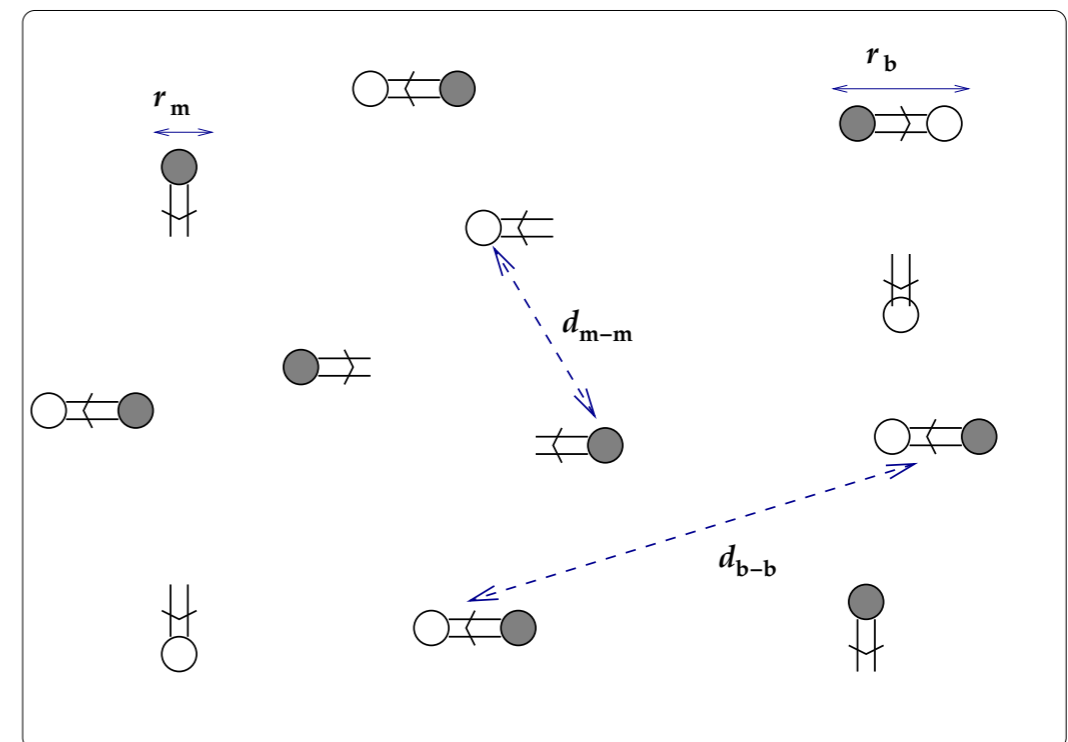


Supersymmetry and neutral bions: hints about deconfinement?

Erich Poppitz



works with

Thomas Schäfer Mithat Ünsal **NCSU** 1205.0290 1212.1238

Mohamed Anber Brett Teeple **Toronto** 1406.1199

will also mention work with Tin Sulejmanpasic **Regensburg** 1307.1317

While the LHC continues the search for variants of weak-scale supersymmetry: “natural”, “compressed”, “(super)split” or “flavorful”, among others
- and may or may not find evidence for it -
I will discuss another, less direct, less mainstream, and more recent, use of supersymmetry in particle theory... albeit one that will not be seen at the LHC...

main message:

It has been realized that studies of supersymmetric gauge theories in the late 1990's, when properly interpreted, lead to insights whose relevance transcends supersymmetry.

...this is really a talk about the “inner working” of QFT, not so much about nuclear theory, applications, or about comparison with real experimental data...

I will illustrate this use of supersymmetry by an example that may have to do with the microscopic description of the thermal **deconfinement** transition in pure YM.

A host of strange **topological molecules** will be seen to be the major players in the dynamics.

Interesting connections emerge, between topology, “condensed-matter” gases of electric and magnetic charges (not this talk!), and attempts to make sense of the divergent perturbation series (also not this talk!).

Outline:

- 1. The “*SYM*/thermal YM-continuity*” conjecture**
- 2. Evidence for conjecture: calculable *SYM** vs lattice**
- 3. Novel topological excitations and their role.**
Why this seems to work the way it does?

$\mathbb{R}^3 \times S^1$ compactifications of SYM*

(non-) thermal

early remarks in Unsal, Yaffe 1006.2101
[Schaefer, Unsal, EP 1205.0290, 1212.1238
Anber 1302.2641; Sulejmanpasic, EP 1307.1317;
Anber, EP 1406.1199]

DEFINITIONS:

1

super YM = "SYM" = YM + massless quark, an adjoint Weyl "gaugino"

fields: gauge bosons + gauginos $Z_{(2N)}$ chiral symmetry for $SU(N)$
[$Z_{(2c_2)}$ chiral symmetry for arbitrary G (cover group)]

2

SYM* = SYM + mass for the triplet quark, i.e. with a "gaugino mass" m

supersymmetry and chiral symmetry **explicitly broken** by m

we study SYM* on $\mathbb{R}^3 \times S^1_L$ with periodic (**supersymmetric, non-thermal**)
boundary condition for gaugino

there are only two parameters to vary: L and m

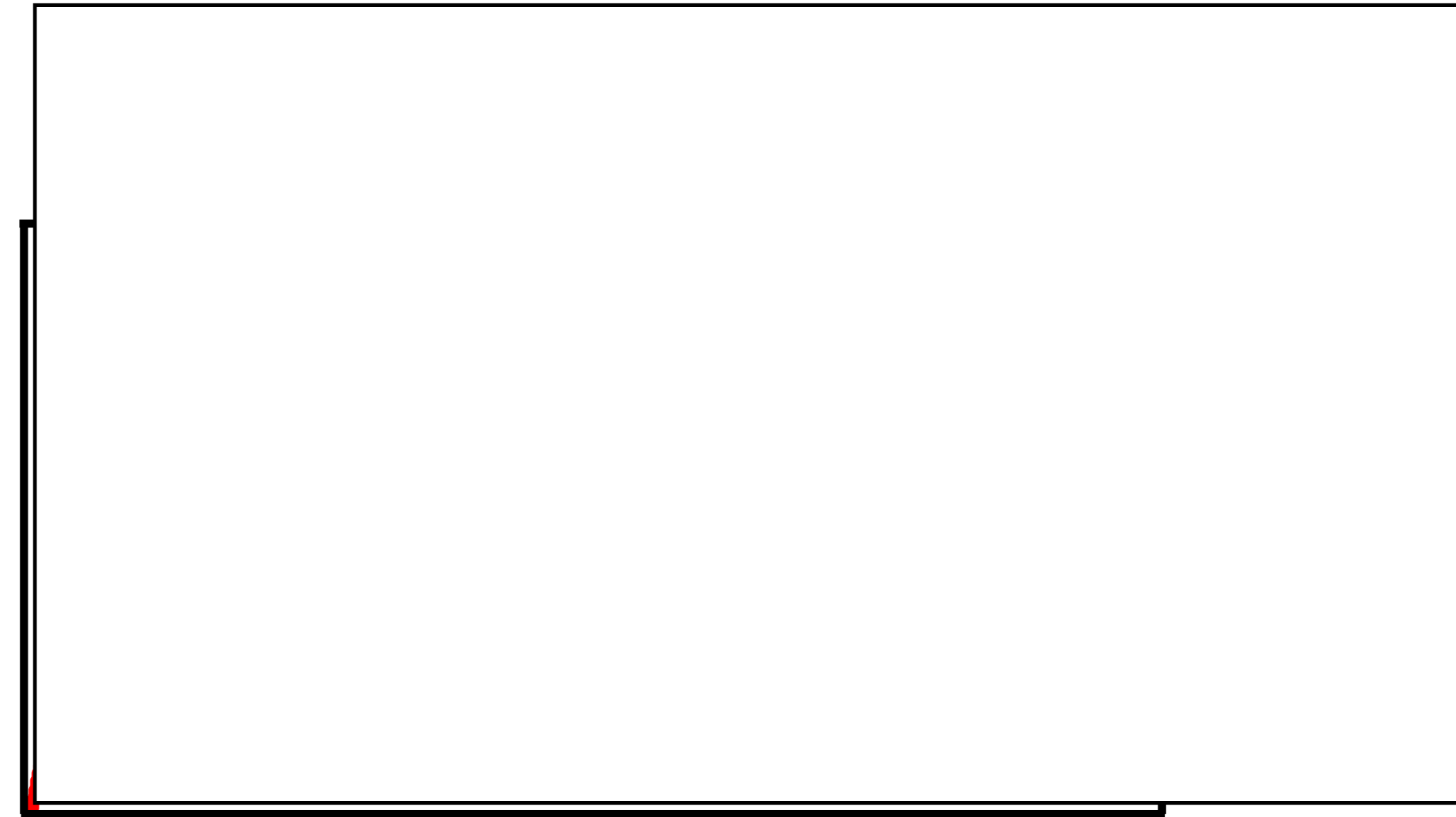
the theory is asymptotically free with a strong scale Λ

$$\left(\frac{m}{\Lambda} \quad \Lambda L \right)$$

$R^3 \times S^1$ compactifications of SYM*

(non-) thermal

size of circle



0

gaugino mass m



SYM on $R^3 \times S^1$:

