Chirality, particle physics, and “theory space”

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Toronto
Pasteur (1848):

different rotation of light polarization due to “chiral” crystals

- inferred existence of “L-/R- handed” molecules

Lord Kelvin’s definition (1904):

“I call any geometrical figure, or group of points, chiral, and say it has chirality, if its image in a plane mirror, ideally realized, cannot be brought to coincide by itself.”
“chiral”: an object that is non-superposable on its mirror image

χειρ = hand "handedness"
Objects with left- or right-handedness commonly occur in the **macroscopic** world.

In fact, some substances only occur naturally in left-handed form, while others occur only in right-handed form. 

aka “homochirality” (left-handed amino acids and right-handed sugars) already observed by Pasteur

The difference between objects with left- or right-handedness is common in the **macroscopic** world.
e.g., we metabolize only right-handed glucose

Alice to Kitty:

“...Perhaps looking-glass milk isn't good to drink....”
In the **microscopic** world of elementary particles and the fundamental forces between them, however, the symmetry between L-(eft) and R-(ight) holds **almost universally**.

the four fundamental forces:

- gravity
- electromagnetism
- weak interactions
- strong interactions

are blind to “chirality” - **except the weak interactions**

Lee, Yang (theory, 1956)  
Wu (experiment, 1957)
Cobalt $\rightarrow$ Nickel + electron + anti-neutrino

- Cobalt: 27 protons, 33 neutrons
- Nickel: 28 protons, 32 neutrons

In the mirror, the nuclei rotate in the opposite direction. Symmetry with mirror image would require equal number of electrons going up and down.
Cobalt $\rightarrow$ Nickel + electron + anti-neutrino

In the mirror, the nuclei rotate in the opposite direction. Symmetry with mirror image would require equal number of electrons going up and down.

But this is not what was found!

Wu (experiment, 1957)

Thus, weak interactions are "chiral"
microscopic world: only weak interactions “chiral,” but “chirality” is common in the macroscopic world

it’s natural to ask: could there be any connection?

(I think) it is hard to imagine how the small chirality violation in the weak interactions could bias chemical/bio processes to lead to the observed “homochirality” of macro world

- recall electromagnetism, most relevant for chemistry/bio is L-R symmetric

- energy splitting between L- vs. R- molecules due to weak interactions is tiny $E/kT \sim 10^{-17}$ ($kT = 0.025$ eV, splitting between l- and d-amino acids)

but, needless to say, speculations trying to amplify this tiny effect exist
- and are subject of debate... (“aminoacidgenesis” vs “baryogenesis”?)
Wu’s discovery led Gell-Mann (1958) to postulate the chiral (“V-A”) structure of weak interactions, and ultimately, led to the establishment of the Standard Model of particle physics of Glashow, Salam, Weinberg (~1970).

It is not my purpose to review it in great detail today - but explain that it is a “chiral” (L-/R- asymmetric) model.

What, more precisely, do we mean by “chirality” in the particle physics world?

What are the “building blocks” of the Standard Model?

gauge bosons of spin-1: photon, gluons, W, Z

fermions of spin-1/2: electron, muon, tau, neutrinos, quarks
electron, muon, tau, quarks, (neutrinos) are all “made of”

“L-handed” fermions

massless fermions move with speed of light - projection of spin on direction of motion is a characteristic independent of observer - called the particle’s “chirality”

“R-handed” fermions

and

- which are mirror images of each other

massive fermions, on the other hand, do not have definite chirality, which can look different to different observers

\[ v' = v - w \]
e.g., at the fundamental level, the electron is “made of”

left-handed electron

\[ e^L \]

its anti-particle:

right-handed positron

\[ e^* \]

right-handed electron

\[ e^R \]

its anti-particle:

left-handed positron

\[ e^{*L} \]

for neutrinos, we do not know yet if R-handed ones exist

for all the rest: muon, tau, quarks (u,d,c,s,b,t) simply put the appropriate value of charge in place of +/-

thus, apart from the neutrinos, the particle content of the Standard Model is L-R symmetric - we call it “vectorlike” or “Dirac”
but, electron not massless - recall:

\[ \text{chirality} \quad \rightarrow \quad v = c \quad (\text{massless}) \]

\[ \text{(massive)} \quad v < c \quad \rightarrow \quad \text{chirality} \]

there are two ways that a fermion can be “slowed down”
clearly, both must involve chirality violation

