Deconfinement in 4d QCD(adj), electric-magnetic Coulomb gases, and affine 2d XY spin models

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to appear, sometime in Winter 2011/12

Work inspired by many sources spanning diverse ages and topics:

2007-: M. Unsal w/ one of L. Yaffe, M. Shifman, E.P., D. Simic, or P. Argyres	gauge theory dynamics on $R^{1,2}$ x S^1 (spatial circle)
early 2000's: G. Dunne, A. Kovner, B. Tekin	deconfinement transition in 3d Polyakov model
late 1990's: P. Yi, K. Lee; P. v. Baal	monopoles/instantons on compactified D-branes
mid 1980's: 2d CFT work of many	solutions of 2d critical theories
late 1970's: J. Jose, L. Kadanoff, S. Kirkpatrick, D. Nelson	lattice Coulomb gases and 2d spin models
D. Nelson; +w/ B. Halperin	theory of melting of 2d crystal on triangular lattice
G. 't Hooft	phases of gauge theories; order-disorder algebra ('t Hooft loop)
mid 1970's: A. Polyakov	monopole-instanton induced confinement in 3d Georgi-Glashow model (=compact U(1)) (Polyakov model)

How are these connected?

gauge theory dynamics on $R^{1,2} \times S^1$ (spatial circle)

lattice Coulomb gases and 2d spin models

theory of melting of 2d crystal on triangular lattice

4d SU(N) gauge theory with n_f massless adjoint Weyl fermions on spatial circle (L)

$\longleftrightarrow$ at finite T, near deconfinement transition is dual to

2d "affine" XY spin models

One side of duality - 4d gauge theory with massless fermions. Difficult to study by any means, including on the lattice.

$\longleftrightarrow$

The other side-2d spin models (known, or generalization of known, models).
Both analytical and numerical progress should be possible.

OUTLINE:

To illustrate, will give two examples to describe our results.
One is well-understood by now.
The second - less so, but in progress.

In the remaining time, will try to give a picture of how it comes about.

How are these connected?

gauge theory dynamics on $R^{1,2}$ x S^1 (spatial circle)

lattice Coulomb gases and 2d spin models

theory of melting of 2d crystal on triangular lattice

4d SU(N) gauge theory with n_f massless adjoint Weyl fermions on spatial circle (L)

at finite T, near deconfinement transition ⟷ is dual to

2d "affine" XY spin models

Ex.1: SU(2) QCD(adj) dual to XY model with Z_4 preserving perturbation

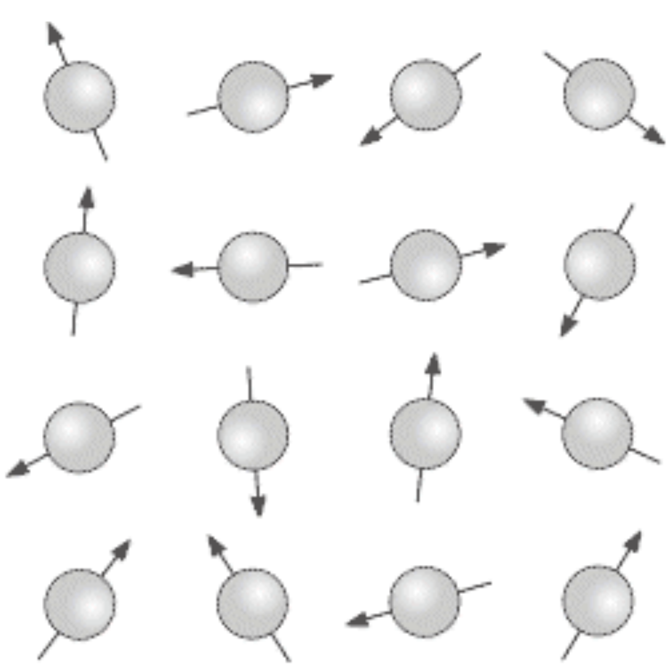
lattice spacing a ~ L

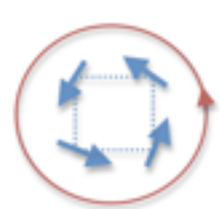
chiral Z_2 and topological Z_2 (dual to center Z_2)

$$-\beta H = \sum_{x;\hat{\mu}=1,2} \frac{\kappa}{2\pi} \cos(\theta_{x+\hat{\mu}} - \theta_x) + \sum_x \tilde{y} \cos 4\theta_x$$

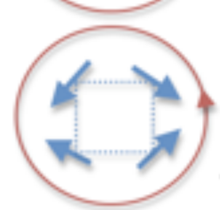
$$\kappa = \frac{g_4^2(L)}{2\pi L T} \qquad \tilde{y} \sim \xi_{bion} a^2 = \frac{e^{-\frac{8\pi^2}{g_4^2(L)}(1+cg_4)}}{LT g_4^{14-2n_f}}$$

at y=0 – Berezinskii–Kosterlitz–Thouless transition at kappa=4





small kappa:
vortices proliferate,
disorder system–mass gap



large kappa:
vortices suppressed,
algebraic long–range
order

adding nonzero y:
rotation–U(1) breaking
"crystal field"
non–BKT transition of
finite order

Ex.1: SU(2) QCD(adj) dual to XY model with Z_4 preserving perturbation
lattice spacing a ~ L
chiral Z_2 and topological Z_2 (dual to center Z_2)

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in the SU(2) dual, behavior near T_c can be understood analytically in some detail
(our work & others')

	$T < T_c$	$T = T_c$	$T > T_c$
$Q_m = \frac{q}{2}$ 't Hooft loop : $\langle e^{iq\theta(x)} e^{-iq\theta(0)} \rangle \big\|_{\|x\| \to \infty}$	1	$\frac{1}{\|x\|^{\frac{q^2}{4}(1+\mathcal{O}(y))}}$	$e^{-\frac{\tilde{\sigma}_q(T)}{T}\|x\|}$
$Q_e = 1$ Polyakov loop : $\langle e^{i\tilde{\theta}(x)} e^{-i\tilde{\theta}(0)} \rangle \big\|_{\text{“}\|x\| \to \infty\text{”}}$	$e^{-\frac{\sigma(T)}{T}\|x\|}$	$\frac{1}{\|x\|^{\frac{1}{4}(1+\mathcal{O}(y))}}$	1
	$\mathbb{Z}_2^{d\chi}, \mathbb{Z}_2^{top}$ broken	c = 1 CFT	

(dual) string tension critical exponent -

$$\frac{\sigma_q(T)}{T} \sim \frac{\tilde{\sigma}_q(T)}{T} \sim \xi^{-1} \sim |T - T_c|^{\nu} = |T - T_c|^{\frac{1}{16\pi\sqrt{y_0 \tilde{y}_0}}}$$

notice high-T/low-T “el.-magn.” duality

TO DO: equation of state? non-static properties?

