Lattice supersymmetry, superfields and renormalization

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based on hep-th/0407135 [*JHEP09(2004)029*]
and work in progress

Motivation for studying lattice supersymmetry:

many supersymmetric theories play a role in particle theory models as well as in string theory

nonperturbative effects well understood via holomorphy and symmetries, but not all desired aspects under control

the lattice is the only* known nonperturbative definition of a general field theory and it would be of interest to have (a useful) one for supersymmetric theories

*apart from string theory, constructive field theory...

What is the problem with preserving supersymmetry on the lattice?

$$S = \int dx d\theta ... d\theta' \; F(\Phi)$$

generic SUSY action:
integral over superspace of a function of superfields

$$Q = \frac{\partial}{\partial \theta} + \theta \Gamma \frac{\partial}{\partial x}$$

$$\delta_\epsilon \Phi = \epsilon Q \Phi$$

SUSY generators:
differential operators acting on superfields

SUSY variation of the action:

$$\begin{aligned} \delta_\epsilon S &= \int dx d\theta ... d\theta' \left[F(\Phi + \epsilon Q \Phi) - F(\Phi) \right] \\ &= \int dx d\theta ... d\theta' \; \epsilon Q F(\Phi) \\ &= \int dx d\theta ... d\theta' \; \epsilon \left(\frac{\partial}{\partial \theta} + \theta \Gamma \frac{\partial}{\partial x} \right) F(\Phi) \quad = 0 \end{aligned}$$

used the Leibnitz
rule for spacetime
derivatives

Dondi, Nicolai, 1977

$$\delta_\epsilon S \quad = \quad \int dx d\theta ... d\theta' \, \epsilon \left(\frac{\partial}{\partial \theta} + \theta \Gamma \frac{\partial}{\partial x} \right) F(\Phi) \; = 0$$

Moral: if the supersymmetry generator was simply

(should sound familiar to the (SUSY) (de)constructionists amongst us!)

$$Q = \frac{\partial}{\partial \theta}$$

the Leibnitz rule would not be needed for supersymmetry of the *interaction lagrangian* (it is really easy to have a *free* supersymmetric lattice theory!)

Hence, we could simply replace continuum coordinate by a set of discrete points, without destroying the nilpotent supersymmetry of the action.

this talk is about:

1. How to do this?

how to write lattice actions invariant under nilpotent supersymmetries

2. What is it good for?

when does nilpotent supersymmetry help restore the entire supersymmetry algebra in the continuum limit without, or with little, fine-tuning

What has been achieved so far?

a (useful) lattice formulation with the same amount of lattice supersymmetry as the target continuum theory does not exist

the best [so far] are lattice models where supersymmetry is recovered in the continuum limit due to:

fine tuning

e.g., N=1 4d SYM with Wilson gauginos
theoretically palatable; in practice, however, prohibitively difficult (in more general cases)

finiteness/superrenormalizability

N=1 2d Wess-Zumino model, say

as an accidental symmetry

N=1 4d SYM with overlap gauginos, say

- or some combination of the above

1980s: Elitzur, Rabinovici, Schwimmer/Sakai, Sakamoto/Curci, Veneziano/Banks, Windey/Golterman, Petcher

1990s: Kaplan, Schmaltz/Kogut, Vranas/Nishimura

2000s: Catterall, Karamov, Gregory, Ghadab/(de)constructionists: Cohen, Kaplan, Katz, Unsal/Sugino`03/`04

What are **we** going to do?

1.
develop a general formalism for constructing Euclidean lattice theories invariant under nilpotent anticommuting supercharges

2.
use it to write down all possible terms consistent with the "lattice supersymmetry" and with other global symmetries

3.
is the lattice action with the desired supersymmetric continuum limit generic?

If yes, we're done...

(Go simulate!)

If no, how do non-generic actions renormalize?

Is fine-tuning needed?

If no

If yes, how much?

If too much, go think.

