HATCH
Holography: Applications to Technicolor, Condensed matter and Hadrons
INR RAS, Moscow, June 11-15, 2012

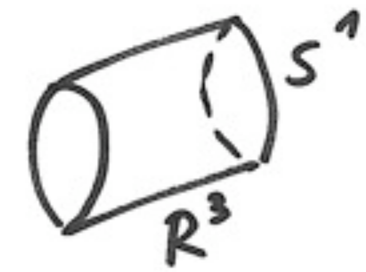
Holography, large-N volume independence, and continuity

Erich Poppitz — U of Toronto

review of work since 2008 with Mithat Ünsal — SLAC/Stanford/San Francisco State

The theme of my talk is about inferring properties of infinite-volume gauge theories by studying (arbitrarily) small-volume dynamics.

The small volume may be



← most of this talk

or

of characteristic size "L"

... is this crazy? desperate?

To guide you through my talk, an outline:

Eguchi-Kawai (EK) reduction (aka large-N volume independence)

- an exact result in QFT most have not heard about...

How is EK supposed to work?

- a holographic example in N=4 SYM: how two exact results fit together [EP, Unsal 2010]
- failure of original EK and some recent developments "resurrecting" it [Unsal, Yaffe ... 2007-]
- potential uses thereof [many refs.]

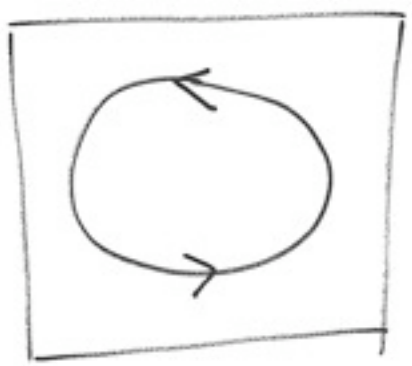
Complementary regimes:
volume independence vs. volume dependence

- continuity and semiclassical studies of confinement, chiral symmetry breaking, conformality, deconfinement...

[Unsal w/ one of Yaffe, Shifman, EP, Schafer, Argyres... 2007-...]

To put my talk in context, some relevant history:

Eguchi and Kawai (1982) showed that the infinite set of loop (Schwinger-Dyson) equations for Wilson loops in pure Yang-Mills theory is identical in small-V and infinite-V theory, to leading order in 1/N:



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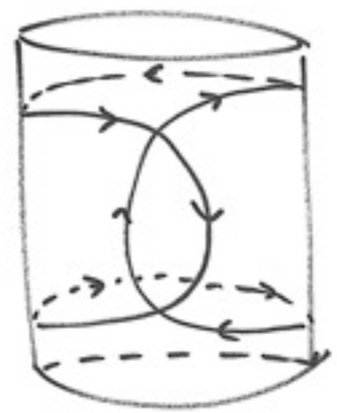


+ O(1/N)

provided all topologically nontrivial (w/ arbitrary winding) Wilson loops have vanishing expectation value
(= unbroken center)

expectation value of any Wilson loop at infinite-L

expectation value of (folded) Wilson loop at small-L

"EK reduction" or "large-N reduction" or "large-N volume-independence"

Note: this is an exact result in QFT (one of the few!).

... potentially exciting, since:

1) **simulations may be cheaper** (use single-site lattice ?)

2) **raises theorist's hopes** (that small-L easier to solve ?)

To put my talk in context, some relevant history:

From a modern point of view EK reduction is a large-N orbifold with respect to the group of translations.

Volume-independence viewed as an orbifold helps establish that VEVs and correlators of operators that are center-neutral and carry momenta quantized in units of 1/L (in compact direction) are the same on, say,

, and in infinite-L theory, to leading order in 1/N.

Kovtun, Unsal, Yaffe (2004)

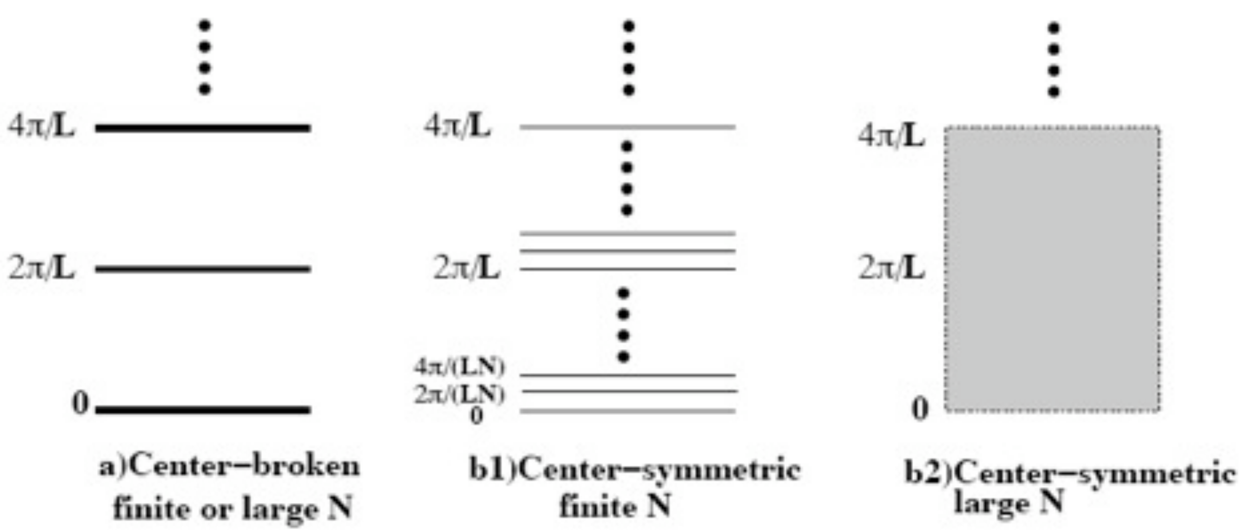
Thus, a working example of EK would be good for

- calculating vevs (symmetry breaking)
  - even if all dimensions small
- calculating spectra (for generic theories/reps)
  - need at least one large dimension

To put my talk in context, some relevant history:

Some intuition of how EK reduction works (note EK valid at any coupling).

in perturbation theory:
from spectra (& Feynman graphs)
in appropriate background



or

at strong coupling:

- use lattice strong-coupling expansion

- use gauge-gravity duality:

an exact correspondence for large-N
N=4 SYM - a conformal field theory;
since EK also exact, it must be that
non-winding Wilson loops & appropriate
correlators are insensitive to box
if center-symmetric vacuum

Since this is a holography workshop,
will first consider a simple example...
[EP, Unsal 2010]

two nonperturbative (*exact*) methods to study large-N gauge dynamics

gauge-gravity duality

$\mathcal{N}=4$ SU(N) SYM

$N \to \infty$

$g_{YM}^2 N$ = λ - large & fixed

believed to be ↕ **exact duality**

weakly-coupled type IIB supergravity on $\mathrm{AdS}_5 \times S_5$

$\lambda \longleftrightarrow R_3^4$ radius in string units (large >>1)

$\frac{1}{N} \longleftrightarrow g_s$ (small, –>0)

large-N volume independence

"Eguchi-Kawai (EK) reduction"

$\mathcal{N}=4$ SU(N) SYM

$N \to \infty$ compactified on

$M_4 = \mathbb{R}^{4-k} \times (\mathbf{S}^1)^k$

↕ **exact for some observables**

$\mathcal{N}=4$ SU(N) SYM on $\mathbb{R}^4$

$N \to \infty$ provided

- translational invariance unbroken
- $(\mathbb{Z}_N)^k$ center symmetry unbroken

-valid at strong coupling, large-N
-calculate correlators (Wilson loops) in dual CFT

-valid at any coupling, large-N
-calculate neutral correlators in large-V theory

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1. do they fit together? how?

2. do we learn anything useful by understanding **1**.?

(apart from testing AdS/CFT...)

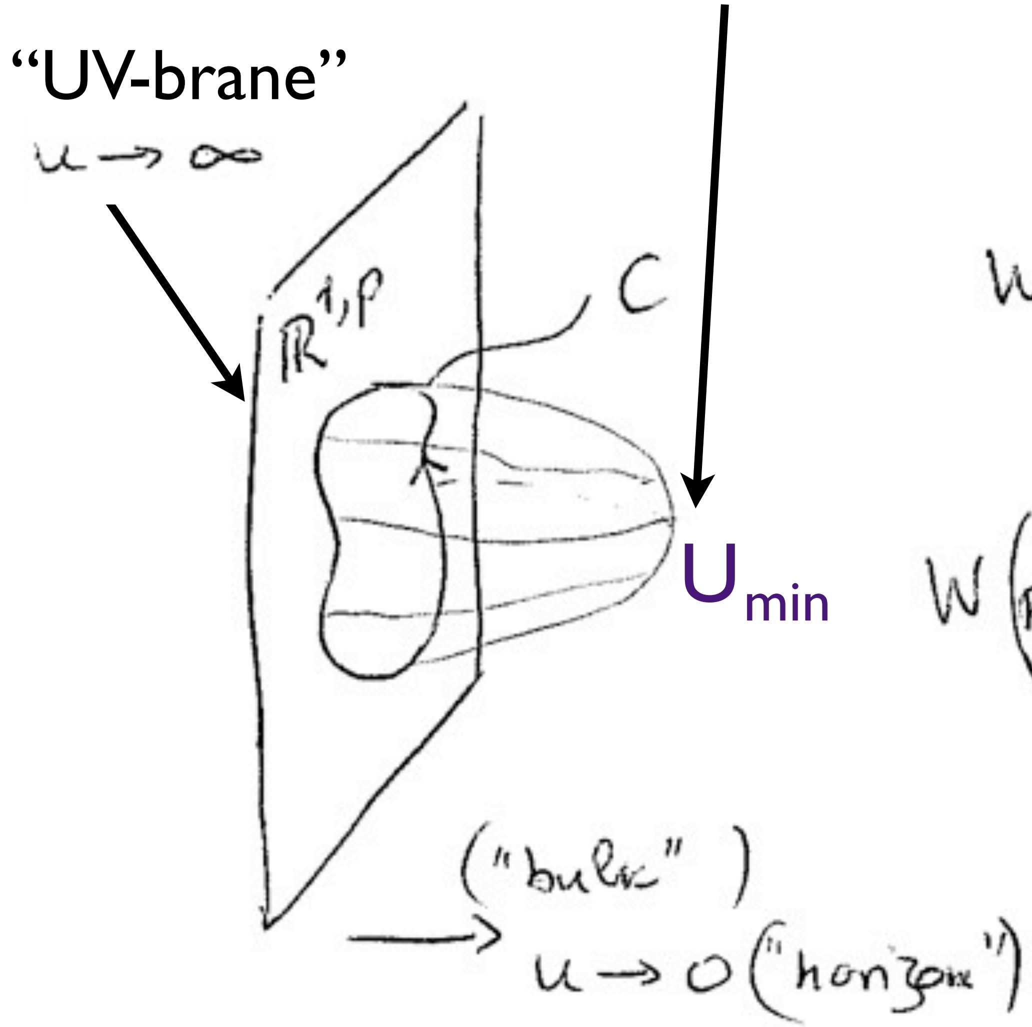
$$ds^2 = \underbrace{\frac{u^2}{R_3^2}\left(-dt^2 + \sum_{i=1}^{2} dx_i^2 + R_0^2\, d\theta^2\right) + \frac{R_3^2}{u^2}\, du^2}_{AdS_5 \text{ (w/ compactified } x_3)} \times \underbrace{+\; R_3^2\, d\Omega_5^2}_{S_5} \qquad R_3 \sim \lambda^{1/4}$$

the token AdS/CFT calculation - Wilson loop vev:

smallest U reached by worldsheet depends on "quark" separation R:

$U_{min} \sim 1/R$: larger loops probe "bulk" geometry deeper

(i.e., away from UV)



$$W(C) \sim e^{-S_{string}(C)}$$

$$W\left(R\,\square^{T}\right) \sim e^{-\#\,\lambda^{1/2}\,\frac{T}{R}}$$

in $\mathcal{N}=4$ SYM <u>for $R_0 \to \infty$</u>

what about finite R_0?

$$ds^2 = \frac{u^2}{R_3^2}\left(-dt^2 + \sum_{i=1}^{2} dx_i^2 + R_0^2\, d\theta^2\right) + \frac{R_3^2}{u^2}\, du^2 + R_3^2\, d\Omega_5^2$$

conical singularity of metric:
KK modes ~ winding modes,
IIB SUGRA not good anymore

proper size of circle ~ string size when $\frac{u R_0}{R_3} \sim 1$ at $u R_0 \sim \lambda^{1/4}$

from $U_{min} \sim 1/R$ ("energy-distance relation") it is clear that larger loops probe deeper (i.e., away from UV) in AdS - thus worldsheet sensitive to singularity - the 4d CFT static quark potential V(R) ~ 1/R should change (even for Wilson loop entirely in the noncompactified directions)

by dimensional reduction, expect:

$R << R_0$ 4d CFT behavior $V \sim 1/R$

$R >> R_0$ 3d behavior $V \sim g_3^2 \log(R)$ at weak coupling

(or $V \sim 1/R^{2/3}$... $1/R$ in various strong D2...M2 regimes)

moral: natural to expect volume dependence

question: how does Volume (In)Dependence show up in the gravity duals?