Ex.2: SU(3) QCD(adj) dual to "affine" XY model with $Z_3 \times Z_3$ preserving perturbation

lattice spacing a ~ L
chiral Z_3 and topological Z_3 (dual to center Z_3)

$$-\beta H = \sum_{x;\hat{\mu}=1,2} \sum_{A=1,2} \frac{\kappa}{2\pi} \cos \alpha_i^A (\theta^i_{x+\hat{\mu}} - \theta^i_x) + \sum_x \tilde{y} \left(\cos 3\theta^1_x + \cos 3\theta^2_x + \cos 3(\theta^1_x - \theta^2_x) \right)$$

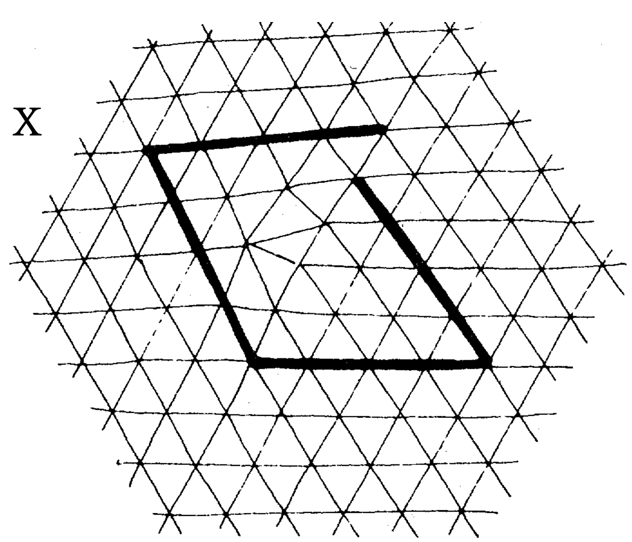
the A-th simple root of SU(3)

$$2\,\kappa = \frac{g_4^2(L)}{2\pi L T} \qquad \tilde{y} \sim \xi_{bion} a^2 = \frac{e^{-\frac{8\pi^2}{g_4^2(L)}(1+c g_4)}}{L T g_4^{14-2n_f}}$$

at y=0 - D. Nelson's theory of melting on 2d triangular (=root) lattice

θ^i_x the distortion (phonon) field at x

κ Lamé coefficient



vortices = dislocations in crystal

winding number = Burger's vector

melting = proliferation of dislocations at small kappa (solid becomes a "fluid of dislocations" upon melting)

Ex.2: SU(3) QCD(adj) dual to "affine" XY model with Z_3 x Z_3 preserving perturbation

lattice spacing a ~ L
chiral Z_3 and topological Z_3
(dual to center Z_3)

$$-\beta H = \sum_{x;\hat\mu=1,2} \sum_{A=1,2} \frac{\kappa}{2\pi} \cos \alpha_i^A (\theta^i_{x+\hat\mu} - \theta^i_x) + \sum_x \tilde{y} \left(\cos 3\theta^1_x + \cos 3\theta^2_x + \cos 3(\theta^1_x - \theta^2_x)\right)$$

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in the SU(3) dual spin model, behavior near T_c is not so well understood
- yet (by us?)...

- just as SU(2), dual has el-m duality (high-T/low-T)

- which - if any - CFT describes critical behavior at self-dual point?

- clearly, however, at small-T (large-kappa) Z_3-chiral x Z_3-topological broken, as appropriate to confining phase

OUTLOOK: we can find out - if all else fails using simulations...

How does this duality come about?

In a class of 4d QCD-like theories - QCD with massless adjoint Weyl fermions, to be precise - the thermal confinement-deconfinement transition can be understood as arising due to a competition of "electric" and "magnetic" degrees of freedom...

"electric": perturbative d.o.f., i.e., gauge bosons

"magnetic": non-perturbative d.o.f.,

in the theories at hand, magnetic monopole-instantons "bound" by fermion zero-mode exchange - the ``magnetic bions" responsible for confinement in QCD(adj) at T=0

This claim, of course, is quite expected - perturbative d.o.f. do not cause confinement, so it must be that the deconfinement transition arises due to a "fight" of perturbative vs. non-perturbative physics.

What's new is that - as should've become clear from my summary:

- I will not attempt to "model" non-perturbative effects,

  i.e., I will not be engaging in "voodoo QCD" - whatever merits this might sometimes have...

  Liao-Shuryak 2006 idea of E-M "competition" near Tc similar, classical E-M gas molecular dynamics, whose relation to underlying gauge theory unclear

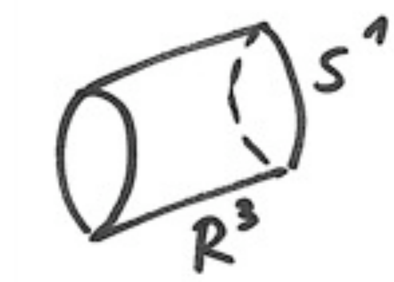
- I will not be using a Svetitsky-Yaffe-type universality - e.g., ZN center symmetry - based effective Landau-Ginsburg theory of the thermal transition our descrition will, of course, reflect symmetries

- Nor will I be doing numerical lattice simulations (yet?)

  ... or AdS/CFT(QCD)

I'll be looking at the dynamics of the theory by studying it in - even arbitrarily - small, but nonzero, volume:

the small volume will be, today,



where spatial circle has size "L"

Is this crazy? What does one hope to learn?

These are very good questions. I have two (and a half) answers:

1.

In the large-N limit, Eguchi-Kawai reduction for QCD(adj) holds, and certain correlators in small-L gauge theories are the same as at infinite-L

- not my topic today - only note this requires N_c L Lambda(QCD)>>1

2.

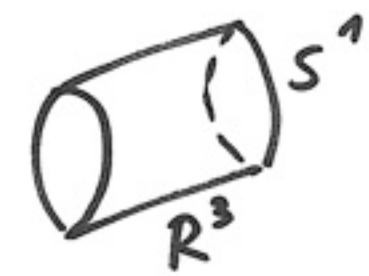
At fixed-L and fixed-N, the dynamics of many 4d QCD like theories in this geometry becomes calculable - usually difficult to study properties, such as confinement and chiral symmetry breaking are semiclassically calculable and under analytical control.

But should one care?

After all, calculability requires taking

$N_c L$ Lambda(QCD) << 1

i.e., rather small-L...

L 

The attitude I take is that, since non-perturbative calculability is not often encountered in the study of gauge dynamics, it may be of interest - ***and is certainly fun!*** - to take this opportunity seriously and "squeeze out" everything we can of this calculable limit.

Recall the numerous efforts in AdS/CFT(QCD): another semiclassically calculable limit - in the (super)gravity regime - where the UV completion is string theory. Here, instead, the UV completion is "ordinary" 4d asymptotically-free QFT and the semiclassical objects are not fundamental strings, but some good old *-and some new-* monopoles, instantons, etc...

The hope is, of course, that some of the insight found at small-L will continue to hold - "morally" if not quantitatively - at large L.

(in some cases, one may contemplate an analytic 1/L-expansion... future...)