Seiberg, Witten 1996

Aharony, Hanany, Intriligator, Seiberg, Strassler 1997

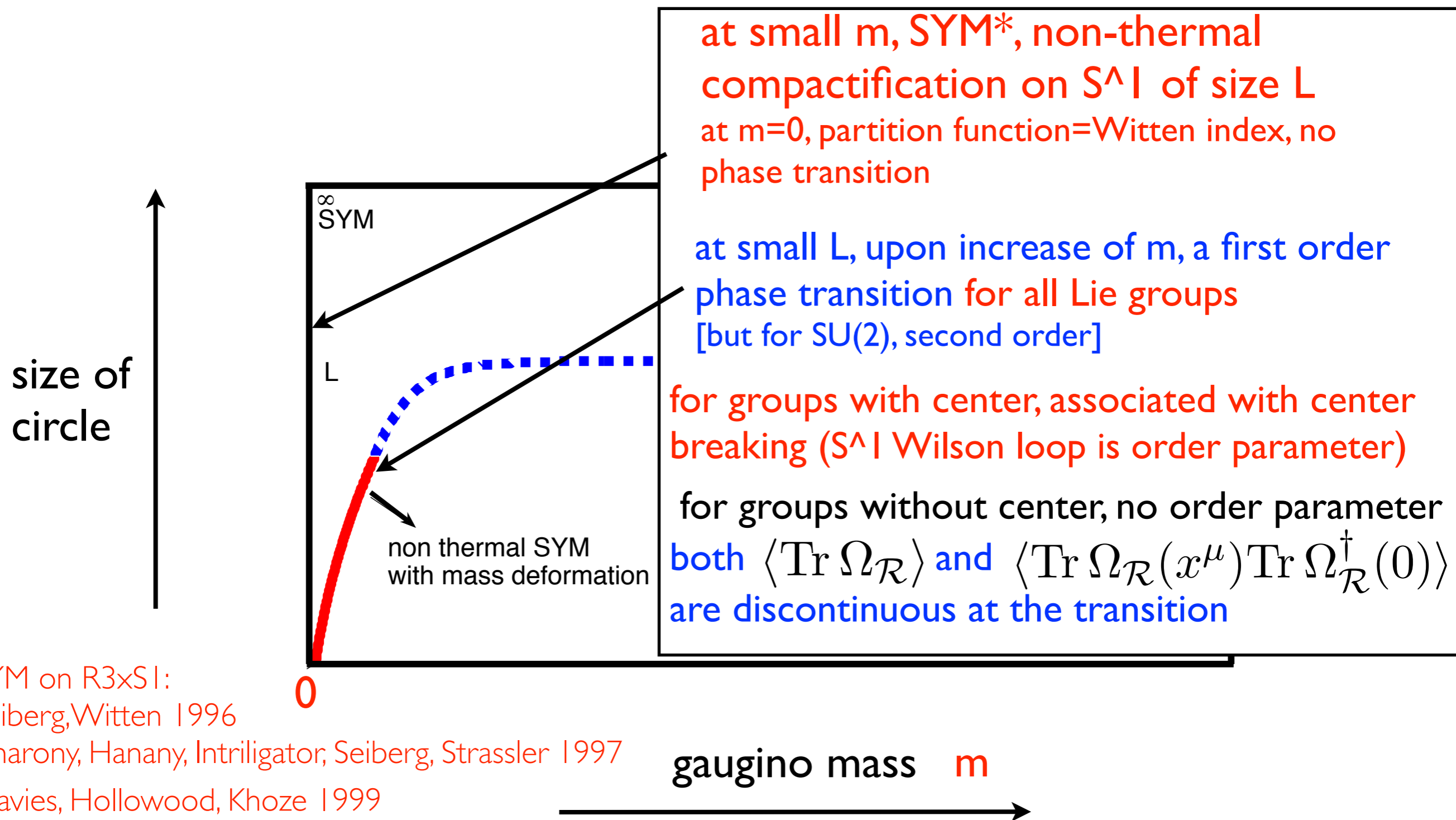
Davies, Hollowood, Khoze 1999

important relevant details of instanton calculation only

EP, Schaefer, Unsal, 2012 + Anber, EP, Teeple 2014

$R^3 \times S^1$ compactifications of SYM*

(non-) thermal



SYM on $R^3 \times S^1$:
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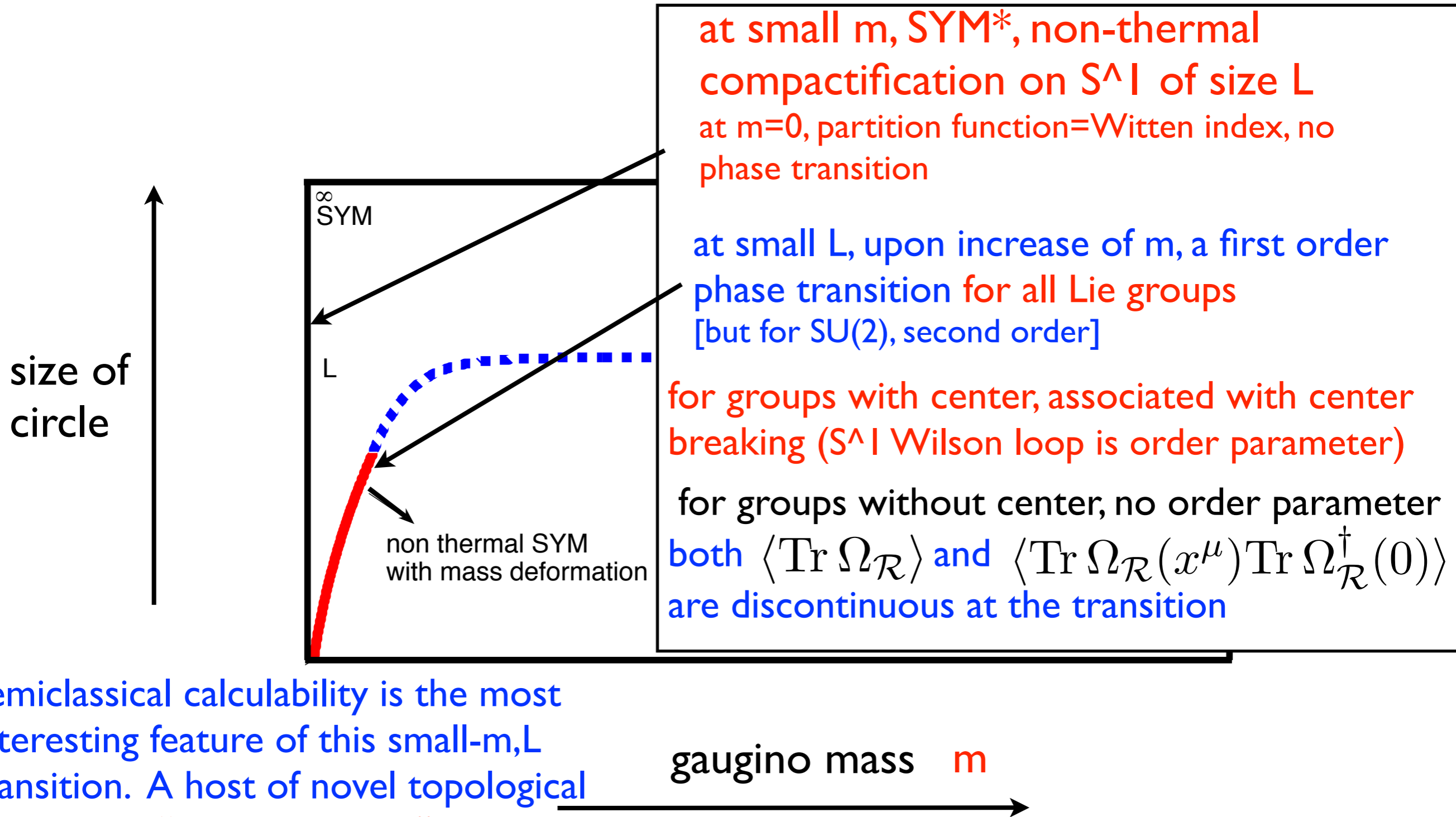
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important relevant details of instanton calculation only

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$R^3 \times S^1$ compactifications of SYM*
 (non-) thermal



at small m , SYM*, non-thermal compactification on S^1 of size L
 at $m=0$, partition function=Witten index, no phase transition

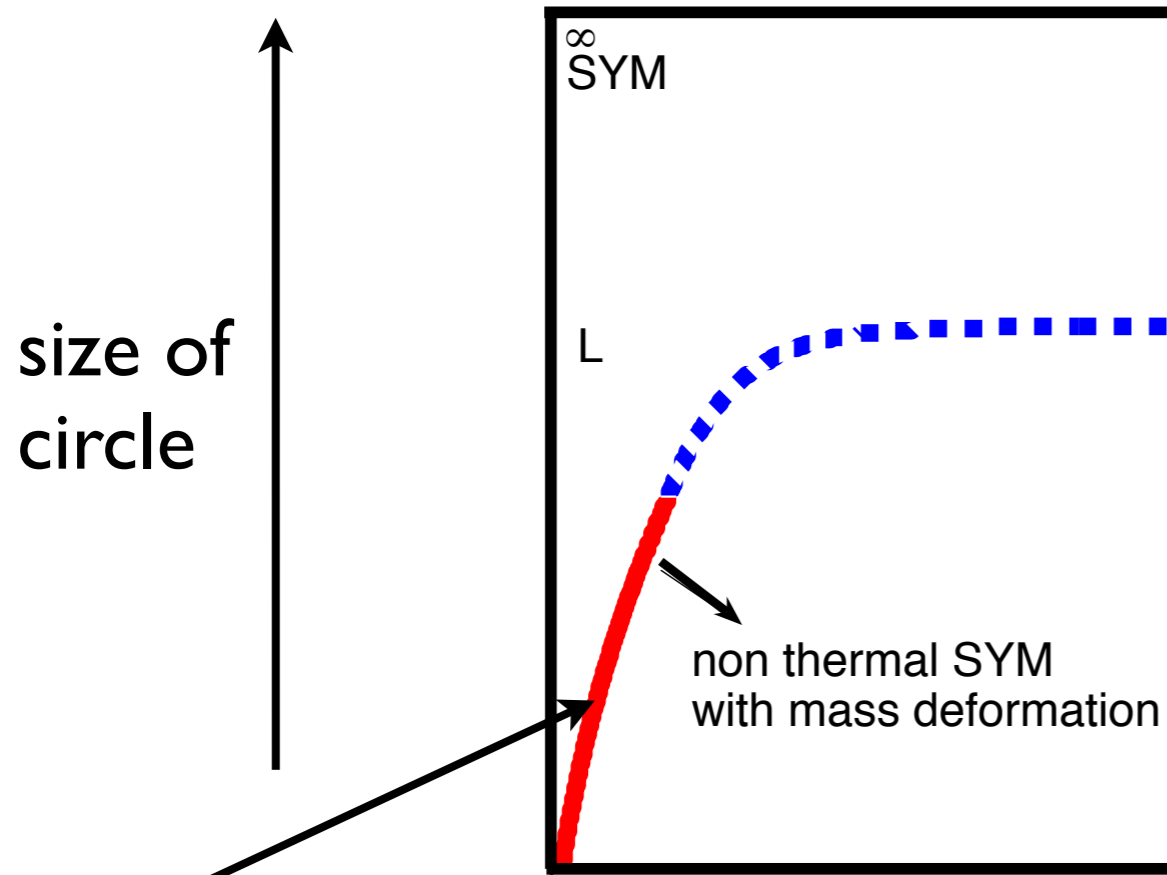
at small L , upon increase of m , a first order phase transition for all Lie groups [but for $SU(2)$, second order]

for groups with center, associated with center breaking (S^1 Wilson loop is order parameter)

for groups without center, no order parameter
 both $\langle \text{Tr } \Omega_{\mathcal{R}} \rangle$ and $\langle \text{Tr } \Omega_{\mathcal{R}}(x^\mu) \text{Tr } \Omega_{\mathcal{R}}^\dagger(0) \rangle$ are discontinuous at the transition

Semiclassical calculability is the most interesting feature of this small- m, L transition. A host of novel topological excitations: “magnetic bions” (Unsal 2007) and “neutral bions” (EP Unsal 2012, Argyres Unsal 2012...) whose raison d’etre runs deep... are responsible for confinement and potential for S^1 holonomy (& center stability, where present)

$R^3 \times S^1$ compactifications of SYM*
 (non-) thermal

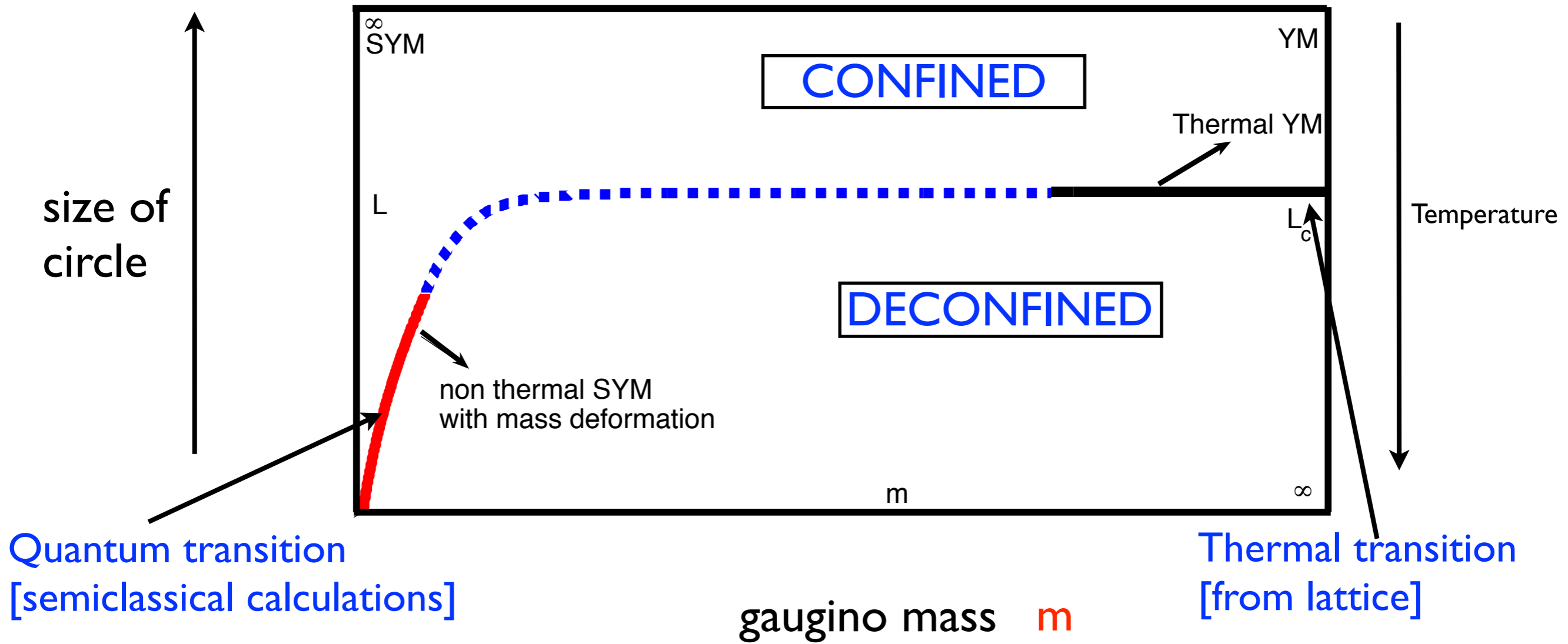


...these effects were already in the 1990's papers I mentioned, but because they relied so much on supersymmetry ($V \sim W'^2$) the generality of the physics, which transcends supersymmetry, was missed!