Euclidean lattice actions invariant under anticommuting nilpotent Q's have been written before using different methods,
obstructing the analysis [never really performed] of genericity, renormalization, and (super)symmetry restoration in the continuum limit.

Elitzur, Rabinovici, Schwimmer
Sakai, Sakamoto (1982/83)
Catterall et al. (2001-3)
Sugino (2003-4)

In this talk, not much discussion of the deconstruction-inspired approach to lattice supersymmetry, only note that it also preserves anticommuting nilpotent supercharges; other properties similar (later).

This work was, in fact, motivated by deconstruction
[a "top down" approach, involving orbifolds of matrix models and dynamically generated lattices,...]
and our desire to study its possible limitations and generalizations.

Here we start building a "bottom-up" approach, which leads to (in many ways) similar supersymmetric lattice theories.

I will only give details of supersymmetric quantum mechanics on a "supersymmetric" lattice, since all relevant technical elements are present there in their simplest form.

I will, in the end, show/discuss our results for various 2d (2,2) theories of scalars and fermions (work on the gauge case (2d, for now) is in progress) and their implication for future developments along these lines.

Supersymmetric quantum mechanics on a Euclidean "supersymmetric" lattice.

$$\{Q_1, Q_2\} = 2i\frac{\partial}{\partial t}$$

two nilpotent real supercharges given as operators in superspace:

$$\begin{aligned} Q_1 &= \frac{\partial}{\partial\theta^1} + i\,\theta^2\,\frac{\partial}{\partial t} = e^{-i\theta^1\theta^2\frac{\partial}{\partial t}}\,\frac{\partial}{\partial\theta^1}\,e^{i\theta^1\theta^2\frac{\partial}{\partial t}} \\ Q_2 &= \frac{\partial}{\partial\theta^2} + i\,\theta^1\,\frac{\partial}{\partial t} = e^{i\theta^1\theta^2\frac{\partial}{\partial t}}\,\frac{\partial}{\partial\theta^2}\,e^{-i\theta^1\theta^2\frac{\partial}{\partial t}} \end{aligned}$$

covariant derivatives anticommute with supercharges and obey:

$$D_1^2 = D_2^2 = 0,\ \{D_1, D_2\} = -2i\frac{\partial}{\partial t}$$

also given as operators

$$\begin{aligned} D_1 &= e^{i\theta^1\theta^2\frac{\partial}{\partial t}}\,\frac{\partial}{\partial\theta^1}\,e^{-i\theta^1\theta^2\frac{\partial}{\partial t}} \\ D_2 &= e^{-i\theta^1\theta^2\frac{\partial}{\partial t}}\,\frac{\partial}{\partial\theta^2}\,e^{i\theta^1\theta^2\frac{\partial}{\partial t}} \end{aligned}$$

acting on real superfields: $\Phi(t, \theta^1, \theta^2) = x(t) + \theta^1\psi(t) + \theta^2\chi(t) + \theta^1\theta^2 F(t)$

The action can be written as an integral over superspace:

$$
\begin{aligned}
S &= \int dt\, d\theta^2 d\theta^1 \left(\frac{1}{2} D_1\Phi D_2\Phi - h(\Phi)\right) \\
&= \int dt \left(\frac{1}{2}\dot{x}^2 + \frac{1}{2}F^2 - i\psi\,\dot{\chi} - h'(x)F + h''(x)\,\psi\,\chi\right)
\end{aligned}
$$

(call this "the N=2 1d Wess-Zumino model")

Our goal is to discretize time in a manner that preserves one of the two Q's, for example:

$$Q_1 = e^{-i\theta^1\theta^2\frac{\partial}{\partial t}}\,\frac{\partial}{\partial\theta^1}\,e^{i\theta^1\theta^2\frac{\partial}{\partial t}}$$