For simplicity, will consider only 4d SU(2) theories with n_f adjoint Weyl fermions

"applications": $\longrightarrow$

$n_f = 1$ is N=1 SUSY YM

$n_f = 4$
- "minimal walking technicolor"
- happens to be N=4 SYM without the scalars

consider theory on $R^3 \times S^1$

$$S^1 : \quad x^4 \sim x^4 + L$$

with periodic ("susy") boundary conditions on fermions

then Polyakov loop $\quad W = P e^{i \oint_{S_1} A_4 \, dx^4}$

acquires a center-symmetric expectation value ($n_f > 1$, Unsal-Yaffe 2007)

$$\langle W \rangle = \begin{pmatrix} e^{i\pi/2} & \\ & e^{-i\pi/2} \end{pmatrix} \qquad \mathrm{tr} \langle W \rangle = 0$$

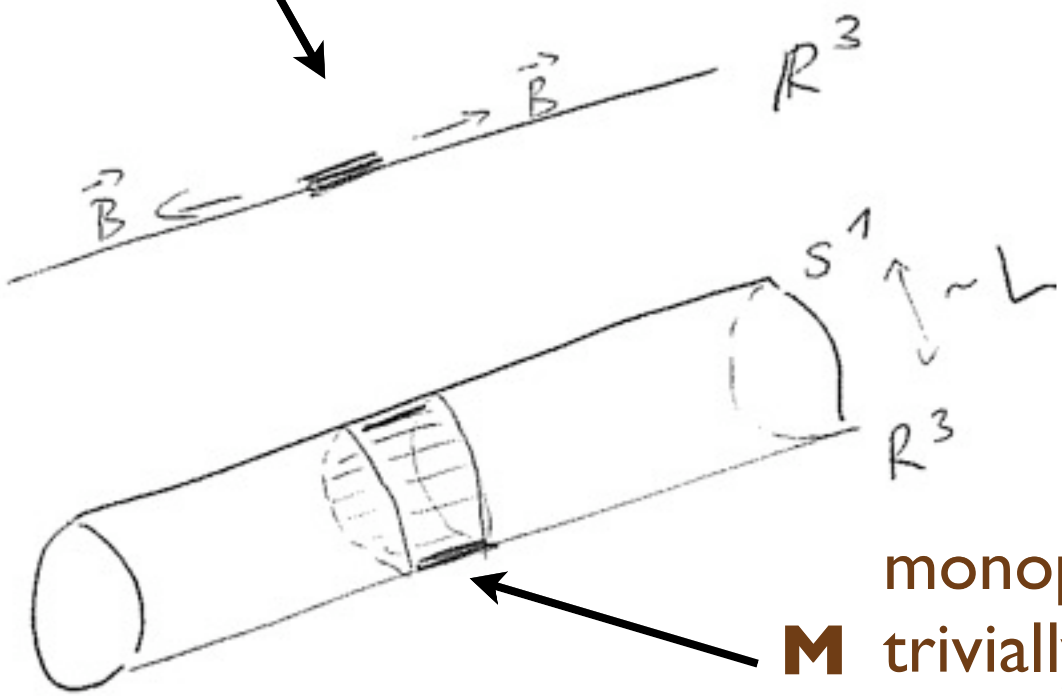
theory dynamically "abelianizes", i.e. $SU(2) \xrightarrow{\frac{1}{L}} U(1)$

clearly, weakly coupled if L << inverse strong scale

Despite weak coupling, nonperturbative dynamics is not trivial: since SU(2) broken to U(1), there are "monopole-instanton" solutions

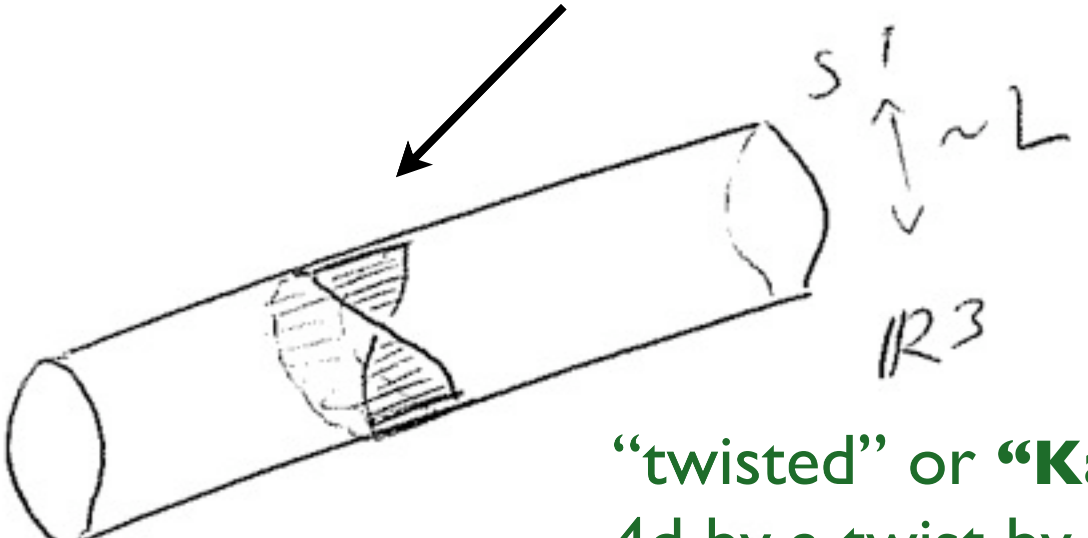
(should be called instantons, since finite action Euclidean, but keeping with tradition will stick with "monopoles")

"monopole instanton" in 3d



M monopole trivially embedded in 4d

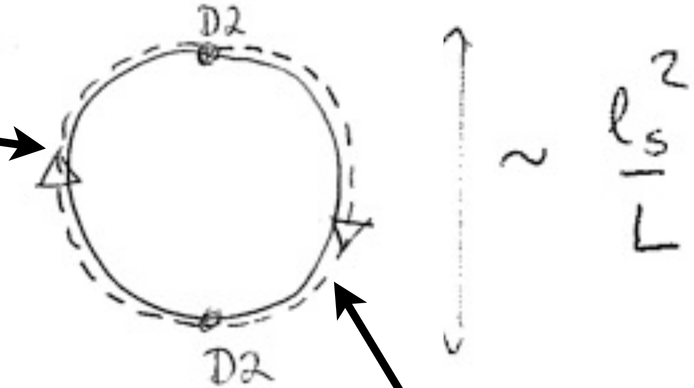
KK



"twisted" or **"Kaluza-Klein":** monopole embedded in 4d by a twist by a "gauge transformation" periodic up to center - in 3d limit not there! (infinite action)

KK discovered by K. Lee, P. Yi, 1997, as *"Instantons and monopoles on partially compactified D-branes"*

