Recent developments in lattice supersymmetry

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Joel Giedt, E.P., hep-th/0407135 [*JHEP09(2004)029*]
Joel Giedt, Roman Koniuk, Tzahi Yavin, E.P., hep-lat/0410041 [*JHEP12(2004)033*]
work in progress

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Catterall (2001-)
[Cohen], Kaplan, Katz, Unsal (2002-);
Sugino (2004-)

closely related older work:

Dondi, Nicolai (1977)
Elitzur, Rabinovici, Schwimmer (1982/3)
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WHY?

supersymmetric theories play a role in:

particle theory models

- supersymmetric extensions of the standard model (MSSM, GUTs)
- typically weakly coupled at TeV, but not always
- hidden sectors used for dynamically breaking supersymmetry

models for studying strong-coupling phenomena

- confinement, chiral symmetry breaking, and duality
- 2d and 3d supersymmetric critical systems (e.g., tricritical Ising model)

string theory

- including fermions requires world-sheet supersymmetry, even if space time theory nonsupersymmetric - 2d supersymmetry important

WHY?

in many of these examples the supersymmetric theories are at strong coupling
generally, in supersymmetry,

nonperturbative effects are well understood via holomorphy and symmetries,

but not all desired aspects are under theoretical control

e.g.:

control over D-terms important for finding low-energy spectrum of strongly coupled supersymmetric theories, with or without dynamical supersymmetry breaking

numerous conjectures are based on symmetries and nonrenormalization,
but few explicit checks? proofs?

WHY?

to address these issues, one requires a tool to study strong coupling:

- a nonperturbative definition...

note: a supersymmetric regulator ≠ nonperturbative definition!

- ways to extract nonperturbative information...

the lattice is the only* known nonperturbative definition of a general field theory and it would be of interest, and perhaps even useful, to have one for supersymmetric theories

*apart from
...constructive field theory
...string theory

Outline:

1. Problems and approaches

2. The essence of the recent developments in a nutshell:

   supersymmetric quantum mechanics on a "naive" vs. "supersymmetric" lattice

3. General criteria and lessons:

   what higher-dimensional theories do we expect to be able to study similarly?
   with how much effort?

4. Outlook

main problem:

- supersymmetry is a space-time symmetry
- a lattice generally breaks space-time symmetries

restoration of Euclidean rotation symmetry in the continuum does not guarantee supersymmetry restoration,

e.g., in supersymmetric theories with scalars, no symmetry, other than supersymmetry itself, can forbid relevant supersymmetry breaking operators (``soft” scalar masses)

only 4d theory without scalars:
pure SYM, where chiral symmetry forbids the **single** relevant operator (gaugino mass)

Kaplan, Schmaltz/
Kogut, Fleming, Vranas, 1990s

more specifically - important if we're to make progress:

variation of a supersymmetric action under supersymmetry is a total derivative iff the Leibnitz rule for spacetime derivative holds (in general, needed in interacting theories only)

however, **there exist no (ultra) local lattice derivatives that obey the Leibnitz rule!**

$$\Delta_- A_i = \frac{A_i - A_{i-1}}{a} \quad \rightarrow \quad \Delta_-(AB)_i = \Delta_- A_i \, B_i + A_i \, \Delta_- B_i + a \, \Delta_- A_i \, \Delta_- B_i$$

[Bartels and Bronzan, 1983, susy algebra with infinite range ``derivatives'' on an infinite lattice...]

