Monopoles, bions, and other oddballs in confinement or conformality

Erich Poppitz — U of Toronto

with Mithat Ünsal — SLAC/Stanford

This talk is about gauge dynamics.

There are many things one would like to understand about any gauge theory:

- does it confine?
- does it break its (super) symmetries?
- is it conformal?
- what are the spectrum, interactions...?

These are tough to address, in almost all theories.

But interesting for:

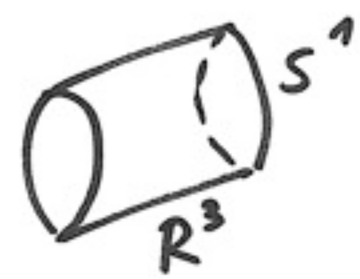
satisfying curiosity
QCD
SUSY extensions of the Standard Model
non-SUSY extensions of the Standard Model

pure YM	- "formal" but see www.claymath.org/millennium/
SUSY	- very "friendly" to theorists beautiful - exact results
QCD-like (vectorlike)	- hard, leave it to lattice folks (m, a, V, $)
non-SUSY chiral gauge theories	- poorly understood strong dynamics ...almost nobody talks about them anymore

What I'll talk about applies to all of these theories...

The theme of my talk is about inferring properties of infinite-volume theory by studying (arbitrarily) small-volume dynamics.

The small volume may be



most of this talk

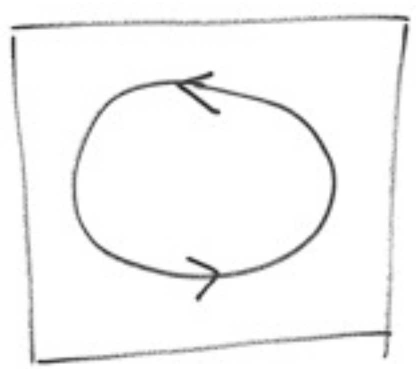
or

of characteristic size "L"

To put my talk in context, some relevant history:

Eguchi and Kawai (1982) showed that loop (Schwinger-Dyson) equations for Wilson loops in pure Yang-Mills theory are identical in small-V and infinite-V theory, to leading order in 1/N, **provided**:

- "center-symmetry" unbroken
- translational symmetry unbroken (see Yaffe, 1982)



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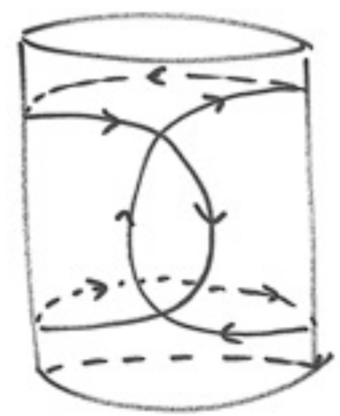


+ O(1/N) **provided**

expectation value of any Wilson loop at infinite-L

expectation value of (folded) Wilson loop at small-L

topologically nontrivial (winding) Wilson loops have vanishing expectation value (= unbroken center)

"EK reduction" or "large-N reduction" or "large-N volume-independence"

(Note: this is an exact result in QFT - so long as unbroken center.)

It could be **potentially exciting**, for:

1) **simulations may be cheaper** (use single-site lattice?)
2) **raises theorist's hopes** (that small-L easier to solve?)

To put my talk in context, some relevant history:

From a “modern” point of view EK reduction is a large-N orbifold with respect to the group of translations.

Kovtun, Unsal, Yaffe (2004)

Volume-independence viewed as an orbifold helps establish that VEVs and correlators of operators that are center-neutral and carry momenta quantized in units of 1/L (in compact direction) are the same on, say

as in infinite-L theory, to leading order in 1/N.

Thus, a working example of EK would be good for

- calculating vevs (symmetry breaking)
  - OK, even if all dimensions small
- calculating spectra (for generic theories/reps)
  - need at least one large dimension

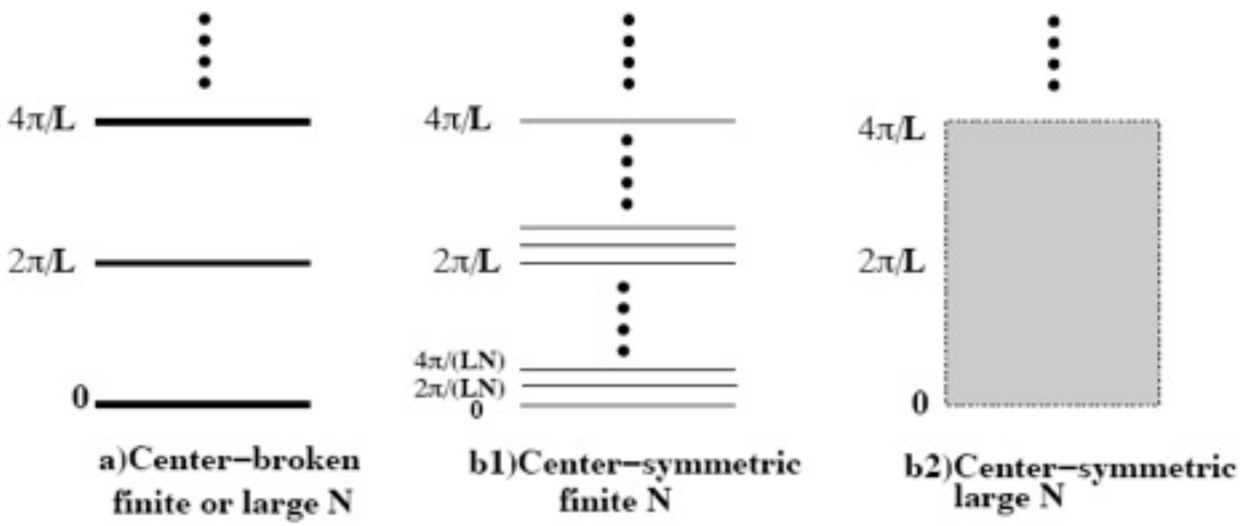
(... scattering for LHC - all large dimensions)

To put my talk in context, some relevant history:

Some intuition of how EK reduction works (valid at any coupling).

in perturbation theory: or

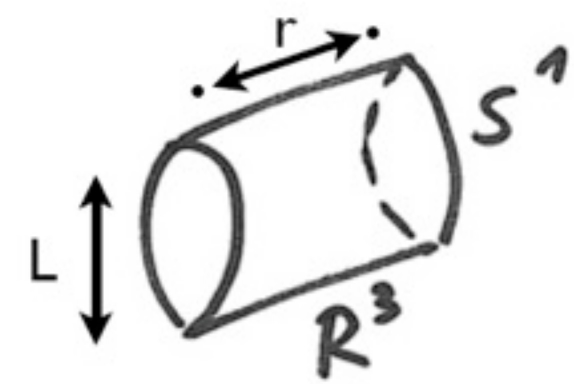
from spectra (& Feynman graphs) in appropriate backgrounds



at strong coupling:

gravity dual of N=4 SYM - a conformal field theory - Wilson loops, appropriate correlators - insensitive to box **if** center-symmetric

V(r) ~ 1/r : CFT result obtains
in center-symmetric vacuum
for any r (<L or >L)
insensitive to box size



Unsal, EP 2010

However, Bhanot, Heller, Neuberger (1982) noticed immediate problem

- center symmetry breaks for $L < L_c$ and thus invalidates EK reduction

remedies: e.g., Gonzales-Arroyo, Okawa (1982) - TEK... + others
later argued to have problems
.... see recent "twists" on TEK ?

To put my talk in context, some relevant history:

Unsal, Yaffe 2008

Remedies proposed: reduction valid to arbitrarily small L (single-site) if:

adjoint fermions (more than one Weyl) - no center breaking, so reduction holds at all L

double-trace deformations: deform measure to prevent center breaking; deformation "drops out" of loop equations at infinite-N

used for current **lattice studies** of "minimal walking technicolor"

is 4 ...3,5... Weyl adjoint theory conformal or not?

small-L(=1) large-N simulations (2009-)
Hietanen-Narayanan; Bringoltz-Sharpe; Catterall et al

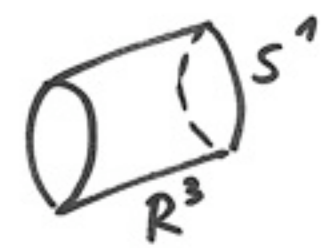
small-N large-L simulations (2007-)
Catterall et al; del Debbio et al; Hietanen et al...

(many issues to still be resolved...)

