Supersymmetry and neutral bions: hints about deconfinement?

Erich Poppitz
University of Toronto

works with
Thomas Schäfer  Mithat Ünsal  NCSU  1205.0290  1212.1238
Mohamed Anber  Brett Teeple  Toronto  1406.1199

will also mention work with Tin Sulejmanpasic  Regensburg  1307.1317
While the LHC continues the search for variants of weak-scale supersymmetry: “natural”, “compressed”, “(super)split” or “flavorful”, among others - and may or may not find evidence for it - I will discuss another, less direct, less mainstream, and more recent, use of supersymmetry in particle theory... albeit one that will not seen at the LHC...

main message:

*It has been realized that studies of supersymmetric gauge theories in the late 1990's, when properly interpreted, lead to insights whose relevance transcends supersymmetry.*

...this is really a talk about the “inner working” of QFT, not so much about nuclear theory, applications, or about comparison with real experimental data...
I will illustrate this use of supersymmetry by an example that may have to do with the microscopic description of the thermal deconfinement transition in pure YM.

A host of strange topological molecules will be seen to be the major players in the dynamics.

Interesting connections emerge, between topology, “condensed-matter” gases of electric and magnetic charges (not this talk!), and attempts to make sense of the divergent perturbation series (also not this talk!).
Outline:

1. The “SYM*/thermal YM-continuity” conjecture
2. Evidence for conjecture: calculable SYM* vs lattice
   Why this seems to work the way it does?
$R^3 \times S^1$ compactifications of $\text{SYM}^*$

(non-) thermal

early remarks in Unsal, Yaffe 1006.2101
[ Schaefer, Unsal, EP 1205.0290, 1212.1238
Anber 1302.2641; Sulejmanpasic, EP 1307.1317;
Anber, EP 1406.1199]

**DEFINITIONS:**

1. super $\text{YM} = \text{“SYM”} = \text{YM} + \text{massless quark, an adjoint Weyl “gaugino”}$

   fields: gauge bosons + gauginos  $Z_{(2 \ N)}$ chiral symmetry for $\text{SU}(N)$
   [Z_{(2 \ c_2)} chiral symmetry for arbitrary $G$ (cover group)]

2. $\text{SYM}^* = \text{SYM} + \text{mass for the triplet quark, i.e. with a “gaugino mass” } m$

   supersymmetry and chiral symmetry *explicitly broken* by $m$

we study $\text{SYM}^*$ on $R^3 \times S^1_L$ with periodic (supersymmetric, non-thermal) boundary condition for gaugino

there are only two parameters to vary: $L$ and $m$

the theory is asymptotically free with a strong scale $\Lambda$

$\frac{m}{\Lambda}$ $\Lambda L$
$R^3 \times S^1$ compactifications of SYM*

(with) thermal

size of circle

SYM on $R^3 \times S^1$:
Seiberg, Witten 1996
Aharony, Hanany, Intriligator; Seiberg, Strassler 1997
Davies, Hollowood, Khoze 1999
important relevant details of instanton calculation only
R\(^3\times S^1\) compactifications of SYM*  

(non-) thermal  

at small m, SYM*, non-thermal compactification on S\(^1\) of size L  
at m=0, partition function=Witten index, no phase transition  

at small L, upon increase of m, a first order phase transition for all Lie groups  
[but for SU(2), second order]  

for groups with center, associated with center breaking (S\(^1\) Wilson loop is order parameter)  
for groups without center, no order parameter  
both \(\langle \text{Tr} \Omega_R \rangle\) and \(\langle \text{Tr} \Omega_R (x^\mu) \text{Tr} \Omega_R^\dagger (0) \rangle\) are discontinuous at the transition  

SYM on R\(^3\times S^1\):  
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important relevant details of instanton calculation only  
Semiclassical calculability is the most interesting feature of this small-\(m\),\(L\) transition. A host of novel topological excitations: “magnetic bions” (Unsal 2007) and “neutral bions” (EP Unsal 2012, Argyres Unsal 2012...) whose raison d'être runs deep... are responsible for confinement and potential for \(S^1\) holonomy (& center stability, where present)
$R^3 \times S^1$ compactifications of SYM*

(non-) thermal

...these effects were already in the 1990's papers I mentioned, but because they relied so much on supersymmetry ($V \sim W'|^2$) the generality of the physics, which transcends supersymmetry, was missed!