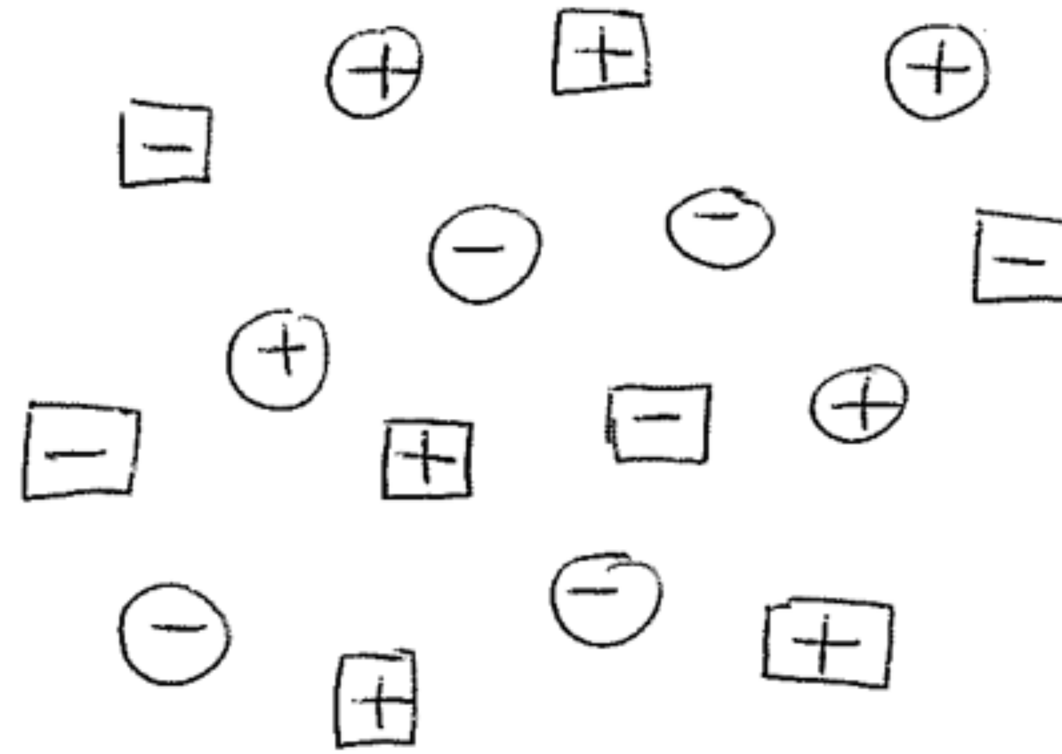
M
Euclidean
D0-brane

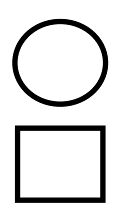


KK
Euclidean
D0-brane

D-brane picture, despite all of its SUSY, is the best - easiest, fastest - way to learn about the properties of M, KK,...!

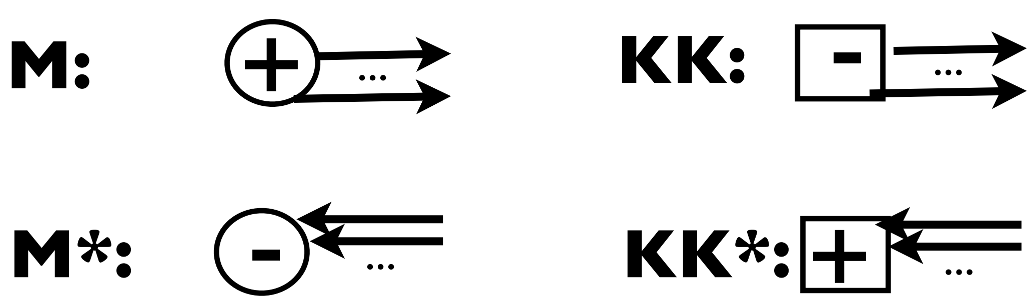
4d QCD(adj) dilute instanton gas of M,M*,KK,KK*
at small L





= M(+)/M*(-)

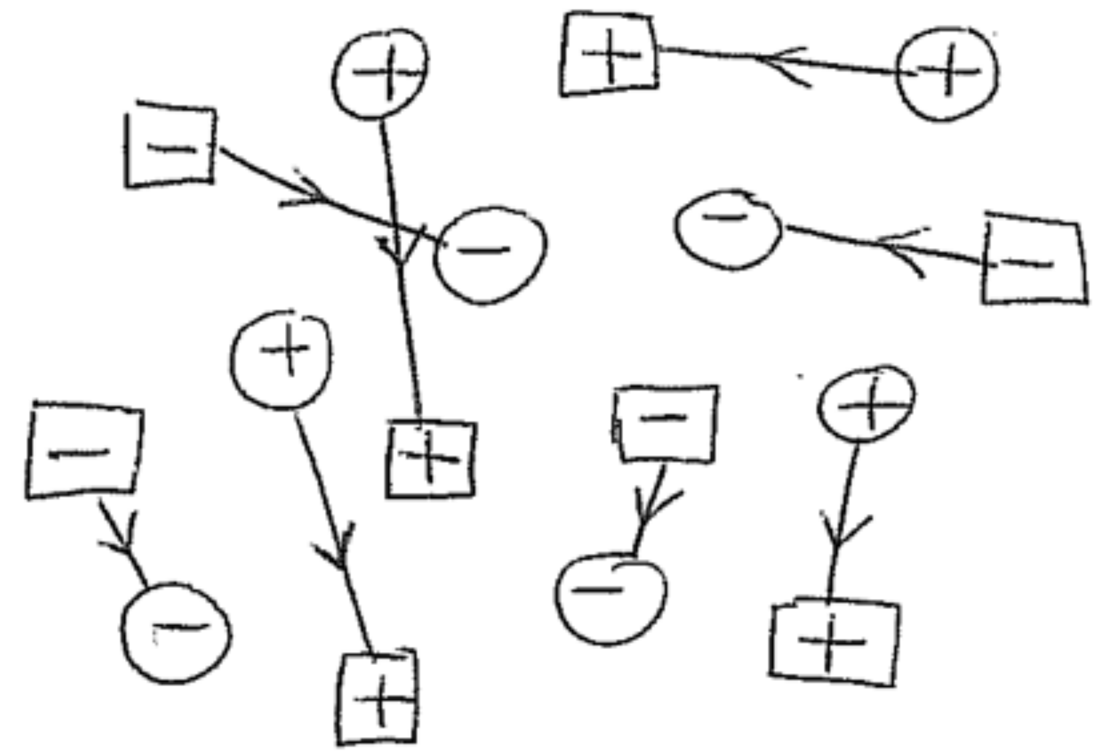
= KK(-)/KK*(+)

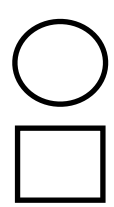


Index theorem:
Nye, Singer 2000
Unsal, E.P. 2008

4d QCD(adj) fermion attraction M-KK at small-L*

Unsal 2007





= M(+)/M*(-)

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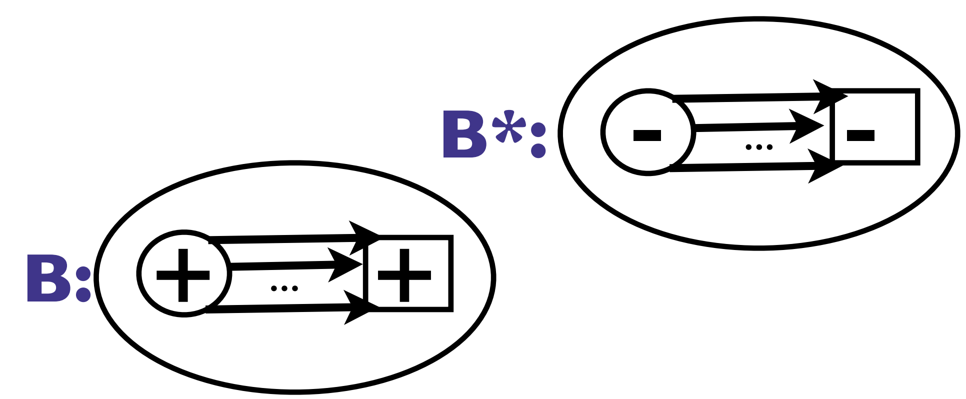