THIS TALK:

theoretical studies

Unsal;
Unsal-Yaffe;
Unsal-Shifman;
Unsal-EP 2007-10



fix-N, take L-small: **semiclassical studies of confinement** due to novel strange "oddball" (nonselfdual) topological excitations, whose nature depends on fermion content

- for vectorlike or chiral theories, with or without supersymmetry

- **a complementary regime to that of volume independence, which requires infinite N - a (calculable!) shadow of the 4d "real thing".**

The plan is to tell you,
largely in pictures,
what this story amounts to.

time

Index theorem for topological excitations on R*3 x S*1 and Chern-Simons theory
JHEP 0903:027,2009; 0812.2085, 29pp

Chiral gauge dynamics and dynamical supersymmetry breaking
JHEP 0907:060,2009; 0905.0634, 31pp

Conformality or confinement: (IR)relevance of topological excitations
JHEP 0909:050,2009; 0906.5156, 42pp

Conformality or confinement (II): One-flavor CFTs and mixed-representation QCD
JHEP 0912:011,2009; 0910.1245, 33pp

inf{rigour}

All by M. Unsal and E.P. + work in progress on relation to Seiberg-Witten confinement

First, the key players:

3d Polyakov model & "monopole-instanton"-induced confinement — Polyakov, 1977

"monopole-instantons" on $R^3 \times S^1$ — K. Lee, P. Yi, 1997; P. van Baal, 1998

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center-symmetry on $R^3 \times S^1$ - adjoint fermions or double-trace deformations — Shifman, Unsal, 2008; Unsal, Yaffe, 2008

"bions", "triplets", "quintets"... - new non-self-dual topological excitations and confinement — Unsal, 2007; Unsal, EP, 2009

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Polyakov, 1977

continuum picture: 3d Georgi-Glashow [on the lattice - compact U(1)]

$$L \sim \frac{1}{g_3^2}\left(F^a_{\mu\nu} F^a_{\mu\nu} + D_\mu \phi^a D^\mu \phi^a\right) \qquad \mu, \nu = 1, 2, 3$$

$$[A_\mu] = [\phi] = 1 \qquad [g_3^2] = 1$$

due to some Higgs potential $\langle \phi \rangle = (0, 0, v)$

$SU(2) \xrightarrow{v} U(1)$ at low energies, $E \ll m_W \sim v$

free U(1) theory $A^3_\mu \equiv A_\mu$

$$\mathcal{L}_{eff} = \frac{1}{g_3^2} F^2_{\mu\nu} + \ldots$$

"..." are perturbatively calculable & not very interesting

$$B_\mu = \epsilon_{\mu\nu\lambda} F_{\nu\lambda}$$

"magnetic field"
topologically conserved current of **"emergent topological U(1) symmetry"** responsible for conservation of magnetic charge

$$B_\mu = g_3^2 \, \partial_\mu \sigma$$

3d photon dual to scalar (as one polarization only)

Abelian duality

$$\partial_\mu B_\mu = 0 \qquad \longleftrightarrow \qquad \partial_\mu^2 \sigma = 0$$

Bianchi identity $\qquad\qquad$ equation of motion

$$\mathcal{L}_{eff} = \frac{1}{g_3^2} F_{\mu\nu}^2 + \ldots \qquad\qquad \mathcal{L}_{eff} = g_3^2 (\partial_\mu \sigma)^2 + \ldots$$

topological U(1) symmetry = shift of "dual photon"

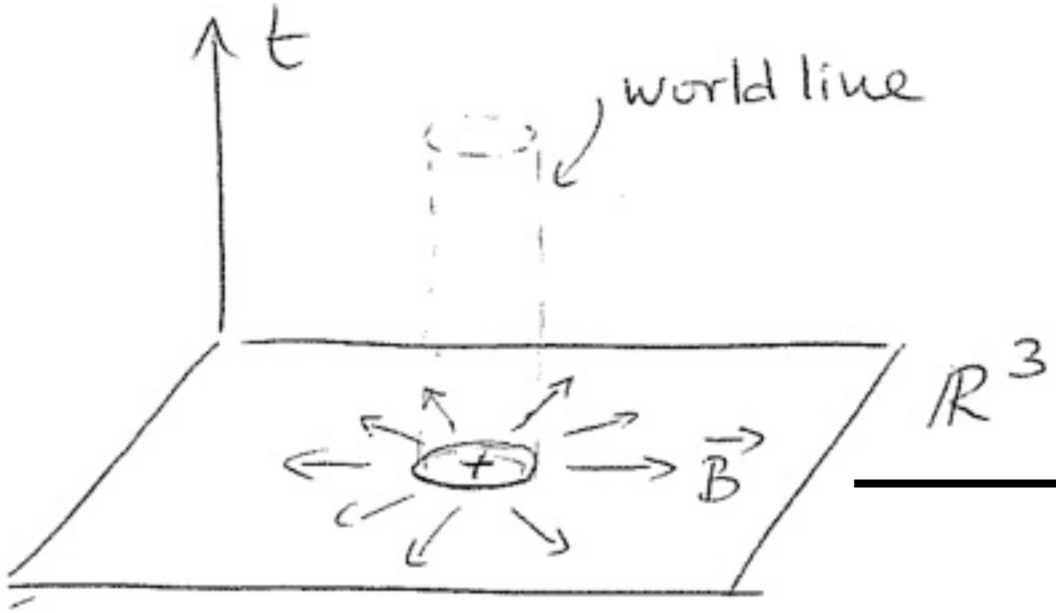
a rather ***"boring-boring" duality*** - if not for the existence of monopoles:

monopoles $\partial_\mu B_\mu$ = quantized magnetic charge - shift symmetry broken

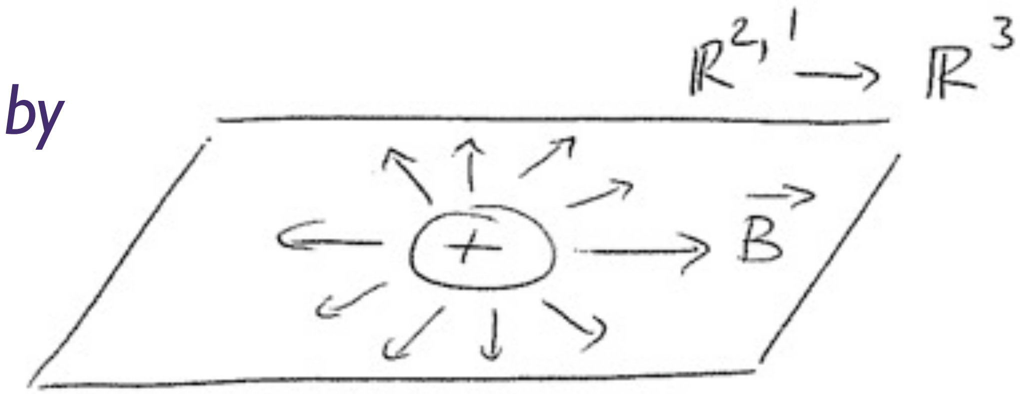
- dual photon gains mass & electric charges confined

how?
...in pictures:

" 't Hooft-Polyakov monopole" - ***static finite energy solution of Georgi-Glashow model in 4d***



get Euclidean 3d by "forgetting time"



solution of Euclidean eqns. of motion of finite action: a "monopole-instanton"

$$E_M = \frac{4\pi v}{g_4^2} \longrightarrow S_0 = \frac{4\pi v}{g_3^2}$$

$$e^{-S_0} \xrightarrow[g_3^2/v \to 0]{} 0$$

M-M* pairs give exponentially suppressed (at weak coupling) "semiclassical" contributions to the vacuum functional

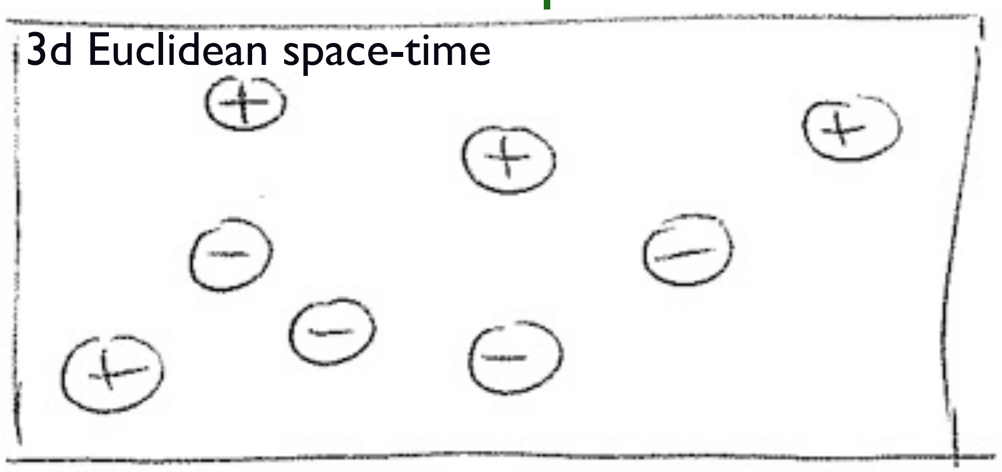
vacuum "is" a dilute monopole-antimonopole plasma

number of M's per unit volume ~ $v^3 e^{-S_0}$

(analogous to B+L violation in electroweak model; at T=0 exponentially small)

vacuum is a dilute M-M* plasma - but interacting, unlike instanton gas in 4d (in say, electroweak theory)

in pictures & in formulae



really meaning grand partition function of classical 3d M-M* plasma

Z =
Z(perturbative)x
Z(charged plasma with Coulomb interactions)

physics is that of Debye screening - analogy:

electric fields are screened in a charged plasma ("Debye mass for photon"), so in the monopole-antimonopole plasma, the dual photon obtains mass from screening of magnetic field:

$$\mathcal{L}_{eff} = g_3^2 (\partial\sigma)^2 + (\#)\, v^3 e^{-S_0}\left(e^{i\sigma} + e^{-i\sigma}\right) + ..$$

"(anti-)monopole operators"

aka **"disorder operators"** - not locally expressed in terms of original gauge fields

(Kadanoff-Ceva; 't Hooft - 1970s)