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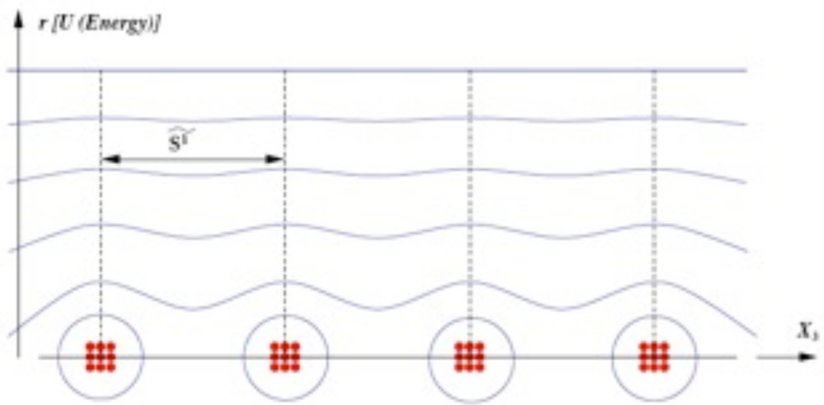
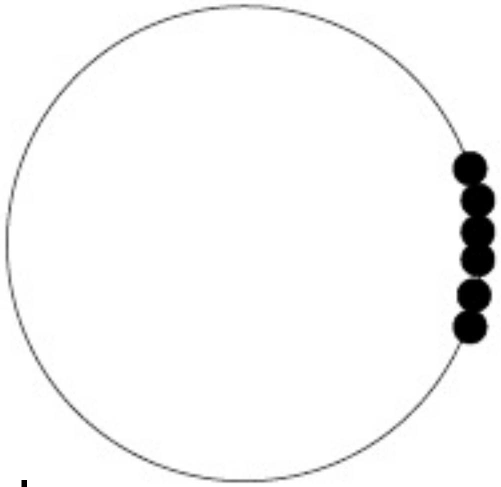


corresponds to



N D2 all at the same point on dual circle

now, locations of D2 on dual circle = eigenvalues of Polyakov loop
thus, if all N on top of each other, all eigenvalues identical (say =1):

$$\langle \frac{\mathrm{tr}}{N} \Omega^k \rangle = 1$$ and center symmetry is broken

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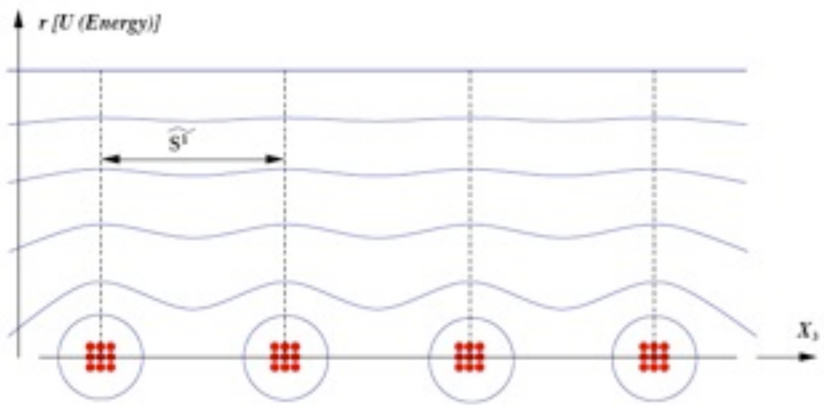

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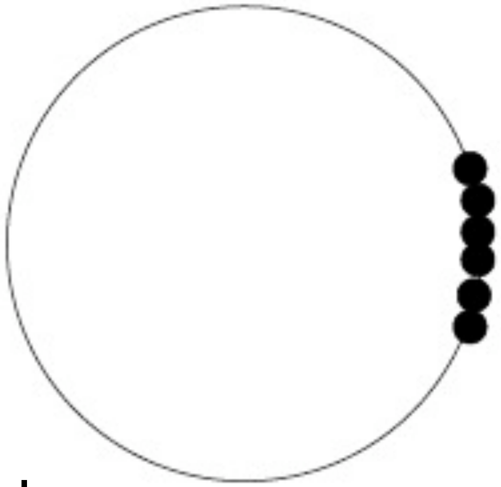

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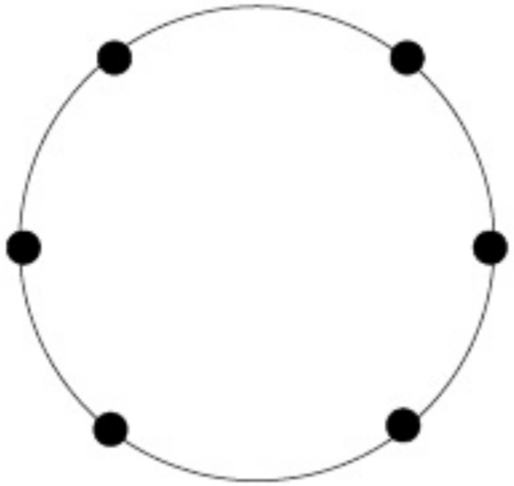
unbroken center symmetry vacuum $\langle \frac{\mathrm{tr}}{N} \Omega^k \rangle = 0$ = equidistant distribution of N D2 branes on circle



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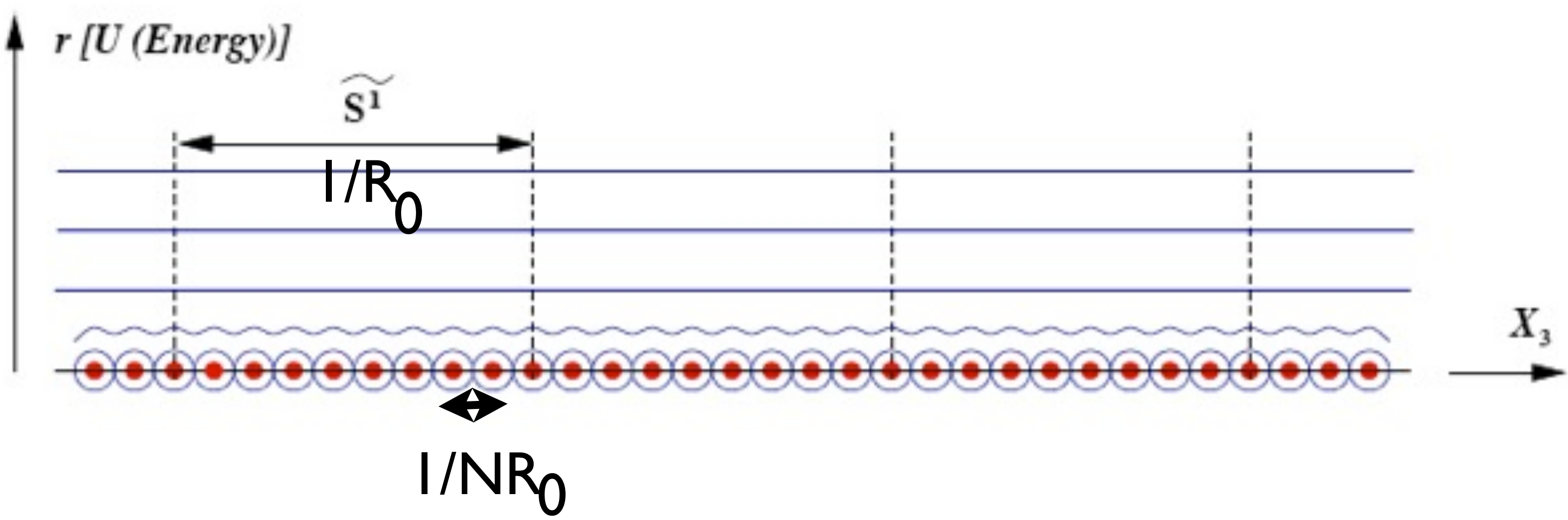
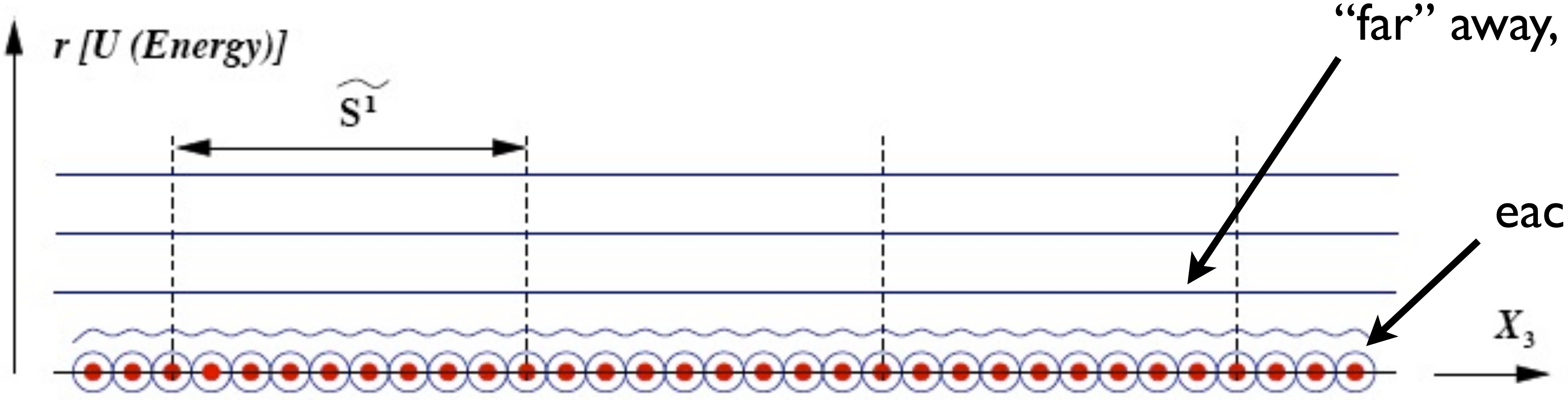