...similar excitations exist in non-SUSY theories (QCD(adj)) and can even be identified in pure thermal YM (if a holonomy expectation value is assumed)

Semiclassical calculability is the most interesting feature of this small-$m, L$ transition. A host of novel topological excitations: “magnetic bions” (Unsal 2007) and “neutral bions” (EP Unsal 2012, Argyres Unsal 2012...) whose raison d’etre runs deep... are responsible for confinement and potential for $S^1$ holonomy (& center stability, where present)
In what follows, we shall compare behavior of
\[ \langle \text{Tr} \, \Omega_R \rangle, \langle \text{Tr} \, \Omega_R (x^\mu) \text{Tr} \, \Omega_R^\dagger (0) \rangle \] (and other quantities)
at the two transitions and find striking similarities...
At small $m, L$, the transition can be studied in a theoretically controlled manner. Novel topological excitations and perturbative contributions yield competing effects, resulting in a transition as dimensionless parameter $\frac{m}{L^2 \Lambda^3}$. 

\[ R^3 \times S^1 \text{ compactifications of SYM}^* \]

(non-) thermal

“continuity conjecture” = this phase diagram

Quantum transition [semiclassical calculations]

Thermal transition [from lattice]

size of circle

Temperature

gaugino mass $m$

non thermal SYM with mass deformation

Thermal YM

SYM

CONFINED

DECONFINED

$\L_\infty$

$\L$

Symmetry breaking quantum phase transition was shown to occur as the dimensionless parameter $\frac{m}{L^2 \Lambda^3}$.
1. The “SYM*/thermal YM-continuity” conjecture

“continuity conjecture” = this phase diagram

Next:
2. Evidence for conjecture: calculable SYM* vs lattice

First, review a few facts about thermal theories...
Thermal partition function is (without fermions):

\[ Z(\beta) = \text{tr}[e^{-\beta H}], \quad \beta = 1/T = \text{radius of } S^1 \]

\[ \mathbb{R}^3 \times S^1 \]

\[ \mathbb{R}^3 \]

high-T:
Quark Gluon Plasma

\[ \Lambda^{-1} \]

low-T:
Confined Phase

strong scale

a static quark probe
\( \Omega = \text{tr} \mathcal{P} \exp[i \int_{S^1} A_4 dx^4] \)

Wilson/Polyakov loop

\[ \langle \Omega^+(\vec{x}) \Omega(0) \rangle \sim e^{-\frac{V(|\vec{x}|)}{T}} \] 

confined

\[ e^{-\frac{\sigma |\vec{x}|}{T}} \to 0 \quad x \to \infty \]

deconfined

\[ e^{-\frac{e^{-m_\varphi |\vec{x}|}}{|\vec{x}|T}} \to 1 \quad x \to \infty \]

hence

\[ \langle \Omega \rangle = 0 \quad \text{confined} \]

\[ \langle \Omega \rangle \neq 0 \quad \text{deconfined} \]

"infinite F_quark"
in SU(N) theory without fundamentals, deconfinement = breaking of global $Z_N$ center symmetry

$$\Omega \xrightarrow{z \subset Z_N} z\Omega$$

$$\Omega = \text{tr} \mathcal{P} \exp[i \int_{S^1} A_4 dx^4]$$

usual description: high T - “broken center”

this is what lattice sees...

to find $\langle ... \rangle$ in finite volume, usual stat mech tricks

notice center symmetry broken at high-T: counterintuitive, discussion of domains/walls-Euclidean configs, see Smilga ‘94,’98+...

low T - unbroken center

$T >> T_c$ behavior has been understood for 30 years

[\text{Gross, Pisarski, Yaffe, 1981}]

High-T perturbation theory good, gives one-loop $V(\text{pert})$, favors center-broken vacuum, e.g.

$$V_{\text{pert.}}(\Omega) = -\frac{2}{\pi^2 \beta^4} \sum_{n=1}^{\infty} \frac{1}{n^4} |\text{tr} \Omega^n|^2 (1 + O(g^2)) = \text{coinciding eigenvalues}$$

$$\Omega = \frac{1}{2} \text{Tr} \left( \begin{array}{cc} e^{i\nu} & 0 \\ 0 & e^{-i\nu} \end{array} \right)$$
in SU(N) theory without fundamentals, deconfinement = breaking of global $\mathbb{Z}_N$ center symmetry

$$\Omega_{\text{fund}} \xrightarrow{z \subset \mathbb{Z}_N} z \Omega_{\text{fund}} \quad \Omega = \text{tr} \mathcal{P} \exp[i \int_{S^1} A_4 dx^4]$$

usual description: high $T$ - “broken center”

perhaps more physical: quark-antiquark (probe) potential

this is also what lattice sees...