... similar excitations exist in non-SUSY theories (QCD(adj)) and can even be identified in pure thermal YM (if a holonomy expectation value is assumed)

Semiclassical calculability is the most interesting feature of this small- m, L transition. A host of novel topological excitations: “magnetic bions” (Unsal 2007) and “neutral bions” (EP Unsal 2012, Argyres Unsal 2012...) whose raison d’etre runs deep... are responsible for confinement and potential for S^1 holonomy (& center stability, where present)

$R^3 \times S^1$ compactifications of SYM*
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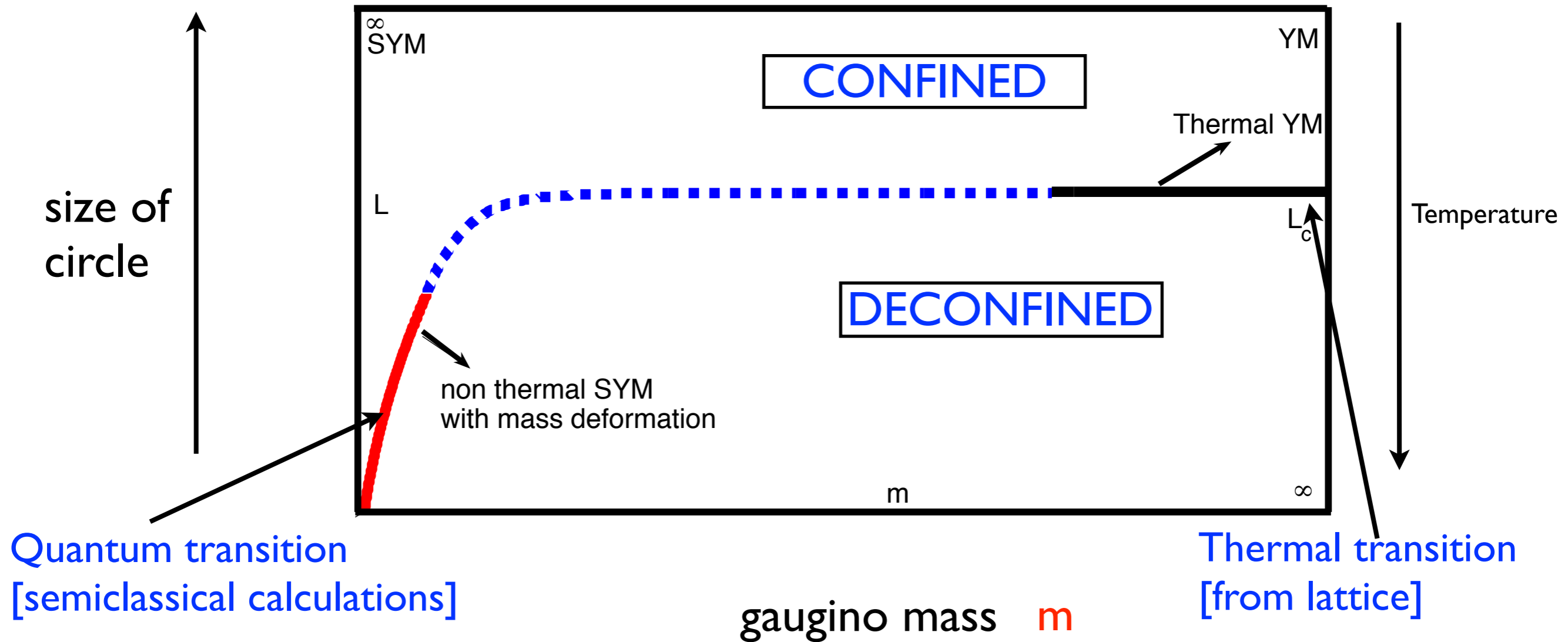
Quantum transition
 [semiclassical calculations]

Thermal transition
 [from lattice]

In what follows, we shall compare behavior of
 $\langle \text{Tr } \Omega_{\mathcal{R}} \rangle$, $\langle \text{Tr } \Omega_{\mathcal{R}}(x^\mu) \text{Tr } \Omega_{\mathcal{R}}^\dagger(0) \rangle$ (and other quantities)
 at the two transitions and find striking similarities...

$R^3 \times S^1$ compactifications of SYM*
 (non-) thermal

“continuity conjecture” = this phase diagram

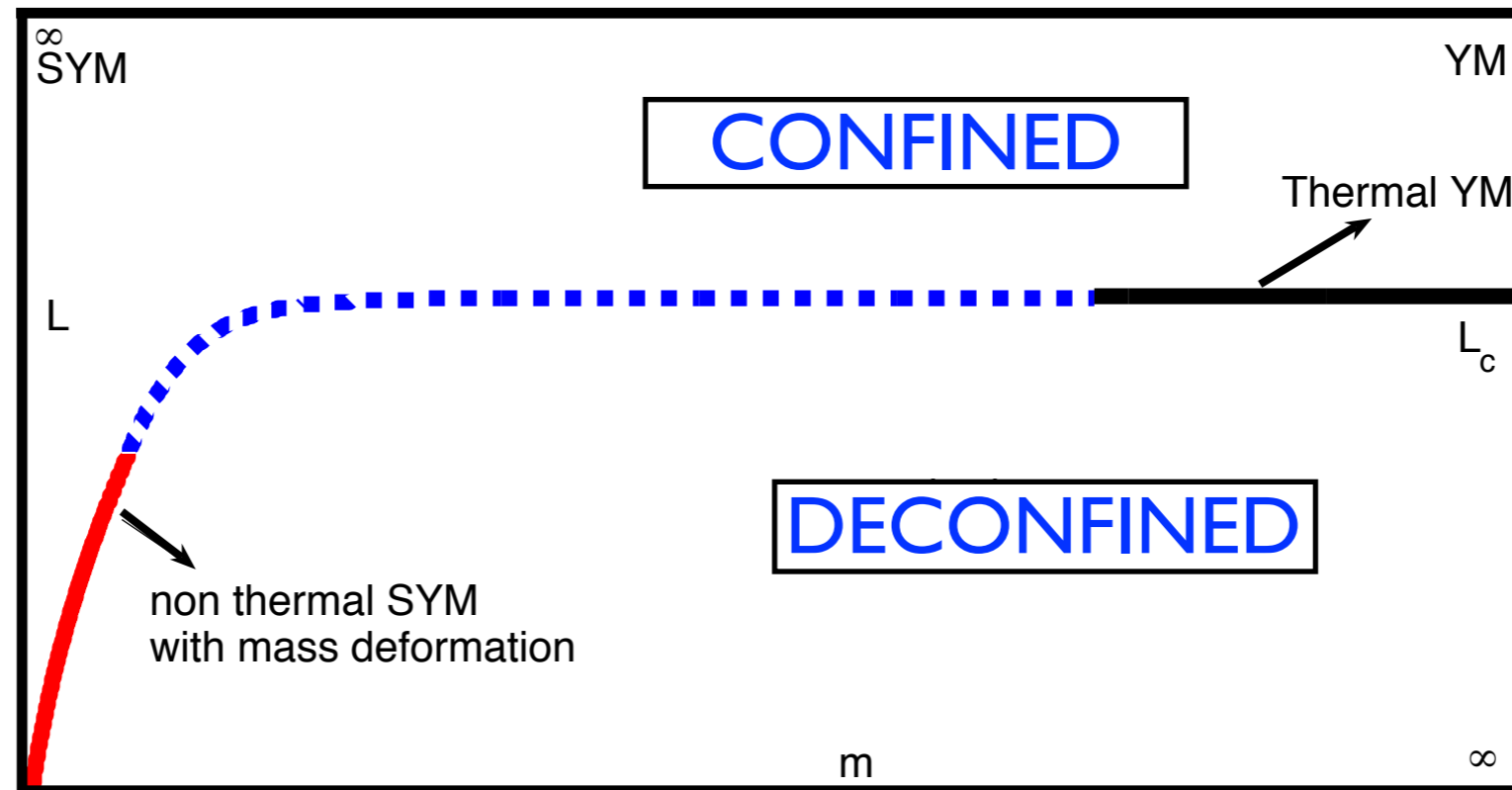


At small m, L , the transition can be studied in a theoretically controlled manner. Novel topological excitations and perturbative contributions yield competing effects, resulting in a transition as dimensionless parameter varies $\frac{m}{L^2 \Lambda^3}$.

End of:

I. The “SYM*/thermal YM-continuity” conjecture

“continuity conjecture” = this phase diagram



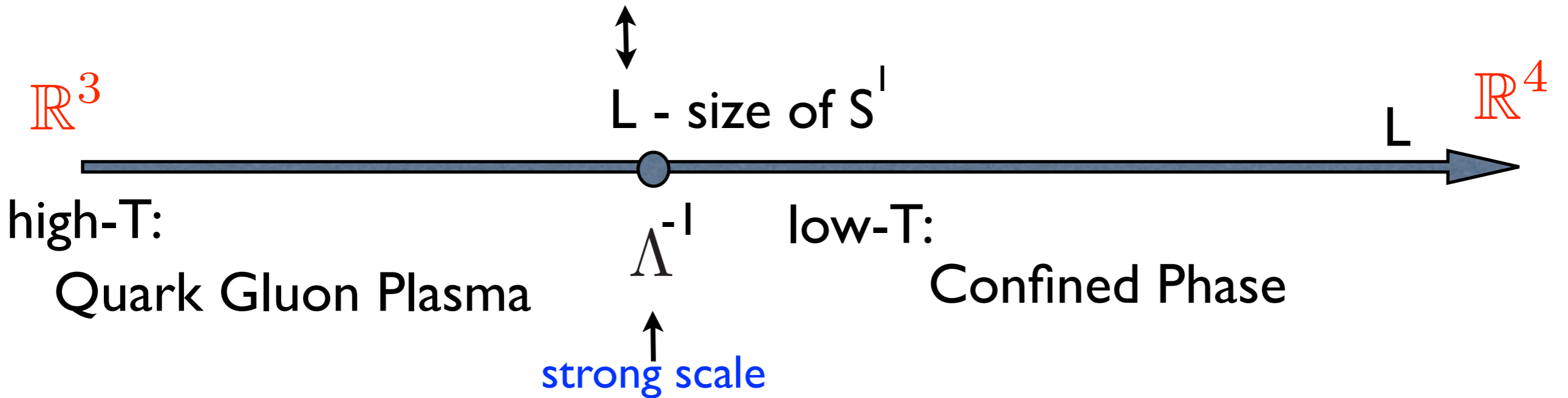
Next:

2. Evidence for conjecture: calculable SYM* vs lattice

First, review a few facts about thermal theories...