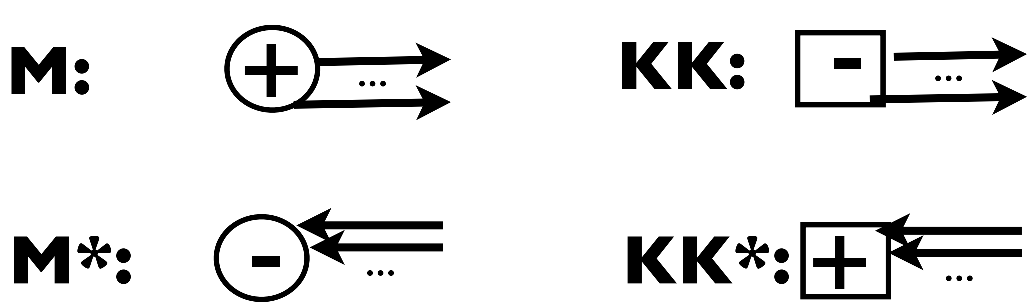

Index theorem:
Nye, Singer 2000
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4d QCD(adj) bion plasma at small-L
Unsal 2007

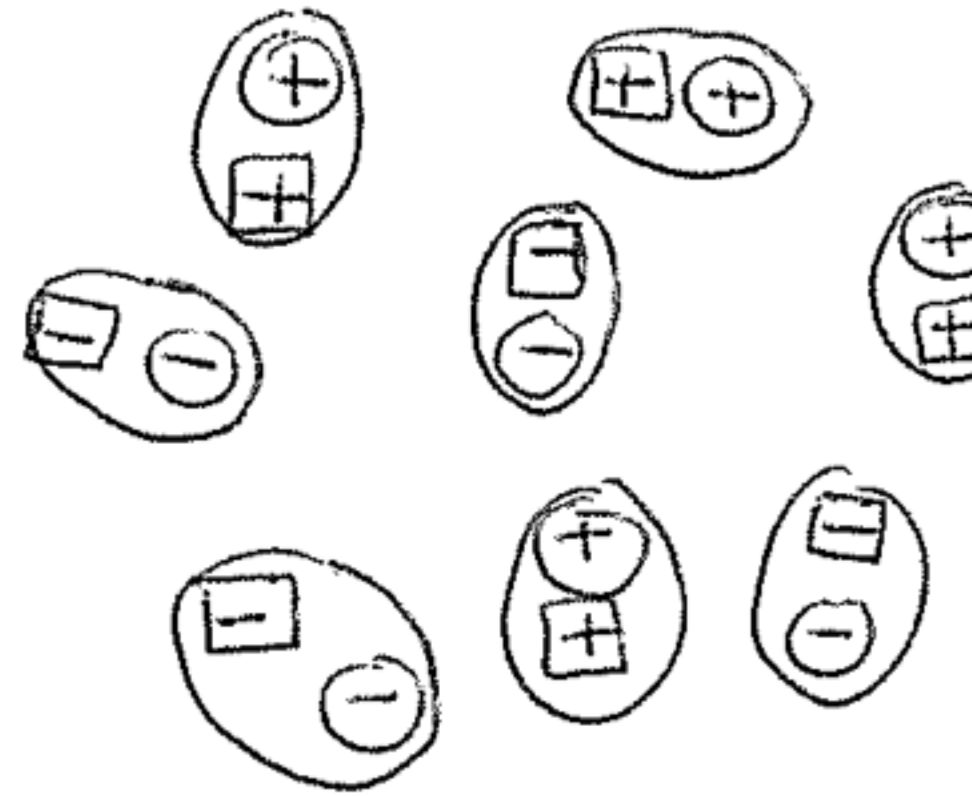
M + KK* = B - magnetic "bions" -

-carry 2 units of magnetic charge

-no topological charge (non self-dual)

locally 4d nature crucial: no KK in 3d

- bion/antibion plasma screening generates mass for dual photon ~ confining string tension



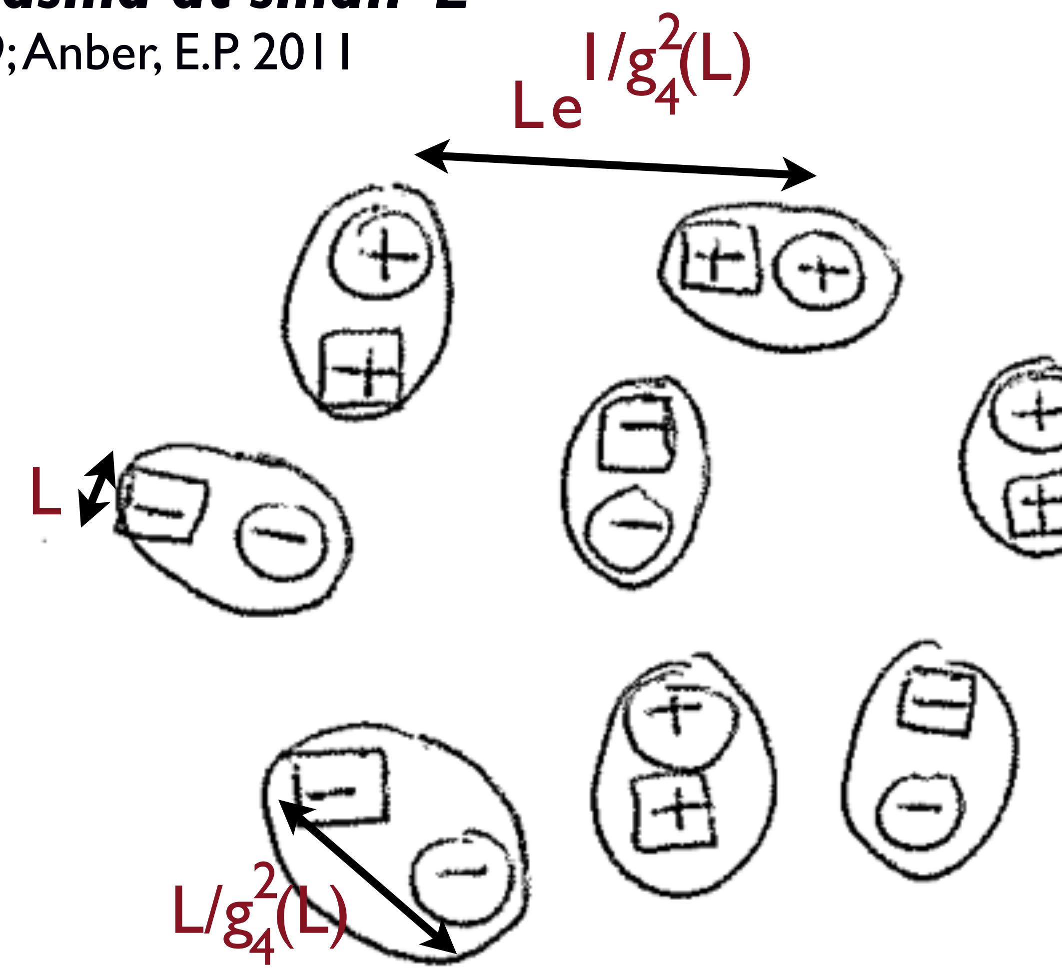
○ = M(+)/M*(-)