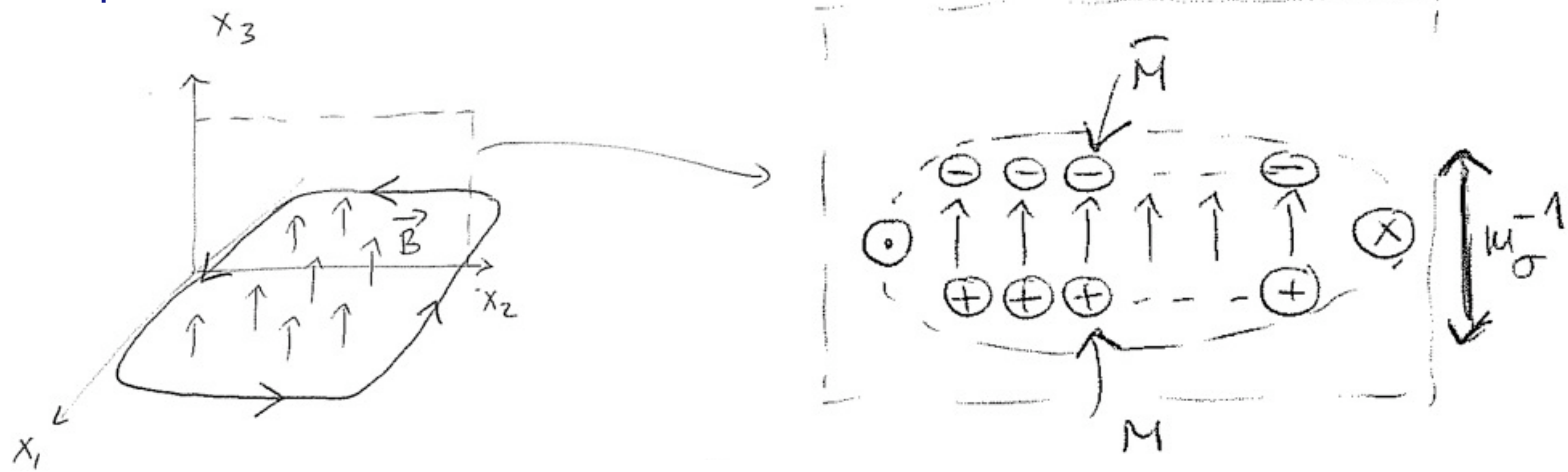
also by analogy with Debye mass:

dual photon mass^2 ~ M-M* plasma density

$$m_\sigma \sim v\, e^{-S_0/2} = v\, e^{-\frac{4\pi v}{2 g_3^2}}$$

next:
dual photon mass
~ confining string tension...

in pictures:

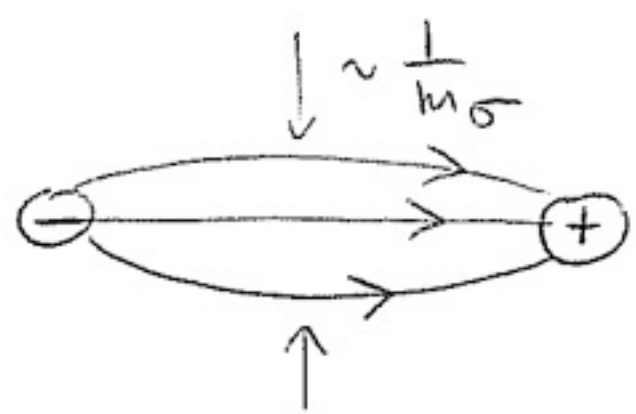


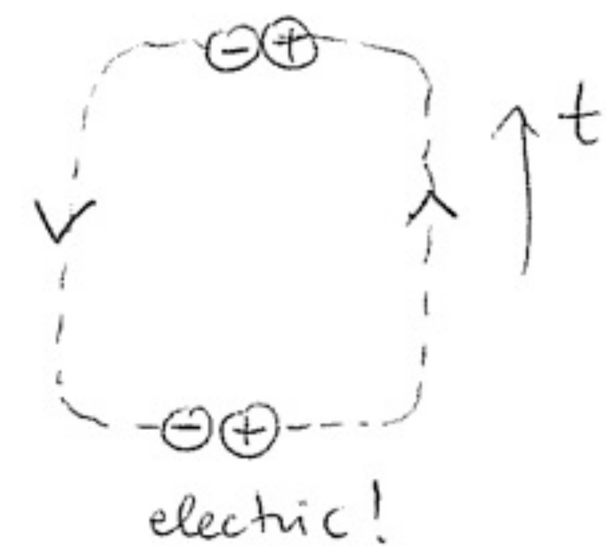
screening of magnetic field in plasma
= Wilson loop area law:

$$\left\langle e^{i\oint A\,dx} \right\rangle \sim e^{-(\mathrm{Area})\, m_\sigma g_3^2}$$

Minkowski space interpretation of Wilson loop: x_1 - time

electric field





confining flux tube: **tension^{-1} ~ thickness ~ inverse dual photon mass**

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"bions", "triplets", "quintets"... - new non-self-dual topological excitations and confinement — Unsal, 2007; Unsal, EP, 2009

First, the key players:

we want to go to 4d - by "growing" a compact dimension: $S^1: \quad x^4 \sim x^4 + L$

"monopole-instantons" on $R^3 \times S^1$

K. Lee, P.Yi, 1997
P. van Baal, 1998

A_4 is now an adjoint 3d scalar Higgs field $\partial_4 + A_4 \longrightarrow \frac{2\pi n}{L} + A_4$

but it is a bit unusual -
a compact Higgs field: $\langle A_4 \rangle \sim \langle A_4 \rangle + \frac{2\pi}{L}$ such shifts of A_4 vev absorbed into shift of KK number "n"

thus, natural scale of "Higgs vev" is $\langle A_4 \rangle \sim \frac{\pi}{L}$ leading to $SU(2) \xrightarrow{\frac{1}{L}} U(1)$

(clearly, semiclassical and weakly coupled if L << inverse strong scale)

A_4 - adjoint 3d scalar Higgs field;
a gauge-covariant description:

$$W = P\, e^{i \oint_{S_1} A_4\, dx^4}$$

"holonomy" around circle
(or "Polyakov loop")
- a unitary gauge-group element
- eigenvalues lie on unit circle
- trace of Polyakov loop is gauge invariant

if the expectation values are

$$\langle W \rangle = \begin{pmatrix} e^{i\pi/2} & \\ & e^{-i\pi/2} \end{pmatrix}$$

then $\mathrm{tr} \langle W \rangle = 0$

and we say that *"center symmetry is preserved"*

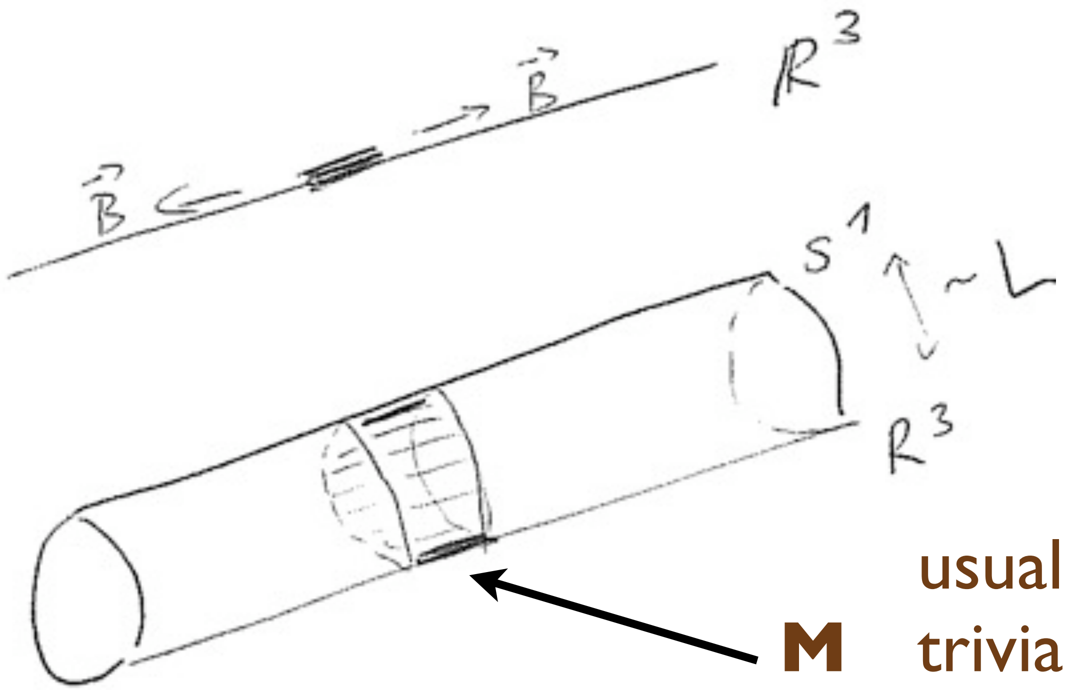
$$\mathrm{tr}\, W \to e^{i\pi}\, \mathrm{tr}\, W$$

for SU(N): $e^{i\frac{2\pi}{N}}$

we are interested in **unbroken center**:
where <trW>=0 and SU(2) broken to U(1) at scale 1/L

$$\langle W \rangle = \begin{pmatrix} e^{i\pi/2} & \\ & e^{-i\pi/2} \end{pmatrix}$$

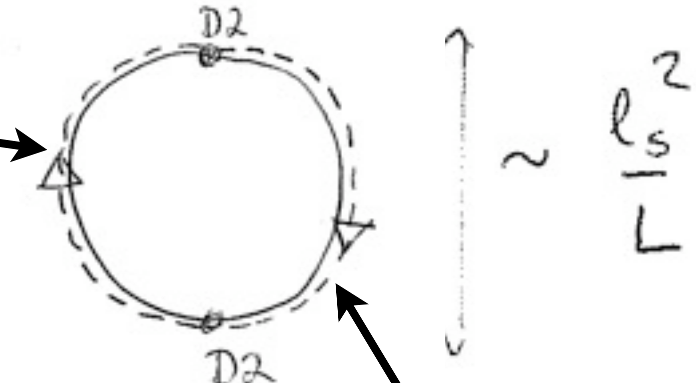
breaks SU(2) to U(1) so there are monopoles:



KK discovered by K. Lee, P. Yi, 1997, as *"Instantons and monopoles on partially compactified D-branes"*