“far” away, however, “potential” is $1/r^4$

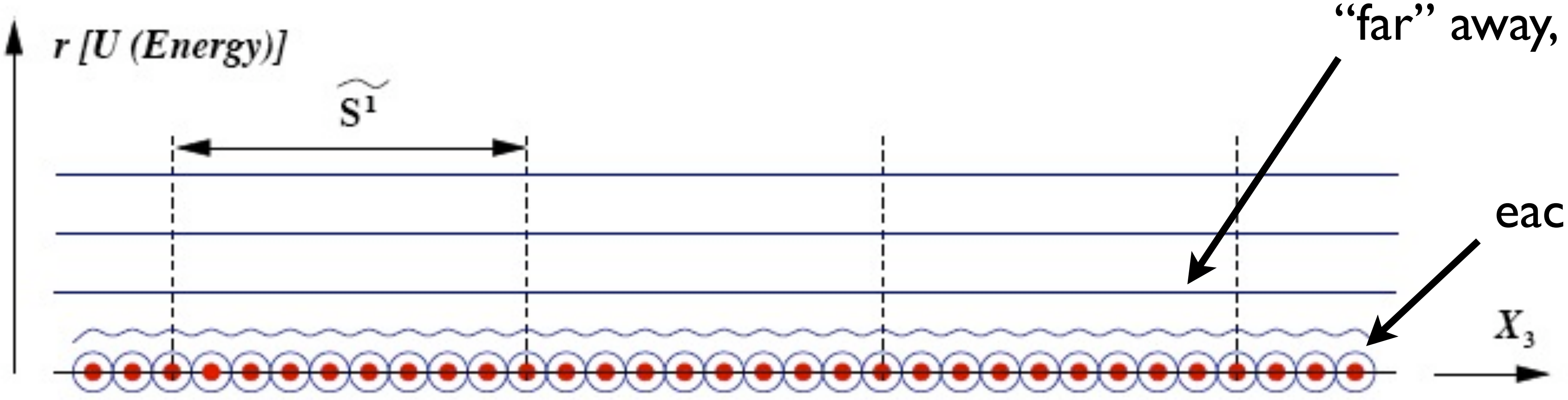
each brane sources $1/r^5$ “potential”



it is clear from picture that x_3 isometry restored at u >> $1/NR_0$

instead of $1/R_0$ as in center-broken case: $H_2^{\mathrm{sym}}(r, x_3) = \frac{\hat{R}_3^4}{(l_s u)^4} \left\{ 1 + \sum_{m=1}^{\infty} (muNR_0)^2 K_2(muNR_0) \cos(mu_3 N R_0) \right\}$

near each of the N D2 branes SUGRA badly breaks down (large curvatures),
however a distance $1/NR_0$ away, SUGRA is valid



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thus, D2 on dual circle at ANY u (in the large-N limit)

at $u > 1/NR_0 \to 0$

$$ds^2 = l_s^2\left[\frac{u^2}{\hat{R}_3^2}\Big(-dt^2 + \sum_{i=1}^{2} dx_i^2\Big) + \frac{\hat{R}_3^2}{R_0^2 u^2} d\theta^2 + \frac{\hat{R}_3^2}{u^2} du^2 + \hat{R}_3^2 d\Omega_5^2\right]$$

recall D3 on original circle:

$$ds^2 = l_s^2\left[\frac{u^2}{\hat{R}_3^2}\Big(-dt^2 + \sum_{i=1}^{2} dx_i^2 + R_0^2 d\theta^2\Big) + \frac{\hat{R}_3^2}{u^2} du^2 + \hat{R}_3^2 d\Omega_5^2\right]$$

center-symmetric metrics are T-dual give same W(C) for ANY SIZE LOOP

Thus: Volume Independence also arises naturally, once a center-symmetric vacuum is considered. D-branes nicely geometrize volume independence in this simple example.

Notice the appearance of "effective volume" N R_0 and that in N=4 center is a choice...

1. do volume independence and AdS/CFT fit together? how?

gravity dual gives an explicitly solvable realization of volume independence (first one above 2d)

2. do we learn anything useful from **1**.?

apart from testing AdS/CFT... brings about one point:

by usual EK argument, loop equations for W(C) in reduced and "original" theory are the same - hence their solution should be as well (modulo ambiguities...see Yaffe '1980s)

in gravity dual, solving loop equations for W(C) is tantamount to finding the appropriate worldsheet

as we saw, finding string worldsheet, and hence W(C), is the same problem in reduced and infinite-V theories

analytic use of EK in general non-SUSY theories still waiting for new techniques/ideas - large-N matrix/model or QM vs large-N YM?

After giving example how EK reduction and AdS/CFT fit together, and seeing the role of center symmetry, back to...

some relevant history:

However, Bhanot, Heller, Neuberger (1982) noticed immediate problem with EK in pure YM:

simplest argument - on S1, center symmetry breaks for $L < L_c$ (e.g. deconfinement transition) and thus invalidates EK reduction

(recall **"in N=4 center is a choice..."**)

Older proposed remedies: e.g., Gonzalez-Arroyo, Okawa (1982) - TEK... + others later argued to have problems (Bringoltz/Sharpe 2009) (some recent "twists" on TEK ?)

A more recent cure is argued to allow reduction valid to arbitrarily small L (e.g., single-site) if one adds either

- periodic adjoint fermions **aka "twisted partition function"** $\mathrm{tr} e^{-\beta H}(-1)^F$
(in SUSY = Witten index; it will become clear later why these help)

or

- appropriate double-trace deformations Unsal, Yaffe 2008
"good samaritan" deformations [Veneziano]

To put my talk in context, some relevant history:

Unsal, Yaffe 2008

Remedies proposed: reduction valid to arbitrarily small L (single-site) if:

periodic adjoint fermions (more than one Weyl) - no center breaking, so reduction holds at all L

used for current **lattice studies**

is 4 ...3,5... Weyl adjoint theory conformal or not?

small-L(=1) large-N (~20 or more...) simulations (2009-)
Hietanen-Narayanan; Bringoltz-Sharpe; Catterall et al
small-N large-L simulations (2007-)
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- "minimal walking TC"
- related by an "orientifold" large-N equivalence to theories with antisymmetric tensor matter, another large-N limit of SU(3) QCD

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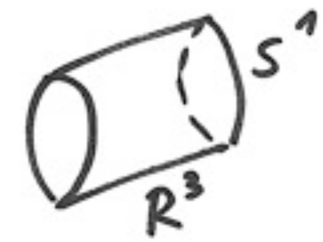
double-trace deformations:
deform measure to prevent center breaking
at infinite-N, deformation does not affect (connected correlators of "untwisted") ***observables***

theoretical studies

Unsal;
Unsal-Yaffe;
Unsal-Shifman;
Unsal-EP 2007-



fix-N, take L-small: **semiclassical studies of confinement** due to novel strange "oddball" (nonselfdual) topological excitations, whose nature depends on fermion content

- for vectorlike or chiral theories, with or without supersymmetry

- a complementary regime to that of volume independence, which requires infinite N - a (calculable!) shadow of the 4d "real thing".

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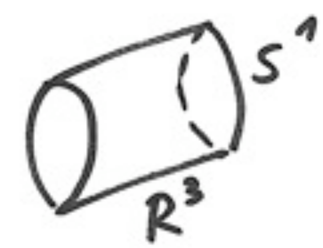
REST OF THIS TALK:

theoretical studies

Unsal;
Unsal-Yaffe;
Unsal-Shifman;
Unsal-EP 2007-



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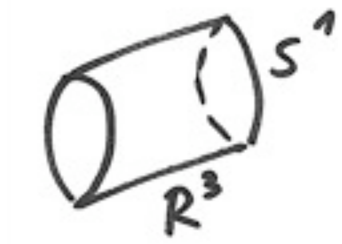

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In 4d theories with periodic adjoint fermions, for small-L, dynamics is semiclassically calculable (including confinement).

Polyakov's 3d mechanism of confinement by "Debye screening" in the monopole-anti-monopole plasma extends to (locally) 4d theories.

However, the "Debye screening" is now due to composite objects, the "magnetic bions" ***of the title.***

For this talk only consider 4d SU(2) theories

with N_W = multiple adjoint ***Weyl fermions***

"applications":

N_W=1 is N=1 SUSY YM ~ Seiberg-Witten theory with soft-breaking mass

N_W=4

- "minimal walking technicolor"
- happens to be N=4 SYM without the scalars

N_W=5.5 asymptotic freedom lost

theoretical studies

Unsal;
Unsal-Yaffe;
Shifman-Unsal;
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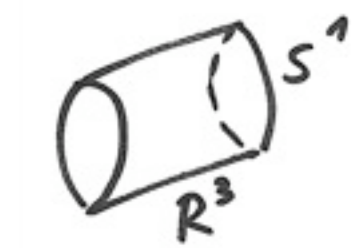

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In 4d theories with periodic adjoint fermions, for small-L, confining dynamics is semiclassically calculable.

$$S^1: \quad x^4 \sim x^4 + L$$

A_4 is now an adjoint 3d scalar Higgs field $\quad \partial_4 + A_4 \longrightarrow \frac{2\pi n}{L} + A_4$

but it is a bit unusual -
a compact Higgs field: $\quad \langle A_4 \rangle \sim \langle A_4 \rangle + \frac{2\pi}{L}$

such shifts of A_4 vev absorbed into shift of KK number "n" $\quad A_4 \to A_4 + \partial_4\left(\frac{2\pi x_4}{L}\right)$

"large" gauge transform

thus, natural
scale of "Higgs vev" is $\quad \langle A_4 \rangle \sim \frac{\pi}{L}$ leading to

$$SU(2) \xrightarrow{\frac{1}{L}} U(1)$$

hence, semiclassical if L << inverse strong scale

exactly this happens in theories with more than one periodic Weyl adjoints

follows from two things, without calculation:

1.) existence of deconfinement transition in pure YM and 2.) supersymmetry

in pure YM, at small L (high-T), Veff min at A_4=0 & max at pi/L (Gross,Pisarsky,Yaffe 1980s)

in SUSY Veff=0, so one Weyl fermion contributes the negative of gauge boson Veff

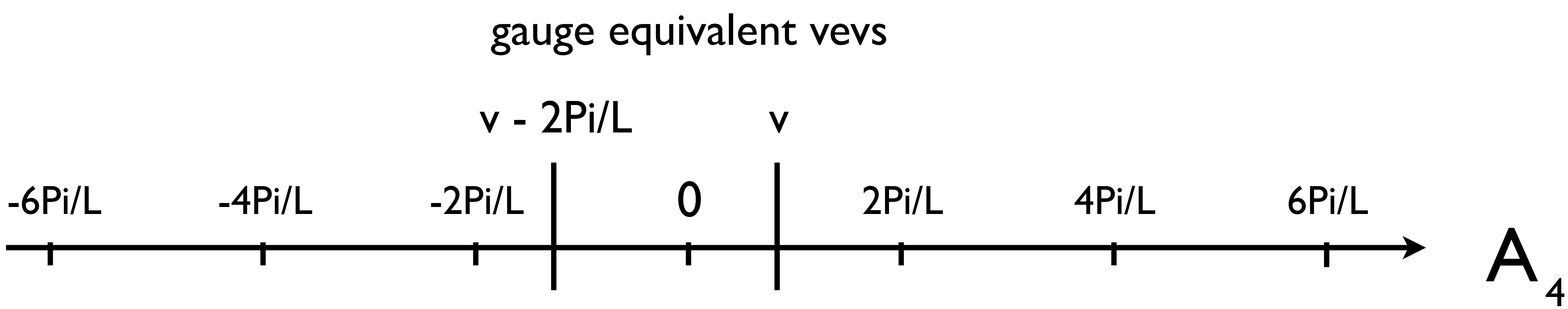
Q.E.D.