string tension discontinuously jumps to zero [for theories with center only]

both discontinuities - of the trace of Polyakov loop or of its two point function - are seen also in the semiclassical SYM* quantum transition

(for theories with nontrivial center)
calculable SYM* vs lattice

Both discontinuities - of the trace of Polyakov loop or of its two point function - are seen also in the semiclassical SYM* quantum transition for all theories with nontrivial center: SU(N), Sp(2N), Spin(N), E_6, E_7 we have for m<m_c

\[
\langle \text{Tr } \Omega(x) \text{Tr } \Omega^\dagger(0) \rangle \bigg|_{r \gg m_0^{-1}} \simeq e^{-\frac{\hat{\sigma} m_0}{R} r R} \equiv e^{-\sigma r R} \quad \text{(and a constant at m>m_c)}
\]

\[
\text{string tension}
\]

For the trace of the Polyakov loop, we have, for all groups with center:

Lattice only SU(N) and Sp(4) [latter case motivated by “Z2 universality”]

\[
\text{Tr} \langle \Omega \rangle = 0 \quad \text{m<m_c}
\]

\[
\text{Tr} \langle \Omega \rangle \neq 0 \quad \text{m>m_c}
\]
**calculable SYM* vs lattice**

for all theories without center: G_2, F_4, E_8

Lattice only G_2 ↔ SYM*: all transitions discontinuous

\[
\langle \text{Tr} \Omega(x) \text{Tr} \Omega(y) \rangle = \begin{cases} 
0.0056 \left( \frac{g^2}{4\pi} \right)^2 \\
0.0056 \left( \frac{g^2}{4\pi} \right)^2 \\
11.3 \left( \frac{g^2}{4\pi} \right)^2 
\end{cases}
\]

- m=0 value (SYM)
- below transition
- above transition

numbers from Anber, EP, Teeple 1406.1199
correct omission of quantum-corrected monopole-instanton vertex in 1212.1238 Schaefer, EP, Unsal

**SYM*** jump of Polyakov loop trace: \( \langle \text{Tr} \Omega \rangle \)

\[
\frac{g^2}{4\pi} 0.0746, \quad \frac{g^2}{4\pi} 3.437.
\]

**lattice jump of Polyakov loop trace:**

[Pepe, Wiese 2006; Cossu et al. 2007]
careful study of FSS, 1st order!

Figure 4: Polyakov loop probability distributions in the region of the deconfinement

it does not make sense to compare numerical values - very different regimes -
**calculable SYM* vs lattice**

1. Both discontinuities - of the trace of Polyakov loop or of its two point function - are seen also in the semiclassical SYM* quantum transition

2. For all theories, with or without center, discontinuous transition seen on lattice as well as in SYM* (SU(2) continuous in both)

3. Theta dependence of:
   - critical temperature
   - discontinuity of Polyakov loop [lattice prompted by Anber SYM* 2013!]
   - string tension (not shown, decreases with theta increase; lattice Del Debbio et al 2006)
   in each case qualitatively agrees with lattice.

4. In case you wonder about quarks (as there are in the real world), a weak-coupling semiclassical description of non-abelian chiral symmetry breaking has not been achieved (no surprise). But if you add massive quarks to SYM* you can see two things that agree with what lattice with massive quarks sees - Polyakov loop crossover and string breaking at distances $\sim 2/\text{mass}$ [EP Tin Sulejmanpasic 1307.1317]

*End of: 2. Evidence ... calculable SYM* vs lattice*

Why this seems to work the way it does?

I will now tell you how this part of the phase diagram comes about.

What is the role of SUSY?

theory is weakly coupled at small L - abelian!, not just asymptotic freedom thus allows us to have calculable non-perturbative effects and calculable perturbative effects - also suppressed by m - so the two can compete and result in a calculable transition

major players: monopole-instanton “BPS” and twisted “KK” [Piljin Yi, Kimeyong Lee, 1997] and various “topological molecules made thereof”


roughly $\sim e^{-\frac{O(1)}{g^2}}$

roughly $\sim g^2 m$
I will attempt to describe the physics for SU(2)
(for a general group see paper, it is fun)

