Monopoles, bions, and other oddballs in confinement or conformality

Erich Poppitz
with Mithat Ünsal

University of Toronto
SLAC/Stanford
This talk is about gauge dynamics.

There are many things one would like to understand about any gauge theory:

- does it confine?
- does it break its (super) symmetries?
- is it conformal?
- what are the spectrum, interactions...?

These are tough to address, in almost all theories.
But interesting for:

- satisfying curiosity
- QCD
- SUSY extensions of the Standard Model
- non-SUSY extensions of the Standard Model

**pure YM**

- “formal” but see [www.claymath.org/millennium/](http://www.claymath.org/millennium/)

**SUSY**

- very “friendly” to theorists
  - beautiful - exact results

**QCD-like (vectorlike)**

- hard, leave it to lattice folks
  - \( m, a, V, \$ \)

**non-SUSY chiral gauge theories**

- poorly understood strong dynamics
  - ...almost nobody talks about them anymore
What I’ll talk about applies to all of these theories...

The theme of my talk is about inferring properties of infinite-volume theory by studying \textbf{(arbitrarily)} small-volume dynamics.

The small volume may be

\begin{itemize}
\item most of this talk of characteristic size “L”
\item or
\end{itemize}
To put my talk in context, some relevant history:

Eguchi and Kawai (1982) showed that loop (Schwinger-Dyson) equations for Wilson loops in pure Yang-Mills theory are identical in small-V and infinite-V theory, to leading order in $1/N$, provided:

- “center-symmetry” unbroken
- translational symmetry unbroken (see Yaffe, 1982)

```
\[
\text{expectation value of any Wilson loop at infinite-L} = \text{expectation value of (folded) Wilson loop at small-L} + O(1/N) \text{ provided topologically nontrivial (winding) Wilson loops have vanishing expectation value (unbroken center)}
\]
```

“EK reduction” or “large-N reduction” or “large-N volume-independence”

(Note: this is an exact result in QFT - so long as unbroken center.)

It could be potentially exciting, for:

1) simulations may be cheaper (use single-site lattice?)
2) raises theorist’s hopes (that small-L easier to solve?)
To put my talk in context, some relevant history:

From a “modern” point of view EK reduction is a large-N orbifold with respect to the group of translations.

Kovtun, Unsal, Yaffe (2004)

Volume-independence viewed as an orbifold helps establish that VEVs and correlators of operators that are center-neutral and carry momenta quantized in units of 1/L (in compact direction) are the same on, say $T^4$ as in infinite-L theory, to leading order in 1/N.

Thus, a working example of EK would be good for

- calculating vevs (symmetry breaking)
  - OK, even if all dimensions small
- calculating spectra (for generic theories/reps)
  - need at least one large dimension
(... scattering for LHC  - all large dimensions)
To put my talk in context, some relevant history:

Some intuition of how EK reduction works (valid at any coupling).

in perturbation theory:  or at strong coupling:  
from spectra (& Feynman graphs)  
in appropriate backgrounds  

<table>
<thead>
<tr>
<th>4π/L</th>
<th>4π/L</th>
<th>4π/L</th>
</tr>
</thead>
<tbody>
<tr>
<td>2π/L</td>
<td>2π/L</td>
<td>2π/L</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

| a) Center–broken finite or large N | b1) Center–symmetric finite N | b2) Center–symmetric large N |

gravity dual of N=4 SYM - a conformal field theory - Wilson loops, appropriate correlators  
- insensitive to box if center-symmetric

V(r) ~ 1/r : CFT result obtains in center-symmetric vacuum for any r (<L or >L) insensitive to box size

Unsal, EP 2010

However, Bhanot, Heller, Neuberger (1982) noticed immediate problem

- center symmetry breaks for L < L_c and thus invalidates EK reduction

remedies:  e.g., Gonzales-Arroyo, Okawa (1982) - TEK... + others later argued to have problems  
.... see recent “twists” on TEK ?
To put my talk in context, some relevant history:

Remedies proposed: reduction valid to arbitrarily small L (single-site) if:

- adjoint fermions (more than one Weyl) - no center breaking, so reduction holds at all L
- double-trace deformations: deform measure to prevent center breaking; deformation “drops out” of loop equations at infinite-N

used for current lattice studies of “minimal walking technicolor”

is 4 ...3,5... Weyl adjoint theory conformal or not?

small-L(=1) large-N simulations (2009-)
  Hietanen-Narayanan; Bringoltz-Sharpe; Catterall et al

small-N large-L simulations (2007-)
  Catterall et al; del Debbio et al; Hietanen et al...

(many issues to still be resolved...)

fix-N, take L-small: semiclassical studies of confinement due to novel strange “oddball” (nonselfdual) topological excitations, whose nature depends on fermion content

- for vectorlike or chiral theories, with or without supersymmetry

- a complementary regime to that of volume independence, which requires infinite N - a (calculable!) shadow of the 4d “real thing”.