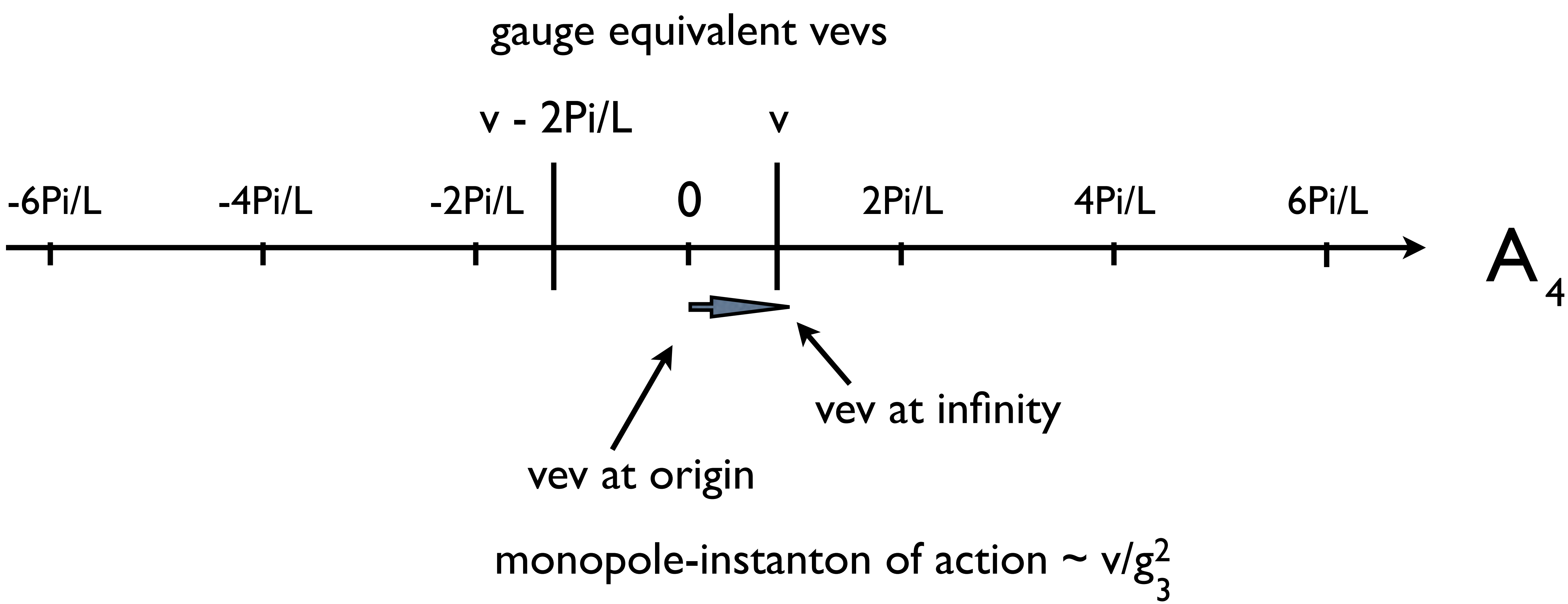
Polyakov's 3d mechanism of confinement by "Debye screening" in the monopole-anti-monopole plasma extends to (locally) 4d theories.

However, the "Debye screening" is now due to composite objects, the "magnetic bions" ***of the title.***

since SU(2) broken to U(1) at scale 1/L (for SU(N) W mass is 1/NL, so validity of Abelian description is1/NL >> strong scale)

there are monopole-instanton solutions of finite Euclidean action, constructed as follows:


gauge equivalent vevs
v - 2Pi/L
v
-6Pi/L
-4Pi/L
-2Pi/L
0
2Pi/L
4Pi/L
6Pi/L
A 4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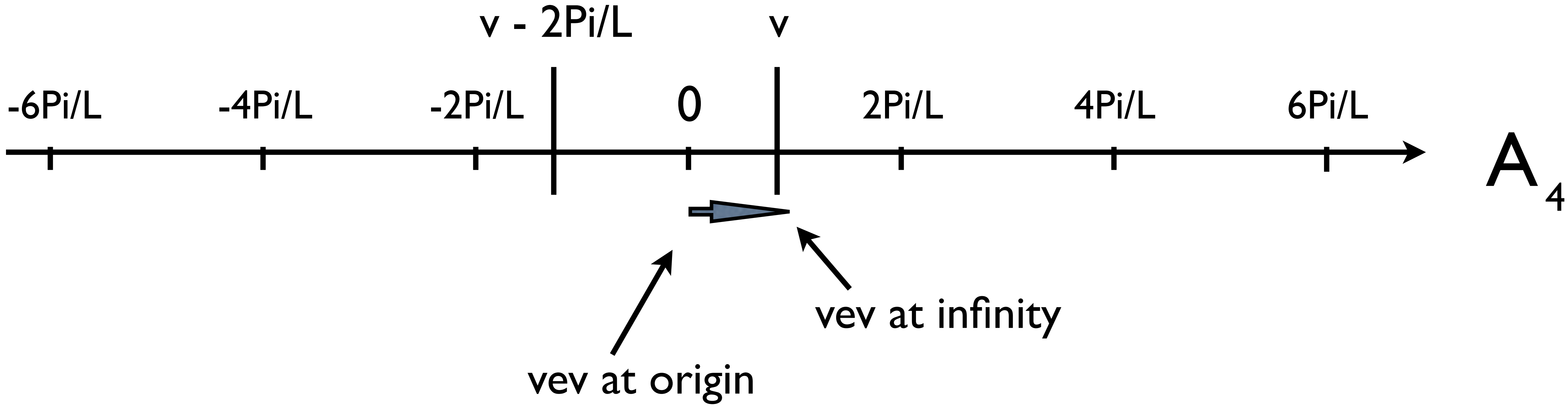
A 4
vev at infinity
vev at origin
monopole-instanton of action ~ v/g$_3^2$

gauge equivalent vevs



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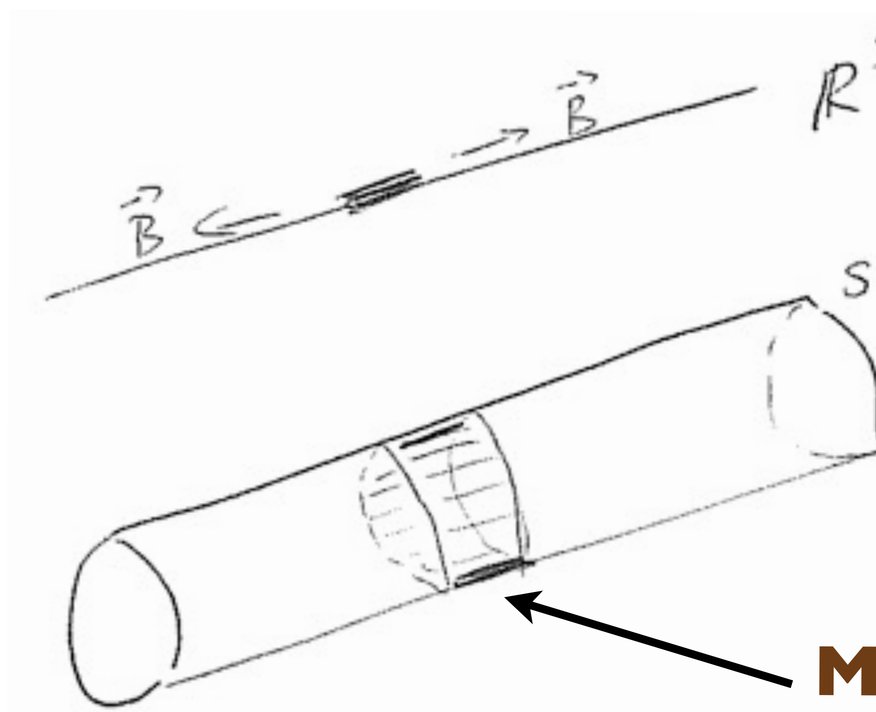
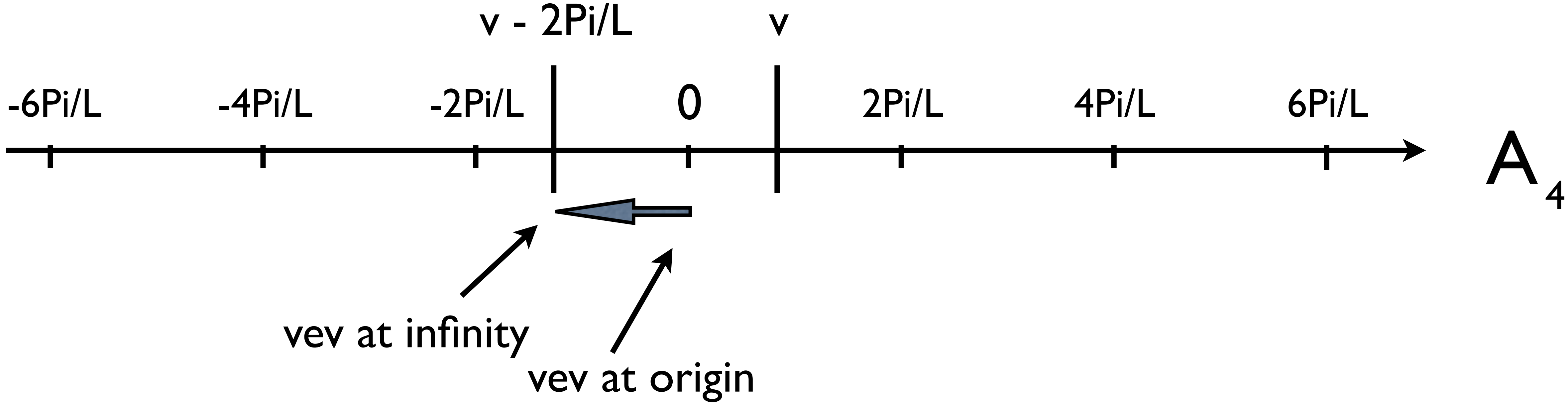


monopole trivially embedded in 4d

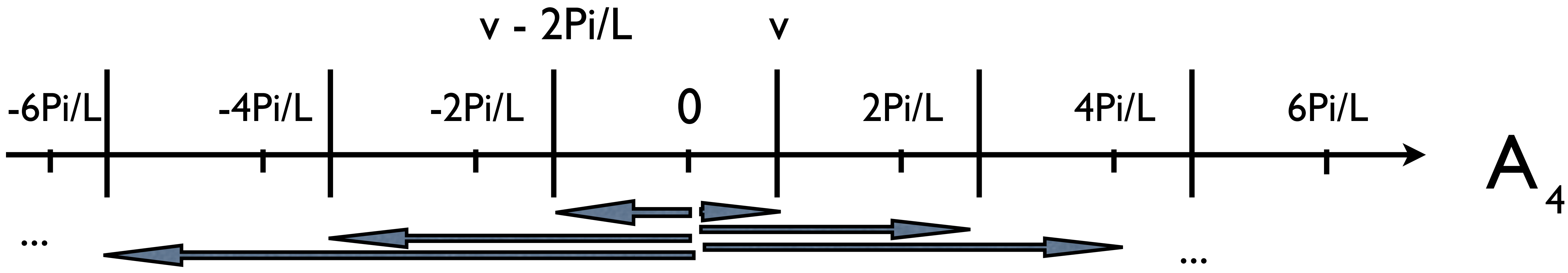
gauge equivalent vevs



monopole-instanton of action ~ $|2\,\mathrm{Pi}/L - v|/g_3^2$

- use a large gauge transformation to make vev at infinity = v
- action does not change
- x_4-dependence is induced, hence called “twisted”

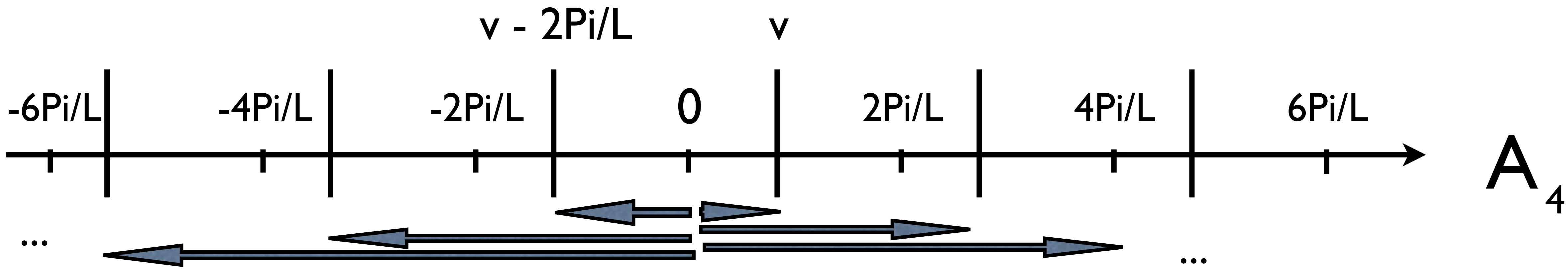
gauge equivalent vevs



monopole-instanton tower; action ~ $|2\ k\ Pi/L - v|/g_3^2$

the lowest action member of the tower can be pictured like this (as opposed to the no-twist):