□ = KK(-)/KK*(+)

"blobs" = Bions(++)/Bions*(--)

4d QCD(adj) bion plasma at small-L

Unsal 2007; Unsal, E.P. 2009; Anber, E.P. 2011



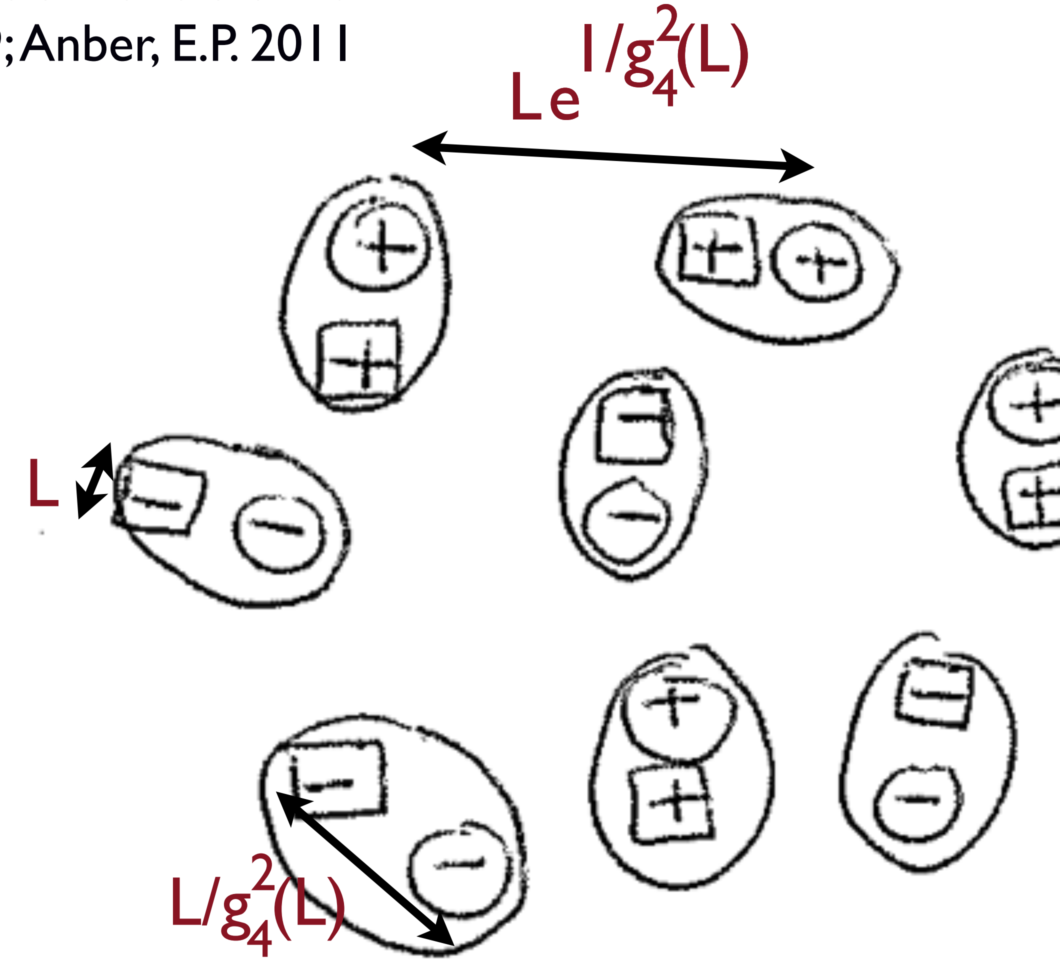
"magnetic bion confinement" operates at small-L in any theory with massless Weyl adjoints, including N=1 SYM **(& N=1 from Seiberg-Witten theory)**

it is "automatic": no need to "deform" theory other than small-L

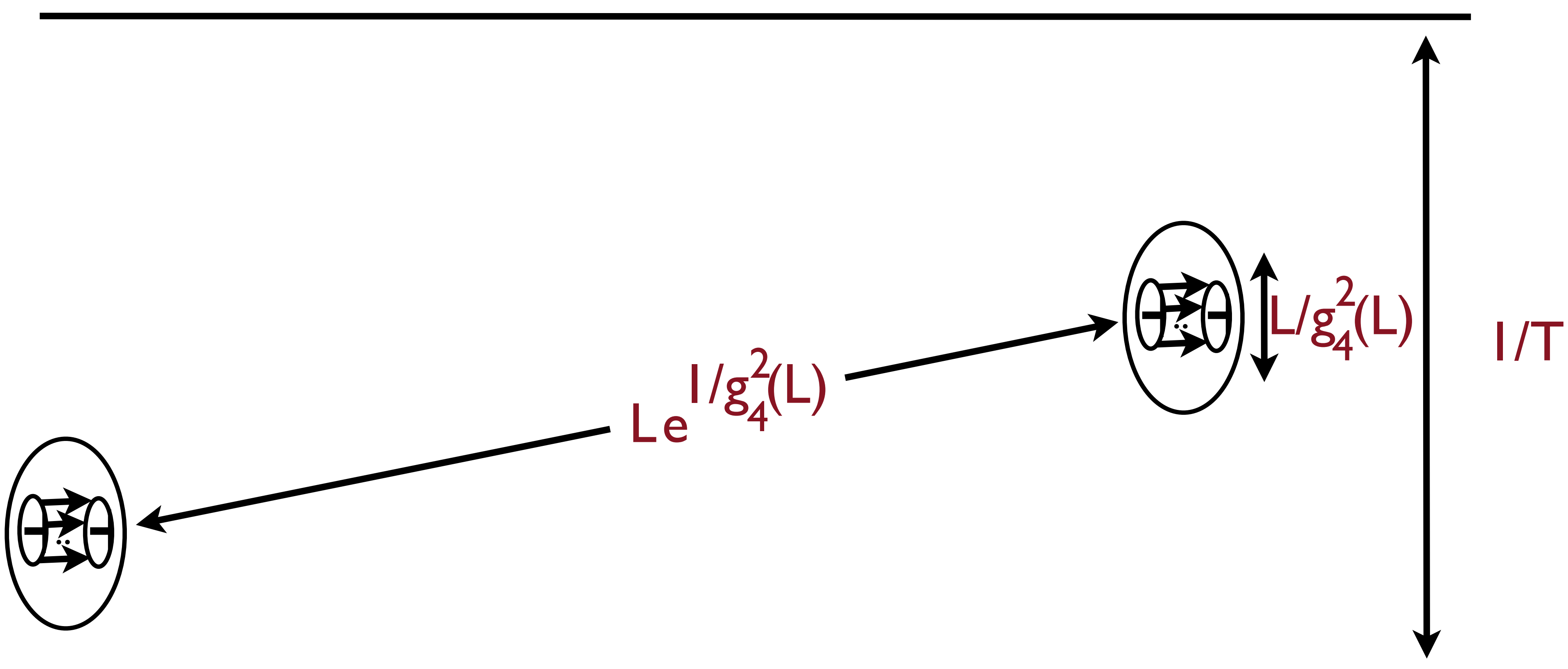
first time confinement analytically shown in a non-SUSY, continuum, **locally** 4d theory

4d QCD(adj) bion plasma at small-L

Unsal 2007; Unsal, E.P. 2009; Anber, E.P. 2011



Next, we turn on finite temperature...