“Dirac” fermion mass - leptons, quarks in Standard Model

interaction with the “Higgs field” condensate “flips” chirality

“Majorana” fermion mass

e.g., if R-handed electron didn’t exist - charge would not be conserved - but a possibility for neutrinos
thus, while the spectrum of the SM is L-R symmetric ("vectorlike" or "Dirac"), some of the interactions - the ones responsible for the nuclear beta decay - are not L-R asymmetric ("chiral")

\[ W \text{ couples only to } L! \]

\[
\begin{align*}
W & \quad e_L \quad u_L \\
\gamma_L & \quad d_L
\end{align*}
\]

L-electron only is produced, so its velocity correlates with spin of W, which in turn "remembers" spin of neutron - part of spin-polarized Cobalt nucleus (since the nuclear beta decay interactions are very weak, it took many years before parity violation was seen)
in summary, the Standard Model incorporates three of the fundamental forces

- electromagnetism

- weak interactions

- strong interactions + gluon self-interactions

as the names may suggest - all but the strong ones are “weak”, in the sense that we can study most of their relevant aspects in “perturbation theory” in some small $g$:

$$H = H_0 + g H_1$$
$H = H_0 + g H_1$

$H_0$ - represents something we can solve (free fields)

$H_1$ - represents a “small” perturbation

(use expansion in $g < 1$ to approximate true answer)

“perturbation theory” approach familiar from mechanics/quantum mechanics - e.g. anharmonic oscillator

works OK for low-lying states

breaks down for highly-excited ones
For a theorist, the Standard Model is just a collection of "gauge theories":

- electromagnetism — "vectorlike", weak

- weak interactions — "chiral", weak

- strong interactions — "vectorlike", strong "in the IR"

an arena for a host of non-perturbative techniques:

- models (quark models, instanton "liquid", SD eqns...)

- first principles - large-N ideas, lattice

  e.g., solve $H$ without expanding in $g$: $H = H_0 + g H_1$
The need for such non-perturbative studies of the strong interactions arises because of a peculiarity of their dynamics - “asymptotic freedom” - interactions are weak at short distances but become strong at long-distances (or, as we say, in the “IR” infrared):

- a flux tube of “glue” (“QCD string”) stretches between quarks and gives rise to a linearly-rising “confining” potential between quarks

“quarks” and “gluons” are useful to describe the short-distance behavior of the theory - but fail to capture its large-distance properties - new “emergent” degrees of freedom become relevant.
Much progress in understanding QCD - the theory of the strong interactions - has been achieved by a variety of techniques - e.g., symmetries, effective field theory, lattice, large-$N$... no doubt also aided by the nature’s “analogue computer” power (experiment).

“Asymptotic freedom” is a generic property of nonabelian gauge theories. It leads to a variety of IR behavior, which can be quite distinct from QCD. My focus is on asymptotically free theories different from those describing the strong interactions:

- why is this interesting?

- what is “theory space”?

- how do we study the dynamics?
The reason is that, despite the spectacular agreement of Standard Model predictions with experiment, we’re somewhat at a loss...

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<th>LEP</th>
<th>LEP</th>
<th>LEP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_Z$ [GeV]</td>
<td>91.1876 ± 0.0021</td>
<td>91.1874 ± 0.0021</td>
<td>0.1</td>
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<td>$\Gamma_Z$ [GeV]</td>
<td>2.4952 ± 0.0023</td>
<td>2.4972 ± 0.0011</td>
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<td>$\Gamma^{(\text{inv})}$ [MeV]</td>
<td>499.0 ± 1.5</td>
<td>501.74 ± 0.15</td>
<td>—</td>
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<td>$\sigma_{\text{had}}$ [nb]</td>
<td>41.541 ± 0.037</td>
<td>41.470 ± 0.010</td>
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<td>$R_e$</td>
<td>20.804 ± 0.050</td>
<td>20.753 ± 0.012</td>
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<td>$R_\mu$</td>
<td>20.785 ± 0.033</td>
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<tr>
<td>$R_\tau$</td>
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<td>20.799 ± 0.012</td>
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</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>LEP + SLD</th>
<th>LEP + SLD</th>
<th>LEP + SLD</th>
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<td>$A_{FB}(b)$</td>
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<td>$A_c$</td>
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<td>0.6681 ± 0.0005</td>
<td>0.1</td>
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<table>
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<th>$A_{LR}$ (hadrons)</th>
<th>SLD</th>
<th>SLD</th>
<th>SLD</th>
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<tbody>
<tr>
<td></td>
<td>0.15138 ± 0.00216</td>
<td>0.1478 ± 0.0012</td>
<td>1.6</td>
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</table>

