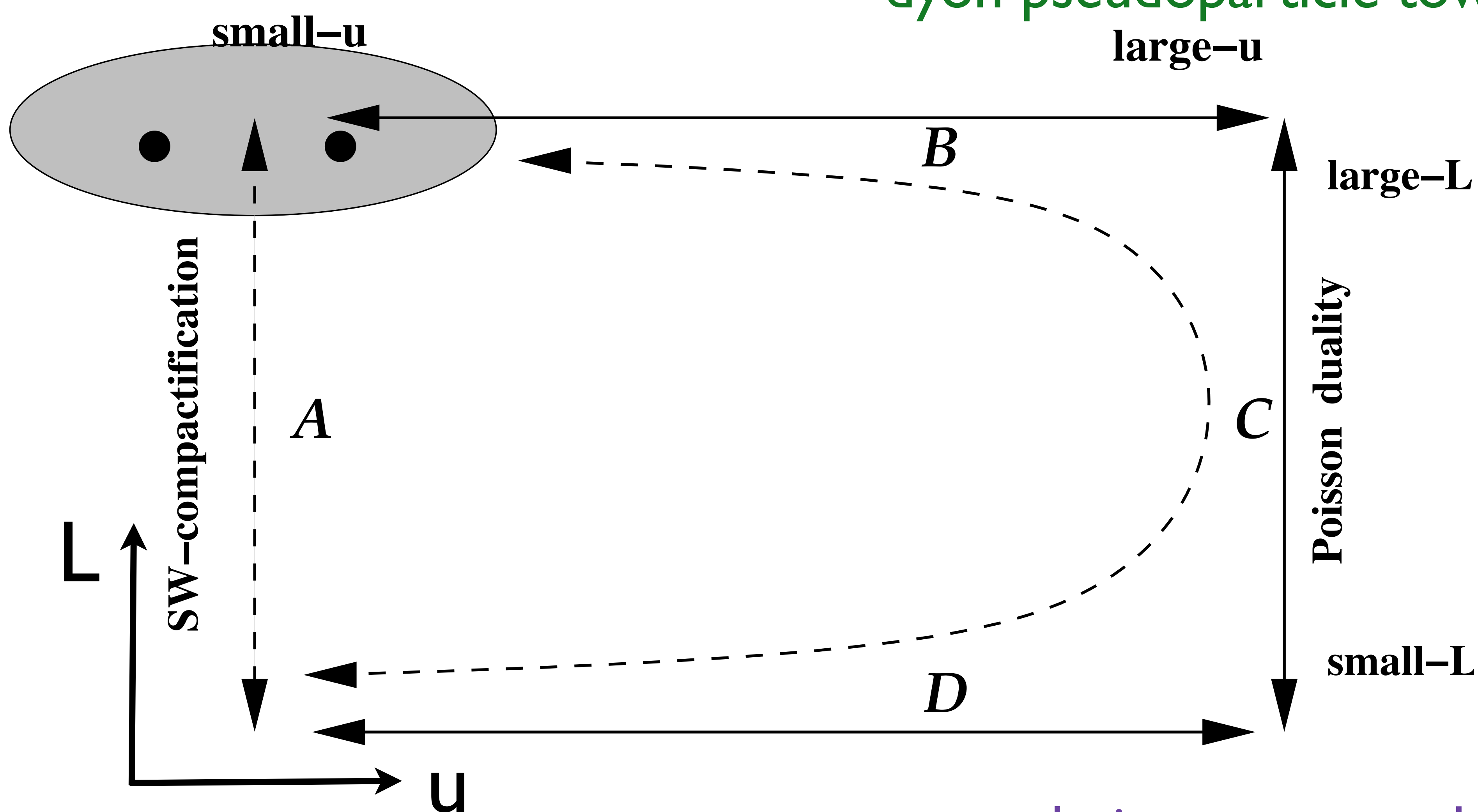
= dyon pseudoparticle tower



monopole-instantons and twisted monopole-instantons

=“KK tower” described earlier

dyon tower (sum over electric charges) of particles with Euclidean worldlines around S^1
= dyon pseudoparticle tower



monopole-instantons and twisted monopole-instantons
="KK tower" described earlier

It turns out the sums over the instanton contributions of the two towers are identical and are related by Poisson duality:

Instanton corrections to K, in complex structure:

$$\sigma - i\,\frac{4\pi\omega}{g_4^2}$$

dual photon

L A_4

v = Sqrt[u]

can be inferred from solution by Chen, Dorey, Petunin (2010) of "wall-crossing" equations of Gaiotto, Moore, Neitzke (2008): an iterative solution, obtained at weak-coupling v >> Lambda, but ***arbitrary vL***:

$$K_{dyon} = \frac{1}{\sqrt{2}\pi^{\frac{3}{2}} L^{\frac{3}{2}} |v|^{\frac{1}{2}}} \sum_{n_m=\pm 1} \sum_{n_e \in \mathbb{Z}} \frac{e^{-L|v|\sqrt{\left(\frac{4\pi}{g_4^2}\right)^2 + n_e^2} + i\omega n_e + i\sigma n_m}}{\left[\left(\frac{4\pi}{g_4^2}\right)^2 + n_e^2\right]^{\frac{1}{4}}} \;;$$

large-L sum over electric charges of dyon pseudoparticles

small-L sum over winding numbers of twisted monopole-instantons

$$K_{winding} = K_{dyon} = \frac{1}{\pi L^2 |v|} \sum_{n_m=\pm 1} \sum_{n_w \in \mathbb{Z}} e^{-\frac{4\pi L}{g_4^2}\sqrt{|v|^2 + \left(\frac{\omega + 2\pi n_w}{L}\right)^2} + i\sigma n_m}$$

Nontrivial to check their equivalence by a semiclassical calculation

(for general value of the moduli as fermion zero modes are different and only sums are equivalent)...