Are we stuck, then? ...yes and no...

$$S = \int dx d\theta ... d\theta' \; F(\Phi)$$

generic SUSY action:
integral over superspace of a function of superfields

$$Q = \frac{\partial}{\partial \theta} + \theta \Gamma \frac{\partial}{\partial x}$$

SUSY generators:
differential operators acting on superfields

$$\delta_\epsilon \Phi = \epsilon Q \Phi$$

SUSY variation of the action:

$$\begin{aligned}\delta_\epsilon S &= \int dx d\theta ... d\theta' \left[F(\Phi + \epsilon Q \Phi) - F(\Phi) \right] \\ &= \int dx d\theta ... d\theta' \; \epsilon Q F(\Phi) \\ &= \int dx d\theta ... d\theta' \; \epsilon \left(\frac{\partial}{\partial \theta} + \theta \Gamma \frac{\partial}{\partial x} \right) F(\Phi) \quad = 0\end{aligned}$$

use the Leibnitz
rule for spacetime
derivatives

Dondi, Nicolai, 1977

$$\delta_\epsilon S \;=\; \int dx d\theta ... d\theta' \, \epsilon \left(\frac{\partial}{\partial \theta} + \cancel{\theta \Gamma \frac{\partial}{\partial x}} \right) F(\Phi) \;= 0$$

Moral: if the supersymmetry generator was simply $Q = \dfrac{\partial}{\partial \theta}$

the Leibnitz rule would not be needed for supersymmetry of the *interaction lagrangian* (it is really easy to have a *free* supersymmetric lattice theory!)

Hence, we could simply replace continuum coordinates by a set of discrete points, without destroying the nilpotent supersymmetry of the action.

This realization is not exactly new - early 1980s, but never pushed much, until recently.

...what has happened meanwhile?

...no complete and detailed understanding of perturbative lattice renormalization until Reisz,1988

...avalanche of exact results in supersymmetry: dualities, etc., in early-mid 1990s: calls for more checks!

...“D-branes on orbifolds” or ”deconstruction” approach and its similarities with lattice theories 1997-

Douglas/Moore-...

What's new?

The essence of the modern developments in lattice supersymmetry is the

progress in our ability to write "supersymmetric" lattice actions

in the sense that a set of nilpotent anticommuting supercharges are exact at finite lattice spacing (not the entire algebra, however!)

- the ability to write "supersymmetric" lattice actions for many theories using **a variety of new techniques**
- the detailed understanding of **how and when** the full supersymmetry algebra is restored in the continuum limit *rather than the "one supersymmetry is better than none" attitude...*

a rather technical topic, skip some detail...

...will show results for simplest example and then discuss generalizations and perspectives...

simplest susy theory: susy QM = "1d QFT"

many advantages:

- easy to study both numerically and theoretically - e.g., can prove convergence to continuum limit using transfer matrix techniques, or calculate Witten index at finite lattice spacing
- there are important general lessons
- will show some [new, preliminary...] 2d numerical results as well

"Euclidean" action - "1d QFT" of one boson and fermion:

$$S = \int_0^\beta dt \left[\frac{1}{2}(\dot{x}^2 + h'^2(x)) + \bar{\psi}(\partial_t + h''(x))\psi \right]$$

$$\delta x = \epsilon_1 \psi + \epsilon_2 \bar{\psi} \ , \qquad \delta \bar{\psi} = -\epsilon_1 (\dot{x} + h') \ , \qquad \delta \psi = -\epsilon_2 (\dot{x} - h')$$

supersymmetry generated by two anticommuting parameters

ψ and $\bar{\psi}$ are independent fields in "Euclidean" space

$$h = \frac{1}{2} m x^2 + \tilde{h} \ , \qquad \tilde{h} = \sum_{n>2} \frac{g_n}{n} x^n$$

"superpotential" fixes all interactions

(1d Wess-Zumino model)

useful to write supersymmetry transforms as:

$$Q_1 x = \psi \ , \qquad Q_1 \bar{\psi} = -(\dot{x} + h') \ , \qquad Q_1 \psi = 0 \ ,$$
$$Q_2 x = \bar{\psi} \ , \qquad Q_2 \bar{\psi} = 0 \ , \qquad Q_2 \psi = -(\dot{x} - h') \ ,$$