Generally,

a set of nilpotent anticommuting supercharges can always be simulateneously conjugated to pure θ-derivatives:

$$\frac{\partial}{\partial\theta^1} = e^{i\theta^1\theta^2\frac{\partial}{\partial t}}\,Q_1\,e^{-i\theta^1\theta^2\frac{\partial}{\partial t}}$$

In this new basis, Q is simply a derivative w.r.t. theta, while the real superfield is:

$$\begin{aligned}\Phi'(t,\theta^1,\theta^2) &= e^{i\theta^1\theta^2\frac{\partial}{\partial t}}\,\Phi(t,\theta^1,\theta^2)\,e^{-i\theta^1\theta^2\frac{\partial}{\partial t}} = \Phi(t+i\theta^1\theta^2,\theta^1,\theta^2)\\ &= (x+\theta^1\psi)+\theta^2(\chi-\theta^1(i\dot{x}+F))\ .\end{aligned}$$

$$U(t) = x(t)+\theta\psi(t) \qquad \xi(t) = \chi(t)-\theta\left(F(t)+\dot{x}(t)\right)$$

and thus has two irreducible components w.r.t. Q [Euclidean, beginning from line above].

Q acts as a shift of theta, as a purely "internal" supersymmetry, so replacing continuum time with a lattice $t^i,\ i=1,\ldots N$ does not affect the action of Q.

$$\xi^i = \chi^i - \theta\left(F^i + \frac{x^i - x^{i-1}}{a}\right)$$

$$U^i = x^i + \theta\psi^i$$

$$\xi^i = \chi^i - \theta f^i$$

we denoted $x(t^i) = x^i$,etc.

We can now use these lattice superfields to write supersymmetric actions:

- bosonic
- Q-invariant
- local
- lattice translation invariant

dimensional analysis (time: dim -1; $\partial/\partial\theta$: dim 1/2)

$a\sum_i \int d\theta$ dim -1/2, fermionic

so superspace lagrangian must be dim 1/2 and fermionic, while relevant terms have dimension less than 1/2

$U^i = x^i + \theta\psi^i$ dim -1/2, bosonic

$\xi^i = \chi^i - \theta f^i$ dim 0, fermionic

important: not every real Q-supersymmetric Euclidean lattice action gives rise to a hermitian hamiltonian system in the continuum limit; impose discrete symmetries to ensure desired continuum limit

then, the most general bilinear action consistent with symmetries is:

$$S = -\sum_i a \int d\theta \left(\frac{1}{2}\,\xi^i\,\frac{\partial}{\partial\theta}\xi^i + \xi^i\,\frac{1}{a}\Delta U^i + m\,\xi^i\,U^i \right) \qquad \Delta U^i \equiv U^i - U^{i-1}$$

Thus, we have successfully latticized the free theory; illustrates the general comment made before:

free lattice theories can be arranged to preserve all supersymmetry

$$\begin{aligned}\delta_1 x^i &= \epsilon^1\psi^i,\ \delta_1\psi^i = 0,\ \delta_1\chi^i = -\epsilon^1 f^i,\ \delta_1 f^i = 0\ ,\\ \delta_2 x^i &= \epsilon^2\chi^i,\ \delta_2\psi^i = \epsilon^2\left(f^i - \frac{2}{a}\,\hat{\Delta}x^i\right),\ \delta_2\chi^i = 0,\ \delta_2 f^i = \frac{2}{a}\,\epsilon^2\hat{\Delta}\chi^i\end{aligned}$$

obeying a lattice version of the supersymmetry algebra when acting on linear functions of the fields:

$$[\delta_1, \delta_2] = \frac{2}{a}\,\epsilon^1\epsilon^2\,\hat{\Delta}\ ,\quad \delta_1^2 = \delta_2^2 = 0$$

True in higher-dimensions too: simply use a symmetric lattice derivative to replace derivatives in continuum algebra; then free action, including Wilson terms, is always invariant.

The second supersymmetry is, of course, broken by the interactions.