gauge equivalent vevs



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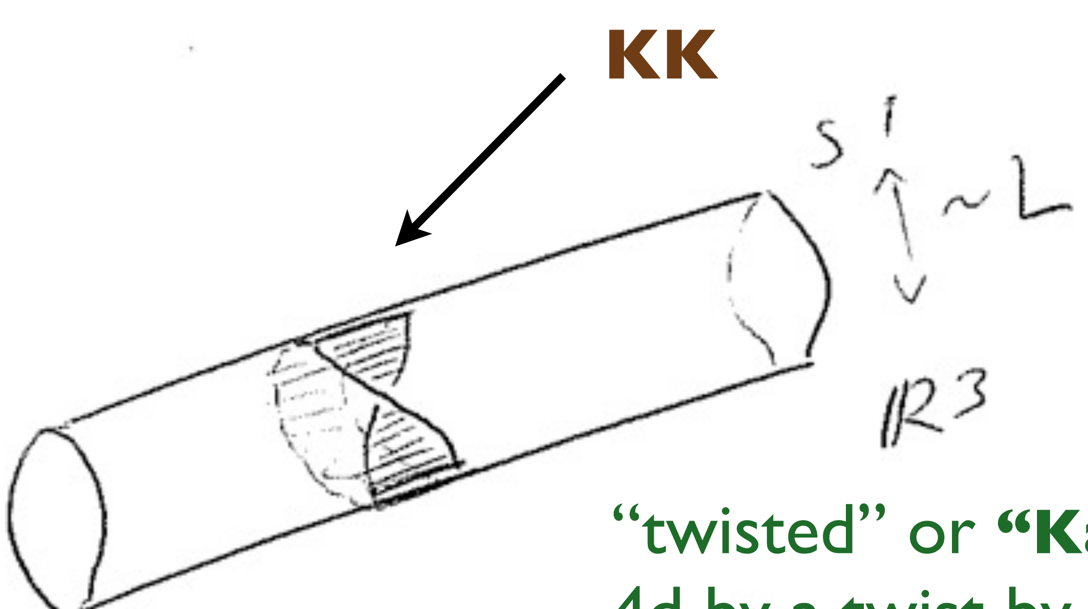
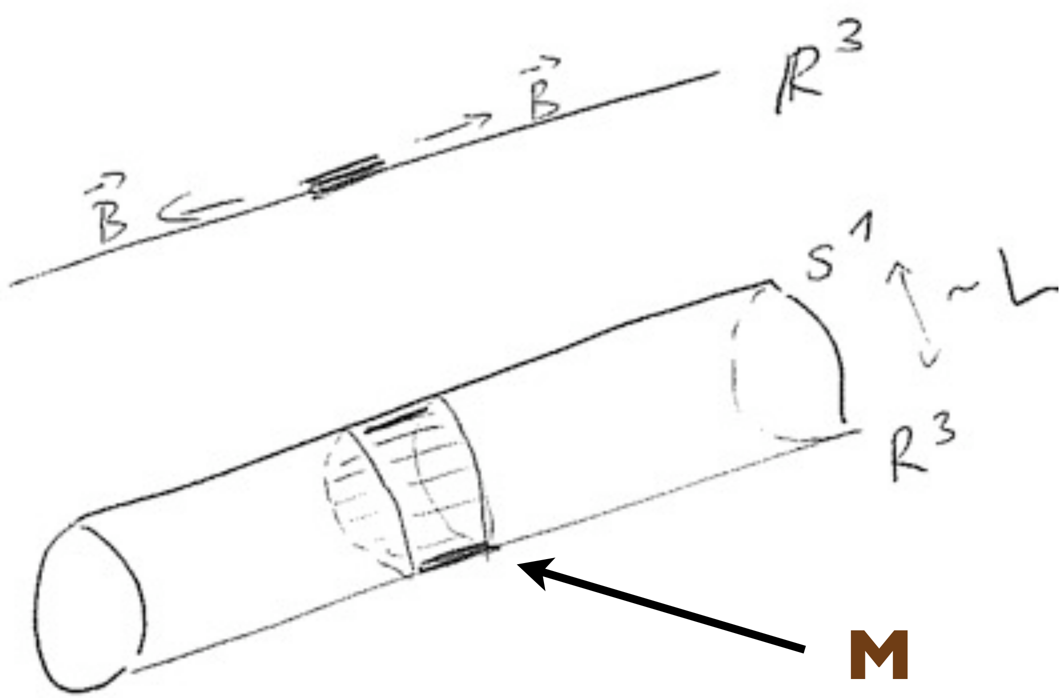
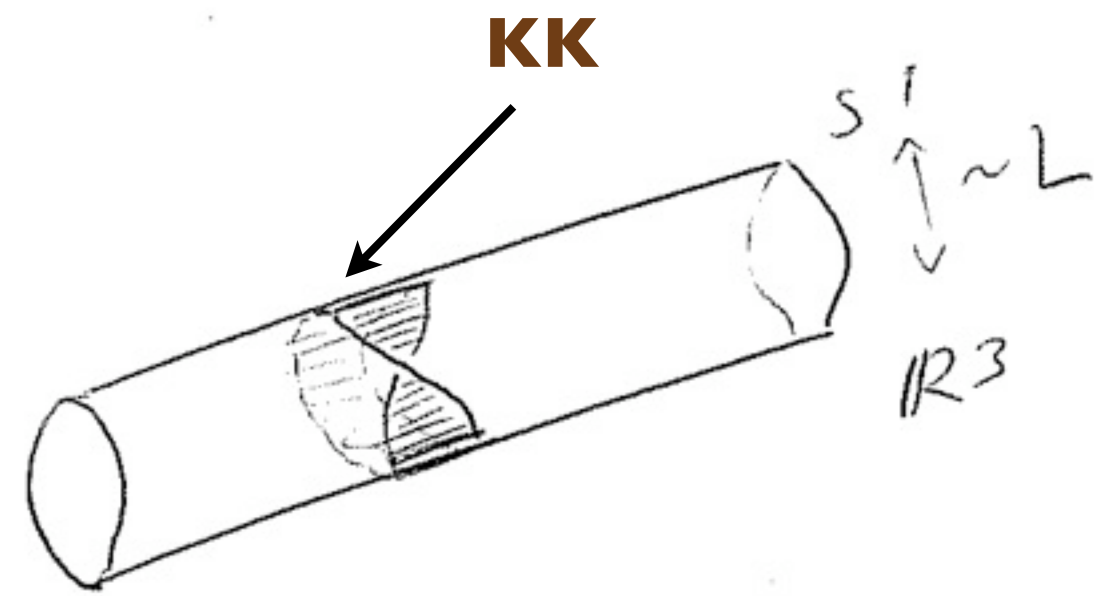
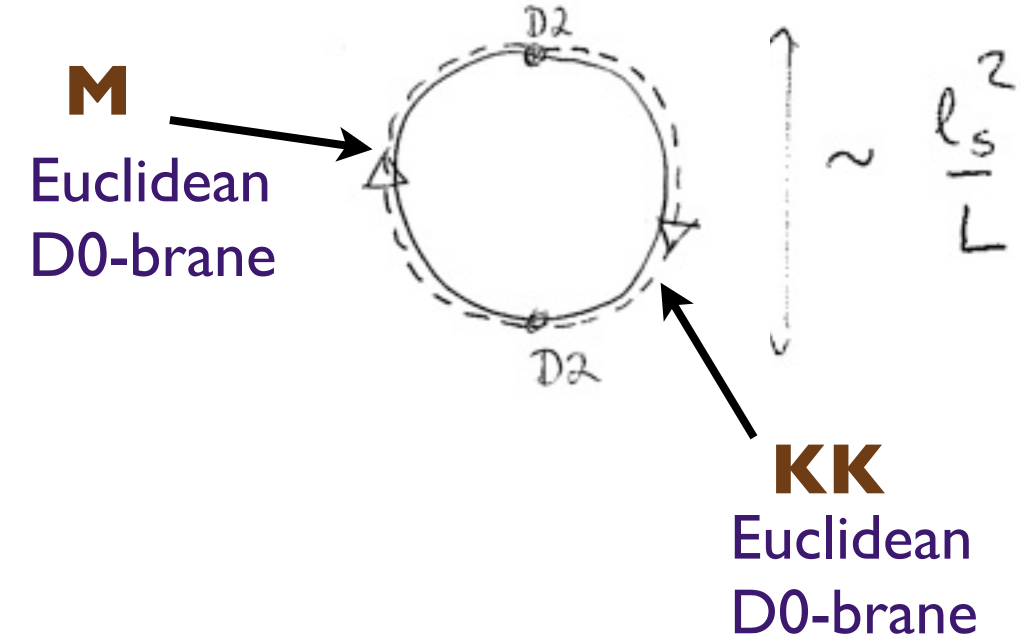


"twisted" or **"Kaluza-Klein":** monopole embedded in 4d by a twist by a "gauge transformation" periodic up to center - in 3d limit not there! (infinite action)





K. Lee, P. Yi, 1997



	magnetic	topological	suppression
M	+1	1/2	e^{-S_0}
KK	−1	1/2	e^{-S_0}
BPST	0	1	e^{-2S_0}

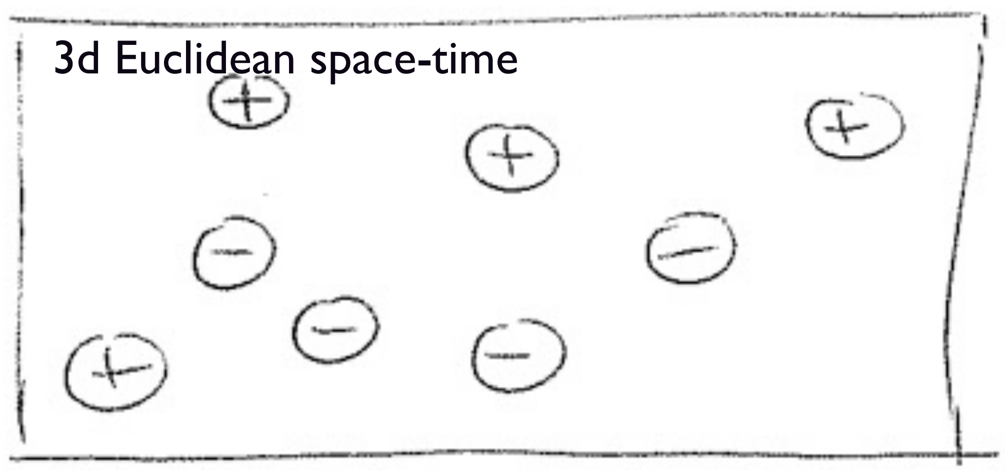
M & KK have 't Hooft suppression given by:

$$e^{-S_0} = e^{-\frac{4\pi v}{g_3^2}} = e^{-\frac{4\pi^2}{L g_3^2}} = e^{-\frac{4\pi^2}{g_4^2(L)}}$$

center-symmetric vev — coupling matching

in SU(N), 1/N-th of the 't Hooft suppression factor

in a purely bosonic theory, vacuum would be a dilute M-M* plasma - but interacting, unlike instanton gas in 4d (in say, electroweak theory)



physics is that of Debye screening

analogy:

electric fields are screened in a charged plasma ("Debye mass for photon")
in the monopole-antimonopole plasma, the dual photon (3d photon ~ scalar) obtains mass from screening of magnetic field:

$$\mathcal{L}_{eff} = g_3^2 (\partial\sigma)^2 + (\#)\, v^3 e^{-S_0}\left(e^{i\sigma} + e^{-i\sigma}\right) + \ldots$$

"(anti-)monopole operators"

aka **"disorder operators"** - not locally expressed in terms of original gauge fields (Kadanoff-Ceva; 't Hooft - 1970s)

also by analogy with Debye mass:

dual photon mass^2 ~ M-M* plasma density

$$m_\sigma \sim v\, e^{-S_0/2} = v\, e^{-\frac{4\pi v}{2 g_3^2}}$$

(for us, v = pi/L)

Polyakov, 1977: ***dual photon mass ~ confining string tension***

"Polyakov model" = 3d Georgi-Glashow model or compact U(1) (lattice)

but our theory has fermions and M and KK have zero modes

each have $2N_W$ zero modes

index theorem
Nye-Singer 2000,

for physicists:
Unsal, EP 0812.2085

disorder operators:

M:

$$e^{-S_0} e^{i\sigma} (\lambda\lambda)^{N_w}$$

KK:

$$e^{-S_0} e^{-i\sigma} (\lambda\lambda)^{N_w}$$

M*:

$$e^{-S_0} e^{-i\sigma} (\bar{\lambda}\bar{\lambda})^{N_w}$$

KK*:

$$e^{-S_0} e^{i\sigma} (\bar{\lambda}\bar{\lambda})^{N_w}$$

chiral symmetry $SU(N_w) \times U(1)$

U(1) anomalous, but $\mathbb{Z}_{4N_w}: \lambda \to e^{i\frac{2\pi}{4N_w}} \lambda \quad \sigma \to \sigma + \pi$ is not

topological shift symmetry is intertwined with exact chiral symmetry

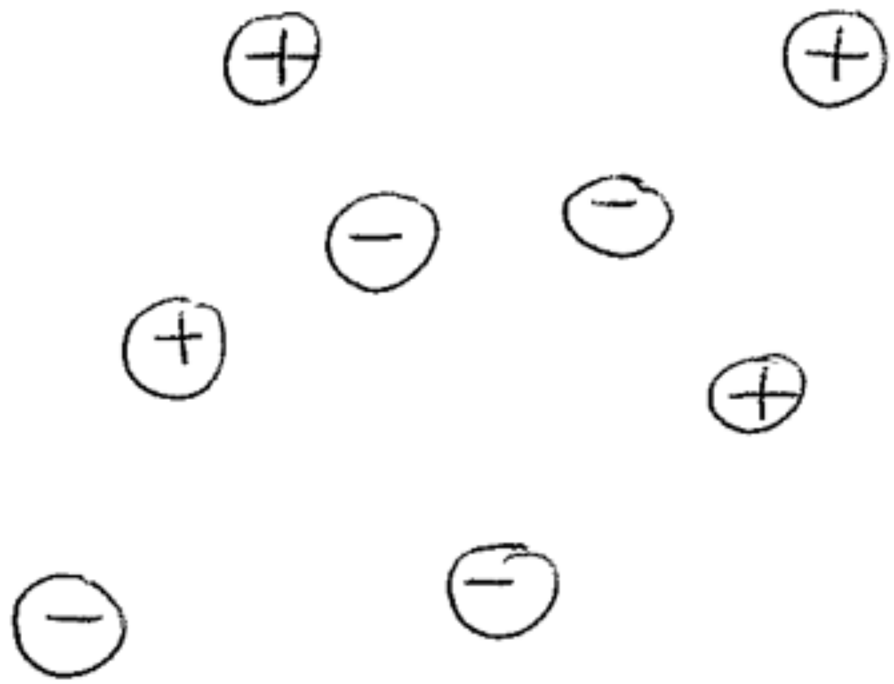
~~$\cos\sigma$~~ $\cos(2\sigma)$ ✓ ...

potential (and dual photon mass) allowed, but what is it due to?

Unsal 2007: **dual photon mass is induced by magnetic "bions" - the leading cause of confinement in SU(N) with adjoints at small L** (including SYM)

3d pure gauge theory vacuum monopole plasma

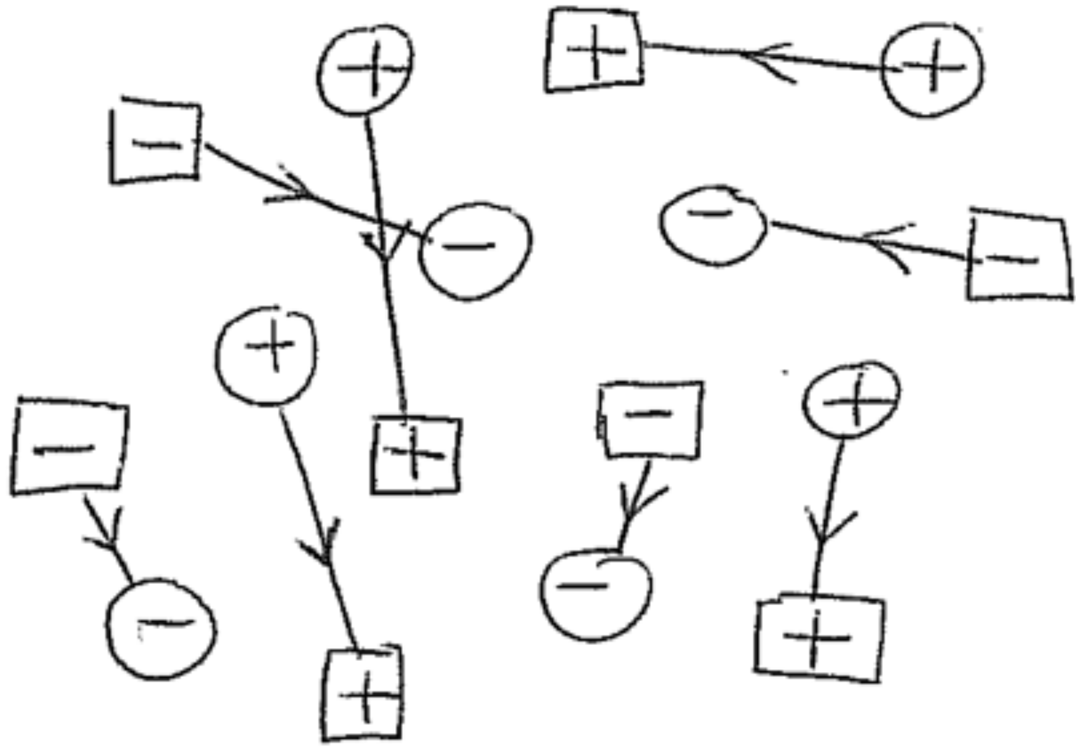
Polyakov 1977



circles = M(+)/M*(-)

4d QCD(adj) fermion attraction M-KK at small-L*

Unsal 2007, ...

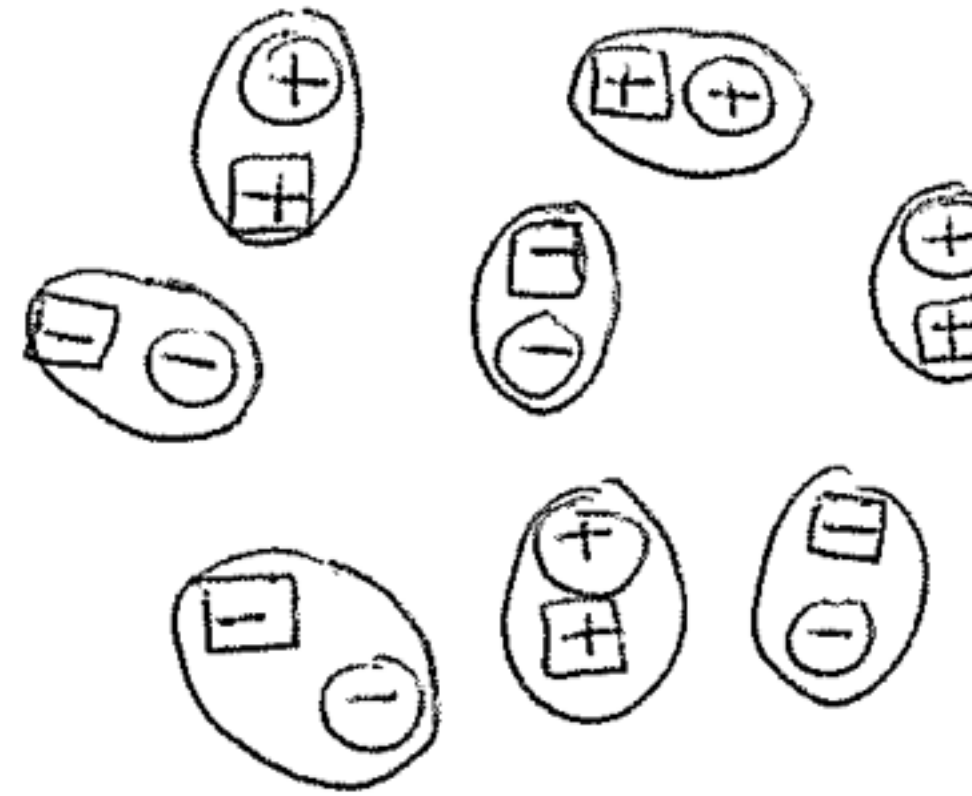


circles = M(+)/M*(-)

squares = KK(-)/KK*(+)

4d QCD(adj) bion plasma at small-L

Unsal 2007, ...

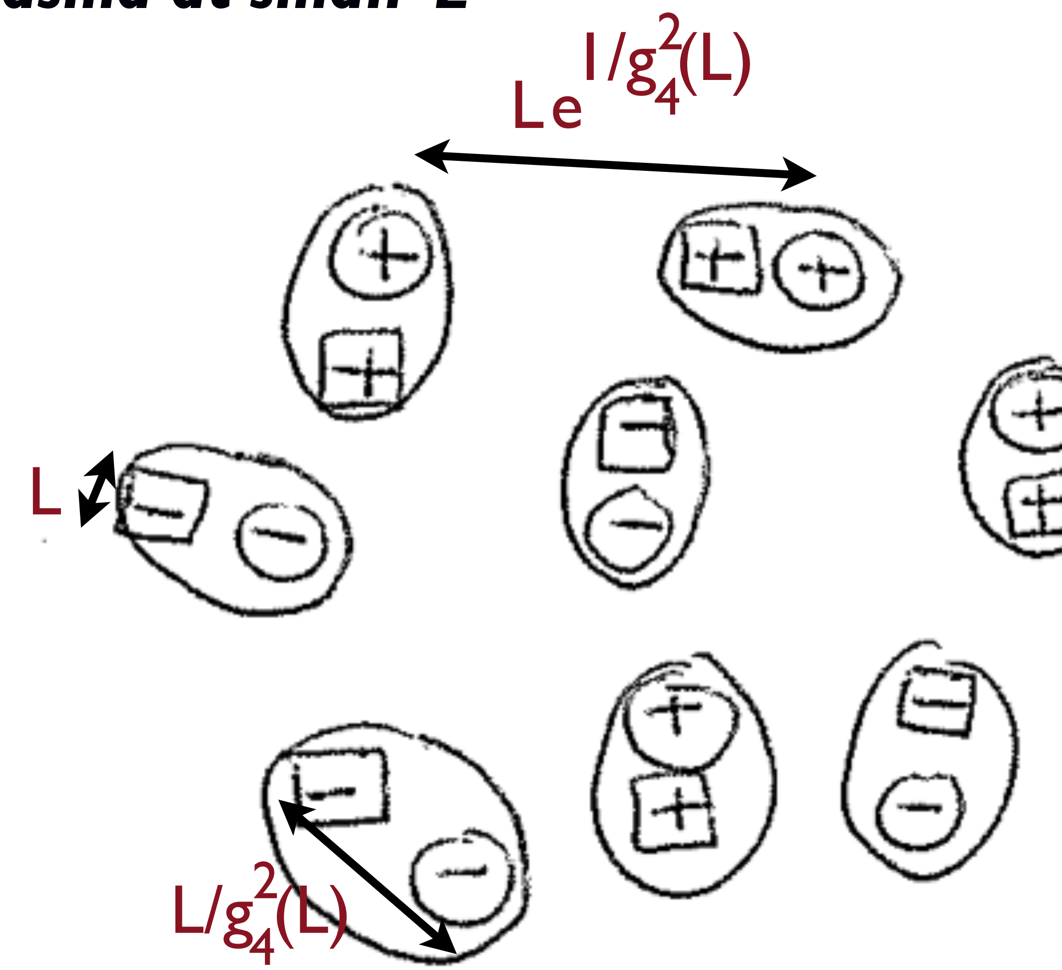


circles = M(+)/M*(-)

squares = KK(-)/KK*(+)

blobs = Bions(++)/Bions*(--)

4d QCD(adj) bion plasma at small-L
Unsal 2007, ...



M + KK* = B - magnetic "bions" -

-carry 2 units of magnetic charge
-no topological charge (non self-dual)
(locally 4d nature crucial: no KK in 4d)

bion stability is due to fermion attraction balancing Coulomb repulsion - results in scales as indicated

- bion/antibion plasma screening generates mass for dual photon

"magnetic bion confinement" operates at small-L in any theory with massless Weyl adjoints, including N=1 SYM (& N=1 from Seiberg-Witten theory)

it is "automatic": no need to "deform" theory other than small-L

first time confinement analytically shown in a non-SUSY, continuum, locally 4d theory

in the last couple of years, many theories have been studied...