M

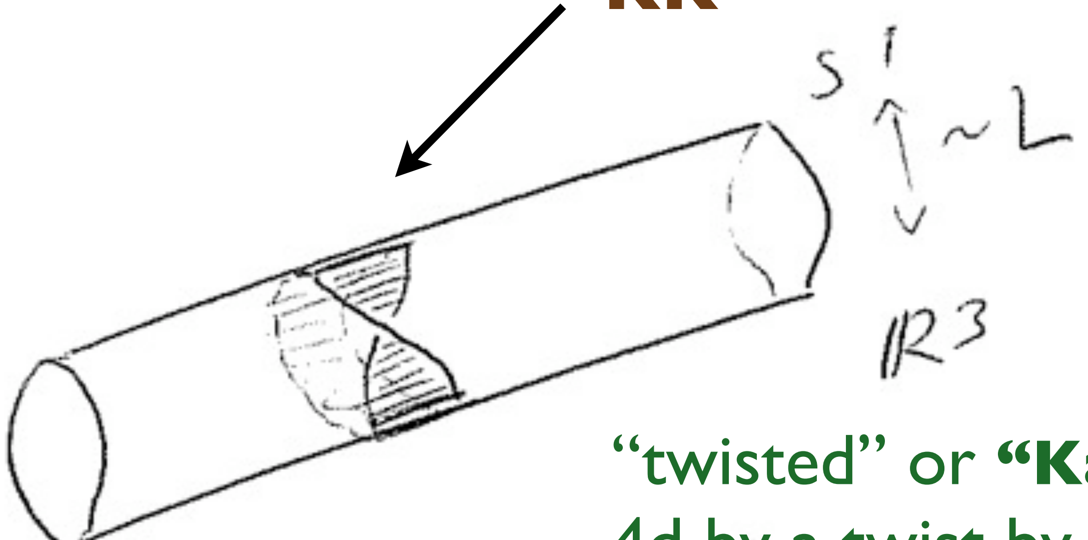
usual monopole trivially embedded in 4d



M
Euclidean D0-brane

KK
Euclidean D0-brane

KK



"twisted" or **"Kaluza-Klein":** monopole embedded in 4d by a twist by a "gauge transformation" periodic up to center - in 3d limit not there! (infinite action)

Summary: "elementary" topological excitations on R3xS1

M & KK both self-dual objects, of opposite magnetic charges

	magnetic charge	topological charge	suppression
M	$+1$	$1/2$	e^{-S_0}
KK	-1	$1/2$	e^{-S_0}
BPST	0	1	e^{-2S_0}

+ their anti-"particles"

- thus, BPST instanton " = M+KK "

(also see P. van Baal, 1998)

$$e^{-S_0} = e^{-\frac{4\pi v}{g_3^2}} = e^{-\frac{4\pi^2}{L g_3^2}} = e^{-\frac{4\pi^2}{g_4^2(L)}}$$

$$SU(N): \quad e^{-S_0} = e^{-\frac{8\pi^2}{g_4^2(L)\,N}}$$

$\downarrow$

(large-N survive!)

M & KK have, in SU(N), 1/N-th of the 't Hooft suppression factor

aka: "fractional instantons", "instanton quarks", "zindons", "quinks", "instanton partons"... [collected by D. Tong]

Next, to understand the role M, KK, M* & KK* play in various theories of interest, need to know what happens to the operators they induce when there are fermions in the theory.

First, the key players:

3d Polyakov model & “monopole-instanton”-induced confinement — Polyakov, 1977

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the relevant index theorem — Nye, Singer, 2000; Unsal, EP, 2008

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“bions”, “triplets”, “quintets”... - new non-self-dual topological excitations and confinement — Unsal, 2007; Unsal, EP, 2009

First, the key players:

the relevant index theorem

Nye, Singer, 2000
Unsal, EP, 2008

- for some theories the answer for the number of zero modes in M or KK background had been guessed (correctly)

  e.g. SUSY YM - Aharony, Hanany, Intriligator, Seiberg, Strassler, 1997

- while studying ISS(henker) proposal for SUSY breaking model [SU(2)+three-index symm. tensor Weyl] Unsal and I needed a general index theorem

- we found this:

Journal of Functional Analysis **177**, 203–218 (2000)
doi:10.1006/jfan.2000.3648, available online at http://www.idealibrary.com on IDEAL®

An L^2-Index Theorem for Dirac Operators on $S^1 \times \mathbb{R}^3$

Tom M. W. Nye and Michael A. Singer

where, in APPENDIX A. ADIABATIC LIMITS OF η-INVARIANTS

we found:

$$\operatorname{ind}(D_{\mathbb{A}}^{+}) = \int_X \operatorname{ch}(\mathbb{E}) + \frac{1}{\mu_0} \sum_{\mu} \epsilon_\mu c_1(E_\mu)[S^2_\infty]$$

$$= \int_X \operatorname{ch}(\mathbb{E}) - \frac{1}{2}\overline{\eta}_{\mathrm{lim}}$$

(last formula in paper)

two obvious questions:

1.) where does this come from?

2.) what number is it equal to in a given topological background (M,KK...) & how does it depend on ratio of radius to holonomy?

for answers & more

see M. Unsal, EP 0812.2085

like on R^3 Callias $\xleftarrow{\text{physicist derivation}}$ E. Weinberg, 1970s, but on $R^3 \times S^1$, so must incorporate anomaly equation, some interesting effects

for this talk it is enough to consider 4d SU(2) theories with N_W adjoint Weyl fermions

"applications":

$N_W=1$ is N=1 SUSY YM

$N_W=4$

- "minimal walking technicolor"
- happens to be N=4 SYM without the scalars

M KK M* KK* each have $2N_W$ zero modes

disorder operators:

M:

$$e^{-S_0} e^{i\sigma} (\lambda\lambda)^{N_W}$$

KK:

$$e^{-S_0} e^{-i\sigma} (\lambda\lambda)^{N_W}$$

M*:

$$e^{-S_0} e^{-i\sigma} (\bar\lambda\bar\lambda)^{N_W}$$

KK*:

$$e^{-S_0} e^{i\sigma} (\bar\lambda\bar\lambda)^{N_W}$$

where:

$$(\lambda\lambda)^{N_W} = \det_{\underset{SU(N_W)}{I,J}} \lambda^{\overset{SU(2)}{a}}_{\alpha I} \lambda^{a}_{\underset{SL(2,\mathbb{C})}{\beta} J} \epsilon^{\alpha\beta}$$

remarks:

- operator due to M+KK = 't Hooft vertex; independent of dual photon
- "our" index theorem interpolates between 3d Callias and 4d APS index thms.

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Shifman, Unsal, 2008
Unsal, Yaffe, 2008

"bions", "triplets", "quintets"... - new non-self-dual topological excitations and confinement
Unsal, 2007
Unsal, EP, 2009

First, the key players:

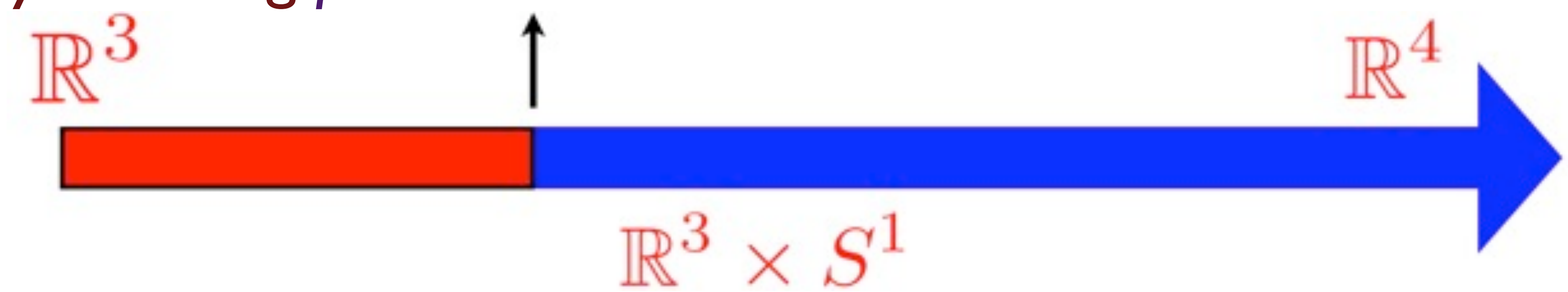
- Abelianization occurs only if there is a nontrivial holonomy (i.e., A_4 has vev)

- upon thermal circle compactifications, gauge theories with fermions do not Abelianize: center symmetry is broken at small circle size - transition to a deconfining phase - A_4 =0, $\langle trW \rangle \neq 0$ - deconfinement - at high-T, 1-loop V_{eff}
  (Gross, Pisarski, Yaffe, early1980s)

center-symmetry on $R^3 \times S^1$ - adjoint fermions or double-trace deformations

Shifman, Unsal, 2008
Unsal, Yaffe, 2008

in other words, in thermal setup, upon decompactification, we have a center-symmetry breaking *phase transition* and no smooth connection to R^4



to ensure calculability at small L and smooth connection to large L in the sense of center symmetry: *can one find ways to avoid phase transition?*

I. non-thermal compactifications - periodic fermions ("twisted partition function")

- with $N_W > 1$ adjoint fermions center symmetry preserved (Unsal, Yaffe 2007) as well as with other, "exotic" fermion reps (Unsal, EP 2009)
- in many supersymmetric theories, can simply choose center-symmetric vev

II. add double-trace deformations: force center symmetric vacuum at small L (also Shifman, Unsal 2008) - connection to large-N volume independence

`In what follows, we assume center-symmetric vacuum - due to either` **I**. **`or`** **II**. - will explicitly discuss only theory where center symmetry is naturally preserved at small L (**I**.)