- small-L theory is abelian: SU(2) breaks to U(1):
1. SYM on S^1 has perturbatively exact flat direction: holonomy
2. assume that it has a vev that breaks SU(2) to U(1)
3. assume that the vev is large, >> strong scale, so coupling is weak
   (at the end, verify self-consistency: L x (strong scale) << 1 is required)

- no light charged states, since breaking SU(2) to U(1) by adjoint,
  so coupling frozen at small value

relevant bosonic fields: A_4 - gauge field in compact direction -
and A_i - 3d gauge field - in the unbroken U(1) of SU(2), equivalent to:

\[
\begin{align*}
  \sigma & \quad - 3d \text{ dual to } A_i = \text{“dual photon”} \quad (\text{potential for magnetic charge}) \\
  \phi & \quad - \text{deviation of } A_4 \text{ from center symmetric value } \text{Tr} \, \hat{\Omega} = 0
\end{align*}
\]

...without taking into account nonpertubative physics, these are FREE...
all (almost) dynamics is due to nonperturbative objects: vacuum of the theory is a dilute 3d “gas” of “molecules” interacting via long-range forces due to (dual) photon, scalar modulus, and fermion zero-mode hopping

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major players: monopole-instanton “BPS” and twisted “KK”

[Piljin Yi, Kimeyong Lee, 1997]

these main “players”, as they interact, can form “molecules” - “correlated tunneling events”
all (almost) dynamics is due to nonperturbative objects: vacuum of the theory is a dilute 3d “gas” of “molecules” interacting via long-range forces due to (dual) photon, scalar modulus, and fermion zero-mode hopping.
all (almost) dynamics is due to nonperturbative objects: vacuum of the theory is a dilute 3d “gas” of “molecules” interacting via long-range forces due to (dual) photon, scalar modulus, and fermion zero-mode hopping.

- **monopole-instantons (M,KK+*)**
  - the ones with arrows: fermion zero modes
  - carry magnetic charge 1

- **magnetic bion “molecules”**
  - carry magnetic charge 2
  - [mass gap; breaking discrete chiral symmetry]

- **neutral bion “molecules”**
  - carry scalar (modulus) charge 2
  - [Z2 center symmetry stabilization]

[aside: BB*~renormalons? ...“resurgence”]
(BPS-KK* “molecules”) “magnetic bions” - confinement!

\[ e^{-2S_0} e^{i2\sigma} \]

\[ e^{-2S_0} e^{-i2\sigma} \]

\( m=0 \) case - physics is that of 3d Debye screening - mass gap and confinement:

**magnetic bion gas: classical**

3d Coulomb plasma

**if nonperturbative saddle points are not summed over...**

- 2d Coulomb potential
(BPS-KK* “molecules”) “magnetic bions” - confinement!

\[ B:\quad BPS \quad KK^* \]
\[ B^*:\quad KK \quad BPS^* \]

\[ e^{-2S_0} e^{+i2\sigma} \]

\[ e^{-2S_0} e^{-i2\sigma} \]

m=0 case - physics is that of 3d Debye screening - mass gap and confinement:

magnetic bion gas: classical 3d Coulomb plasma

... in reality, B-B* plasma screens magnetic field of external probes

“string worldsheet”: B-B* dipole layer

[Polyakov 1977]

“monopole condensation” is due to composite “molecular” objects - this theory does not confine in 3d limit

[Unsal 2007]
We generalize the setup of Ref. [2] Quarks with Dirac mass is closer to the real world QCD compared to those studies, and hence of some interest. Our purpose here is to extend the study of 1 Introduction

2 Quarks with Dirac mass

1 Introduction

Contents

2.1 Taming the perturbative contributions

2.2 Qualitative expectations: quarks with Dirac mass and topological molecules

2.2.1 Brief review of pure SYM case

2.2.2 Adding fundamentals: symmetries and monopole-instanton zero modes

2.2.3 The case

2.2.4 The case

2.3 Remarks on the linear-chiral duality and charges under global chiral symmetries

2.3.1 More of fixing on Coulomb branch

We begin with our naive expectations, starting with adding massive quark supermultiplets.

We generalize the setup of Ref. [2] Quarks with Dirac mass. is closer to the real world QCD compared to those studies, and hence of some interest.

Our interest is in the center $Z_2$ (as chiral $Z_2$ broken at $m>0$) Recall it is the center $Z_2$ which becomes the thermal center symmetry of pure YM when $m$ goes to infinity.

magnetic bion gas: classical 3d Coulomb plasma

magnetic bions: break chiral $Z_2$, mass gap for dual photon

neutral bions: stabilize center $Z_2$, mass gap for modulus ($\phi=0$ - center stable)
I will now tell you how this part of the phase diagram comes about.