(fairly old transparency, agreement has only improved since; point is to show %-level agreement)
“Dirac” fermion mass - leptons, quarks in Standard Model

interaction with the “Higgs field” condensate “flips” chirality

the nature of this “thing” is not known

But it is precisely this unknown Higgs “thing,” which, together with L-R asymmetry of the electroweak theory that links the W, Z-boson masses to the quark, lepton masses - which span some 12 orders of magnitude below W,Z - why?

e.g., is “the Higgs” a fundamental scalar field?
is it a composite object? is there strongly-coupled dynamics involved?
is there supersymmetry? is the theory “natural”? ...
What will the LHC discover?

Blogger Jester says:

Higgs boson.
...
Non-SM Higgs boson.
...
New Beyond SM Particles.
...
Strong Interactions.
...
Dark matter.
...
Little Higgs and friends.
...
Supersymmetry.
...
Dragons.
...
Black Holes.
What will the LHC discover?

Blogger Jester says:

Here are my expectations. The probabilities were computed using all currently available data and elaborated Bayesian statistics.

Higgs boson. Probability 80%
...
Non-SM Higgs boson. Probability 50%
...
New Beyond SM Particles. Probability 50%
...
Strong Interactions. Probability 20%
...
Dark matter. Probability 5%
...
Little Higgs and friends. Probability 1%
...
Supersymmetry. Probability 0.1%
...
Dragons. Probability $\exp(-S(\text{dragon}))$
...
Black Holes. Probability $0.1*\exp(-S(\text{dragon}))$...
What will the LHC discover?

Higgs boson.
... 
Non-SM Higgs boson.
... 
New Beyond SM Particles.
... 
Strong Interactions.
... 
Dark matter.
... 
Little Higgs and friends.
... 
Supersymmetry.
...

In all “scenarios” for “Beyond the Standard Model” physics, new gauge dynamics is invoked, at some scale. (Unless “supersplit supersymmetry” turns out to be nature’s choice.)

In many cases this gauge dynamics is of the “weak interactions” type, e.g., is chiral or L-R asymmetric.

My purpose here is not to discuss “scenarios” (aka “model-building” - a separate and very long subject), but to focus on the “theory space” involved ...
“gauge theory space”

conventional wisdom:

SUSY
- very “friendly” to theorists
- beautiful - exact results

pure YM
- formal
  but see www.claymath.org/millennium/

QCD-like (vectorlike)
- hard, leave it to lattice folks
  (m, V, $)

non-SUSY chiral gauge theories
- poorly understood strong dynamics
  ...(almost) nobody talks about them anymore
“gauge theory space”

**SUSY**
- superpartner masses;
- supersymmetry breaking in chiral SUSY theories

**QCD-like (vectorlike)**
- W, Z-masses: “walking” or “conformal” technicolor

**non-SUSY chiral gauge theories**
- extended technicolor - fermion mass generation;
- quark and lepton compositeness;
- & recent very speculative ideas of W, Z, t masses by monopole condensation

**applications:**
- very “friendly” to theorists
- beautiful - exact results

I left this part because I believe that one of the most important “applications” of supersymmetry is to teach us about the many “weird” things gauge field theories could do - often very much unlike QCD:

- massless monopole/dyon condensation causing confinement and chiral symmetry breaking
- “magnetic free phases” - dynamically generated gauge fields and fermions
- chiral-nonchiral dualities
- last but not least: gauge-gravity dualities
“gauge theory space”

nonperturbative tools:

**SUSY**
- ‘t Hooft anomaly matching
- “power of holomorphy”
- mass and flat direction “deformations”
- semiclassical expansions
- strings/branes and gauge-gravity dualities
  (my older work, as well as recent/upcoming Unsal-EP 2009/10)