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"bions", "triplets", "quintets"... - new non-self-dual topological excitations and confinement — Unsal, 2007; Unsal, EP, 2009

First, the key players:

ready to study the dynamics of theories with massless fermions on a small circle

in a vacuum with A_4 vev, Abelianization:

- in SU(2): (dual) photon massless + fermion components w/out mass from vev (neutral)
- monopoles + KK monopoles are the basic topological excitations

is there magnetic field screening in the vacuum?

the answer would appear to be "no":

M and KK have fermion zero modes

monopole operators do not generate potential for dual photon

so, **no screening & no confinement... ?**

"bions", "triplets", "quintets"... - new non-self-dual topological excitations and confinement

Unsal, 2007
Unsal, EP, 2009

but take a look at the symmetries first:

as an example, again consider 4d SU(2) theories with N_w adjoint Weyl fermions

classical global chiral symmetry is

$SU(N_w) \times U(1)$

but 't Hooft vertex $(\lambda\lambda)^{2N_w} e^{-\frac{8\pi^2}{g_4^2}}$ only preserves $\mathbb{Z}_{4N_w}: \lambda \to e^{i\frac{2\pi}{4N_w}} \lambda$

so, quantum-mechanically we have only $SU(N_w) \times Z_{4N_w}$ exact chiral symmetry

now **M, KK**(+*) operators all look like: $e^{-S_0} e^{i\sigma} (\lambda\lambda)^{N_w}$

hence $(\lambda\lambda)^{N_w} \xrightarrow{\mathbb{Z}_{4N_w}} e^{i\pi} (\lambda\lambda)^{N_w}$

invariance of **M**, **KK**(+*) operators under exact chiral symmetry means that

dual photon must transform under the exact chiral symmetry

i.e., topological shift symmetry is intertwined with chiral symmetry:

$$\mathbb{Z}_{4N_w}: \quad \sigma \to \sigma + \pi$$

$\sigma \to \sigma + \pi$ $\qquad \cancel{\cos\sigma} \qquad \cos(2\sigma)$ ✓

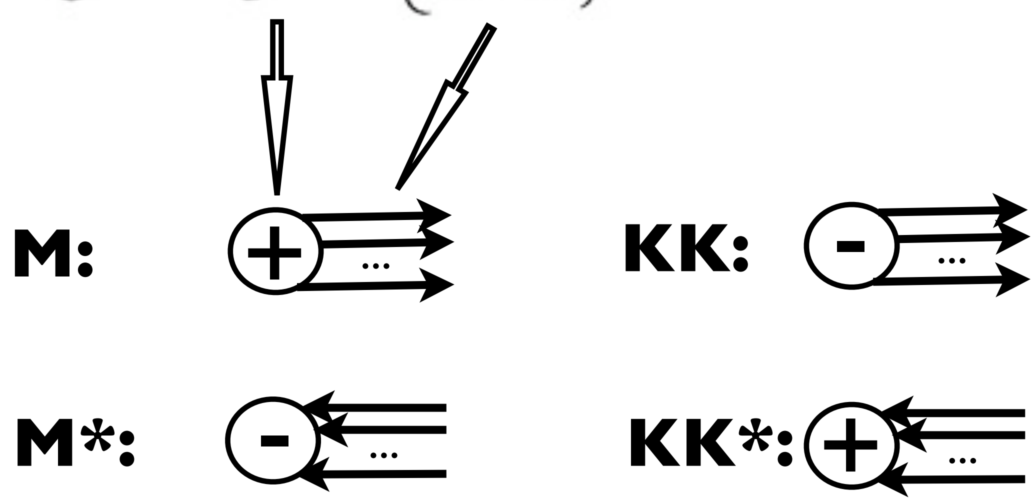
so the exact chiral symmetry allows a potential - **but what is it due to?**

to generate $\cos(2\sigma)$ must have

i. magnetic charge 2

ii. no zero modes

$$e^{-S_0}\, e^{i\sigma}\, (\lambda\lambda)^{N_w}$$



M + KK* bound state? (Unsal, 2007)

- same magnetic charge ~ 1/r-repulsion
- fermion exchange ~ log(r)-attraction

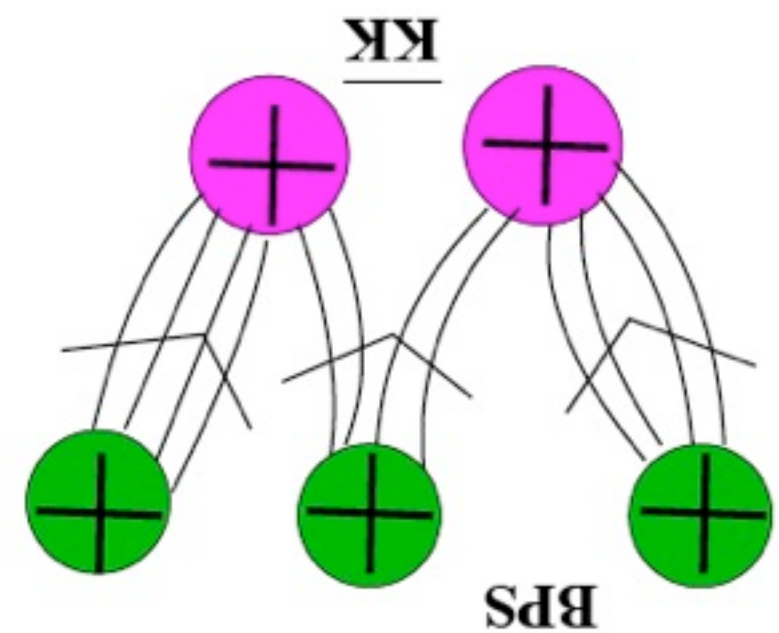
M + KK* = B - magnetic "bion"

size ~ $L/g_4^2(L) \gg L$ (our "lattice spacing")

$$e^{-2S_0}\left(e^{2i\sigma} + e^{-2i\sigma}\right)$$

dual photon mass is induced by magnetic "bions"- the leading cause of confinement in SU(N) with adjoints at small L (incl. SYM) to summarize, in QCD(adj),

cartoon of a "quintet":



M + KK* = B - magnetic "bions" -

-carry magnetic charge

-noo topological charge (non self-dual)

(locally 4d nature crucial: no KK in 4d)

-generate "Debye" mass for dual photon

main tools used:

- intertwining of topological shift symmetry & chiral symmetry
- index theorem

topological objects generating magnetic screening depend on massless fermion content (not usually thought that fermions relevant)

using these tools, one can analyze any theory...

in the last couple of years, many theories have been studied...

v e c t o r l i k e

c h i r a l

Theory all SU(N)	Confinement mechanism on $\mathbb{R}^3 \times S^1$	Index for monopoles $[\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_N]$ Nye-M.Singer '00; PU '08	Index for instanton $I_{inst.} = \sum_{i=1}^{N} I_i$ Atiyah-Singer	$(\text{Mass Gap})^2$ units $\sim 1/L^2$
YM Y,U '08	monopoles	$[0, \ldots, 0]$	0	e^{-S_0}
QCD(F) S,U '08	monopoles	$[2, 0, \ldots, 0]$	2	e^{-S_0}
SYM U '07 /QCD(Adj)	magnetic bions	$[2, 2, \ldots, 2]$	$2N$	e^{-2S_0}
QCD(BF) S,U '08	magnetic bions	$[2, 2, \ldots, 2]$	$2N$	e^{-2S_0}
QCD(AS) S,U '08	bions and monopoles	$[2, 2, \ldots, 2, 0, 0]$	$2N - 4$	e^{-2S_0}, e^{-S_0}
QCD(S) P,U '09	bions and triplets	$[2, 2, \ldots, 2, 4, 4]$	$2N + 4$	e^{-2S_0}, e^{-3S_0}
SU(2)YM $I = \frac{3}{2}$ P,U '09	magnetic quintets	$[4, 6]$ SUSY version: ISS(henker) model of SUSY [non-]breaking	10	e^{-5S_0}
chiral S,U '08 $[SU(N)]^K$	magnetic bions	$[2, 2, , \ldots, 2]$	$2N$	e^{-2S_0}
$AS + (N-4)\overline{\text{F}}$ S,U '08	bions and a monopole	$[1, 1, , \ldots, 1, 0, 0]$ + $[0, 0, \ldots, 0, N-4, 0]$	$(N-2)\text{AS} + (N-4)\overline{\text{F}}$	e^{-2S_0}, e^{-S_0},
$S + (N+4)\overline{\text{F}}$ P,U '09	bions and triplets	$[1, 1, , \ldots, 1, 2, 2]$ + $[0, 0, \ldots, 0, N+4, 0]$	$(N+2)\text{S} + (N+4)\overline{\text{F}}$	e^{-2S_0}, e^{-3S_0},

name codes:

U=Unsal
S=Shifman
Y=Yaffe
P=the speaker

Table 1. Topological excitations which determine the mass gap for gauge fluctuations and chiral symmetry realization in vectorlike and chiral gauge theories on $\mathbf{R}^3 \times \mathbf{S}^1$. Unless indicated otherwise,

+ SO(N),SP(N) - S. Golkar 0909.2838; for mixed-representation/higher-index reps. SU(N) - P,U 0910.1245

So, I have now introduced all the key players:

3d Polyakov model & "monopole-instanton"-induced confinement	Polyakov, 1977
"monopole-instantons" on $R^3 \times S^1$	K. Lee, P. Yi, 1997 P. van Baal, 1998
the relevant index theorem	Nye, Singer, 2000 Unsal, EP, 2008
center-symmetry on $R^3 \times S^1$ - adjoint fermions or double-trace deformations	Shifman, Unsal, 2008 Unsal, Yaffe, 2008
"bions", "triplets", "quintets"... - new non-self-dual topological excitations and confinement	Unsal, 2007 Unsal, EP, 2009

The upshot is the **dual lagrangian of QCD(adj)** on a circle of size L:

$$\frac{g^2(L)}{2L}(\partial\sigma)^2 - \frac{b}{L^3}\, e^{-2S_0}\cos 2\sigma + i\bar{\lambda}^I\gamma_\mu\partial_\mu\lambda_I + \frac{c}{L^{3-2N_f}}\, e^{-S_0}\cos\sigma(\det_{I,J}\lambda^I\lambda^J + \text{c.c.})$$

B, B*

M, KK+*

leading-order perturbation theory; perturbative corrections ~ $g_4(L)^2$ omitted

$$m_\sigma \sim \frac{1}{L} e^{-S_0} = \frac{1}{L} e^{-\frac{8\pi^2}{N_c g_4^2(L)}}$$

$$(\Lambda L)^{\beta_0} = e^{-\frac{8\pi^2}{g_4^2(L)}}$$

$$\beta_0 = \frac{11}{3} N_c - \frac{2}{3} N_W N_c$$

$$m_\sigma = \frac{1}{L}(\Lambda L)^{\frac{\beta_0}{N_c}} = \Lambda(\Lambda L)^{\frac{\beta_0}{N_c}-1} = \Lambda(\Lambda L)^{\frac{8-2N_W}{3}}$$