Including interactions in a Q-invariant way in our formalism is now trivial:

$$S = -\sum_i a \int d\theta \left(\frac{1}{2}\,\xi^i\,\frac{\partial}{\partial\theta}\xi^i + \xi^i\,\frac{1}{a}\Delta U^i + m\,\xi^i\,U^i \right)$$

to include superpotential replace: $mU^i \rightarrow h'(U^i)$

Then get Q-supersymmetric lattice version of N=2 1d WZ model:

$$S = \sum_{i=1}^{N} a \left(\frac{1}{2} \left(h'(x^i) + \frac{x^i - x^{i-1}}{a} \right)^2 + \chi^i h''(x^i)\psi^i + \chi^i\,\frac{\psi^i - \psi^{i-1}}{a} \right)$$

cross term becomes $\frac{\partial}{\partial t}h$
in continuum limit

violates reflection positivity of the
Euclidean lattice theory at finite N,a

specific choice of fermion derivative: r=1
Wilson term for both bosons and fermions;
any r is OK with the lattice supersymmetry

finally, define the supersymmetric lattice partition function:

$$Z_N = -c^N \prod_{i=1}^{N} \int d\chi^i d\psi^i \int_{-\infty}^{\infty} dx^i \, e^{-S}$$

In 1d, discretized path integral will converge to the desired continuum limit, independent on whether any supersymmetry is preserved at finite lattice spacing, *provided a finite number of one-loop subtractions are performed.* [Giedt, Koniuk, E.P., Yavin, hep-lat/0410nnn]

Using the Q-invariant Z_N above liberates us from having to do subtractions. Moreover, the exact supersymmetry leads to some desirable properties of the "improved" lattice partition function:

1. Despite lack of reflection positivity and nonhermiticity of the transfer matrix, one can show that:

$$\lim_{N\to\infty, a\to 0} Z_N = \mathrm{tr}\,(-1)^F e^{-\beta H_{SQM}} \;, \qquad \beta \equiv Na \;-\; \text{fixed}$$

i.e., the Q-supersymmetric Euclidean lattice partition function defines a hermitean hamiltonian system, and converges to the Witten index in the continuum limit
(fermions in Z_N are also periodic).

II. Exact Q-invariance allows one, using deformation invariance, to exactly calculate Z_N *at finite* N,a, *and for any superpotential*:

$$Z_N = \left(\sum_{x^*} \frac{h''(x^*)}{|h''(x^*)|} \right)^N \quad = +/- 1 \text{ or } 0$$

which equals, for odd N, the Witten index of the continuum quantum mechanics.

III. We can also use this formalism to give a Q-supersymmetric ("in the bulk") lattice version of the finite temperature partition function:

$$Z_N(\beta) = \mathrm{tr}\ e^{-\beta H_{SQM}}$$

IV. Numerically, convergence to supersymmetric continuum limit is much faster with the supersymmetric lattice partition function. Order-a improved actions also possible...

Catterall et al., 2001
Giedt, Koniuk, E.P., Yavin, hep-lat/0410nnn

The supersymmetric lattice partition function has been written before, using different methods. [Catterall et al., 2001]

However, properties (I-III) as well as some impotant points of (IV) have not been studied before.

Also, we believe that our approach is more general.

For example, it can be used in cases where no local Nicolai map is known, e.g., supersymmetric quantum mechanics on Riemannian manifolds ("1d NLSM") [Giedt, E.P., in progress]

Now, on to higher goals (dimensions)...

For what supersymmetric theories does one expect to be able to write a lattice interaction lagrangian preserving nilpotent anticommuting supersymmetries?

I

Clearly, it is necessary that such nilpotent anticommuting* charges exist.

not enough!

II

If some interactions are given by integrals over restricted superspace (e.g., **chiral),** *there must exist a linear combination of nilpotent anticommuting Q's* **such that** *the Q-variation of these interactions is not a total derivative.*

*anticommuting is not a must; central charges are allowed on the r.h.s. of the anticommutator, but not derivatives [in gauge theories in WZ gauge: all up to gauge transforms]

Criterion II. above (severely) limits possibilities...