THIS TALK: theoretical studies

Unsal; Unsal-Yaffe; Unsal-Shifman; Unsal-EP 2007-10

Unsal, Yaffe 2008
The plan is to tell you, largely in pictures, what this story amounts to.

- **Index theorem for topological excitations on $\mathbb{R}^3 \times S^1$ and Chern-Simons theory**
  JHEP 0903:027, 2009; 0812.2085, 29pp

- **Chiral gauge dynamics and dynamical supersymmetry breaking**
  JHEP 0907:060, 2009; 0905.0634, 31pp

- **Conformality or confinement: (IR)relevance of topological excitations**
  JHEP 0909:050, 2009; 0906.5156, 42pp

- **Conformality or confinement (II): One-flavor CFTs and mixed-representation QCD**
  JHEP 0912:011, 2009; 0910.1245, 33pp

All by M. Unsal and E.P. + work in progress on relation to Seiberg-Witten confinement
First, the key players:

3d Polyakov model & “monopole-instanton”-induced confinement

“monopole-instantons” on $\mathbb{R}^3 \times S^1$

the relevant index theorem

center-symmetry on $\mathbb{R}^3 \times S^1$ - adjoint fermions or double-trace deformations

“bions”, “triplets”, “quintets”... - new non-self-dual topological excitations and confinement

Polyakov, 1977

K. Lee, P. Yi, 1997
P. van Baal, 1998

Nye, Singer, 2000
Unsal, EP, 2008

Shifman, Unsal, 2008
Unsal, Yaffe, 2008

Unsal, 2007
Unsal, EP, 2009
First, the key players:

3d Polyakov model & “monopole-instanton”-induced confinement

Polyakov, 1977

continuum picture: 3d Georgi-Glashow [on the lattice - compact U(1)]

\[ L \sim \frac{1}{g_3^2} \left( F_{\mu\nu}^a F_{\mu\nu}^a + D_\mu \phi^a D^\mu \phi^a \right) \quad \mu, \nu = 1, 2, 3 \]

\[ [A_\mu] = [\phi] = 1 \quad [q_3^2] = 1 \]

due to some Higgs potential

\[ \langle \phi \rangle = (0, 0, \nu) \]

\[ SU(2) \xrightarrow{\nu} U(1) \]

at low energies, \( E \ll m_w \sim \nu \)

free U(1) theory

\[ A_3^\mu = A_\mu \]

\[ L_{\text{eff}} = \frac{1}{g_3^2} F_{\mu\nu}^2 + ... \]

“..." are perturbatively calculable & not very interesting
“magnetic field”
topologically conserved current of “emergent topological U(1) symmetry” responsible for conservation of magnetic charge

3d photon dual to scalar (as one polarization only)

Abelian duality

Bianchi identity

equation of motion

topological U(1) symmetry = shift of “dual photon”

a rather “boring-boring” duality - if not for the existence of monopoles:

monopoles \[ \partial_\mu B_\mu = \text{quantized magnetic charge} \] - shift symmetry broken

- dual photon gains mass & electric charges confined

how?

...in pictures:
"'t Hooft-Polyakov monopole" - static finite energy solution of Georgi-Glashow model in 4d

get Euclidean 3d by "forgetting time"

solution of Euclidean eqns. of motion of finite action: a "monopole-instanton"

\[ E_M = \frac{4\pi \nu}{g^2} \]

\[ S_0 = \frac{4\pi \nu}{g^3} \]

M-M* pairs give exponentially suppressed (at weak coupling) "semiclassical" contributions to the vacuum functional

vacuum "is" a dilute monopole-antimonopole plasma

number of M's per unit volume \( \sim \sqrt{3} e^{-S_0} \)

(analogous to B+L violation in electroweak model; at T=0 exponentially small)
vacuum is a dilute M-M* plasma - but interacting, unlike instanton gas in 4d (in say, electroweak theory)

\[ Z = Z_{\text{perturbative}} \times Z_{\text{charged plasma with Coulomb interactions}} \]

really meaning grand partition function of classical 3d M-M* plasma

physics is that of Debye screening - analogy:

electric fields are screened in a charged plasma (“Debye mass for photon”), so in the monopole-antimonopole plasma, the dual photon obtains mass from screening of magnetic field:

\[ L_{\text{eff}} = g_3^2 (\partial \sigma)^2 + \left( \# \right) \nu^3 e^{-S_0} (e^{i\sigma} + e^{-i\sigma}) + \ldots \]