Theory all SU(N)	Confinement mechanism on $\mathbb{R}^3 \times S^1$	Index for monopoles $[\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_N]$ Nye-M.Singer '00; PU '08	Index for instanton $I_{inst.} = \sum_{i=1}^{N} I_i$ Atiyah-Singer	$(\text{Mass Gap})^2$ units ~1/L^2
YM Y,U '08	monopoles	$[0, \ldots, 0]$	0	e^{-S_0}
QCD(F) S,U '08	monopoles	$[2, 0, \ldots, 0]$	2	e^{-S_0}
SYM /QCD(Adj) U '07	magnetic bions	$[2, 2, \ldots, 2]$	$2N$	e^{-2S_0}
QCD(BF) S,U '08	magnetic bions	$[2, 2, \ldots, 2]$	$2N$	e^{-2S_0}
QCD(AS) S,U '08	bions and monopoles	$[2, 2, \ldots, 2, 0, 0]$	$2N - 4$	e^{-2S_0}, e^{-S_0}
QCD(S) P,U '09	bions and triplets	$[2, 2, \ldots, 2, 4, 4]$	$2N + 4$	e^{-2S_0}, e^{-3S_0}
SU(2)YM $I = \frac{3}{2}$ P,U '09	magnetic quintets	$[4, 6]$ SUSY version: ISS(henker) model of SUSY [non-]breaking	10	e^{-5S_0}
chiral $[SU(N)]^K$ S,U '08	magnetic bions	$[2, 2, , \ldots, 2]$	$2N$	e^{-2S_0}
$AS + (N-4)\overline{\text{F}}$ S,U '08	bions and a monopole	$[1, 1, , \ldots, 1, 0, 0]$ + $[0, 0, \ldots, 0, N-4, 0]$	$(N-2)\text{AS} + (N-4)\overline{\text{F}}$	$e^{-2S_0}, e^{-S_0},$
$S + (N+4)\overline{\text{F}}$ P,U '09	bions and triplets	$[1, 1, , \ldots, 1, 2, 2]$ + $[0, 0, \ldots, 0, N+4, 0]$	$(N+2)\text{S} + (N+4)\overline{\text{F}}$	$e^{-2S_0}, e^{-3S_0},$

vectorlike | chiral

Table 1. Topological excitations which determine the mass gap for gauge fluctuations and chiral symmetry realization in vectorlike and chiral gauge theories on $\mathbf{R}^3 \times \mathbf{S}^1$. Unless indicated otherwise,

name codes:

U=Unsal
S=Shifman
Y=Yaffe
P=the speaker

+ SO(N),SP(N),G2,... - Argyres, Unsal - to appear; mixed-/higher-index reps.-P,U 0910.1245

in the last couple of years, many theories have been studied...

v e c t o r l i k e | c h i r a l

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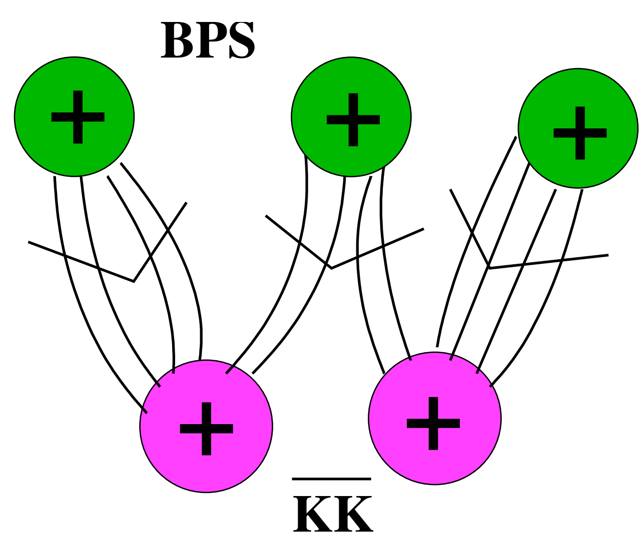

cartoon of the "magnetic quintet:" the leading cause of mass gap for the dual photon in non-SUSY chiral SU(2) with I=3/2

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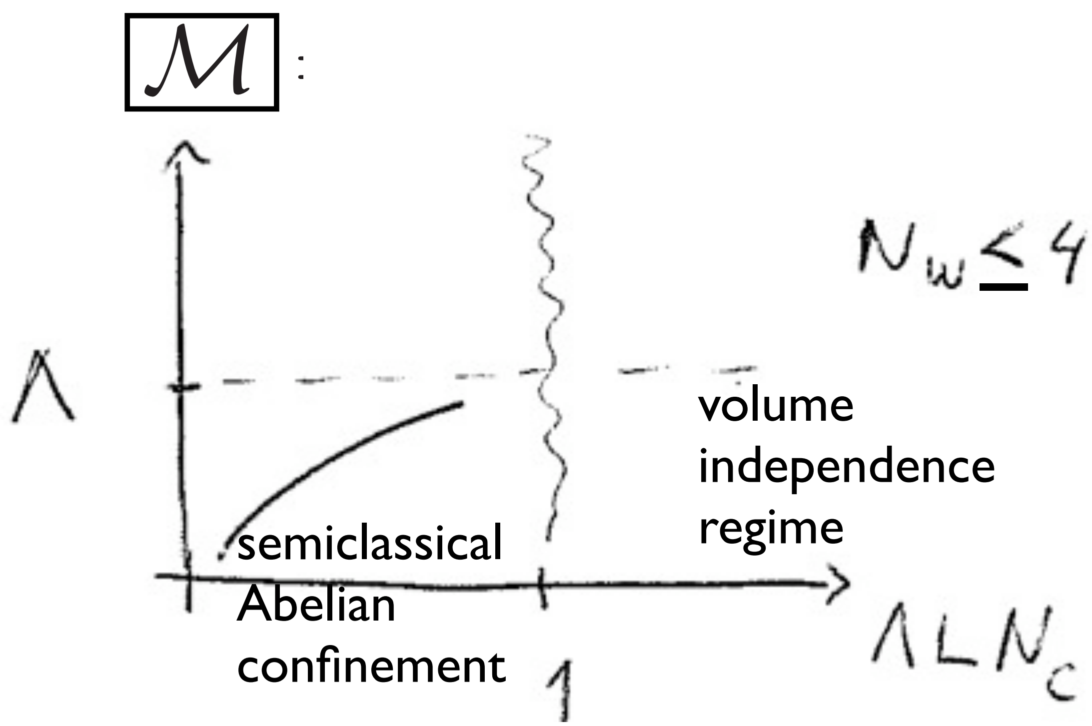
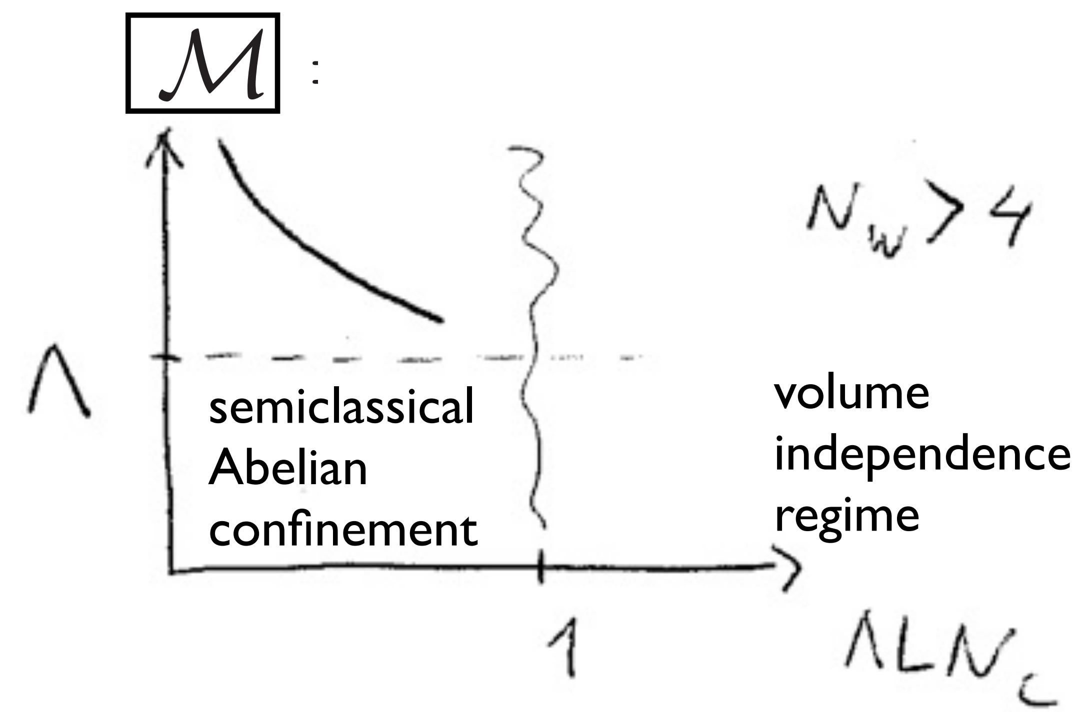
Unsal, EP 2009, Anber, EP 2011

$$\frac{\mathcal{M}}{\Lambda} \sim (\Lambda L)^{\frac{8-2n_w}{3}} e^{-2\pi\tilde{c}\left(\log\frac{1}{\Lambda L}\right)^{1/2}} \times (\text{less relevant contributions})$$

strong scale

O(1), positive





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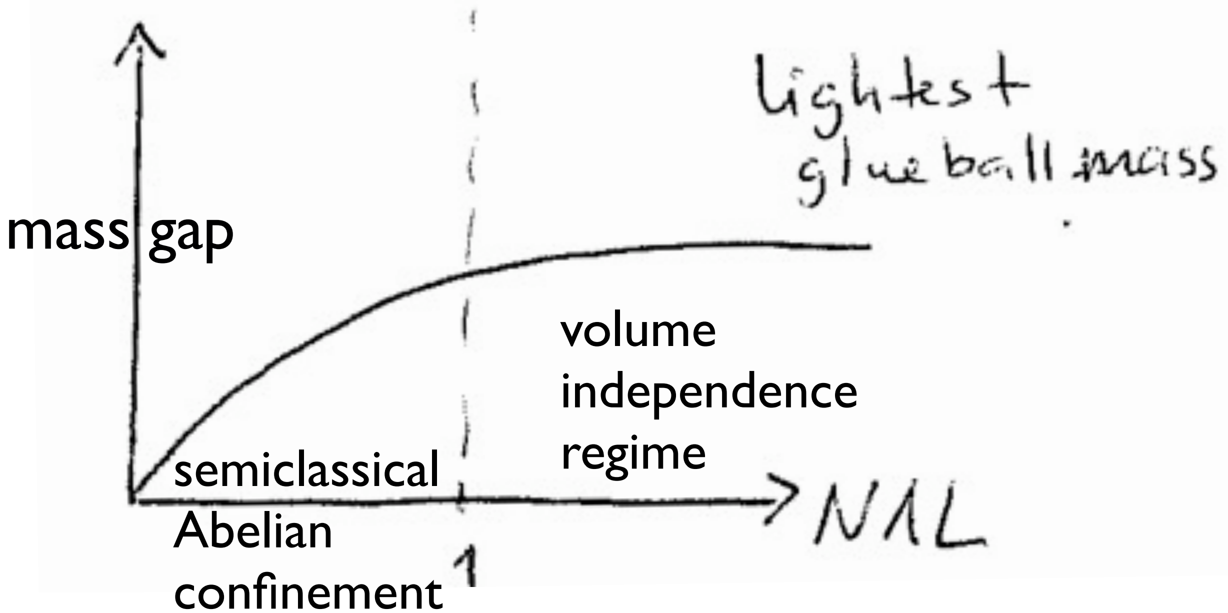
strong scale