Thermal partition function is (without fermions):

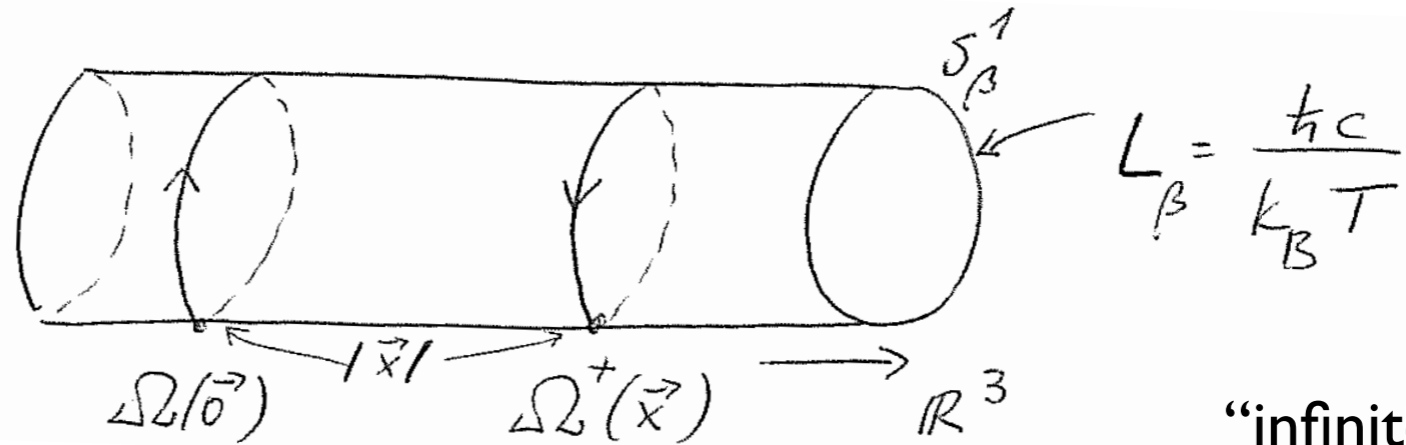
$$Z(\beta) = \text{tr}[e^{-\beta H}], \quad \beta = 1/T = \text{radius of } S^1 \quad \mathbb{R}^3 \times S^1$$



a static quark probe

$$\Omega = \text{tr} \mathcal{P} \exp\left[i \int_{S^1} A_4 dx^4\right]$$

Wilson/Polyakov loop



\bar{q} at \vec{x} q at $\vec{0}$

$$\langle \Omega^\dagger(\vec{x}) \Omega(0) \rangle \sim e^{-\frac{V(|\vec{x}|)}{T}}$$

confined $e^{-\frac{\sigma|\vec{x}|}{T}} \rightarrow 0$ as $x \rightarrow \infty$

deconfined $e^{-\frac{e^{-m_e|\vec{x}|}}{|\vec{x}|T}} \rightarrow 1$ as $x \rightarrow \infty$

hence $\langle \Omega \rangle = 0$ confined

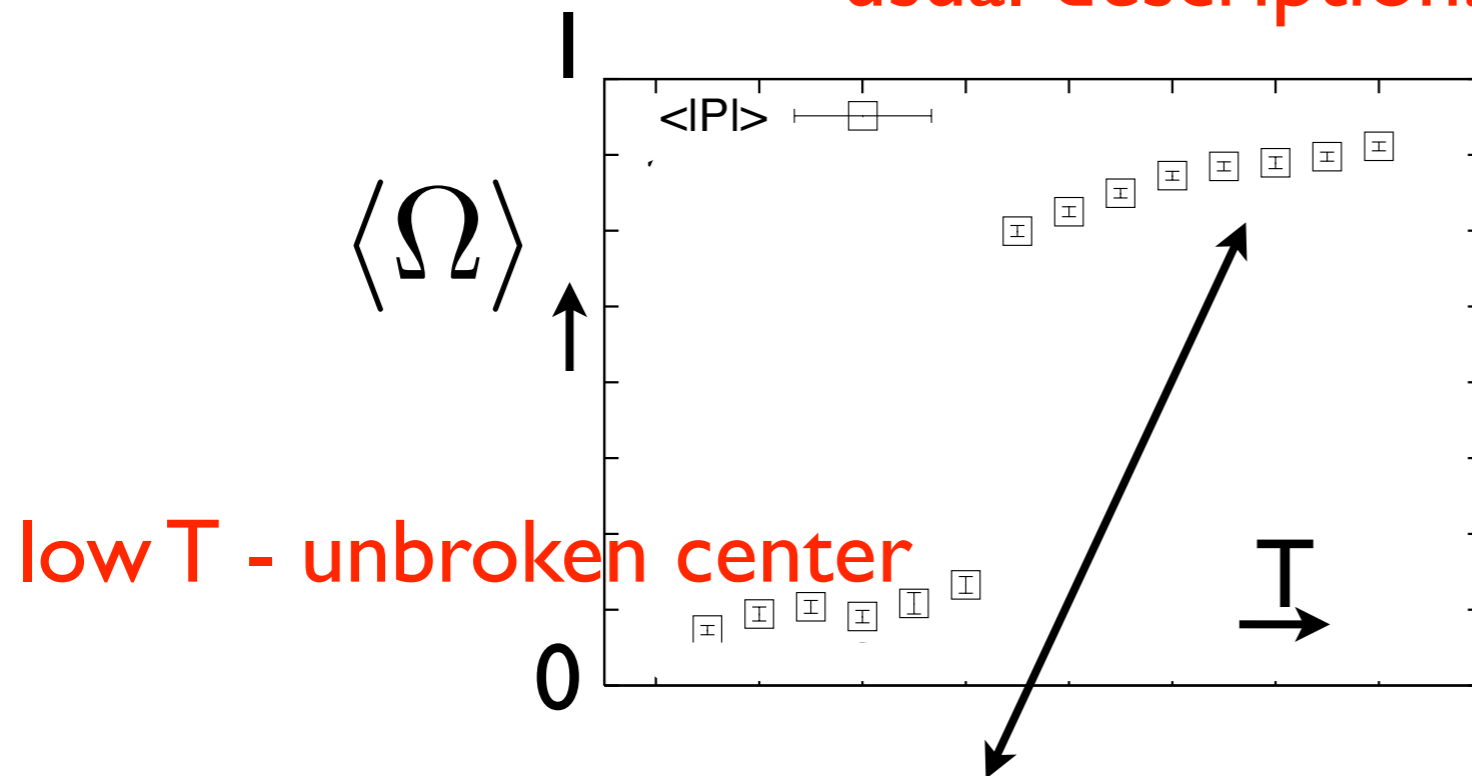
$\langle \Omega \rangle \neq 0$ deconfined

“infinite F_quark”

in SU(N) theory without fundamentals, deconfinement = breaking of global Z_N center symmetry

$$\Omega_{\text{fund}} \xrightarrow{z \in Z_N} z \Omega_{\text{fund}} \quad \Omega = \text{tr} \mathcal{P} \exp \left[i \int_{S^1} A_4 dx^4 \right]$$

usual description: high T - “broken center”



this is what lattice sees...

to find $\langle \dots \rangle$ in finite volume, usual stat mech tricks

notice center symmetry broken at high-T: counterintuitive, discussion of domains/walls-Euclidean configs, see Smilga '94,'98+...

$T \gg T_c$ behavior has been understood for 30 years

[Gross, Pisarski, Yaffe, 1981]

High-T perturbation theory good, gives one-loop $V(\text{pert})$, favors center-broken vacuum, e.g.

$$V_{\text{pert.}}(\Omega) = -\frac{2}{\pi^2 \beta^4} \sum_{n=1}^{\infty} \frac{1}{n^4} |\text{tr} \Omega^n|^2 (1 + O(g^2)).$$

=coinciding eigenvalues

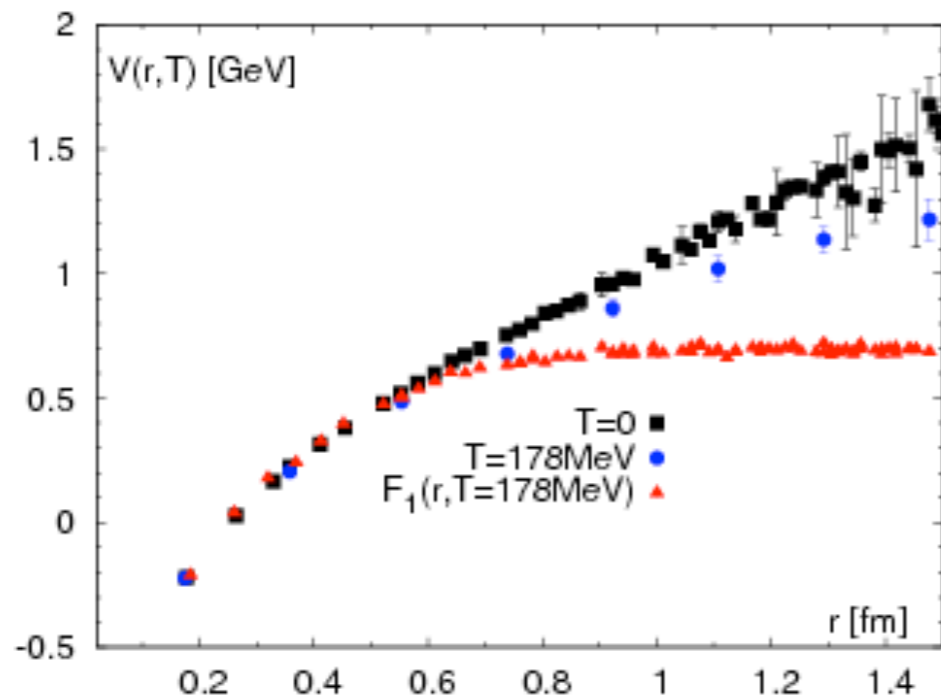
$$\Omega = \frac{1}{2} \text{Tr} \begin{pmatrix} e^{i\pi\nu} & 0 \\ 0 & e^{-i\pi\nu} \end{pmatrix}$$

in $SU(N)$ theory without fundamentals, deconfinement =
breaking of global Z_N center symmetry

$$\Omega_{\text{fund}} \xrightarrow{z \in Z_N} z \Omega_{\text{fund}} \quad \Omega = \text{tr} \mathcal{P} \exp\left[i \int_{S^1} A_4 dx^4\right]$$

usual description: high T - “broken center”

perhaps more physical:
quark-antiquark (probe) potential



this is also what lattice sees...

string tension discontinuously
jumps to zero [for theories
with center only]

$$\langle \Omega^\dagger(\vec{x}) \Omega(0) \rangle \sim e^{-\frac{V(|\vec{x}|)}{T}}$$

both discontinuities - of the trace of Polyakov loop or of its two point
function - are seen also in the semiclassical SYM* quantum transition

(for theories with nontrivial center)

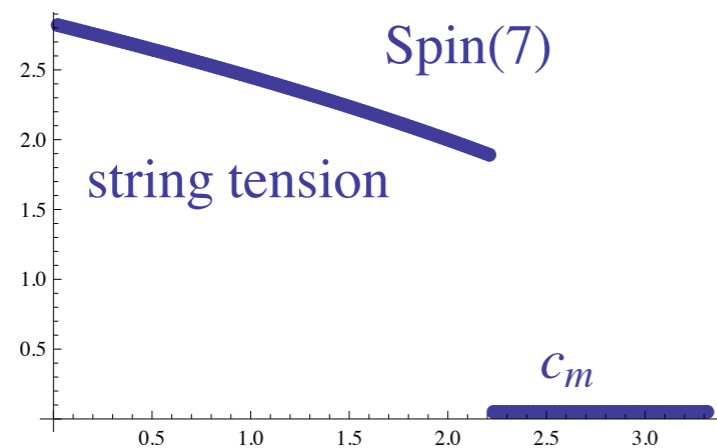
calculable SYM* vs lattice

Both discontinuities - of the trace of Polyakov loop or of its two point function - are seen also in the semiclassical SYM* quantum transition