“(anti-)monopole operators” aka “disorder operators” - not locally expressed in terms of original gauge fields (Kadanoff-Ceva; ’t Hooft - 1970s)

also by analogy with Debye mass:

dual photon mass\(^2 \sim M-M^*\) plasma density

\[ m_\sigma \sim v e^{-S_0/2} = v e^{-\frac{4\nu^2}{2g_3^2}} \]

next:
dual photon mass ~ confining string tension...
Minkowski space interpretation of Wilson loop:

confining flux tube: tension$^{-1}$ ~ thickness ~ inverse dual photon mass
### First, the key players:

<table>
<thead>
<tr>
<th>3d Polyakov model &amp; “monopole-instanton”-induced confinement</th>
<th>Polyakov, 1977</th>
</tr>
</thead>
<tbody>
<tr>
<td>“monopole-instantons” on $\mathbb{R}^3 \times S^1$</td>
<td>K. Lee, P.Yi, 1997</td>
</tr>
<tr>
<td></td>
<td>P. van Baal, 1998</td>
</tr>
<tr>
<td>the relevant index theorem</td>
<td>Nye, Singer, 2000</td>
</tr>
<tr>
<td></td>
<td>Unsal, EP, 2008</td>
</tr>
<tr>
<td>center-symmetry on $\mathbb{R}^3 \times S^1$ - adjoint fermions or double-trace deformations</td>
<td>Shifman, Unsal, 2008</td>
</tr>
<tr>
<td></td>
<td>Unsal, Yaffe, 2008</td>
</tr>
<tr>
<td></td>
<td>Unsal, EP, 2009</td>
</tr>
</tbody>
</table>
First, the key players:

we want to go to 4d - by “growing” a compact dimension:

\[ S^4 : x^4 \sim x^4 + L \]

“monopole-instantons” on \( R^3 \times S^1 \)

K. Lee, P. Yi, 1997
P. van Baal, 1998

\[ A_4 \] is now an adjoint 3d scalar Higgs field

\[ \partial_4 + A_4 \longrightarrow \frac{2 \pi n}{L} + A_4 \]

but it is a bit unusual - a compact Higgs field:

\[ \langle A_4 \rangle \sim \langle A_4 \rangle + \frac{2 \pi n}{L} \]

such shifts of \( A_4 \) vev absorbed into shift of KK number “n”

thus, natural scale of “Higgs vev” is

\[ \langle A_4 \rangle \sim \frac{\pi n}{L} \]

leading to

\[ SU(2) \rightarrow U(1) \]

(clearly, semiclassical and weakly coupled if \( L \ll \) inverse strong scale)
$A_4$ - adjoint 3d scalar Higgs field;
a gauge-covariant description:

$$W = P e^{i \oint A_4 \, dx^4}$$

if the expectation values are

$$\langle W \rangle = \begin{pmatrix} e^{i \pi/2} \\ e^{-i \pi/2} \end{pmatrix}$$

then

$$\text{tr} \langle W \rangle = 0$$

and we say that “center symmetry is preserved”

$$\text{tr} W \rightarrow e^{i \frac{\pi}{N}} \quad \text{tr} W \quad \text{for SU}(N): \quad e^{i \frac{2\pi}{N}}$$

we are interested in unbroken center:
where $\langle \text{tr} W \rangle = 0$ and SU(2) broken to U(1) at scale $1/L$
breaks SU(2) to U(1) so there are monopoles:

\[ \langle w \rangle = \left( \begin{array}{c} e^{i \pi/2} \\ e^{-i \pi/2} \end{array} \right) \]

**M** usual monopole trivially embedded in 4d

**KK** discovered by K. Lee, P. Yi, 1997, as "Instantons and monopoles on partially compactified D-branes"

“twisted” or **Kaluza-Klein**: monopole embedded in 4d by a twist by a “gauge transformation” periodic up to center - in 3d limit not there! (infinite action)
Summary: “elementary” topological excitations on $\mathbb{R}^3 \times S^1$

M & KK both self-dual objects, of opposite magnetic charges

<table>
<thead>
<tr>
<th>Magnetic Charge</th>
<th>Topological Charge</th>
<th>Suppression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$ +1</td>
<td>$\frac{1}{2}$</td>
<td>$e^{-s_0}$</td>
</tr>
<tr>
<td>$KK$ -1</td>
<td>$\frac{1}{2}$</td>
<td>$e^{-s_0}$</td>
</tr>
<tr>
<td>BPST 0</td>
<td>1</td>
<td>$e^{-2s_0}$</td>
</tr>
</tbody>
</table>

+ their anti-”particles”

- thus, BPST instanton “ = M+KK ”

(also see P. van Baal, 1998)

\[
e^{-s_0} = e^{-\frac{4\pi\nu}{g_3}} = e^{-\frac{4\pi^2}{Lg_3^2}} = e^{-\frac{4\pi^2}{g_4^2(L)}}
\]

M & KK have, in SU(N), $1/N$-th of the ‘t Hooft suppression factor aka:

“fractional instantons”, “instanton quarks”, “zindons”,
“quinks”, “instanton partons”... [collected by D. Tong]

**Next**, to understand the role M, KK, M* & KK* play in various theories of interest, need to know what happens to the operators they induce when there are fermions in the theory.
First, the key players:

<table>
<thead>
<tr>
<th>3d Polyakov model &amp; “monopole-instanton”-induced confinement</th>
<th>Polyakov, 1977</th>
</tr>
</thead>
</table>
| “monopole-instantons” on $R^3 \times S^1$                  | K. Lee, P. Yi, 1997  
P. van Baal, 1998 |
| the relevant index theorem                                  | Nye, Singer, 2000  
Unsal, EP, 2008 |
| center-symmetry on $R^3 \times S^1$ - adjoint fermions or double-trace deformations | Shifman, Unsal, 2008  
Unsal, Yaffe, 2008 |
Unsal, EP, 2009 |
First, the key players:

The relevant index theorem

- for some theories the answer for the number of zero modes in M or KK background had been guessed (correctly)
  
  e.g. SUSY YM - Aharony, Hanany, Intriligator, Seiberg, Strassler, 1997

- while studying ISS(henker) proposal for SUSY breaking model \([SU(2)+}\text{three-index symm. tensor Weyl}\] Unsal and I needed a general index theorem

- we found this:
An $L^2$-Index Theorem for Dirac Operators on $S^1 \times \mathbb{R}^3$

Tom M. W. Nye and Michael A. Singer

where, in appendix A. adiabatic limits of $\eta$-invariants

we found:

$$\text{ind} \left( D_A^+ \right) = \int_X \text{ch}(\mathcal{E}) + \frac{1}{\mu_0} \sum_\mu \epsilon_\mu c_1(E_\mu)[S^2_\infty]$$

$$= \int_X \text{ch}(\mathcal{E}) - \frac{1}{2} \eta_{\text{lim}}$$

two obvious questions:

1.) where does this come from?

2.) what number is it equal to in a given topological background (M, KK...) & how does it depend on ratio of radius to holonomy?
for this talk it is enough to consider 4d SU(2) theories with $N_w$ adjoint Weyl fermions

$M$, $KK$, $M^*$, $KK^*$ each have $2N_w$ zero modes

disorder operators:

$M$:
$$ e^{-s_0} e^{i\sigma \lambda} N_w $$

$M^*$:
$$ e^{-s_0} e^{-i\sigma \lambda} N_w $$

$KK$:
$$ e^{-s_0} e^{i\sigma \lambda} N_w $$

$KK^*$:
$$ e^{-s_0} e^{-i\sigma \lambda} N_w $$

where:

- operator due to $M+KK$ = ‘t Hooft vertex; independent of dual photon
- “our” index theorem interpolates between 3d Callias and 4d APS index thms.

“applications”:

$N_w = 1$ is $N=1$ SUSY YM

$N_w = 4$
- “minimal walking technicolor”
- happens to be $N=4$ SYM without the scalars

remarks:

- see M. Unsal, EP 0812.2085

like on $R^3$ Callias E. Weinberg, 1970s, but on $R^3 \times S^1$, so must incorporate anomaly equation, some interesting effects
**First, the key players:**

<table>
<thead>
<tr>
<th>3d Polyakov model &amp; “monopole-instanton”-induced confinement</th>
<th>Polyakov, 1977</th>
</tr>
</thead>
</table>
| “monopole-instantons” on $\mathbb{R}^3 \times S^1$ | K. Lee, P. Yi, 1997  
| | P. van Baal, 1998 |
| the relevant index theorem | Nye, Singer, 2000  
| | Unsal, EP, 2008 |
| center-symmetry on $\mathbb{R}^3 \times S^1$ - adjoint fermions or double-trace deformations | Shifman, Unsal, 2008  
| | Unsal, Yaffe, 2008 |
| | Unsal, EP, 2009 |
First, the key players:

- Abelianization occurs only if there is a nontrivial holonomy (i.e., $A_4$ has vev)

- upon thermal circle compactifications, gauge theories with fermions do not Abelianize: center symmetry is broken at small circle size - transition to a deconfining phase - $A_4 = 0$, $<\text{tr} W> \neq 0$ - deconfinement - at high-$T$, 1-loop $V_{\text{eff}}$

  (Gross, Pisarski, Yaffe, early 1980s)

center-symmetry on $\mathbb{R}^3 \times S^1$ - adjoint fermions or double-trace deformations

Shifman, Unsal, 2008
Unsal, Yaffe, 2008
in other words, in thermal setup, upon decompactification, we have a center-symmetry breaking *phase transition* and no smooth connection to $\mathbb{R}^4$

\[ \mathbb{R}^3 \rightarrow \mathbb{R}^3 \times S^1 \rightarrow \mathbb{R}^4 \]

to ensure calculability at small $L$ and smooth connection to large $L$ in the sense of center symmetry: *can one find ways to avoid phase transition?*