O(1), positive

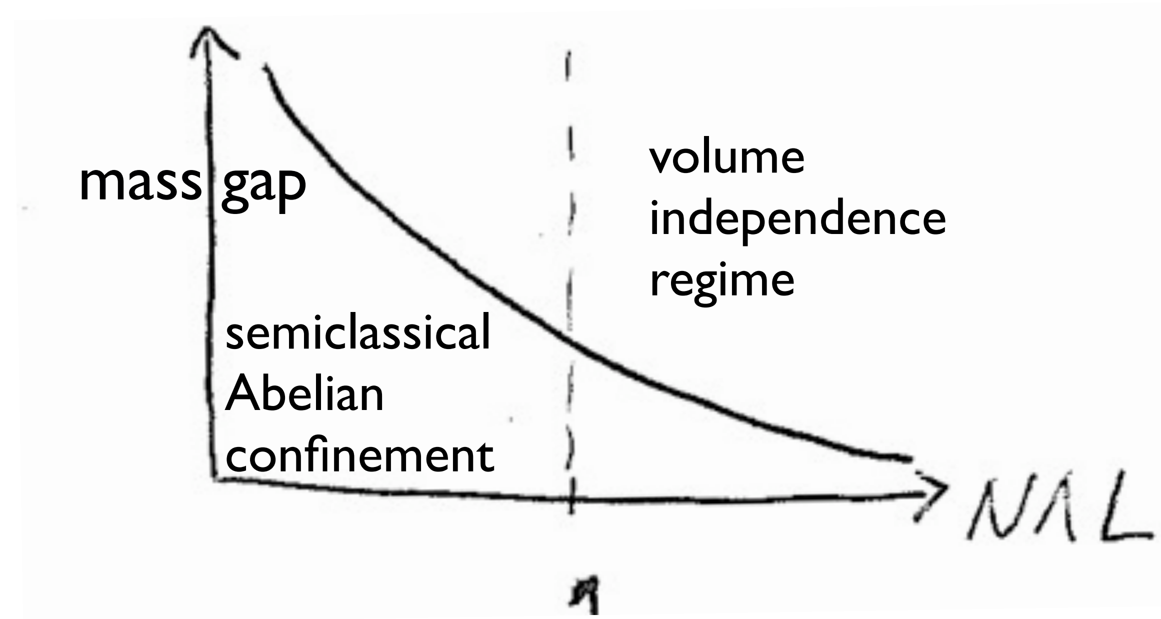
... how **dare** you study non-protected quantities?

Back to SU(N) with Weyl adjoints [no deformation needed]:

less than 4 Weyl adjoints (and =4, Anber,EP, 5/2011)



>4 Weyl adjoints
mass gap = 0 at infinite L: conformal?



The idea is that if theory does not confine, topological excitations causing confinement should dilute away, causing vanishing of mass gap. Prehaps most defensible for 5 adjoints ~ "Banks-Zaks-ish"...

taken from Ryttov, Sanino:

	our "estimate"	gap equation	beta function gamma=2/1	AF lost	"experiment"
any N	4	4.15	2.75/3.66	5.5	**4 ?**

Catterall et al; del Debbio, et al; Hietanen et al. all 2007-

all theoretical "determinations" rely on un-controlled extrapolations
hence, "error bars" unknown

lattice will eventually tell us whether curves really continue like this...
(but it may take a long time!)

meanwhile, compare estimates for other models from theory and lattice:

comparing theory "estimates" of critical number of flavors for SU(N)

Weyl adjoints [no deformation needed]

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"experiment"

4 ? e.g.: *Catterall et al; del Debbio, Patella,Pica; Hietanen et al.*

Dirac 2-index (anti)symmetric tensor [deformation needed; but large-N equiv!]

N	our estimate	gap eqn	beta function gamma=2/1	AF lost
3	2.40	2.50	1.65/2.2	3.30
4	2.66	2.78	1.83/2.44	3.66
5	2.85	2.97	1.96/2.62	3.92
10	3.33	3.47	2.29/3.05	4.58
∞	4	4.15	2.75/3.66	5.5

2 ? e.g.: *DeGrand,Shamir, Svetitsky; Fodor et al; Kogut, Sinclair*

Dirac fundamentals [deformation needed]

N	our estimate (a/c)	gap eqn	functional RG	beta function gamma=2/1	AF lost
2	5/8	7.85	8.25	5.5/7.33	11
3	7.5/12	11.91	10	8.25/11	16.5
4	10/16	15.93	13.5	11/14.66	22
5	12.5/20	19.95	16.25	13.75/18.33	27.5
10	25/40	39.97	n/a	27.5/36.66	55
∞	$2.5N/4N$	$4N$	$\sim(2.75-3.25)N$	$2.75N/3.66N$	$5.5N$

12 ? e.g.: *Appelquist,Fleming, Neal; Deuzemann, Lombardo,Pallante; Iwasaki et al; Fodor et al; Jin, Mahwinney; A. Hasenfratz*

gap equation and lattice - only vectorlike theories;
beta function

in chiral gauge theories with multiple "generations" our estimates were the only known ones until Sannino's recent 0911.0931 via the proposed exact beta function

comparing theory "estimates" of critical number of flavors for SU(N)

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"experiment"

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QUICK REVIEW:
What other insights has the semiclassically calculable volume-dependent regime given us?

- A FEW RECENT EXAMPLES -

1.) let's go back to SUSY:

We argued that "magnetic bions" are responsible for confinement in N=1 SYM at small L - a particular case of our Weyl adjoint theory - a "Polyakov like" confinement. This remains true if N=1 obtained from N=2 by soft breaking.

On the other hand, we know monopole and dyon condensation is responsible for confinement in N=2 softly broken to N=1 at large L (Seiberg, Witten `94)

So, in different regimes we have different pictures of confinement in softly broken N=2 SYM. Both regimes are Abelian and quantitatively understood. Turns out they connect via Poisson resummation.

[EP, Unsal 2011]

small-L physics well described by a few twisted monopole-instantons (as we'd already done) - or an infinite sum over charged 4d dyons

large-L physics well described by a few dyons - or an infinite sum over twisted monopole instantons

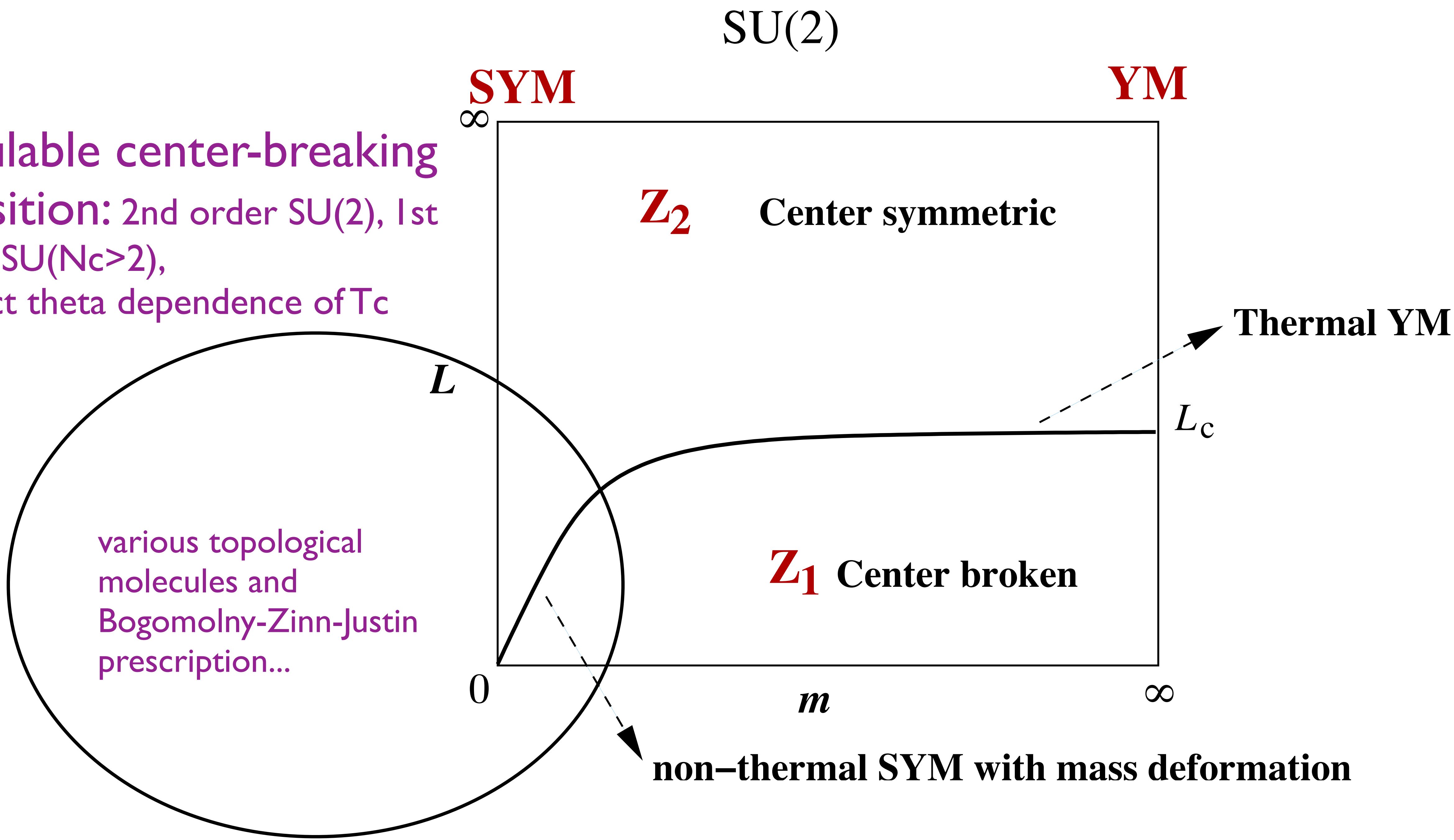
(some wall-crossing results useful)

2.) SUSY w/ gaugino mass and deconfinement in pure YM:

[EP, Schaefer, Unsal 2012]

pure SYM with gaugino mass on a (non-)thermal S^1 is a theory lab allowing study of deconfinement transition in a controlled setting

calculable center-breaking transition: 2nd order SU(2), 1st order SU(Nc>2), correct theta dependence of Tc



3.) Thermodynamics of deformed YM and QCD(adj) at small L:

$R^2 \times S^1 \times S^1$ compactifications [Simic, Unsal 2010 & Anber, EP, Unsal, 2011]

non-thermal

thermal

"deformed" pure-YM "QCD(adj)"= YM + Nf massless adjoint fermions

at small S^1, map 4d thermal gauge theory to a 2d spin system - "affine" XY spin models related to cond. mat. systems studying, e.g., 2d triangular lattice crystal melting for SU(3)(adj)

abelian (de-) confinement only-**nonetheless, (I think) fascinating systems:** **2d "gases" of el. and m. charged particles, with Aharonov-Bohm interactions, inheriting the symmetries of their respective 4d gauge theories and showing a deconfinement transition**

4.) Bogomolny-Zinn-Justin, resurgent series, and semiclassical QFT...

[Unsal, Argyres...2012.xxx]

5+.) omitted older stuff - chiral gauge theories and the like...

SUMMARY - *two regimes in finite volume studies:*

$N_c \Lambda L \gg 1$ ***(can be holographic)***

studying gauge dynamics at finite L can yield exact results for infinite L theory at large-N if EK can be "made to work"

- it appears that there are working examples of large-N volume independence now
- analytic approaches await developments/new ideas
  - from AdS/CFT example, problem appears equally hard (in my uneducated opinion)
- numerical efforts just beginning, appear promising...

$N_c \Lambda L \ll 1$ ***(non-holographic)***

the volume-dependent regime yields semiclassically calculable nonperturbative dynamics of 4d gauge theories

- confinement, deconfinement, chiral symmetry breaking - a host of difficult phenomena can be described semiclassically - ***non-gravitationally...*** - with a clear connection to the well-defined microscopic theory
- the dynamics is very rich on its own, and offers fascinating connections to, e.g., condensed matter systems - "melting"-deconfinement in QCD(adj)

the hope is that, apart from being fun, there is a continuous connection to the 4d "real thing" - in some cases seems to appear, e.g., large-nf conformality, deconfinement...