for all theories with nontrivial center: SU(N), Sp(2N), Spin(N), E_6, E_7

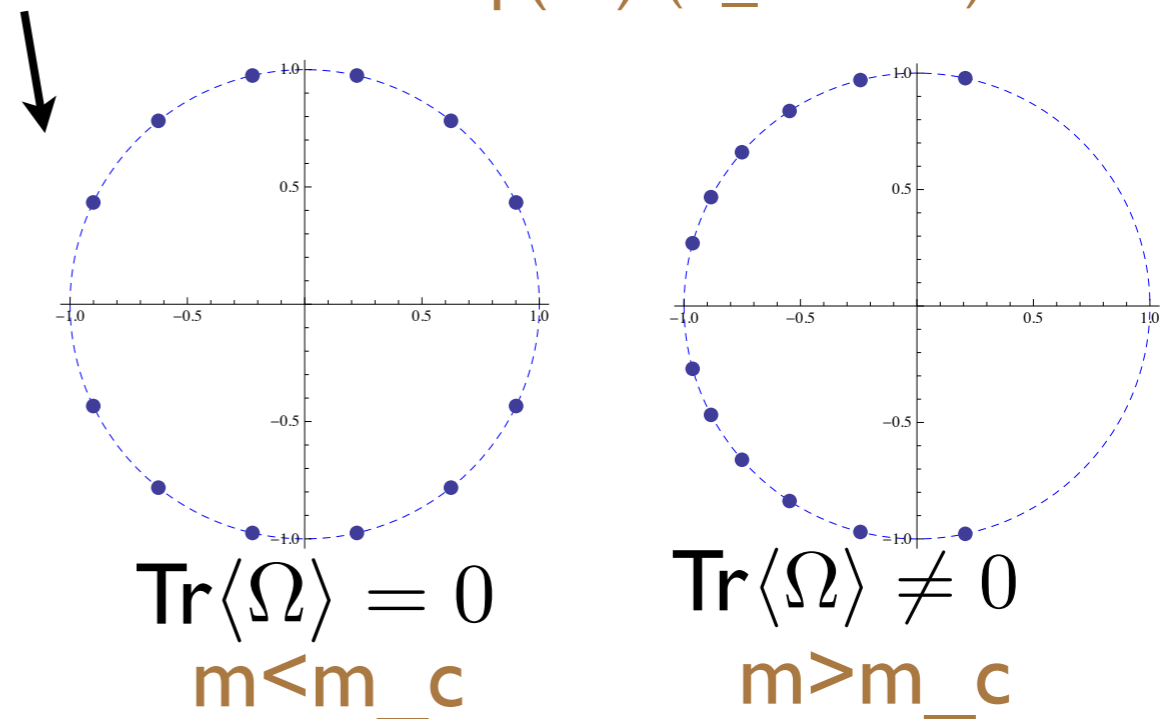
we have for $m < m_c$

$$\langle \text{Tr } \Omega(x) \text{Tr } \Omega^\dagger(0) \rangle \Big|_{r \gg m_0^{-1}} \simeq e^{-\frac{\hat{\sigma} m_0}{R} r R} \equiv e^{-\sigma r R} \quad (\text{and a constant at } m > m_c)$$



← e.g., probes in the spinor of SO(7)

e.g., eigenvalues of Polyakov loop in fundamental of Sp(12) (Z_2 center)



For the trace of the Polyakov loop, we have, for all groups with center:

Lattice only SU(N) and Sp(4)
[latter case motivated by “Z2 universality”]

calculable SYM* vs lattice

for all theories without center: G_2, F_4, E_8

Lattice only G_2 ↔ SYM*: all transitions discontinuous

$$\langle \text{Tr } \Omega(x) \text{Tr } \Omega^\dagger(y) \rangle = \begin{cases} 0.0056 \left(\frac{g^2}{4\pi} \right)^2 & \leftarrow m=0 \text{ value (SYM)} \\ 0.0056 \left(\frac{g^2}{4\pi} \right)^2 & \leftarrow \text{below transition} \\ 11.3 \left(\frac{g^2}{4\pi^2} \right)^2 & \leftarrow \text{above transition} \end{cases}$$

numbers from Anber, EP, Teple 1406.1199

correct omission of quantum-corrected monopole-instanton vertex in 1212.1238 Schaefer, EP, Unsal

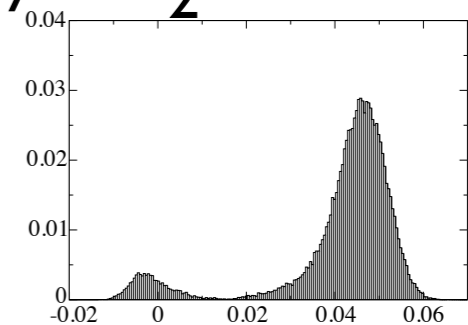
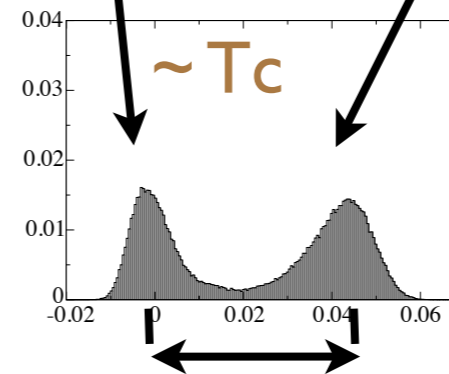
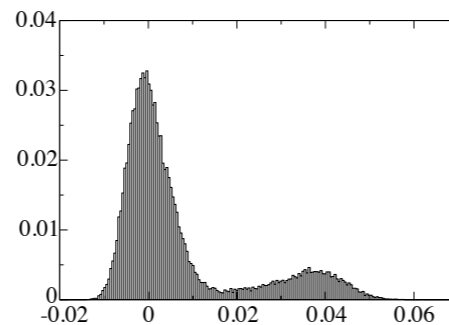
SYM* jump of Polyakov loop trace: $\langle \text{Tr } \Omega \rangle$

$$\frac{g^2}{4\pi} 0.0746 \quad \frac{g^2}{4\pi} 3.437$$

lattice jump of Polyakov loop trace:

[Pepe, Wiese 2006;
Cossu et al. 2007]

careful study of FSS, 1st order!



$\langle \text{tr } \Omega \rangle$

Figure 4: Polyakov loop probability distributions in the region of the deconfinement

it does not make sense to compare numerical values - very different regimes -

calculable SYM* vs lattice

1. Both discontinuities - of the trace of Polyakov loop or of its two point function - are seen also in the semiclassical SYM* quantum transition

2. For all theories, with or without center, discontinuous transition seen on lattice as well as in SYM* (SU(2) continuous in both)

3. Theta dependence of:

- critical temperature

- discontinuity of Polyakov loop [lattice prompted by Anber SYM* 2013!]

- string tension (not shown, decreases with theta increase; lattice Del Debbio et al 2006)

in each case qualitatively agrees with lattice.

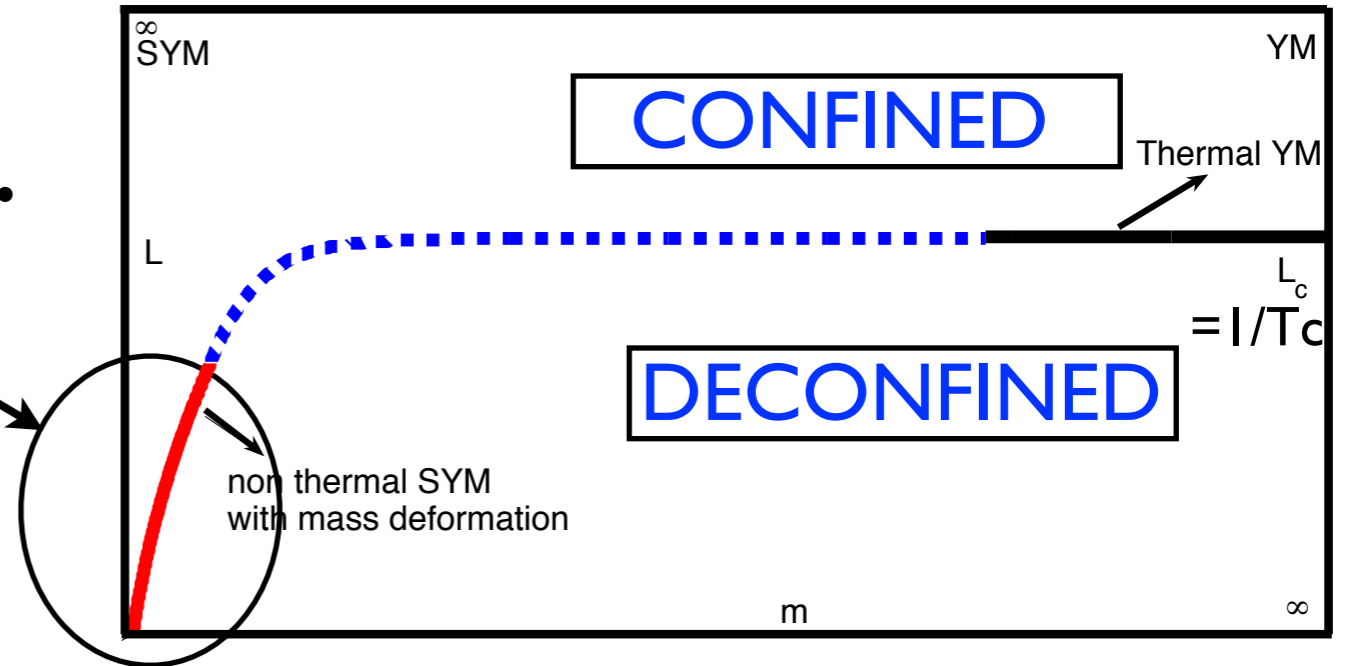
4. In case you wonder about quarks (as there are in the real world), a weak-coupling semiclassical description of non-abelian chiral symmetry breaking has not been achieved (no surprise). But if you add massive quarks to SYM* you can see two things that agree with what lattice with massive quarks sees - Polyakov loop crossover and string breaking at distances $\sim 2/\text{mass}$ [EP Tin Sulejmanpasic 1307.1317]

End of: 2. Evidence ... calculable SYM* vs lattice

3. Novel topological excitations and their role.

Why this seems to work the way it does?

I will now tell you how this part of the phase diagram comes about.



What is the role of SUSY?

theory is weakly coupled at small L - abelian!, not just asymptotic freedom

thus

allows us to have calculable non-perturbative effects

$$\text{roughly} \sim e^{-\frac{\mathcal{O}(1)}{g^2}}$$

and

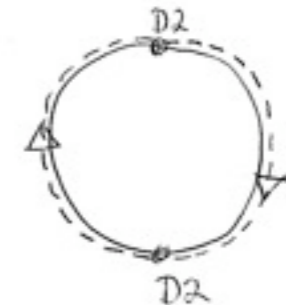
calculable perturbative effects - also suppressed by m -

$$\text{roughly} \sim g^2 m$$

so the two can compete and result in a calculable transition

major players: monopole-instanton “BPS” and twisted “KK” [Piljin Yi, Kimeyong Lee, 1997]

and various “topological molecules made thereof”



[Unsal 2007, Unsal EP 2011, Argyres Unsal 2012...]

I will attempt to describe the physics for SU(2)

(for a general group see paper, it is fun)

- small-L theory is abelian: SU(2) breaks to U(1):

1. SYM on S^1 has perturbatively exact flat direction: holonomy

2. assume that it has a vev that breaks SU(2) to U(1)

3. assume that the vev is large, \gg strong scale, so coupling is weak

(at the end, verify self-consistency: $L \times (\text{strong scale}) \ll 1$ is required)

- no light charged states, since breaking SU(2) to U(1) by adjoint, so coupling frozen at small value

relevant bosonic fields: A_4 - gauge field in compact direction -
and A_i - 3d gauge field - in the unbroken U(1) of SU(2), equivalent to:

σ - 3d dual to A_i = “dual photon” (potential for magnetic charge)
 ϕ - deviation of A_4 from center symmetric value $\text{Tr } \Omega = 0$

...without taking into account nonperturbative physics, these are FREE...