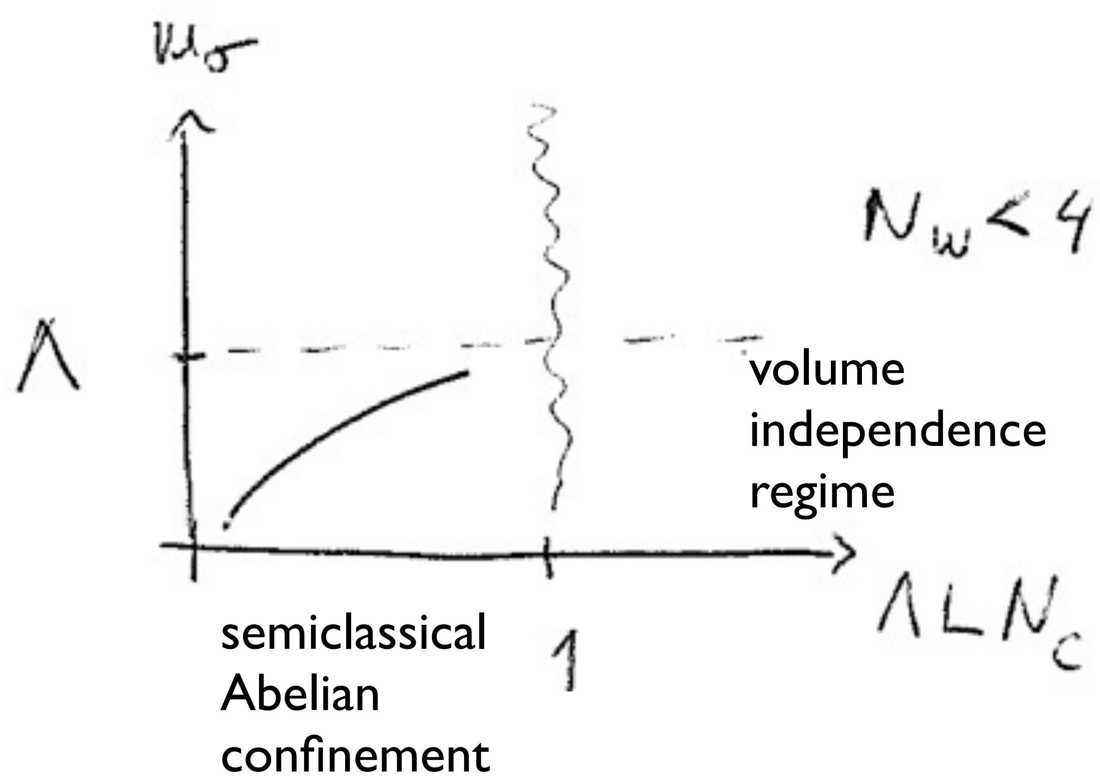
mass gap ~ string tension behaves in an interesting way

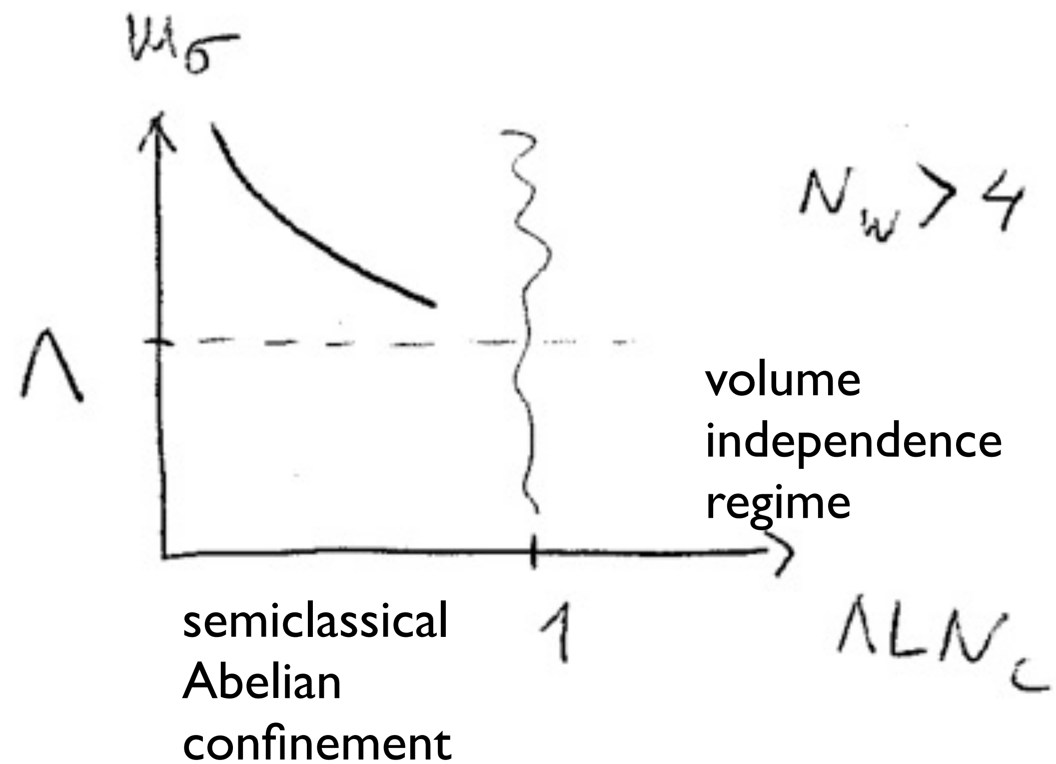
as L changes at fixed Λ ... $N_W^* = 4$?

region of validity of semiclassical analysis:

$$\Lambda L \ll 1 \qquad (N_c \Lambda L \ll 1 \text{, really})$$ as mass of W ~ 1/(NL)

$$m_\sigma \sim \Lambda (\Lambda L)^{(8-2N_w)/3}$$





analysis shows that this switch of behavior as number of fermion species is increased occurs in all theories - vectorlike or chiral alike

in each case we obtain a value for the critical number of "flavors" or "generations"... N_f^*

like $N_w^* = 4$ for QCD(adj)

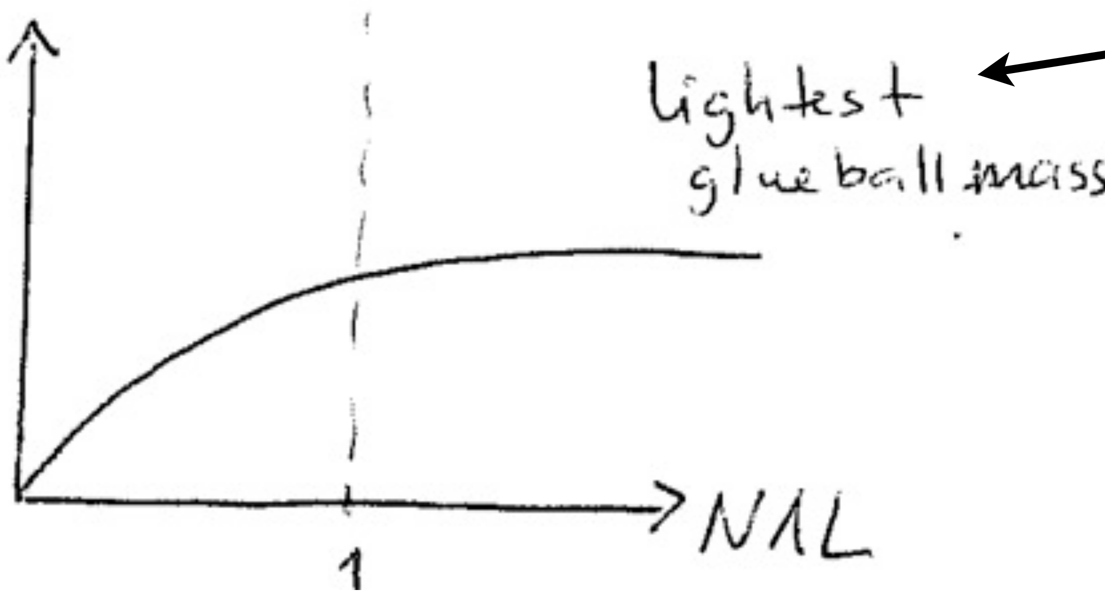
does it tell us anything about R^4?

(what follows is the promised not-so-rigorous part) ⟶

I know I am in danger of being arrested...

... how **dare** you study non-protected quantities?