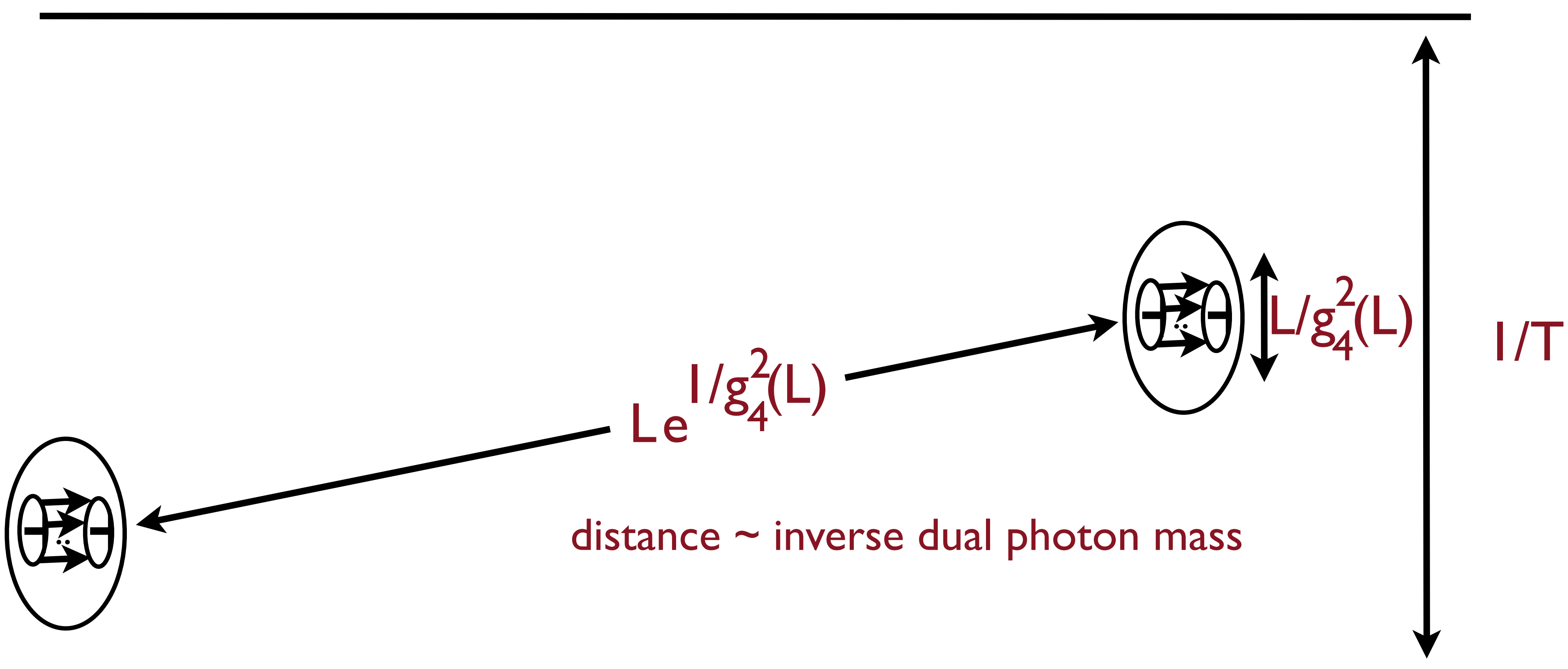


for temperatures in the range

dual photon mass << T << inverse bion size

bion gas is essentially 2-d Coulomb gas

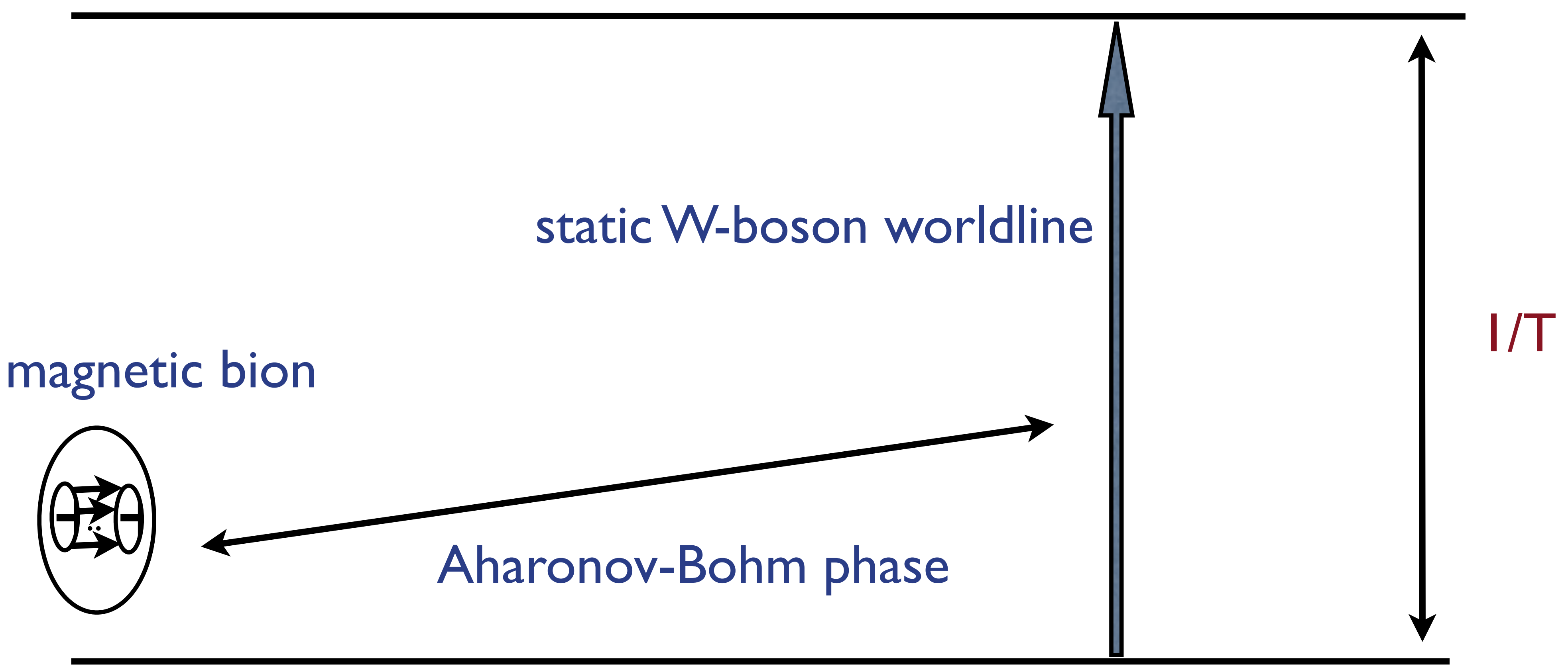
bions have not yet "dissociated"



for temperatures in the range

dual photon mass << T << inverse bion size << 1/L

W bosons, of mass ~ 1/L, can not be ignored in this range
- Boltzmann suppressed, but as important as bions