"naive" lattice action - simply discretize action:

$$a^{-1} S = \frac{1}{2} \Delta^- x_i \Delta^- x_i + \frac{1}{2} h'_i h'_i + \bar{\psi}_i (\Delta^W(r)_{ij} + h''_i \delta_{ij}) \psi_j$$

and discretize supersymmetry transforms (both procedures non-unique!):

$$Q_1 x_i = \psi_i, \qquad Q_1 \bar{\psi}_i = -(\Delta^+ x_i + h'_i), \qquad Q_1 \psi_i = 0$$
$$Q_2 x_i = \bar{\psi}_i, \qquad Q_2 \bar{\psi}_i = 0, \qquad Q_2 \psi_i = -(\Delta^+ x_i - h'_i)$$

Now, of course, these "lattice" Qs are not symmetries (or any two Qs, as per general argument) the Q-variation of the lattice action is nonzero:

$$Q_A S = a\mathcal{Y}_A \equiv a \sum_i a Y_{A,i} \to \int_0^\beta dt\, a Y_A(t) \to 0\ , \quad A = 1,2$$

but vanishes in the classical continuum limit

What happens quantum mechanically?

one finds that $\langle Q_A S \rangle = a \langle \mathcal{Y}_A \rangle \sim a \cdot a^{-1} \to \mathrm{finite}$

(despite the fact that this 1d theory has no UV divergences)

The violation of continuum supersymmetry Ward identity found numerically by Catterall in 2001 and attributed to "*large nonperturbative supersymmetry-breaking renormalization*"...

...will address shortly, but before that, describe construction of "supersymmetric" lattice action:

two **nilpotent** real supercharges given as operators in superspace:

$$\{Q_1, Q_2\} = 2i\frac{\partial}{\partial t}$$

$$\begin{aligned} Q_1 &= \frac{\partial}{\partial \theta^1} + i\,\theta^2\,\frac{\partial}{\partial t} = e^{-i\theta^1\theta^2\frac{\partial}{\partial t}}\,\frac{\partial}{\partial \theta^1}\,e^{i\theta^1\theta^2\frac{\partial}{\partial t}} \\ Q_2 &= \frac{\partial}{\partial \theta^2} + i\,\theta^1\,\frac{\partial}{\partial t} = e^{i\theta^1\theta^2\frac{\partial}{\partial t}}\,\frac{\partial}{\partial \theta^2}\,e^{-i\theta^1\theta^2\frac{\partial}{\partial t}} \end{aligned}$$

acting on real superfields: $\Phi(t, \theta^1, \theta^2) = x(t) + \theta^1\psi(t) + \theta^2\chi(t) + \theta^1\theta^2 F(t)$

[action is of the form discussed above, $S = \int dx d\theta ... d\theta'\;F(\Phi)$]

goal is to discretize time in a manner that preserves one of the two Q's, for example, Q_1. Generally, a set of nilpotent anticommuting supercharges can always be simulateneously conjugated to pure θ-derivatives:

$$\frac{\partial}{\partial \theta^1} = e^{i\theta^1\theta^2\frac{\partial}{\partial t}}\;Q_1\;e^{-i\theta^1\theta^2\frac{\partial}{\partial t}}$$

in the conjugated basis, $Q = \dfrac{\partial}{\partial \theta}$, while the real superfield is:

II. The essence... on the SUSY QM example...

$$\Phi'(t,\theta^1,\theta^2) = e^{i\theta^1\theta^2\frac{\partial}{\partial t}}\,\Phi(t,\theta^1,\theta^2)\,e^{-i\theta^1\theta^2\frac{\partial}{\partial t}} = \Phi(t+i\theta^1\theta^2,\theta^1,\theta^2)$$
$$= (x+\theta^1\psi)+\theta^2(\chi-\theta^1(i\dot{x}+F))\ .$$

$$U(t) = x(t)+\theta\psi(t) \qquad \xi(t) = \chi(t)-\theta\,(F(t)+\dot{x}(t))$$

and thus has two irreducible components w.r.t. Q [Euclidean, beginning from line above]. Q acts as a shift of theta, as a purely "internal" supersymmetry, so replacing continuum time with a lattice does not affect the action of Q.