A reasonable expectation of what happens at very small or very large number of "flavors" is this:



topological excitations become non-dilute with increase of L, cause confinement, M, KK+* operators

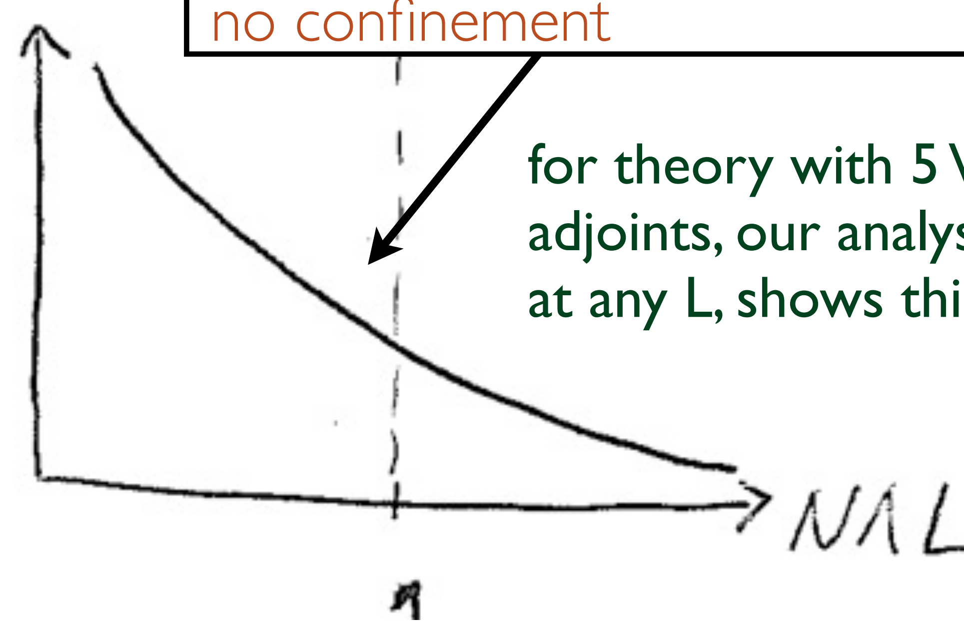
$$e^{-S_0}\cos\sigma(\det_{I,J}\lambda^I\lambda^J + \text{c.c.})$$

become strong, can cause chiral symmetry breaking (whenever the confining theories break their nonabelian chiral symmetries)

sufficiently small # fermion species
confining theories

sufficiently large # fermion species
fixed point at weak coupling
conformal in IR, no mass gap

topological excitations that cause confinement dilute with increase of L, no confinement



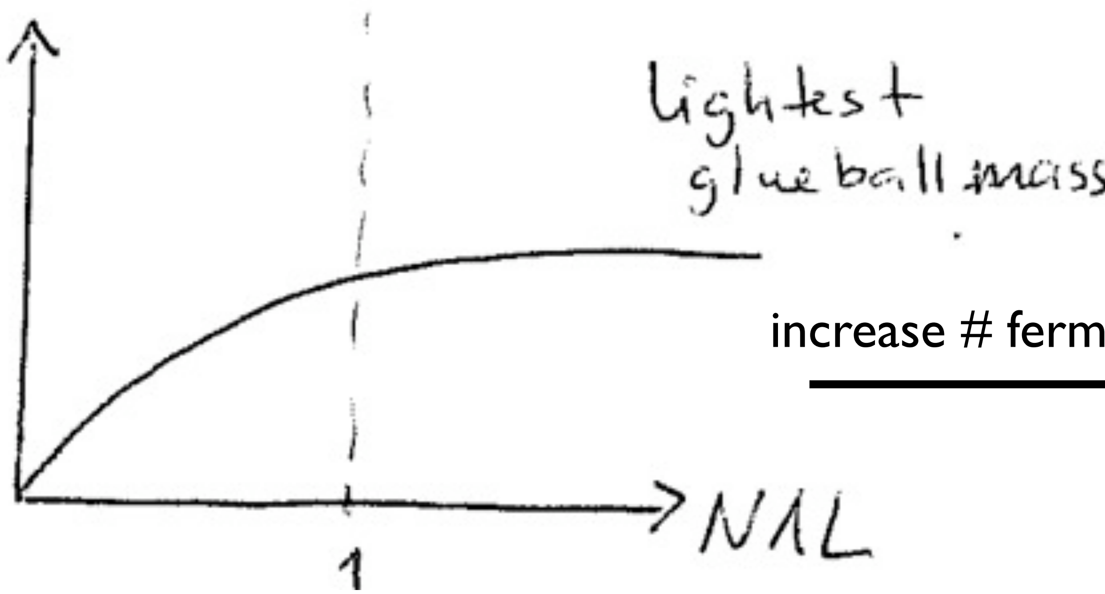
for theory with 5 Weyl adjoints, our analysis is valid at any L, shows this behavior

but where does the transition **really** occur?
is it at our value N_f*?

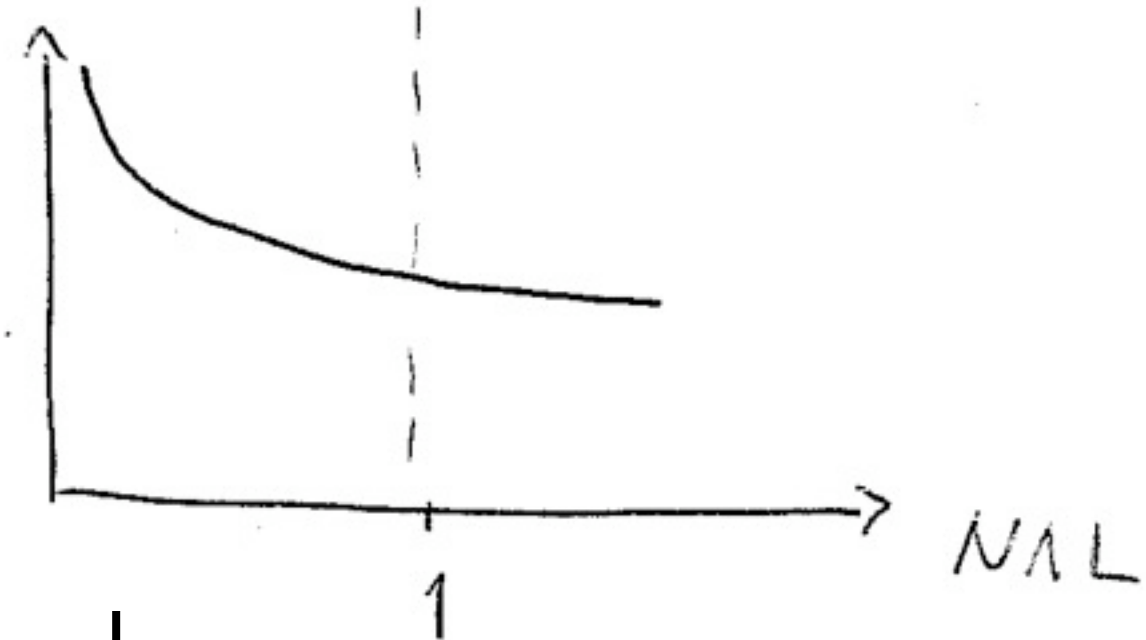
there appear to be three possibilities
(in any given class of theories, only one is realized)

A.) our N_f^* is the true critical value N_{crit} [theory that may be in this class: QCD(adj), experiment (lattice)]

B.) if, as # species is increased above N_f^*

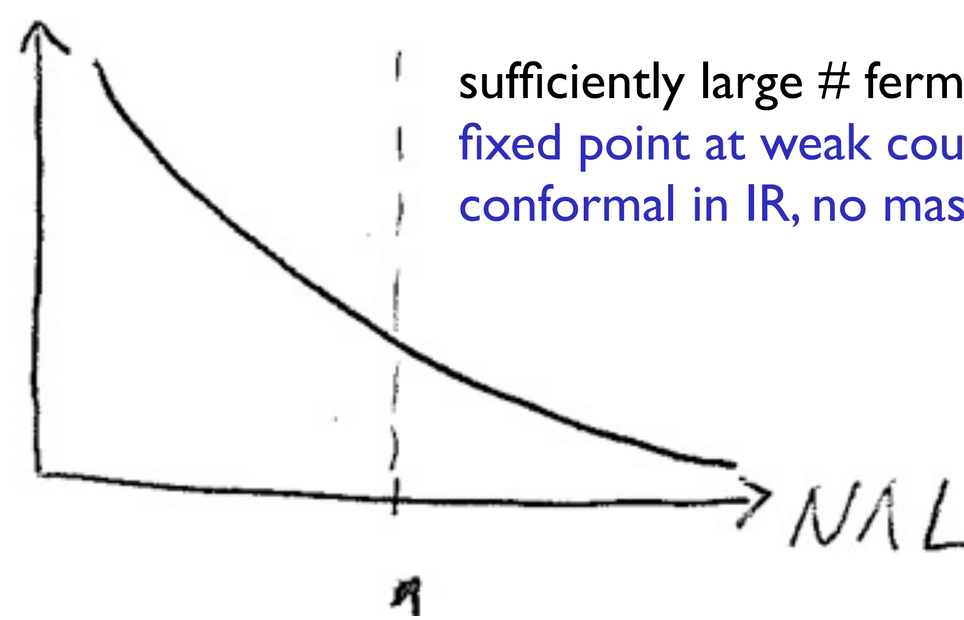


increase # fermions



sufficiently small # fermion species
confining theories

increase # fermions



sufficiently large # fermion species
fixed point at weak coupling
conformal in IR, no mass gap

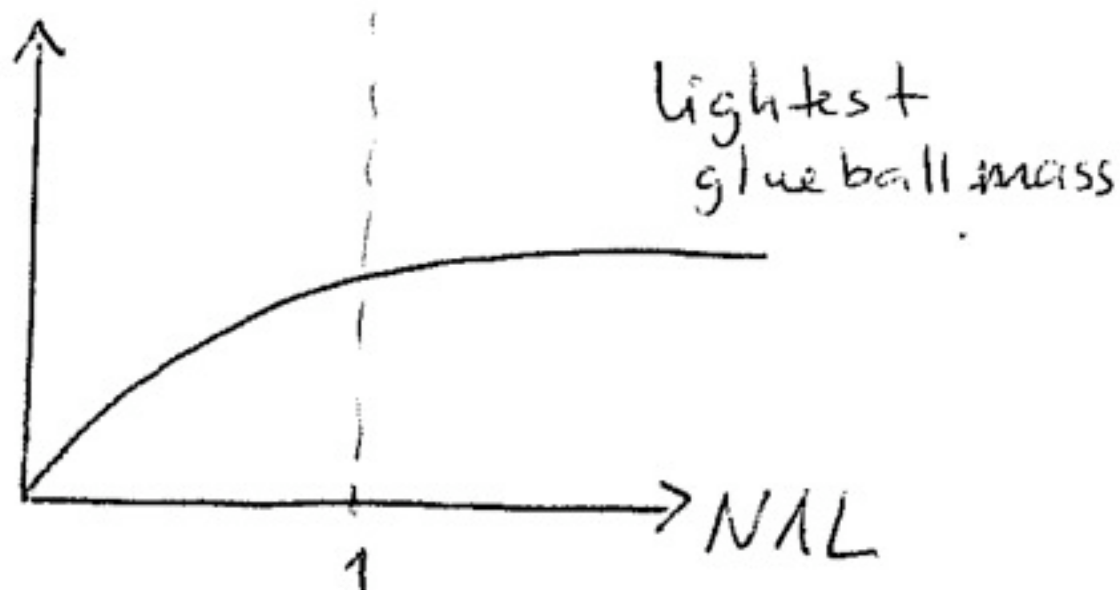
then, $N_{crit} > N_f^*$

true value of critical # “flavors”