1. extra nonperturbative contributions from monopole-instantons (no fermion zero modes)

2. extra perturbative Gross-Pisarski-Yaffe-like contribution (small since $m$ is small)

Small SUSY breaking “$m$” allows us to have perturbative and nonperturbative contributions compete while under theoretical control, resulting in a center-breaking transition as $\frac{m}{L^2 \Lambda^3}$ becomes $O(1)$ (2nd order for SU(2); 1st for SU(N)...) $\Rightarrow T_c \sim \Lambda$. 

$m > 0$ case: breaks chiral symmetry, yields:

- monopole-instantons (BPS,KK+*)
- magnetic bion “molecules” [breaking of discrete chiral symmetry]
- neutral bion “molecules” [stability of Z2 center symmetry [non-thermal]]
Quantum phase transition, second order for SU(2), first order in all other gauge groups, with causes that are well understood and under theoretical control - “fight” between topological molecules and perturbative contribution to holonomy potential - appears continuously connected to thermal deconfinement transition.

“fight” of nonperturbative vs. perturbative in SYM*, e.g. in SU(2):

\[
\frac{1}{L^3} e^{-\frac{8\pi^2}{g^2(L)}} (\cosh 2\phi - \cos 2\sigma) + \frac{m}{L^2} e^{-\frac{4\pi^2}{g^2(L)}} (\cosh \phi \cos \sigma) - \frac{m^2}{L} \phi^2
\]

center-stabilizing “bions” - II and I  
center-breaking (sigma=Pi is min) “monopole-instantons”  
dimensionless parameter controlling the transition

For a general gauge group, potential looks like this (will not explain notation):

\[
\sum_{a=0, b=0}^{r} k_a^* k_b^* \alpha_a^* \cdot \alpha_b^* e^{-(\alpha_a^* + \alpha_b^*) \cdot b} \cos ((\alpha_a^* - \alpha_b^*) \cdot \sigma') - c_m \sum_{a=0}^{r} k_a^* e^{-\alpha_a^* \cdot b} \cos (\alpha_a^* \cdot \sigma' + \frac{\theta + 2\pi u}{c_2})
\]
instead of formulae, plot of potential due to “neutral bions” for SU(3):
Z3-symmetric vs Z3-breaking as \( \frac{m}{L^2 \Lambda^3} \) increases (deviation of \( \Omega \) EVs from Z3).

End of: Novel topological excitations and their role.
Why this seems to work the way it does?

Honestly, I do not know.

Some indications:
Why this seems to work the way it does?

Honestly, I do not know for sure. Some thoughts:

Same objects that were identified in SYM also exist in pure thermal YM. What is lost is the theoretical control - but not all are bothered ... the(ir) logic:

1. Lattice data show that the Tr(Polyakov loop) is not =1 immediately after the transition, but is quite a bit smaller (and nonzero, of course).

2. Assuming a semiclassical situation with small fluctuations, this would mean that $A_4$ is nonzero, eigenvalues are not on top of each other, so theory can still be thought as abelianized.

3. Then all the monopoles, KK monopoles pictured above exist. Nonperturbative fluctuations should be important for the dynamics, hence let us model the vacuum as a liquid thereof - not dilute gas.

4. Use some lattice measurements (caloron densities) to fix the density of the BPS and KK monopole-instantons (now a model parameter). Try to compute something to compare with other data.

Shuryak, Sulejmanpasic 2013: instanton-liquid type model of the pure YM deconfinement transition, incorporating “molecular” contributions (neutral bions! - use “excluded volume” not SUSY or BZJ prescription... from old instanton-liquid model of T=0 QCD vacuum). The model gives order-of-magnitude agreement with lattice measurements of electric and magnetic masses.

EP: OK, it is a model; but the lattice data is poor (and gauge dependent) perhaps can improve?
Why this seems to work the way it does?

Honestly, I do not know for sure.

Some thoughts:

Same objects that were identified in SYM also exist in pure thermal YM, assuming...see comments on previous page...

Experiment (lattice) can test the entire phase diagram, using present-day technology, at least sufficiently far from semiclassical regime (that’s hard on the lattice). Since \( m \) is nonzero, no need to take chiral limit for gaugino, so easier than SYM.

Find something that blatantly contradicts continuity assumption.

Is this “Resurgence in action”?