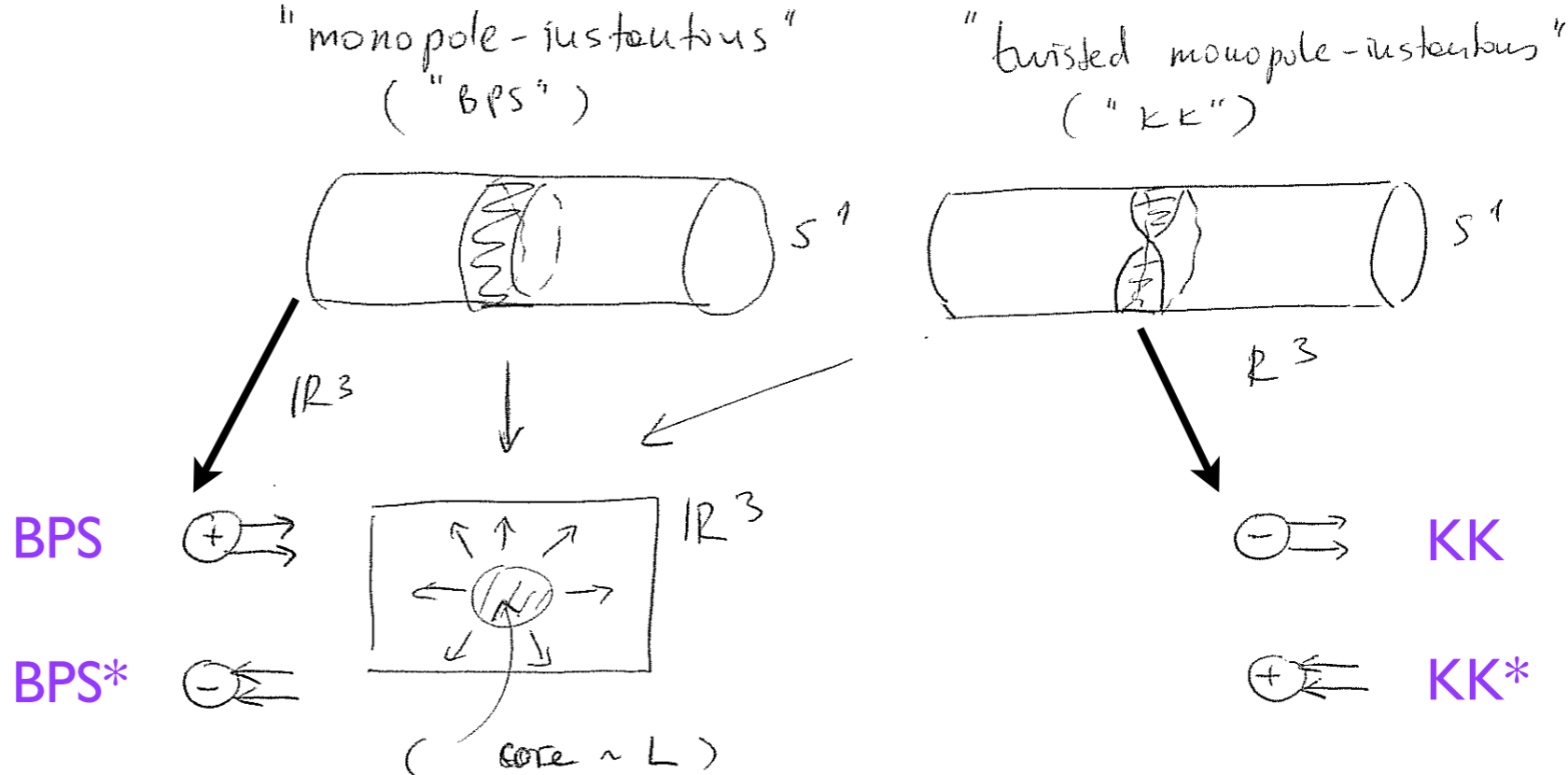
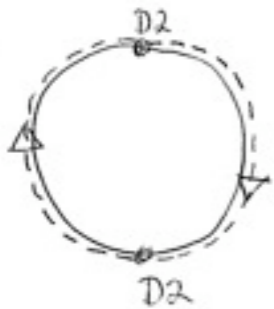
all (almost) dynamics is due to nonperturbative objects: vacuum of the theory is a dilute 3d “gas” of “molecules” interacting via long-range forces due to (dual) photon, scalar modulus, and fermion zero-mode hopping

- σ - 3d dual to $A_i =$ “dual photon” (potential for magnetic charge)
- ϕ - deviation of A_4 from center symmetric value $\text{Tr } \Omega = 0$

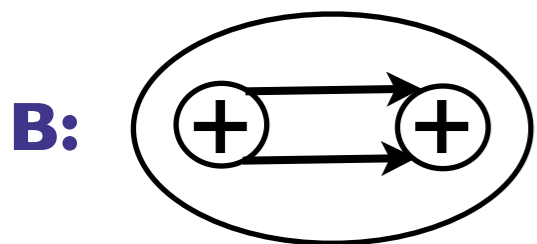
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major players: monopole-instanton “BPS” and twisted “KK”

[Piljin Yi, Kimeyong Lee, 1997]

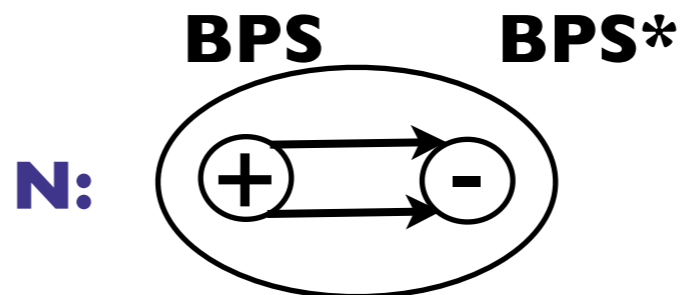


these main “players”, as they interact, can form “molecules” - “correlated tunneling events”

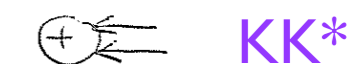
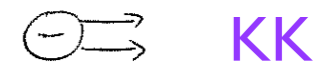


BPS **KK***

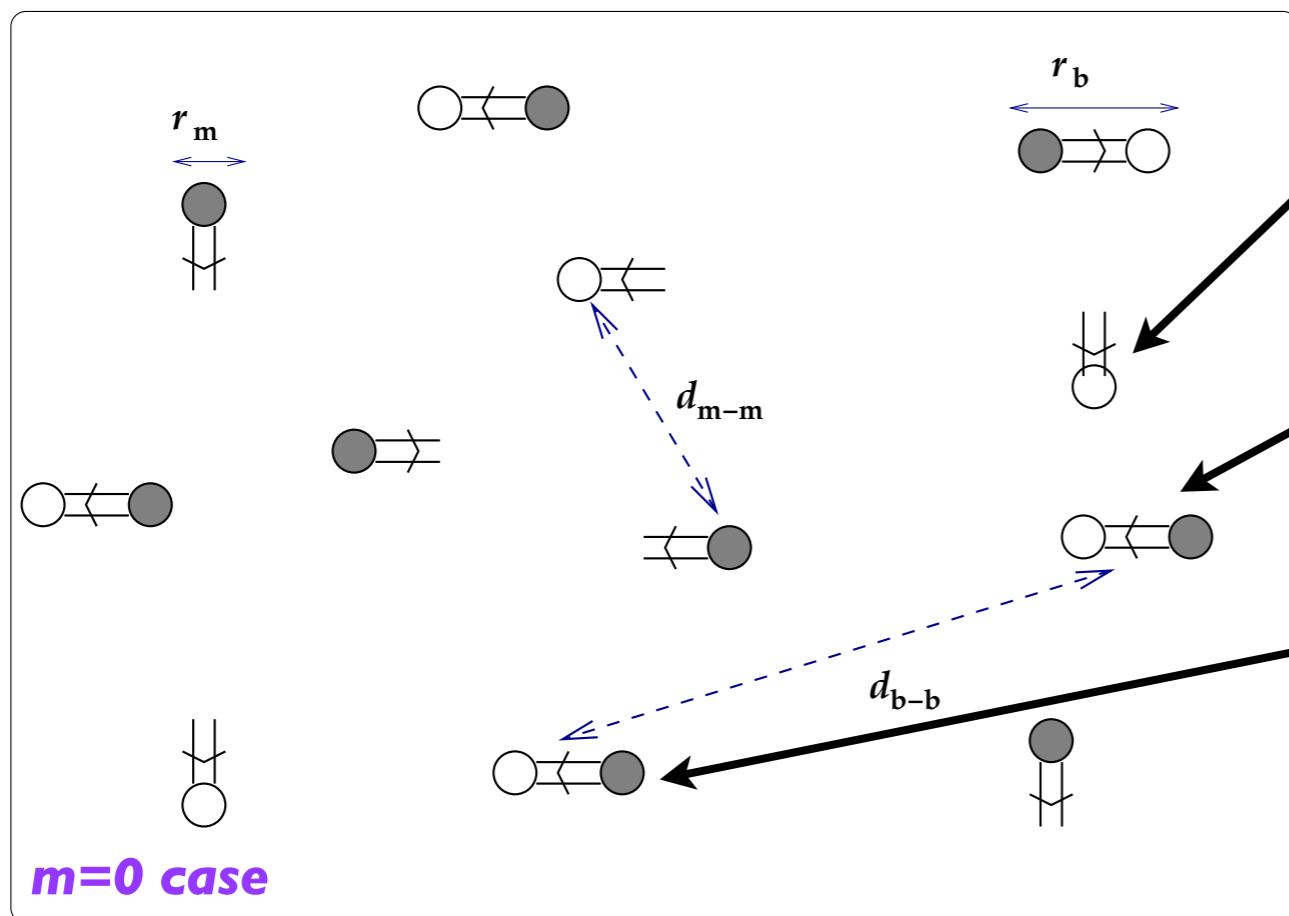
$$e^{-2S_0} e^{+i2\sigma}$$



$$e^{-2S_0} e^{-2\phi}$$



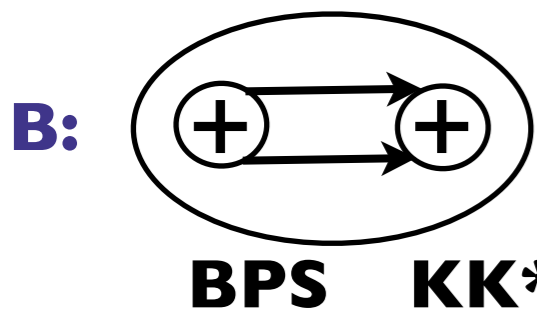
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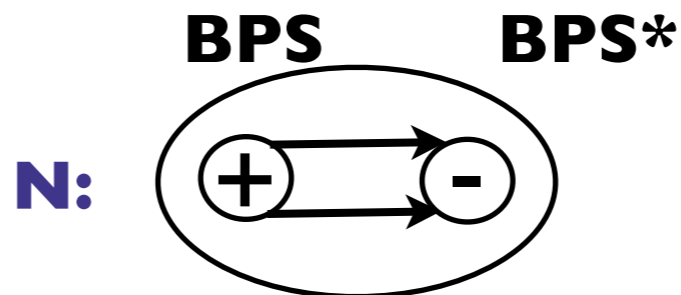
monopole-instantons (M, KK+*)

magnetic bion “molecules”

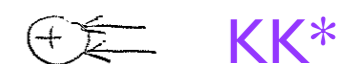
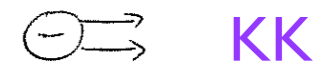
neutral bion “molecules”