Ⅰ. non-thermal compactifications - periodic fermions ("twisted partition function")

- with $N_w > 1$ adjoint fermions center symmetry preserved (Unsal, Yaffe 2007) as well as with other, "exotic" fermion reps (Unsal, EP 2009)
- in many supersymmetric theories, can simply choose center-symmetric vev

Ⅱ. add double-trace deformations: force center symmetric vacuum at small $L$ (also Shifman, Unsal 2008) - connection to large-$N$ volume independence

*In what follows, we assume center-symmetric vacuum - due to either Ⅰ. or Ⅱ.* - will explicitly discuss only theory where center symmetry is naturally preserved at small $L$ (Ⅰ.)
First, the key players:

<table>
<thead>
<tr>
<th>Topic</th>
<th>Authors/Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>3d Polyakov model &amp; “monopole-instanton”-induced confinement</td>
<td>Polyakov, 1977</td>
</tr>
<tr>
<td>“monopole-instantons” on $\mathbb{R}^3 \times S^1$</td>
<td>K. Lee, P. Yi, 1997</td>
</tr>
<tr>
<td></td>
<td>P. van Baal, 1998</td>
</tr>
<tr>
<td>the relevant index theorem</td>
<td>Nye, Singer, 2000</td>
</tr>
<tr>
<td></td>
<td>Unsal, EP, 2008</td>
</tr>
<tr>
<td>center-symmetry on $\mathbb{R}^3 \times S^1$ - adjoint fermions or</td>
<td>Shifman, Unsal, 2008</td>
</tr>
<tr>
<td>double-trace deformations</td>
<td>Unsal, Yaffe, 2008</td>
</tr>
<tr>
<td>topological excitations and confinement</td>
<td>Unsal, EP, 2009</td>
</tr>
</tbody>
</table>
First, the key players:

ready to study the dynamics of theories with massless fermions on a small circle

in a vacuum with $A_4$ vev, Abelianization:

- in SU(2): (dual) photon massless + fermion components w/out mass from vev (neutral)
- monopoles + KK monopoles are the basic topological excitations

is there magnetic field screening in the vacuum?

the answer would appear to be “no”:

M and KK have fermion zero modes

monopole operators do not generate potential for dual photon

so, no screening & no confinement... ?

“bions”, “triplets”, “quintets”... - new non-self-dual topological excitations and confinement

Unsal, 2007
Unsal, EP, 2009
but take a look at the symmetries first:

as an example, again consider 4d SU(2) theories with $N_w$ adjoint Weyl fermions

classical global chiral symmetry is

$$SU(N_w) \times U(1)$$

but 't Hooft vertex

$$\left(\lambda \lambda\right)^{2N_w} e^{-\frac{8\pi^2}{g_4^2}}$$

only preserves

$$Z_{4N_w} : \lambda \rightarrow e^{i\frac{2\pi}{4N_w}} \lambda$$

so, quantum-mechanically we have only

$$SU(N_w) \times Z_{4N_w}$$

exact chiral symmetry

now $M, KK(+\ast)$ operators all look like:

$$e^{-\frac{1}{\kappa_0}} e^{i\sigma} \left(\lambda \lambda\right)^{N_w}$$

hence

$$\left(\lambda \lambda\right)^{N_w} \xrightarrow{Z_{4N_w}} e^{i\frac{\pi}{2N_w}} \left(\lambda \lambda\right)^{N_w}$$

invariance of $M, KK(+\ast)$ operators under exact chiral symmetry means that
dual photon must transform under the exact chiral symmetry

i.e., topological shift symmetry is intertwined with chiral symmetry:

$$Z_{4N_w} : \sigma \rightarrow \sigma + \pi$$
so the exact chiral symmetry allows a potential - **but what is it due to?**

to generate \( \cos(2\sigma) \) must have

i. magnetic charge 2

ii. no zero modes

\[ M + KK^* \text{ bound state? (Unsal, 2007)} \]

- same magnetic charge \( \sim 1/r \)-repulsion

- fermion exchange \( \sim \log(r) \)-attraction

\[ M + KK^* = B \text{ - magnetic “bion”} \]

dual photon mass is induced by magnetic “bions” - the leading cause of confinement in SU(N) with adjoints at small \( L \) (incl. SYM)
M + KK* = B - magnetic “bions” -
- carry magnetic charge
- have no topological charge (non self-dual)
  (locally 4d nature crucial: no KK in 4d)
- generate “Debye” mass for dual photon

main tools used:
- intertwining of topological shift symmetry & chiral symmetry
- index theorem

topological objects generating magnetic screening depend on massless fermion content (not usually thought that fermions relevant)

using these tools, one can analyze any theory...
in the last couple of years, many theories have been studied...