For example, 3d and 4d 4-supercharge theories have two anticommuting nilpotent supersymmetries, e.g.,

$$Q_\alpha \ (\alpha = 1, 2 \text{ is the } SL(2, C) \text{ index})$$

However, "1/3" of the lattice action (W*) will not be supersymmetric - as criterion II. is violated.

In 3d WZ models, one can, perhaps, combine super-renormalizability with supersymmetry of K and W, but not of W*, to argue that fine tuning of counterterms can either be avoided or is one-loop only... future work...

as opposed to 4d WZ, the 3d models have interesting infrared dynamics: 3d supersymmetric "Wilson-Fischer" fixed points, where some anomalous dimensions are predicted by the 3d R-symmetry/anomalous dimension correspondence

In (2,2) 2d theories (=dim reduction of 4d N=1):

$$\{Q_\pm, \bar{Q}_\pm\} = -2i\partial_\pm \ , \quad Q_\pm^2 = \bar{Q}_\pm^2 = 0 \ , \quad Q_\pm^\dagger = \bar{Q}_\pm \ , \quad \{Q_+, Q_-\} = 0$$

$$\partial_\pm = \frac{1}{2}(\partial_0 \pm \partial_1) \ .$$

Two possible choices of anticommuting charges to be preserved on the lattice (up to complex conjugation):

A-type: Q_- and $\bar{Q}_+$ unique to 2d

B-type: $\bar{Q}_+$ and $\bar{Q}_-$ also available in 3d and 4d

Consider "A-type" lattice with chiral superfields; proceed as in QM:

- introduce lattice superfields
- impose global symmetries: Z_4, $U(1)_V$, $U(1)_A$, Z_{2F}, I-involution
- write most general action consistent with symmetries ("D-term")
- include F-term, preserving only one linear combination of supercharges (in accord with our criterion II.)

$\Phi^A = U + \theta^+ \Xi$ in A-type basis

introduce lattice superfields:

$$\begin{aligned}
U_{\vec{m}} &= \phi_{\vec{m}} + \theta^- \psi_{-,\vec{m}} \ , \\
\Xi_{\vec{m}} &= \psi_{+,\vec{m}} - \bar{\theta}^+ i\Delta_z \phi_{\vec{m}} + \theta^- F_{\vec{m}} + i\theta^- \bar{\theta}^+ \Delta_z \psi_{-,\vec{m}} \ , \\
\bar{U}_{\vec{m}} &= \bar{\phi}_{\vec{m}} - \bar{\theta}^+ \bar{\psi}_{+,\vec{m}} \ , \\
\bar{\Xi}_{\vec{m}} &= \bar{\psi}_{-,\vec{m}} - \theta^- i\Delta_z \bar{\phi}_{\vec{m}} - \bar{\theta}^+ \bar{F}_{\vec{m}} + i\theta^- \bar{\theta}^+ \Delta_z \bar{\psi}_{+,\vec{m}} \ .
\end{aligned}$$

impose global symmetries:

$$\begin{aligned}
U(1)_V \ &: \ \theta^- \to e^{-i\beta}\,\theta^-, \ \ \bar\theta^+ \to e^{i\beta}\,\bar\theta^+, \ \ \Xi \to e^{i\beta}\,\Xi, \ \ \bar\Xi \to e^{-i\beta}\,\bar\Xi, \\
&\quad U \to U, \ \ \bar U \to \bar U \ , \\
U(1)_A \ &: \ \theta^- \to e^{i\omega}\,\theta^-, \ \ \bar\theta^+ \to e^{i\omega}\,\bar\theta^+, \ \ \Xi \to e^{i\omega}\,\Xi, \ \ \bar\Xi \to e^{i\omega}\,\Xi, \\
&\quad U \to U, \ \ \bar U \to \bar U \ ,
\end{aligned}$$