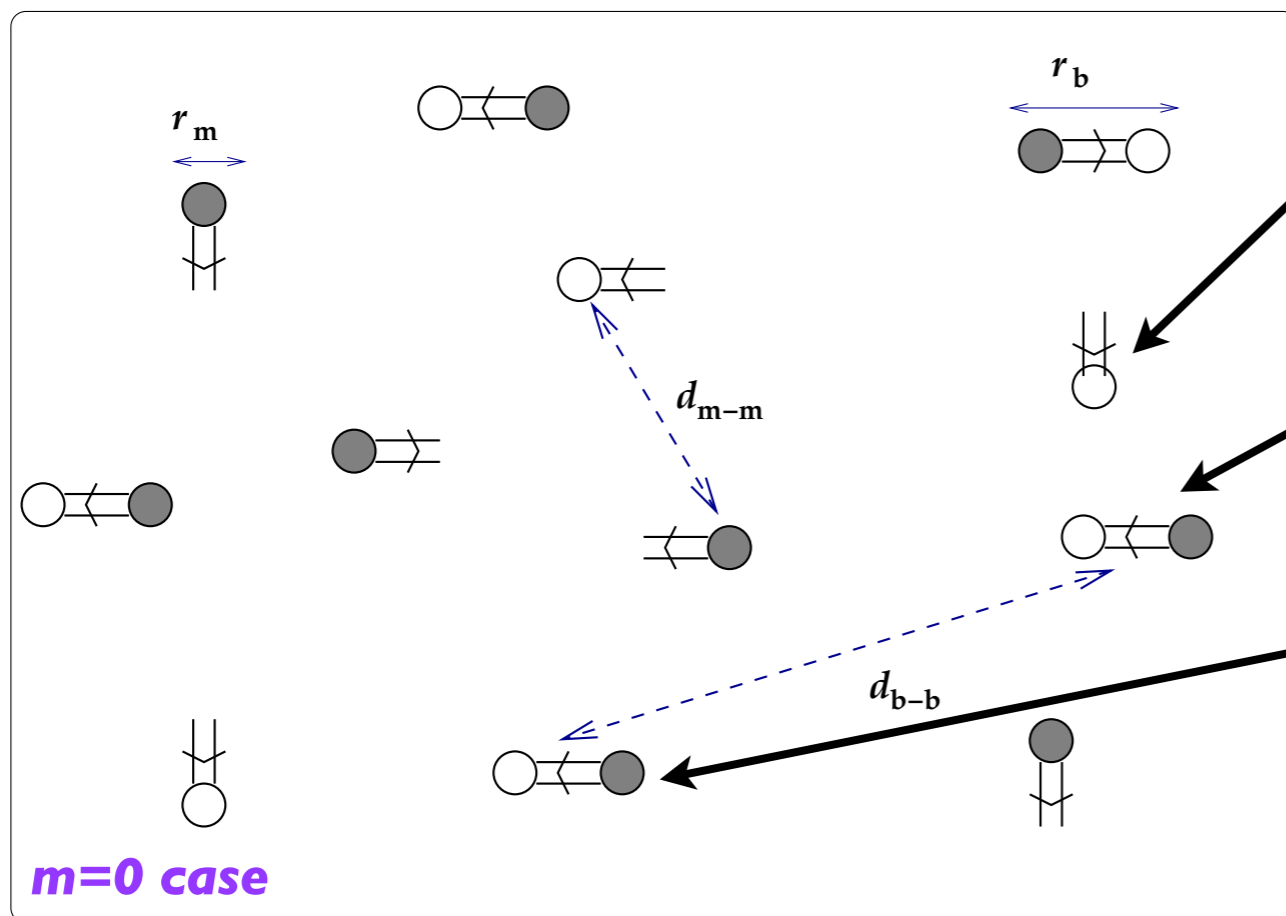
$$e^{-2S_0} e^{+i2\sigma}$$



$$e^{-2S_0} e^{-2\phi}$$



all (almost) dynamics is due to nonperturbative objects: vacuum of the theory is a dilute 3d “gas” of “molecules” interacting via long-range forces due to (dual) photon, scalar modulus, and fermion zero-mode hopping



monopole-instantons (M, KK+*)

the ones with arrows: fermion zero modes carry magnetic charge 1

magnetic bion “molecules”

carry magnetic charge 2

[mass gap; breaking discrete chiral symmetry]

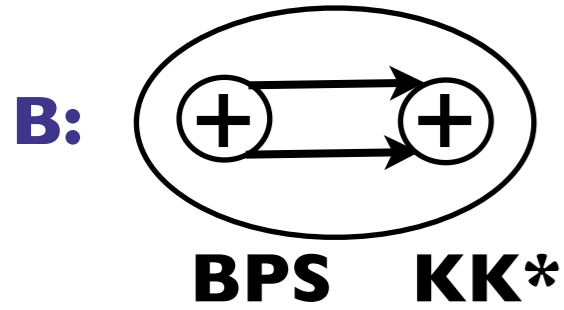
neutral bion “molecules”

carry scalar (modulus) charge 2

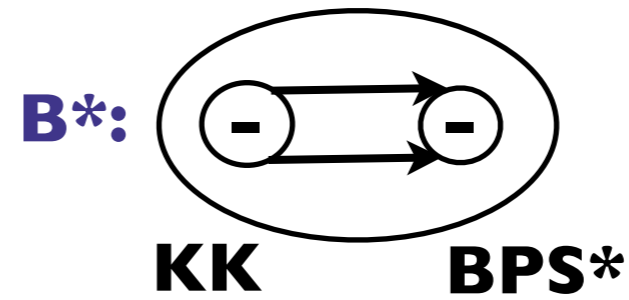
[Z2 center symmetry stabilization]

[aside: $BB^* \sim$ renormalons? ...“resurgence”]

(BPS-KK* “molecules”) “magnetic bions” - confinement!



$$e^{-2S_0} e^{+i2\sigma}$$

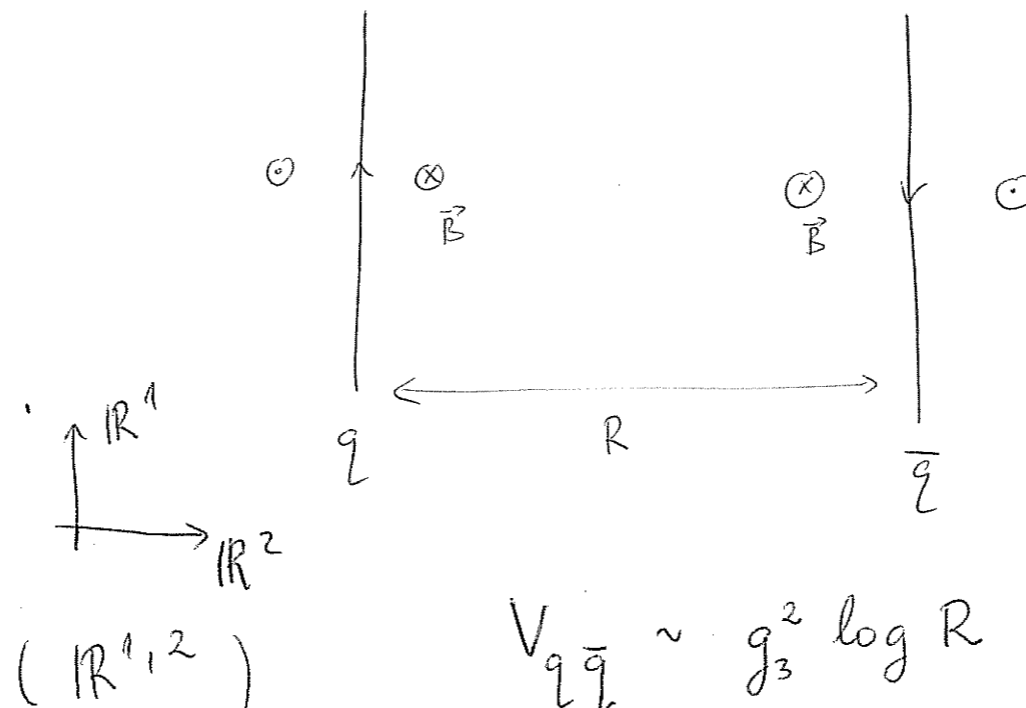
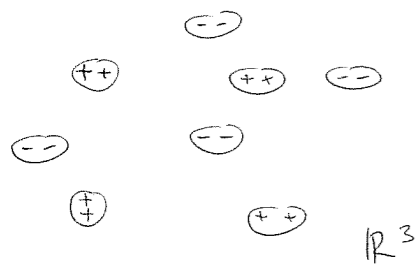


$$e^{-2S_0} e^{-i2\sigma}$$

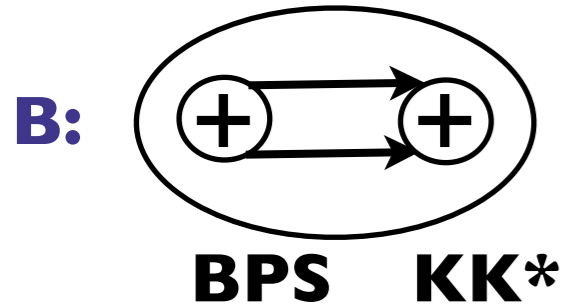
m=0 case - physics is that of 3d Debye screening - mass gap and confinement:

if nonperturbative saddle points are not summed over...

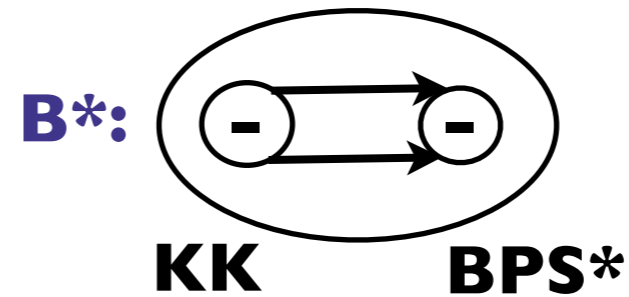
*magnetic bion gas: classical
3d Coulomb plasma*



(BPS-KK* “molecules”) “magnetic bions” - confinement!



$$e^{-2S_0} e^{+i2\sigma}$$

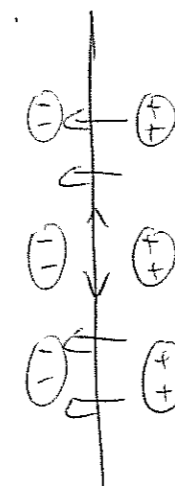
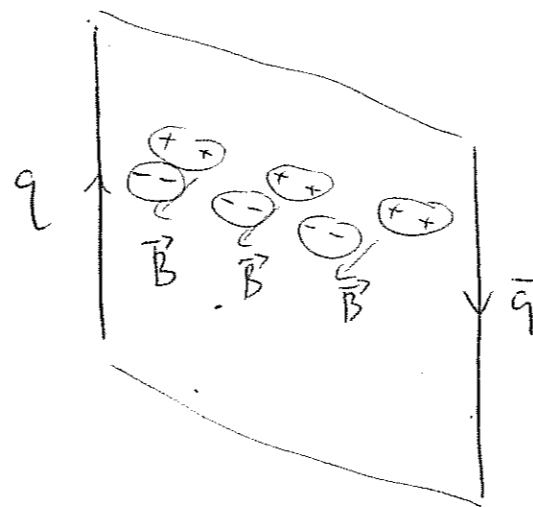
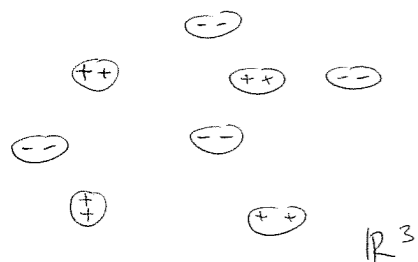


$$e^{-2S_0} e^{-i2\sigma}$$

m=0 case - physics is that of 3d Debye screening - mass gap and confinement:

... in reality, B-B plasma screens magnetic field of external probes*

*magnetic bion gas: classical
3d Coulomb plasma*



*“string worldsheet”:
B-B* dipole layer*

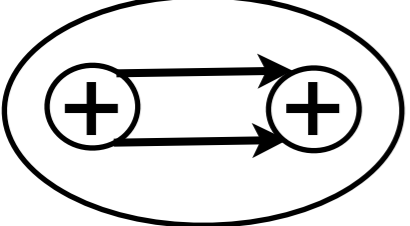
[Polyakov 1977]

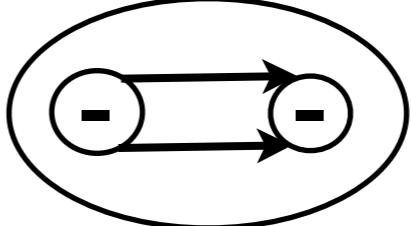
“monopole condensation” is due to composite
“molecular” objects - this theory does not confine in 3d limit

[Unsal 2007]

$$V_{q\bar{q}} \sim g_3^2 \log R \implies \sigma R$$

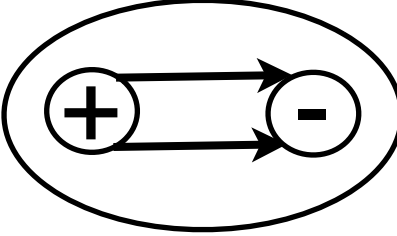
(BPS-KK* “molecules”) “magnetic bions” - confinement!