**QCD-like (vectorlike)**
- most powerful: lattice
- the others mentioned already for QCD

**non-SUSY chiral gauge theories**
- ‘t Hooft anomaly matching
- semiclassical expansions
  (Unsal-Shifman, Unsal-EP 2008/9)

“MAC” (most attractive channel) truncated Schwinger-Dyson equations

tools you don’t really know whether to trust unless confirmed by other means - experiment or the tools on the left - the equivalent of “voodoo QCD” [Intriligator]

* in some cases, must assume that nonperturbatively string theory exists
“gauge theory space” nonperturbative tools:

[SUSY]
- ‘t Hooft anomaly matching
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[QCD-like (vectorlike)]
- most powerful: lattice
- the others mentioned already for QCD

[non-SUSY chiral gauge theories]
- ‘t Hooft anomaly matching
- semiclassical expansions
  (Unsal-Shifman, Unsal-EP 2008/9)

NOTE: “most powerful: lattice” is absent from “SUSY” and “chiral” parts of theory space

- the lattice is the only nonperturbative definition of general field theories we know of (although some vehemently disagree...)
- can it be useful in cases other than QCD and QCD-like theories?
In the remaining time, will briefly tell you two stories that I’ve worked on, unified by the motivational role that string theory has played.

i.) lattice SUSY and D-brane orbifolds

- ‘t Hooft anomaly matching
- “power of holomorphy”
- mass and flat direction “deformations”
- semiclassical expansions
- strings/branes and gauge-gravity dualities

• these are powerful tools - but not “tell all”
• many quantities of interest are not calculable
  - e.g., for dynamical susy breaking model-building of, say, “gauge mediation”
  - or, needless to say, would be nice to know what Seiberg-Witten’s massless monopoles/dyons are “made of”

- the lattice is the only nonperturbative definition of general field theories we know of (although some vehemently disagree...)
- can it be useful in cases other than QCD and QCD-like theories?

...
i.) **lattice SUSY and D-brane orbifolds**

**Ex.:** 1-d lattice SUSY: D-branes on a $\mathbb{Z}_N$ orbifold

$(p+1)$-dim gauge theory with fermions (and scalars) localized on the brane’s world volume
i.) lattice SUSY and D-brane orbifolds

**Ex.:** 1-d lattice SUSY: D-branes on a $\mathbb{Z}_N$ orbifold

- inverse lattice spacing
- number of lattice sites

```
"winding" open strings between images = lattice momentum modes (here: 1 dim)
```

lattice theory preserves supersymmetry, which does not involve translations in the lattice directions - but enhances to full SUSY in continuum limit

- Douglas, Moore ‘96
- Lykken, Trivedi, EP ‘97
- Cohen, Kaplan, Katz, Unsal ‘02
- Giedt, Rozali, EP ‘03
- Kaplan, Unsal ‘06
- (Giedt, EP ‘04, Giedt, Koniuk, Yavin, EP ‘04 and many other non-brane papers)
i.) **lattice SUSY and D-brane orbifolds**

Lykken, Trivedi, EP `97
“Chiral gauge theories from D-branes”

realized this on a 1-dim lattice
in retrospect! - had a non-lattice motivation

**Cohen, Kaplan, Katz, Unsal `02** constructed 2,3,4-dim lattices with SUSY

studies of quantum continuum limit, anomalies, fine-tuning issues, etc....

Giedt, Rozali, EP `03; Giedt, EP `04; Giedt, Koniuk, Yavin, EP `04
and many other brane and non-brane papers

moral/summary of what transpired since:

- branes on orbifolds motivated construction of actions of gauge
  theories that preserve some SUSY; quite nontrivial to come up with in
  field theory (a version of Kaehler-Dirac fermions appears; all unified under
  “topological twisting” umbrella - Giedt, EP `04; Unsal `06)

- actually useful for simulations in low-dim SUSY systems (Catterall,
  KEK group) to, e.g., study aspects of low-dim AdS/CFT

- all of this works only for vectorlike (extended) SUSY; no clue about chiral!
  (but recall interest: superpartner masses; supersymmetry breaking in chiral SUSY theories?)
ii.) non-SUSY chiral lattice gauge theories?