$$\begin{aligned}
Z_4 \ &: \ \theta^- \to e^{-i\frac{\pi}{4}}\theta^-, \ \bar\theta^+ \to e^{i\frac{\pi}{4}}\bar\theta^+, \ \Xi_{m_1,m_2} \to e^{-i\frac{\pi}{4}}\Xi_{m_2,-m_1}, \\
&\quad \bar\Xi_{m_1,m_2} \to e^{i\frac{\pi}{4}}\bar\Xi_{m_2,-m_1}, \ \ U_{m_1,m_2} \to U_{m_2,-m_1}, \\
&\quad \bar U_{m_1,m_2} \to \bar U_{m_2,-m_1} \ . \\
Z_{2F} \ &: \ \theta^- \to -\theta^-, \ \ \bar\theta^+ \to -\bar\theta^+, \ \ \Xi \to -\Xi, \ \ \bar\Xi \to -\bar\Xi, \\
&\quad U \to U, \ \ \bar U \to \bar U \ , \\
I \ &: \ \Xi \to i\,\bar\Xi, \ \ \bar\Xi \to i\,\Xi, \ \ U \to \bar U, \ \ \bar U \to U, \ \ d\theta^- d\bar\theta^+ \to -d\theta^- d\bar\theta^+ \ .
\end{aligned}$$

find the most general - relevant and marginal - lattice action consistent with the symmetries:

$$
\begin{aligned}
S_D &= -a^2 \sum_{\vec{m}} \int d\bar{\theta}^+ d\theta^- K_{I\bar{J}}(U_{\vec{m}}, \bar{U}_{\vec{m}})\, \bar{\Xi}^{\bar{J}}_{\vec{m}}\, \Xi^I_{\vec{m}} \\
&= -a^2 \sum_{\vec{m}} \qquad -K_{I\bar{J}}\, \Delta_z \phi^I_{\vec{m}} \cdot \Delta_{\bar{z}} \bar{\phi}^{\bar{J}}_{\vec{m}} + K_{I\bar{J}}\, F^I_{\vec{m}} \bar{F}^{\bar{J}}_{\vec{m}} \\
&\qquad + \; i K_{I\bar{J}}\, \bar{\psi}^{\bar{J}}{}_{-,\vec{m}} \left[\Delta_z \psi^I{}_{-,\vec{m}} + K^{I\bar{Q}} K_{ML\bar{Q}} \Delta_z \phi^M_{\vec{m}} \cdot \psi^L{}_{-,\vec{m}} \right] \\
&\qquad - \; i K_{I\bar{J}}\, \psi^I_{+,\vec{m}} \left[\Delta_{\bar{z}} \bar{\psi}^{\bar{J}}{}_{+,\vec{m}} + K^{\bar{J}L} K_{L\bar{M}\bar{Q}} \Delta_{\bar{z}} \bar{\phi}^{\bar{M}}_{\vec{m}} \cdot \bar{\psi}^{\bar{Q}}_{+,\vec{m}} \right] \\
&\qquad + \; K_{I\bar{J}\bar{L}} F^I_{\vec{m}} \bar{\psi}^{\bar{L}}{}_{+,\vec{m}} \bar{\psi}^{\bar{J}}{}_{-,\vec{m}} + K_{\bar{I}JL} \bar{F}^{\bar{I}}_{\vec{m}} \psi^J{}_{-,\vec{m}} \psi^L{}_{+,\vec{m}} \\
&\qquad + \; K_{IL\bar{J}\bar{M}} \bar{\psi}^{\bar{M}}{}_{+,\vec{m}} \psi^L{}_{-,\vec{m}} \psi^I{}_{+,\vec{m}} \bar{\psi}^{\bar{J}}{}_{-,\vec{m}} \; .
\end{aligned}
$$

all may appear wonderful, but it is not:

$U(1)_A$ is exact on the lattice...

clearly, there are doublers to be dealt with... will simply state results...