$$t^i,\ i=1,\ldots N$$

$$\xi^i = \chi^i - \theta\left(F^i + \frac{x^i-x^{i-1}}{a}\right)$$

$$U^i = x^i+\theta\psi^i$$

$$\xi^i = \chi^i - \theta f^i$$

denoted $x(t^i) = x^i$,etc.

can now use these lattice superfields to write supersymmetric actions:

- bosonic
- Q-invariant
- local
- lattice translation invariant

[Giedt, E.P., hep-th/0407135]

then, the most general bilinear action consistent with above (+some discrete symmetry) is:

$$S = -\sum_i a \int d\theta \left(\frac{1}{2} \, \xi^i \, \frac{\partial}{\partial\theta} \xi^i + \xi^i \, \frac{1}{a} \Delta U^i + m \, \xi^i \, U^i \right)$$

$$\Delta U^i \equiv U^i - U^{i-1}$$

to include superpotential interactions $mU^i \to h'(U^i)$

- resulting lattice action has one exact nilpotent supersymmetry on the lattice
- "lattice superfields" useful also in more general 1d examples: QM on Riemannian manifolds...[E.P., unpublished]
- works almost like this for 2d WZ (LG) and other (2,2) 2d models: discuss later

[Giedt, E.P., hep-th/0407135]

Now, back to the restoration of supersymmetry in the quantum continuum limit...

$$a^{-1}S = \frac{1}{2}\Delta^- x_i \Delta^- x_i + \frac{1}{2} h'_i h'_i + \bar{\psi}_i (\Delta^W(r)_{ij} + h''_i \delta_{ij})\psi_j$$ "NAIVE"

$$a^{-1}S_{ca} = \frac{1}{2}\,(\Delta^W x_i + h'_i)^2 + \bar{\psi}_i(\Delta^W_{ij} + h''_i \delta_{ij})\psi_j$$ "SUPERSYMMETRIC"

- both actions have the same classical continuum limit
- continuum, as well as $a \to 0$ lattice, theories are finite, so all loop graphs are finite
- only two diagrams with (superficially) nonnegative, D=0, lattice degree of divergence:

(all remaining diagrams have lattice-D<0 and thus by "Reisz's theorem," 1988, approach their continuum values)

For NAIVE action fermion loop contribution on the lattice is twice that of the continuum: doublers do not decouple from D=0 graphs

- doubler contribution needs to be subtracted off via a finite counterterm
- with the counterterm added, the perturbative series on the lattice agrees with the continuum perturbation theory in the $a \to 0$ limit

a nonperturbative proof that the finite counterterm suffices to obtain the quantum continuum limit is given (our paper) via the transfer matrix

moreover, transfer matrix also suggests ways to improve the naive lattice action to $\mathcal{O}(a)$

For SUPERSYMMETRIC action D=0 parts of the graphs cancel between the boson and the fermion [i.e., ``lattice superfgraph" has D<0]

so **no need for counterterms**; a nonperturbative proof that the quantum continuum limit is as desired (our paper) via the transfer matrix

instead of presenting transfer matrix formalism, look at "experiment":