<table>
<thead>
<tr>
<th>Theory</th>
<th>Confinement mechanism on $\mathbb{R}^3 \times S^1$</th>
<th>Index for monopoles $[I_1, I_2, \ldots, I_N]$</th>
<th>Index for instanton $I_{inst.} = \sum_{i=1}^{N} I_i$</th>
<th>(Mass Gap)$^2$ units ~$1/L^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>YM $\gamma$,U '08</td>
<td>monopoles</td>
<td>$[0, \ldots, 0]$</td>
<td>0</td>
<td>$e^{-S_0}$</td>
</tr>
<tr>
<td>QCD(F) S, U '08</td>
<td>monopoles</td>
<td>$[2, 0, \ldots, 0]$</td>
<td>2</td>
<td>$e^{-S_0}$</td>
</tr>
<tr>
<td>SYM U '07 / QCD(Adj)</td>
<td>magnetic bions</td>
<td>$[2, 2, \ldots, 2]$</td>
<td>$2N$</td>
<td>$e^{-2S_0}$</td>
</tr>
<tr>
<td>QCD(BF) S, U '08</td>
<td>magnetic bions</td>
<td>$[2, 2, \ldots, 2]$</td>
<td>$2N$</td>
<td>$e^{-2S_0}$</td>
</tr>
<tr>
<td>QCD(AS) S, U '08</td>
<td>bions and monopoles</td>
<td>$[2, 2, \ldots, 2, 0, 0]$</td>
<td>$2N - 4$</td>
<td>$e^{-2S_0}, e^{-S_0}$</td>
</tr>
<tr>
<td>QCD(S) PU '09</td>
<td>bions and</td>
<td>$[2, 2, \ldots, 2, 4, 4]$</td>
<td>$2N + 4$</td>
<td>$e^{-2S_0}, e^{-3S_0}$</td>
</tr>
<tr>
<td>SU(2)YM $I = \frac{3}{2}$ PU '09</td>
<td>magnetic quintets</td>
<td>$[4, 6]$</td>
<td>10</td>
<td>$e^{-5S_0}$</td>
</tr>
<tr>
<td>chiral SU '08</td>
<td>magnetic bions</td>
<td>$[2, 2, \ldots, 2]$</td>
<td>$2N$</td>
<td>$e^{-2S_0}$</td>
</tr>
<tr>
<td>AS + $(N - 4)\overline{F}$ S, U '08</td>
<td>bions and a monopole</td>
<td>$[1, 1, \ldots, 1, 0, 0] + [0, 0, \ldots, 0, N - 4, 0]$</td>
<td>$(N - 2)AS + (N - 4)\overline{F}$</td>
<td>$e^{-2S_0}, e^{-S_0}$</td>
</tr>
<tr>
<td>$S + (N + 4)\overline{F}$ PU '09</td>
<td>bions and</td>
<td>$[1, 1, \ldots, 1, 2, 2] + [0, 0, \ldots, 0, N + 4, 0]$</td>
<td>$(N + 2)S + (N + 4)\overline{F}$</td>
<td>$e^{-2S_0}, e^{-3S_0}$</td>
</tr>
</tbody>
</table>

Table 1. Topological excitations which determine the mass gap for gauge fluctuations and chiral symmetry realization in vectorlike and chiral gauge theories on $\mathbb{R}^3 \times S^1$. Unless indicated otherwise,

+ $SO(N), SP(N)$ - S. Golkar 0909.2838; for mixed-representation/higher-index reps. SU(N) - PU 0910.1245

name codes:

U=Unsal
S=Shifman
Y=Yaffe
P=the speaker
So, I have now introduced all the key players:

| 3d Polyakov model & “monopole-instanton”-induced confinement | Polyakov, 1977 |
| “monopole-instantons” on $\mathbb{R}^3 \times S^1$ | K. Lee, P. Yi, 1997  
| | P. van Baal, 1998 |
| the relevant index theorem | Nye, Singer, 2000  
| | Unsal, EP, 2008 |
| center-symmetry on $\mathbb{R}^3 \times S^1$ - adjoint fermions or double-trace deformations | Shifman, Unsal, 2008  
| | Unsal, Yaffe, 2008 |
| | Unsal, EP, 2009 |
The upshot is the dual lagrangian of QCD(adj) on a circle of size $L$:

$$
\frac{g^2(L)}{2L} (\partial \sigma)^2 - \frac{b}{L^3} e^{-2S_0} \cos 2\sigma + i \bar{\lambda}^I \gamma_\mu \partial_\mu \lambda^I + \frac{c}{L^{3-2N_f}} e^{-S_0} \cos \sigma (\det \lambda^I \lambda^J + \text{c.c.})
$$

leading-order perturbation theory; perturbative corrections $\sim g_4(L)^2$ omitted