Consider two classes of 2d (2,2) theories with scalars and fermions on the "supersymmetric" lattice:

2d (2,2) nonlinear sigma models - these **are** interesting -

-some are asymptotically free, confinement, mass gap, anomalies

-describe moduli spaces of compactified 4d SYM

-string vacua with 4d N=1 spacetime supersymmetry

can latticize [as shown above] and lift doublers [details not shown]; *however*:

either

- the global symmetries are exact only up to powers of the lattice spacing while one supersymmetry is exact

or

- the lattice supersymmetry is explicitly broken by the Wilson terms but some of the global symmetries are exact

also, in each case, the lattice action not generic; thus, continuum limit with (2,2) susy may require fine tuning (exactly how much:...future work...renormalization of supersymmetric NLSM is nontrivial even in continuum, most models not even superrenormalizable)

2d (2,2) Wess-Zumino (a.k.a. "Landau-Ginzburg") **models** - these **are** also interesting:

- depending on superpotential, flow to N=2 CFT "minimal models" (e.g. cubic superpotential: tricritical Ising model)
- some are mirror duals to particular U(1) gauge theories with matter

can latticize and lift doublers [details not shown]:

- one lattice supersymmetry is exact, as is the $U(1)_A$ R-symmetry

- doublers are lifted by *F*-type Wilson terms

- due to the one exact lattice supersymmetry, the lattice theory is finite at small *a* [more precisely: all integrals contributing to proper vertices have lattice-D<0].

Lattice power counting generally different: new D>0 vertices due to Wilson term. *Reisz's (1988) theorem:* since all lattice integrals have D<0 (due to exact supersymmetry), all approach continuum value.

thus, despite the fact that *the lattice action with the desired (2,2) continuum limit is not generic*, finiteness of the lattice theory ensures that desired limit is achieved at small-a without fine tuning

With regard to the (2,2) LG models we then conclude that:

I.

They can be simulated:

- fermion determinant is positive on a square lattice
- the goal is to verify the predicted values of critical exponents via a lattice simulation: **first direct "proof" of flow to minimal models**!

[Giedt, in progress]

II.

Furthermore, one can use the one exact lattice supersymmetry to analytically study some nonperturbative properties of the lattice partition function, using localization [like the Witten index calculation in lattice supersymmetric QM, only a bit trickier... future work].

What more general lessons can be taken from our study of 2d (2,2) supersymmetric lattice models?

1. We gave a fairly straightforward **general** procedure for writing lattice actions preserving some [subset of the] nilpotent supersymmetries.

2. We learned that lattice actions with the desired supersymmetric Euclidean invariant continuum limit are not generic.

This *non-genericity is likely to be generic* and to persist in higher dimensions and in gauge theories, with two major implications:

non-finite theories will likely require fine tuning... manageable, perhaps, if superrenormalizable

if the lattice theory is finite [in a precise technical sense, if lattice-D<0 for all integrals] will not need tuning:
thus, in 3d and 4d, this approach is more likely to succeed in finite theories...in any case, they are the ones with most nilpotent Qs

So, while the "Q-exact" [or simply Q-closed] approach to lattice supersymmetry has its limitations, there exist interesting theories where it is likely to work *and* that have not been studied yet.

It is interesting to study theories in higher dimensions and gauge theories. To begin with, 2d (2,2) gauge theories are the simplest ones of interest [mirror symmetry, dualities, flows to various interesting sigma models,...].

C(Sh)ould make contact of our approach and closely related proposal of Sugino (2003/4) [in progress]

- study genericity and renormalization of the supersymmetric compact lattice gauge theory actions [any relation to deconstruction?]

- couple to matter and study [U(1)] dualities numerically and analytically(?)

- last but not least, study other interesting 3d and 4d theories...