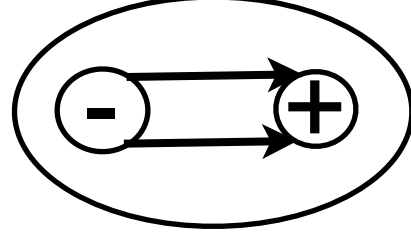
B:  $e^{-2S_0} e^{+i2\sigma}$
BPS **KK***

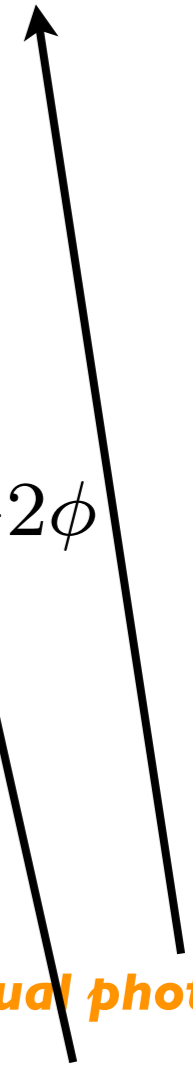
B*:  $e^{-2S_0} e^{-i2\sigma}$
KK **BPS***

(BPS-BPS*, KK-KK* “molecules”) “neutral bions”

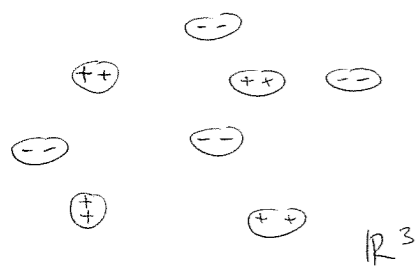
in pure-SYM: center-stabilizing

N:  $e^{-2S_0} e^{-2\phi}$
BPS **BPS***

N*:  $e^{-2S_0} e^{+2\phi}$
KK **KK***



magnetic bion gas: classical 3d Coulomb plasma



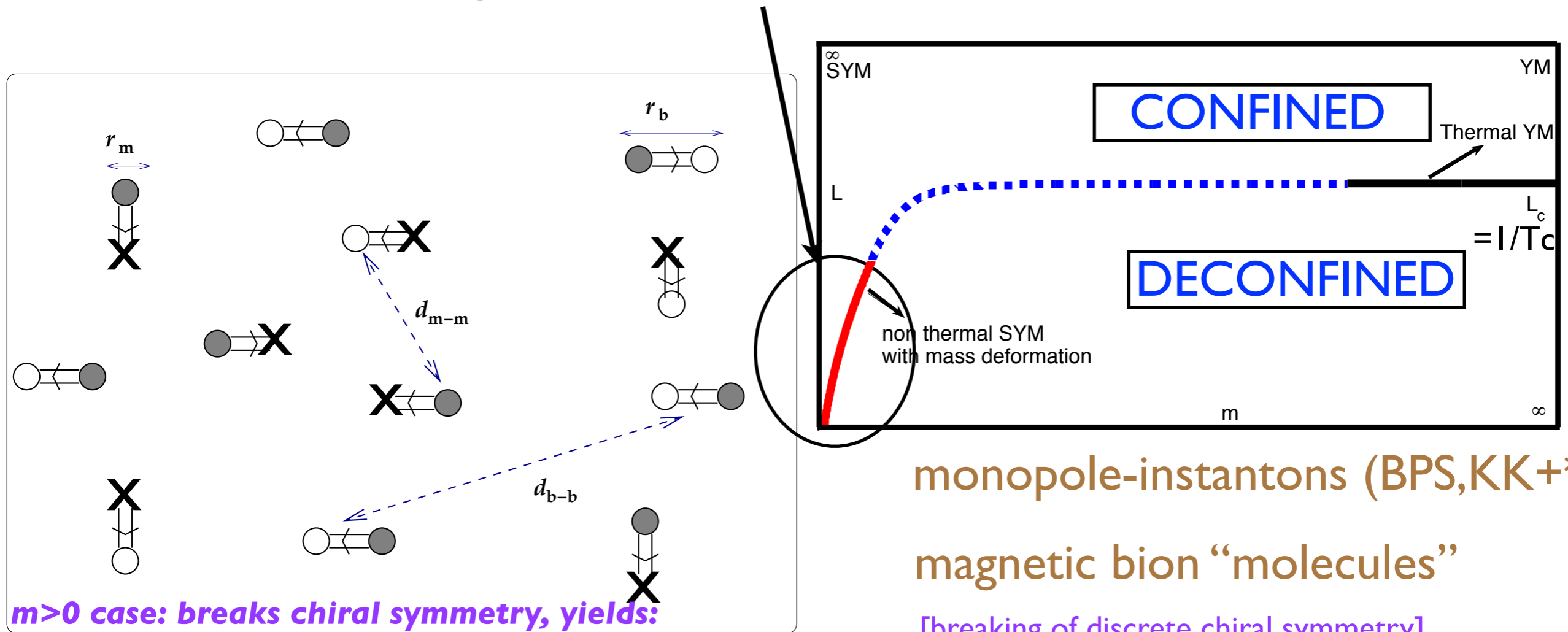
magnetic bions: break chiral Z_2 , mass gap for dual photon

neutral bions: stabilize center Z_2 , mass gap for modulus ($\phi=0$ - center stable)

Our interest is in the center Z_2 (as chiral Z_2 broken at $m>0$)

Recall it is the center Z_2 which becomes the thermal center symmetry of pure YM when m goes to infinity.

I will now tell you how this part of the phase diagram comes about.



monopole-instantons (BPS, KK+*)

magnetic bion “molecules”

[breaking of discrete chiral symmetry]

neutral bion “molecules”

[stability of Z2 center symmetry [non-thermal]]

1. extra nonperturbative contributions from monopole-instantons (no fermion zero modes)

2. extra perturbative Gross-Pisarski-Yaffe-like contribution (small since m is small)

small SUSY breaking “m” allows us to have perturbative and nonperturbative contributions compete while under theoretical control, resulting in a center-breaking transition as $\frac{m}{L^2 \Lambda^3}$ becomes $\mathcal{O}(1)$ (2nd order for SU(2); 1st for SU(N)...)
 --- =8, so if at $m > 5\Lambda$ decoupled, as quarks in QCD, $1/L_c = \Lambda \sqrt{8\Lambda/m} \rightarrow T_c \simeq \Lambda$

main result:

Quantum phase transition, second order for SU(2), first order in all other gauge groups, with causes that are well understood and under theoretical control - “fight” between topological molecules and perturbative contribution to holonomy potential - appears continuously connected to thermal deconfinement transition.

“fight” of nonperturbative vs. perturbative in SYM*, e.g. in SU(2):

$$\frac{1}{L^3} e^{-\frac{8\pi^2}{g^2(L)}} (\cosh 2\phi - \cos 2\sigma) + \frac{m}{L^2} e^{-\frac{4\pi^2}{g^2(L)}} (\cosh \phi \cos \sigma) - \frac{m^2}{L} \phi^2$$

center-stabilizing

“bions” - II and I

center-breaking (sigma=Pi is min)

“monopole-instantons”

center-breaking

GPY potential shown before, expanded for small phi

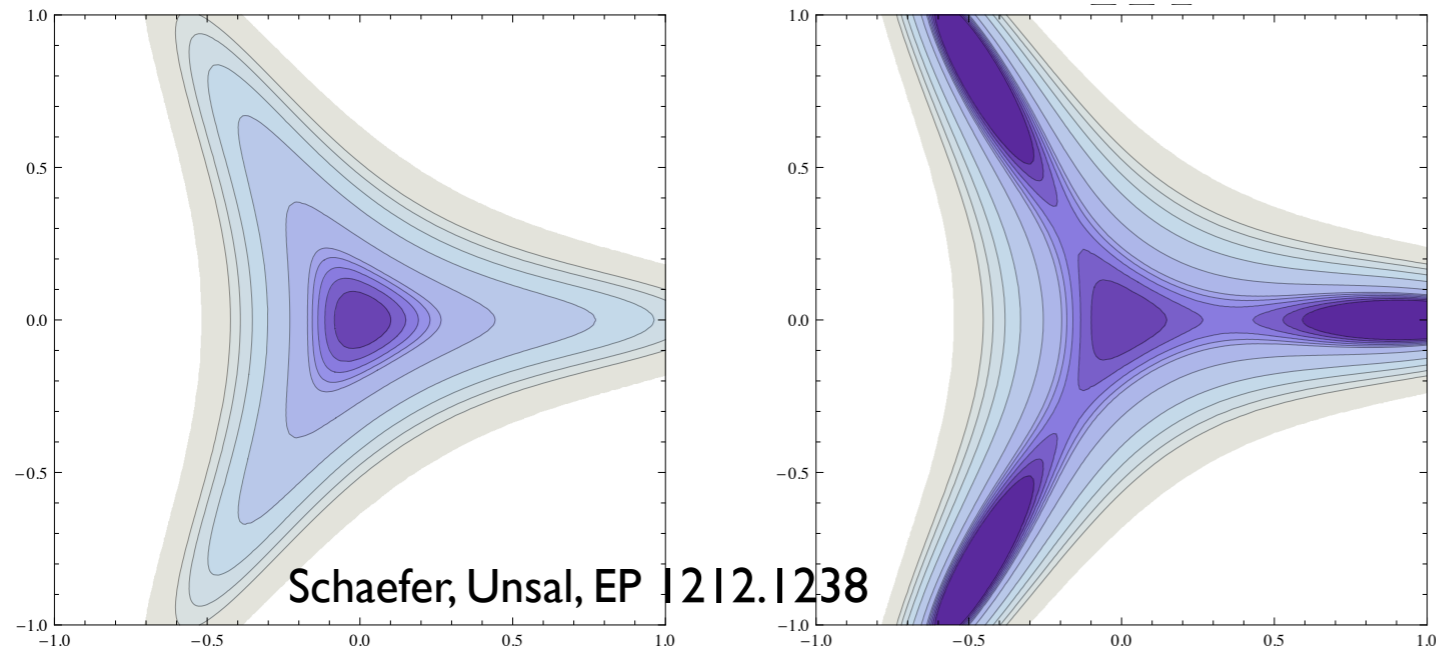
$$\frac{m_{soft}}{L^2 \Lambda^3}$$

dimensionless parameter controlling the transition

For a general gauge group, potential looks like this (will not explain notation):

$$\sum_{a=0, b=0}^r k_a^* k_b^* \alpha_a^* \cdot \alpha_b^* e^{-(\alpha_a^* + \alpha_b^*) \cdot \mathbf{b}} \cos((\alpha_a^* - \alpha_b^*) \cdot \boldsymbol{\sigma}') - c_m \sum_{a=0}^r k_a^* e^{-\alpha_a^* \cdot \mathbf{b}} \cos\left(\alpha_a^* \cdot \boldsymbol{\sigma}' + \frac{\theta + 2\pi u}{c_2}\right)$$

**instead of formulae, plot of potential due to “neutral bions” for $SU(3)$:
 Z_3 -symmetric vs Z_3 -breaking as $\frac{m}{L^2 \Lambda^3}$ increases (deviation of Ω EVs from Z_3)**



**End of: Novel topological excitations and their role.
Why this seems to work the way it does?**

Honestly, I do not know.

Some indications:

Why this seems to work the way it does?

Honestly, I do not know for sure. Some thoughts:

Same objects that were identified in SYM also exist in pure thermal YM.

What is lost is the theoretical control - but not all are bothered ... the(ir) logic:

1. Lattice data show that the $\text{Tr}(\text{Polyakov loop})$ is not $=1$ immediately after the transition, but is quite a bit smaller (and nonzero, of course).

2. Assuming a semiclassical situation with small fluctuations, this would mean that A_4 is nonzero, eigenvalues are not on top of each other, so theory can still be thought as abelianized.

3. Then all the monopoles, KK monopoles pictured above exist. Nonperturbative fluctuations should be important for the dynamics, hence let us model the vacuum as a liquid thereof - not dilute gas.

4. Use some lattice measurements (caloron densities) to fix the density of the BPS and KK monopole-instantons (now a model parameter). Try to compute something to compare with other data.

Shuryak, Sulejmanpasic 2013: **instanton-liquid type model of the pure YM deconfinement transition, incorporating “molecular” contributions** (neutral bions! - use “excluded volume” not SUSY or BZJ prescription... from old instanton-liquid model of $T=0$ QCD vacuum). **The model gives order-of-magnitude agreement with lattice measurements of electric and magnetic masses.**

EP: OK, it is a model; but the lattice data is poor (and gauge dependent) perhaps can improve?

Why this seems to work the way it does?

Honestly, I do not know for sure.

Some thoughts:

Same objects that were identified in SYM also exist in pure thermal YM, assuming...see comments on previous page...

Experiment (lattice) can test the entire phase diagram, using present-day technology, at least sufficiently far from semiclassical regime (that's hard on the lattice). Since m is nonzero, no need to take chiral limit for gaugino, so easier than SYM.

Find something that blatantly contradicts continuity assumption.

Is this “Resurgence in action”?