$$
M_0 \sim \frac{1}{L} e^{-S_0} = \frac{1}{L} e^{-\frac{8\pi^2}{N_c g_4^2(L)}}
$$

$$
(\Lambda L)^{\beta_0} = e^{-\frac{8\pi^2}{g_4^2(L)}}
$$

$$
\beta_0 = \frac{2}{3} N_c - \frac{2}{3} N_w N_c
$$

$$
M_0 = \frac{1}{L} (\Lambda L)^{\beta_0} = \Lambda (\Lambda L)^{\beta_0} = \Lambda (\Lambda L)^{-1} \frac{8 - 2N_w}{3}
$$

mass gap $\sim$ string tension behaves in an interesting way

as $L$ changes at fixed $\Lambda$ ...

$N_w^* = 4$ ?
Region of validity of semiclassical analysis:

\[ \Lambda L \ll 1 \quad \left( N_c \Lambda L \ll 1, \text{ really} \right) \]

\[ m_\sigma \sim \Lambda \left( \Lambda L \right) \]

\[ \Lambda \sim \Lambda N_c \lan \]

\[ \Lambda \sim \Lambda N_c \]

Analysis shows that this switch of behavior as number of fermion species is increased occurs in all theories - vectorlike or chiral alike.

In each case we obtain a value for the critical number of "flavors" or "generations"... \( N_f^* \)

like \( N_w^* = 4 \) for QCD(adj)

does it tell us anything about \( R^4 \)?

(what follows is the promised not-so-rigorous part)
I know I am in danger of being arrested...

...how dare you study non-protected quantities?
A reasonable expectation of what happens at very small or very large number of “flavors” is this:

- **sufficiently small # fermion species**
  - confining theories
  - topological excitations become non-dilute with increase of $L$, cause confinement, $M$, KK+* operators
  - $e^{-S_0} \cos \left( \det \sum_{I,J} \lambda^I \lambda^J + c.c. \right)$
  - become strong, can cause chiral symmetry breaking (whenever the confining theories break their nonabelian chiral symmetries)

- **sufficiently large # fermion species**
  - fixed point at weak coupling
  - conformal in IR, no mass gap
  - topological excitations that cause confinement dilute with increase of $L$, no confinement

but where does the transition **really** occur? is it at our value $N_f^*$?

there appear to be three possibilities (in any given class of theories, only one is realized)

A.) our $N_f^*$ is the true critical value $N_{crit}$ [theory that may be in this class: QCD(adj), experiment (lattice)]
B.) if, as # species is increased above $N_f^*$

sufficiently small # fermion species
confining theories

then, $N_{\text{crit}} > N_f^*$
true value of critical # “flavors”

thus, for such theories $N_f^*$ is a lower bound thereof

[theory believed to be in this class: QCD(F) - arguments using mixed reps., experiment (lattice)]
c.) if, as \( \# \) species has not yet reached \( N_f^* \)

\[ N_f < N_f^* \]

thus, for this class of theories \( N_f^* \) is an upper bound on critical \( \# \) "flavors"

then,

\[ N_{\text{crit}} < N_f^* \]

sufficiently small \( \# \) fermion species confining theories

increase \( \# \) fermions

sufficiently large \( \# \) fermion species fixed point at weak coupling conformal in IR, no mass gap

[only one theory we know is believed to be in this class: SU(2) 4-index symmetric tensor Weyl, theory arguments]
comparing theory estimates of critical number of fermions for SU(N)

**Weyl adjoints** [no deformation needed]

<table>
<thead>
<tr>
<th>N</th>
<th>our estimate</th>
<th>gap eqn</th>
<th>beta function gamma=2/1</th>
<th>AF lost</th>
</tr>
</thead>
<tbody>
<tr>
<td>any N</td>
<td>4</td>
<td>4.15</td>
<td>2.75/3.66</td>
<td>5.5</td>
</tr>
</tbody>
</table>

“experiment”

? e.g.:
Catterall et al; del Debbio,
Patella, Pica; Hietanen et al.

**Dirac 2-index (anti)symmetric tensor** [deformation needed; but large-N equiv!]

<table>
<thead>
<tr>
<th>N</th>
<th>our estimate</th>
<th>gap eqn</th>
<th>beta function gamma=2/1</th>
<th>AF lost</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2.40</td>
<td>2.50</td>
<td>1.65/2.2</td>
<td>3.30</td>
</tr>
<tr>
<td>4</td>
<td>2.66</td>
<td>2.78</td>
<td>1.83/2.44</td>
<td>3.66</td>
</tr>
<tr>
<td>5</td>
<td>2.85</td>
<td>2.97</td>
<td>1.96/2.62</td>
<td>3.92</td>
</tr>
<tr>
<td>10</td>
<td>3.33</td>
<td>3.47</td>
<td>2.29/3.05</td>
<td>4.58</td>
</tr>
<tr>
<td>∞</td>
<td>4</td>
<td>4.15</td>
<td>2.75/3.66</td>
<td>5.5</td>
</tr>
</tbody>
</table>