But, in an appropriate regime, same 4-fermi terms appear, and the "wall-crossing" consequence K(dyon)=K(winding) can be semiclassically tested [Chen et al 2010]

- in this limit, Poisson duality can also be more simply understood, sans wallXing, but no time...

$$e^{-\frac{4\pi v L}{g_4^2}+i\sigma} \sum_{n_w \in \mathbb{Z}} e^{-\frac{1}{2}\frac{4\pi}{Lvg_4^2}(\omega+2\pi n_w)^2} \times (\text{four}-\text{fermion operator})$$

$Lvg_4^2 \ll 1$ small-L physics well described by a few twisted monopole-instantons (as we'd already done)

- or an infinite sum over charged dyons

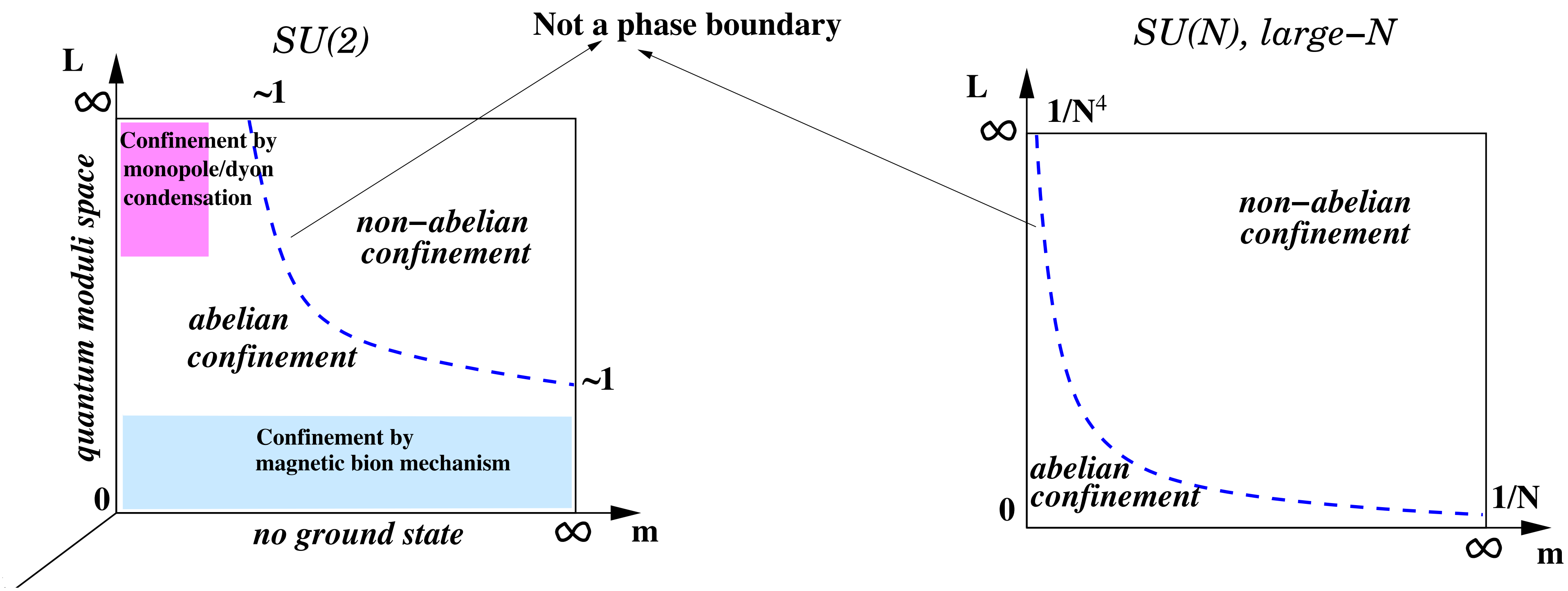
$Lvg_4^2 \gg 1$ large-L physics well described by a few dyons

- or an infinite sum over twisted monopole instantons

$$= e^{-\frac{4\pi v L}{g_4^2}+i\sigma} \sum_{n_e \in \mathbb{Z}} \sqrt{\frac{Lvg_4^2}{8\pi^2}}\, e^{-\frac{1}{2}\frac{vLg_4^2}{4\pi}n_e^2 + in_e\omega} \times (\text{four}-\text{fermion operator})$$

The moral is that the dyons, whose condensation at large-L causes confinement, are related by Poisson resummation to the twisted monopole-instantons that form the small-L magnetic bions - which are responsible for confinement at small L, as was already described.

To conclude, we have found an - albeit indirect - relation between the 4d monopole/dyon condensation confinement of Seiberg and Witten and the small-L magnetic bion-induced "Polyakov-like" confinement.



The magnetic bion mechanism also applies to large classes of non-supersymmetric theories and can be used to study the approach to R^4.

Does the relation between small and large L topological excitations in SUSY have anything to teach us about non-SUSY dynamics?

... it is perhaps early to tell, but the Poisson resummation of nonperturbative effects has interesting implications in finite-T YM

Unsal, EP, 11xx.yyyy - or next CAQCD...