for temperatures in the range dual photon mass << T << inverse bion size << 1/L

$$
\begin{aligned}
&Z_{bion+W} \\
&= \sum_{N_{b\pm},q_a=\pm1}\ \sum_{N_{W\pm},q_A=\pm1} \frac{\xi_{bion}^{N_{b+}+N_{b-}}}{N_{b+}!N_{b-}!}\frac{(2\xi_W)^{N_{W+}+N_{W-}}}{N_{W+}!N_{W-}!}\prod_a^{N_{b+}+N_{b-}}\int d^2R_a \prod_A^{N_{W+}+N_{W-}}\int d^2R_A \\
&\times\ \exp\left[\frac{32\pi LT}{g^2}\sum_{a>b} q_a q_b \ln|\vec{R}_a-\vec{R}_b| + \frac{g^2}{2\pi LT}\sum_{A>B} q_A q_B \ln|\vec{R}_A-\vec{R}_B| + 4i\sum_{a,B} q_B q_a \Theta(\vec{R}_B-\vec{R}_a)\right]
\end{aligned}
$$

bion-bion magnetic interaction

W-W electric interaction

W-bion AB phase interaction

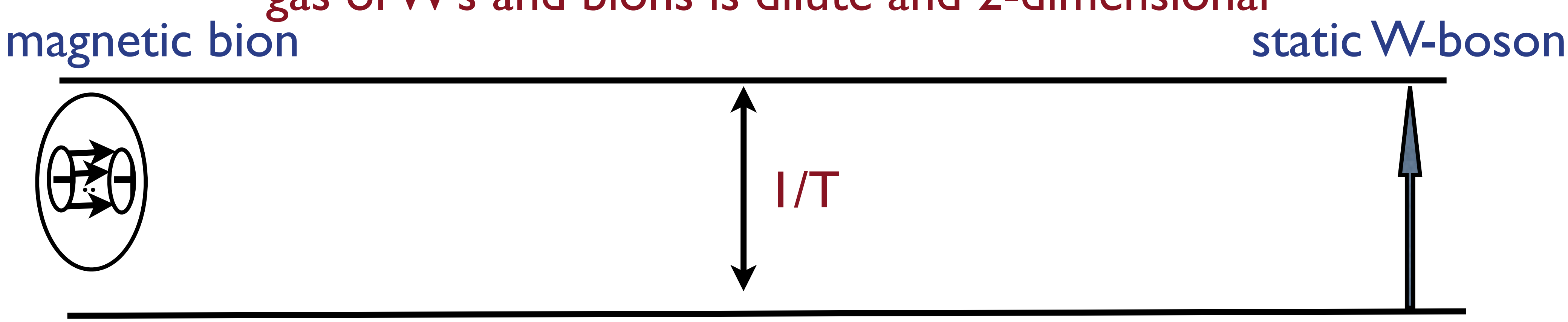
$$\xi_{\rm bion} \sim \frac{e^{-2S_0(1+cg)}}{L^3 T g^{14-8n_f}} \qquad\qquad \xi_W = (2n_f+1)\frac{m_W T}{2\pi}\ e^{-\frac{m_W}{T}}$$

Clearly, bion/W partition function for SU(2) is el.-m. duality invariant

= Kramers-Vannier (low-T/high-T duality)

= 2d T-duality (vortex-charge duality)

gas of W's and bions is dilute and 2-dimensional



for temperatures in the range dual photon mass << T << inverse bion size << 1/L

$$Z_{bion+W}$$

$$= \sum_{N_{b\pm}, q_a = \pm 1} \sum_{N_{W\pm}, q_A = \pm 1} \frac{\xi_{bion}^{N_{b+}+N_{b-}}}{N_{b+}! N_{b-}!} \frac{(2\xi_W)^{N_{W+}+N_{W-}}}{N_{W+}! N_{W-}!} \prod_a^{N_{b+}+N_{b-}} \int d^2 R_a \prod_A^{N_{W+}+N_{W-}} \int d^2 R_A$$

$$\times \exp\left[\frac{32\pi LT}{g^2} \sum_{a>b} q_a q_b \ln|\vec{R}_a - \vec{R}_b| + \frac{g^2}{2\pi LT} \sum_{A>B} q_A q_B \ln|\vec{R}_A - \vec{R}_B| + 4i \sum_{a,B} q_B q_a \Theta(\vec{R}_B - \vec{R}_a) \right]$$

bion-bion magnetic interaction

W-W electric interaction

W-bion AB phase interaction

W's = XY model gas of vortices

$$-\beta H = \sum_{x;\hat{\mu}=1,2} \frac{\kappa}{2\pi} \cos(\theta_{x+\hat{\mu}} - \theta_x) + \sum_x \tilde{y} \cos 4\theta_x$$

for temperatures in the range dual photon mass << T << inverse bion size << 1/L

$$Z_{bion+W}$$

$$= \sum_{N_{b\pm}, q_a = \pm 1} \sum_{N_{W\pm}, q_A = \pm 1} \frac{\xi_{bion}^{N_{b+}+N_{b-}}}{N_{b+}! N_{b-}!} \frac{(2\xi_W)^{N_{W+}+N_{W-}}}{N_{W+}! N_{W-}!} \prod_a^{N_{b+}+N_{b-}} \int d^2 R_a \prod_A^{N_{W+}+N_{W-}} \int d^2 R_A$$

$$\times \exp\left[\frac{32\pi LT}{g^2} \sum_{a>b} q_a q_b \ln |\vec{R}_a - \vec{R}_b| + \frac{g^2}{2\pi LT} \sum_{A>B} q_A q_B \ln |\vec{R}_A - \vec{R}_B| + 4i \sum_{a,B} q_B q_a \Theta(\vec{R}_B - \vec{R}_a)\right]$$

bion-bion magnetic interaction

W-W electric interaction

W-bion AB phase interaction

W's = XY model gas of vortices

bions = XY model gas of charges

$$-\beta H = \sum_{x;\hat{\mu}=1,2} \frac{\kappa}{2\pi} \cos(\theta_{x+\hat{\mu}} - \theta_x) + \sum_x \tilde{y} \cos 4\theta_x$$

In addition to low-T/high-T duals, mentioned above,
there are different (GNO-like) duals appropriate to SU(2) vs SO(3) gauge theories
center Z_2 symmetry for SU(2) maps to topological Z_2 symmetry for SO(3)

Analysis of phase transition & critical indices involves Coulomb gas RGEs and bosonization.

What did I tell you about?

gauge theory dynamics on $R^{1,2} \times S^1$ (spatial circle)

lattice Coulomb gases and 2d spin models

theory of melting of 2d crystal on triangular lattice

4d SU(N) gauge theory with n_f massless adjoint Weyl fermions on spatial circle (L) ⟵ at finite T, near deconfinement transition is dual to ⟶ 2d "affine" XY spin models

One side of duality - 4d gauge theory with massless fermions. Difficult to study by any means, including on the lattice

The other side - 2d spin models, known, or a generalization of known ones. Both analytical and numerical progress should be possible - as I showed you in an example.

CHALLENGES:

Understanding of higher-rank cases is still incomplete.

SU(3) is self dual, should be possible to find order of transition? exponents? (if there's a CFT at Tc?)

No e-m duality for SU(N>3), RGEs flow to strong coupling...

Study of nonequilibrium properties? QGP experiments non-static, really...

Complete phase diagram in L - 1/T plane?