thus, for such theories N_f^* is a lower bound thereof

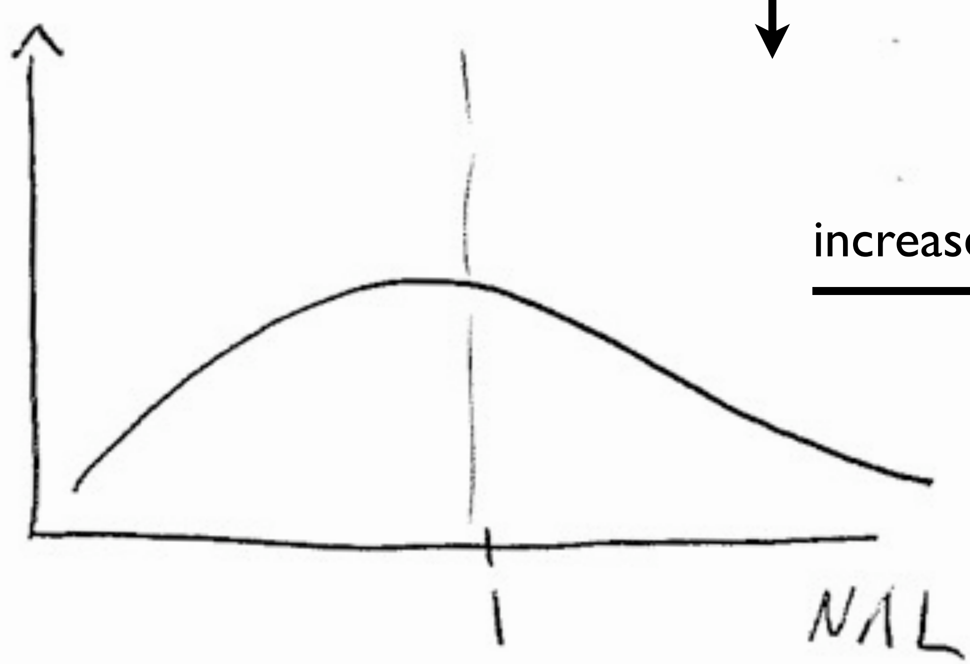
[theory believed to be in this class: QCD(F) - arguments using mixed reps., experiment (lattice)]

c.) if, as # species has not yet reached N_f^*

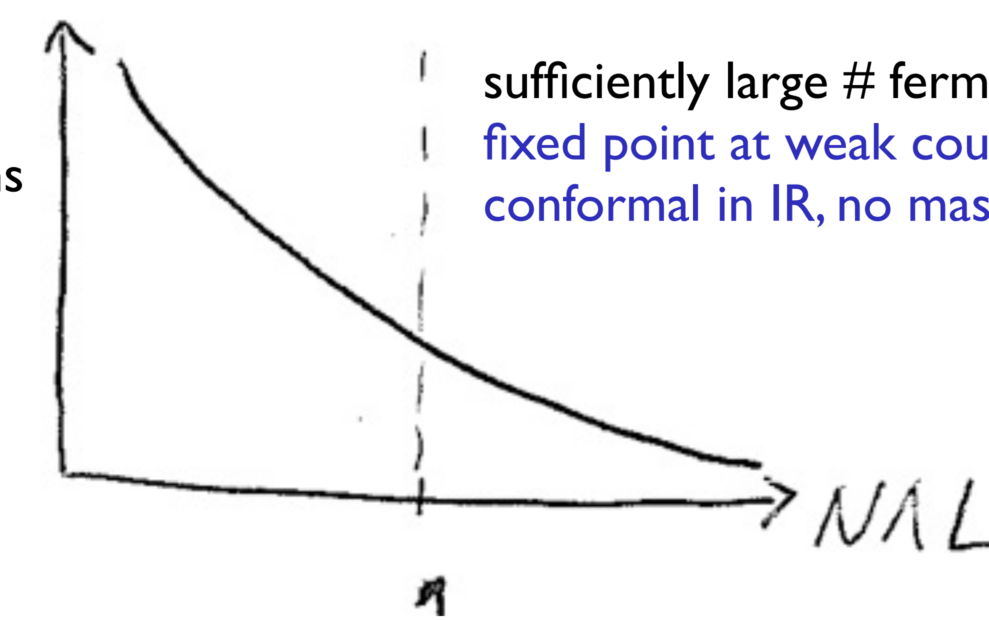


sufficiently small # fermion species
confining theories

increase # fermions



increase # fermions



sufficiently large # fermion species
fixed point at weak coupling
conformal in IR, no mass gap

then,

$N_{crit} < N_f^*$

thus, for this class of theories N_f^* is an upper bound on critical # "flavors"

[only one theory we know is believed to be in this class: SU(2) 4-index symmetric tensor Weyl, theory arguments]

comparing theory estimates of critical number of fermions for SU(N)

"experiment"

Weyl adjoints [no deformation needed]

	our estimate	gap eqn	beta function gamma=2/1	AF lost
any N	4	4.15	2.75/3.66	5.5

4 ? e.g.: *Catterall et al; del Debbio, Patella,Pica; Hietanen et al.*

Dirac 2-index (anti)symmetric tensor [deformation needed; but large-N equiv!]

N	our estimate	gap eqn	beta function gamma=2/1	AF lost
3	2.40	2.50	1.65/2.2	3.30
4	2.66	2.78	1.83/2.44	3.66
5	2.85	2.97	1.96/2.62	3.92
10	3.33	3.47	2.29/3.05	4.58
∞	4	4.15	2.75/3.66	5.5

2 ? e.g.: *DeGrand,Shamir, Svetitsky; Fodor et al; Kogut, Sinclair*

Dirac fundamentals [deformation needed]

N	our estimate (a/c)	gap eqn	functional RG	beta function gamma=2/1	AF lost
2	5/8	7.85	8.25	5.5/7.33	11
3	7.5/12	11.91	10	8.25/11	16.5
4	10/16	15.93	13.5	11/14.66	22
5	12.5/20	19.95	16.25	13.75/18.33	27.5
10	25/40	39.97	n/a	27.5/36.66	55
∞	$2.5N/4N$	$4N$	$\sim(2.75-3.25)N$	$2.75N/3.66N$	$5.5N$

12 ? e.g.: *Appelquist,Fleming, Neal; Deuzemann, Lombardo,Pallante; Iwasaki et al; Fodor et al; Jin, Mahwinney; A. Hasenfratz*

gap equation and lattice - only vectorlike theories;
beta function (Ryttov/Sannino)

in chiral gauge theories with multiple "generations" our estimates were the only known ones until Sannino's recent 0911.0931 via the proposed exact beta function

Conclusions 1:

Compactifying 4d gauge theories on a small circle is a "deformation" where nonperturbative dynamics is under control - dynamics as "friendly" as in SUSY, e.g. Seiberg-Witten.
(regime of validity: Λ LN<<1 complimentary to EK: ΛLN >> 1)

Confinement is due to various "oddball" topological excitations, in most theories non-self-dual.

Polyakov's "Debye screening" mechanism works on R^3 x S^1 also with massless fermions, contrary to what many thought
- KK monopoles and index theorem-crucial ingredients of analysis.

Precise nature - monopoles, bions, triplets, or quintets - depends on the light fermion content of the theory.

U,P; 0812.2085, 0906.5156

Conclusions II:

Didn’t have time for these:

Found chiral symmetry breaking (Abelian) due to expectation values of topological “disorder” operators: occurs in mixed-rep. theories with anomaly-free chiral U(1), broken at any radius

U,P; 0910.1245

Circle compactification gives another calculable deformation of SUSY theories - not yet fully explored -

in I=3/2 SU(2) Intriligator-Seiberg-Shenker model we argued that theory conformal, rather than SUSY-breaking.

U,P; 0905.0634
agreement with
different arguments of
Shifman, Vainshtein `98
Intriligator `05

Conclusions III:

Gave “estimates” of conformal window boundary in vectorlike and chiral gauge theories (OK with “experiment” when available).

Conformality tied to relevance vs irrelevance of topological excitations. Perhaps of interest especially in theories where chiral symmetries do not break.

U,P; 0906.5156

Now, clearly,

on $R^3 \times S^1$ we only see the “shadow” of the real thing...

Conclusions IV: Questions?

Is it so crazy to expect "relevance vs. irrelevance" (with changing Nf) of topological excitations also in R^4?

Lattice studies in pure YM (early ref.: Kronfeld et al, 1987) **have found that confinement appears to be due to topological excitations- center vortices, monopoles** – these are 't Hooft's (1978) "transient particles" that are revealed to us in particular gauges – **and the deconfinement transition at high-T is associated with them becoming irrelevant.**
... huge body of literature (mostly in pure YM) & apparently not much agreement in the details...

To expect that massless fermions would affect the nature of topological excitations is thus quite natural.

What is harder (for me?) is how to make this precise on R^4 .

So, back to SUSY? **- theorists' "safe haven"**

Conclusions V: Questions?

We argued that "bions" are responsible for confinement in N=1 SYM at small L (a particular case of our Weyl adjoint theory).

This remains true if N=1 obtained from N=2 by soft breaking

Monopoles and dyons are responsible for confinement in N=2 softly broken to N=1 at large L. (Seiberg, Witten `94)

So, in different regimes we have different pictures of confinement in N=1 SYM.

Do they connect in an interesting way?

... we think yes (Unsal, EP - 201x)