A fascinating development also influenced by strings (extra dimensions) ultimately led to the understanding of how to explicitly realize exact chiral symmetries on the lattice.

Ginsparg, Wilson `82- forgotten! - ‘til ‘97
Callan, Harvey, ‘85
Kaplan ‘92
Narayanan, Neuberger ‘94
Neuberger ‘97
Hasenfratz, Laliena, Niedermayer ‘98
Neuberger ‘98
Luscher ‘98-‘99

With few exceptions, however, only L-R symmetric (vectorlike) theories with chiral symmetry were nonperturbatively defined on the lattice - great progress for QCD and QCD-like theories, even if a bit $$$ to use.

Where is the challenge?
ii.) non-SUSY chiral lattice gauge theories?

Kaplan’s ’92 construction led to

i.) an infinite 4th dim where Dirac fermion propagates

ii.) chiral zero mode localized on 3-dim defect

to understand “infinity” - make dimension compact - but brings in “anti-wall” with mirror (R-handed) fermion - end up with vectorlike theory. with Neuberger ’97 operator etc...

to an observer who doesn’t resolve dimension, the end result looks like:

(to the experts: apologies for somewhat misleading picture)
ii.) non-SUSY chiral lattice gauge theories?

Where is the challenge? - get rid of the mirror partners

Luscher ‘98-‘99 (also, in part, Neuberger ‘98) - implicitly did so by showing how to define chiral partition function for anomaly free U(1). Mathematically, generalization to, say, SU(N), poses a difficult problem. Can this hard (for me) math be avoided?

In string theory, constructing chiral gauge theories often proceeds from higher-dim theories by “orbifolding” - or a “blown-up” version - of L-R symmetric (vectorlike) theories - in a way, “erasing” the mirrors, or, more prosaically, decoupling them. In each case, more structure than a simple domain wall is required and anomaly cancellation is always built-in the construction (which otherwise “falls apart”).

It is even possible to have chiral-nonchiral “transitions” as parameters (compactification moduli) are varied. (Douglas, Zhou ‘04)

Finally, there are some fascinating examples of chiral-nonchiral Seiberg dualities in N=1 SUSY gauge theories. (Pouliot; Strassler and Pouliot ‘96)

These are perhaps useful hints to think more about.

For now, follow a simple-minded, “experimentalist” approach - which is testable!
ii.) **non-SUSY chiral lattice gauge theories?**

Where is the challenge? - get rid of the mirror partners

**“light” fermions**

Bhattacharya, Martin, EP `06  
Giedt, EP `07  
Shang, EP `07  
Shang, EP `09

earlier idea of similar flavor  
(not pursued & independent)  
Creutz, Rebbi, Tytgat, Xue `96

**“mirror” fermions**

**our strategy:**

- add gauge invariant multi-fermion (or Yukawa) interactions between mirror fermions
- break all anomalous and anomaly free global symmetries of the mirror
- ’t Hooft anomaly matching on the lattice applies to these interactions (Shang, EP `07, `09)
- drive the mirror interactions into a strong-coupling symmetric phase (exists: Giedt, EP `07)
- at strong mirror couplings expect all mirrors to decouple; there is no symmetry reason for light mirrors at strong coupling - a field theory “orbifold” realization!?

note that this story has all the flavor of Eichten, Preskill `86 with one crucial difference: we now have exact chiral symmetries and anomaly matching arguments on the lattice!
I told you our strategy - but “Does it work?”

... hang on - “experiment” is slow, mainly due to Joel Giedt (BlueGene, Rensselaer PI)

but we have not seen reasons to give up -

so far, our results on anomaly matching go in the right direction
- min number of fermions needed to match anomalies remains massless
  when anomalous mirrors (cheap!) are studied -

- at the same time, we don’t know if we have succeeded or “not failed”, yet!

I have not failed. I’ve just found 10,000 ways that won’t work.

Thomas A. Edison
CONCLUSION: studying “gauge theory space” is

a.) fun

and

b.) it may help us understand what the LHC will be trying to tell us about the short-distance properties of nature

SUSY

Pure YM

QCD-like (vectorlike)

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... Non-SM Higgs boson.

... New Beyond SM Particles.

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