2 e.g.:
DeGrand, Shamir,
Svetitsky;
Fodor et al;
Kogut, Sinclair

**Dirac fundamentals** [deformation needed]

<table>
<thead>
<tr>
<th>N</th>
<th>our estimate (a/c)</th>
<th>gap eqn</th>
<th>functional RG</th>
<th>beta function gamma=2/1</th>
<th>AF lost</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5/8</td>
<td>7.85</td>
<td>8.25</td>
<td>5.5/7.33</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>7.5/12</td>
<td>11.91</td>
<td>10</td>
<td>8.25/11</td>
<td>16.5</td>
</tr>
<tr>
<td>4</td>
<td>10/16</td>
<td>15.93</td>
<td>13.5</td>
<td>11/14.66</td>
<td>22</td>
</tr>
<tr>
<td>5</td>
<td>12.5/20</td>
<td>19.95</td>
<td>16.25</td>
<td>13.75/18.33</td>
<td>27.5</td>
</tr>
<tr>
<td>10</td>
<td>25/40</td>
<td>39.97</td>
<td>n/a</td>
<td>27.5/36.66</td>
<td>55</td>
</tr>
<tr>
<td>∞</td>
<td>2.5N/4N</td>
<td>4N</td>
<td>~ (2.75 – 3.25)N</td>
<td>2.75N/3.66N</td>
<td>5.5N</td>
</tr>
</tbody>
</table>

12 e.g.:
Appelquist, Fleming, Neal;
Deuzemman, Lombardo, Pallante;
Iwasaki et al;
Fodor et al;
Jin, Mahwinney;
A. Hasenfratz

gap equation and lattice - only vectorlike theories;
beta function (Ryttov/Sannino)

in chiral gauge theories with multiple “generations” our estimates were the only known ones until Sannino’s recent 0911.0931 via the proposed exact beta function
Conclusions I:
Compactifying 4d gauge theories on a small circle is a “deformation” where nonperturbative dynamics is under control - dynamics as “friendly” as in SUSY, e.g. Seiberg-Witten. (regime of validity: $\Lambda LN << 1$ complimentary to EK: $\Lambda LN >> 1$)

Confinement is due to various “oddball” topological excitations, in most theories non-self-dual.

Polyakov’s “Debye screening” mechanism works on $R^3 \times S^1$ also with massless fermions, contrary to what many thought - KK monopoles and index theorem-crucial ingredients of analysis.

Precise nature - monopoles, bions, triplets, or quintets - depends on the light fermion content of the theory.

U,P; 0812.2085, 0906.5156
Conclusions II:

Didn’t have time for these:

Found chiral symmetry breaking (Abelian) due to expectation values of topological “disorder” operators: occurs in mixed-rep. theories with anomaly-free chiral U(1), broken at any radius

Circle compactification gives another calculable deformation of SUSY theories - not yet fully explored -

in I=3/2 SU(2) Intriligator-Seiberg-Shenker model we argued that theory conformal, rather than SUSY-breaking.
Conclusions III:

Gave “estimates” of conformal window boundary in vectorlike and chiral gauge theories (OK with “experiment” when available).

Conformality tied to relevance vs irrelevance of topological excitations. Perhaps of interest especially in theories where chiral symmetries do not break.

Now, clearly,

on $\mathbb{R}^3 \times S^1$ we only see the “shadow” of the real thing...
Conclusions IV: Questions?

Is it so crazy to expect “relevance vs. irrelevance” (with changing Nf) of topological excitations also in $\mathbb{R}^4$?

Lattice studies in pure YM (early ref.: Kronfeld et al, 1987) have found that confinement appears to be due to topological excitations - center vortices, monopoles - these are ‘t Hooft’s (1978) “transient particles” that are revealed to us in particular gauges - and the deconfinement transition at high-T is associated with them becoming irrelevant.

... huge body of literature (mostly in pure YM) & apparently not much agreement in the details...

To expect that massless fermions would affect the nature of topological excitations is thus quite natural.

What is harder (for me?) is how to make this precise on $\mathbb{R}^4$.

So, back to SUSY? - theorists’ “safe haven”
We argued that “bions” are responsible for confinement in N=1 SYM at small L (a particular case of our Weyl adjoint theory).

This remains true if N=1 obtained from N=2 by soft breaking.

Monopoles and dyons are responsible for confinement in N=2 softly broken to N=1 at large L. (Seiberg, Witten `94)

So, in different regimes we have different pictures of confinement in N=1 SYM.

Do they connect in an interesting way?

... we think yes (Unsal, EP - 201x)