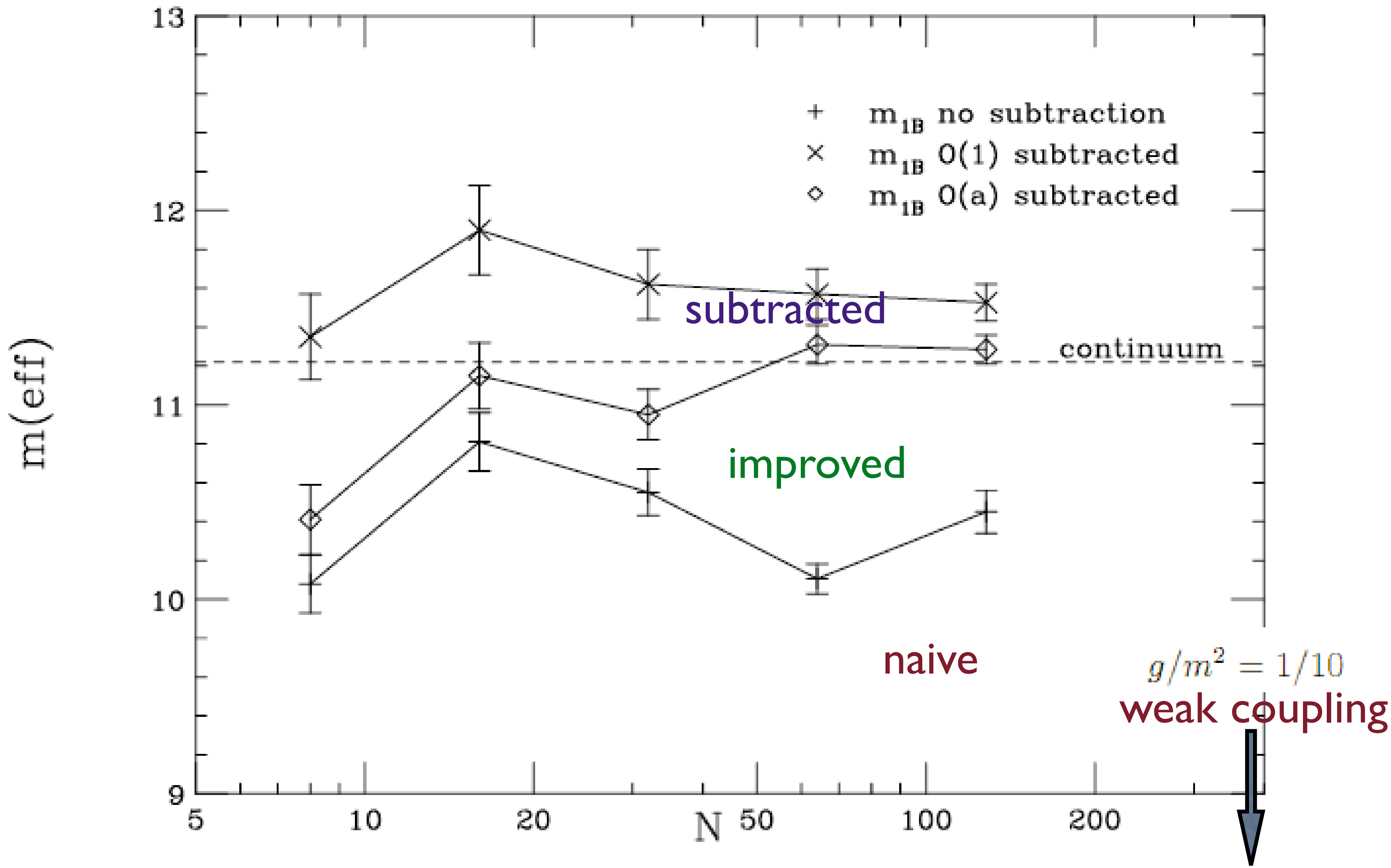


Figure 3: Leading boson mass for various forms of the naive action, with bare parameters $m = g = 10$. Large N corresponds to the continuum limit with $\beta = Na$ held fixed at $\beta = 1$. Lines are drawn to guide the eye.

II. The essence... on the SUSY QM example...

CONTINUUM $$S = \int_0^\beta dt \left[\frac{1}{2}(\dot{x}^2 + h'^2(x)) + \bar{\psi}(\partial_t + h''(x))\psi\right]$$

"NAIVE"

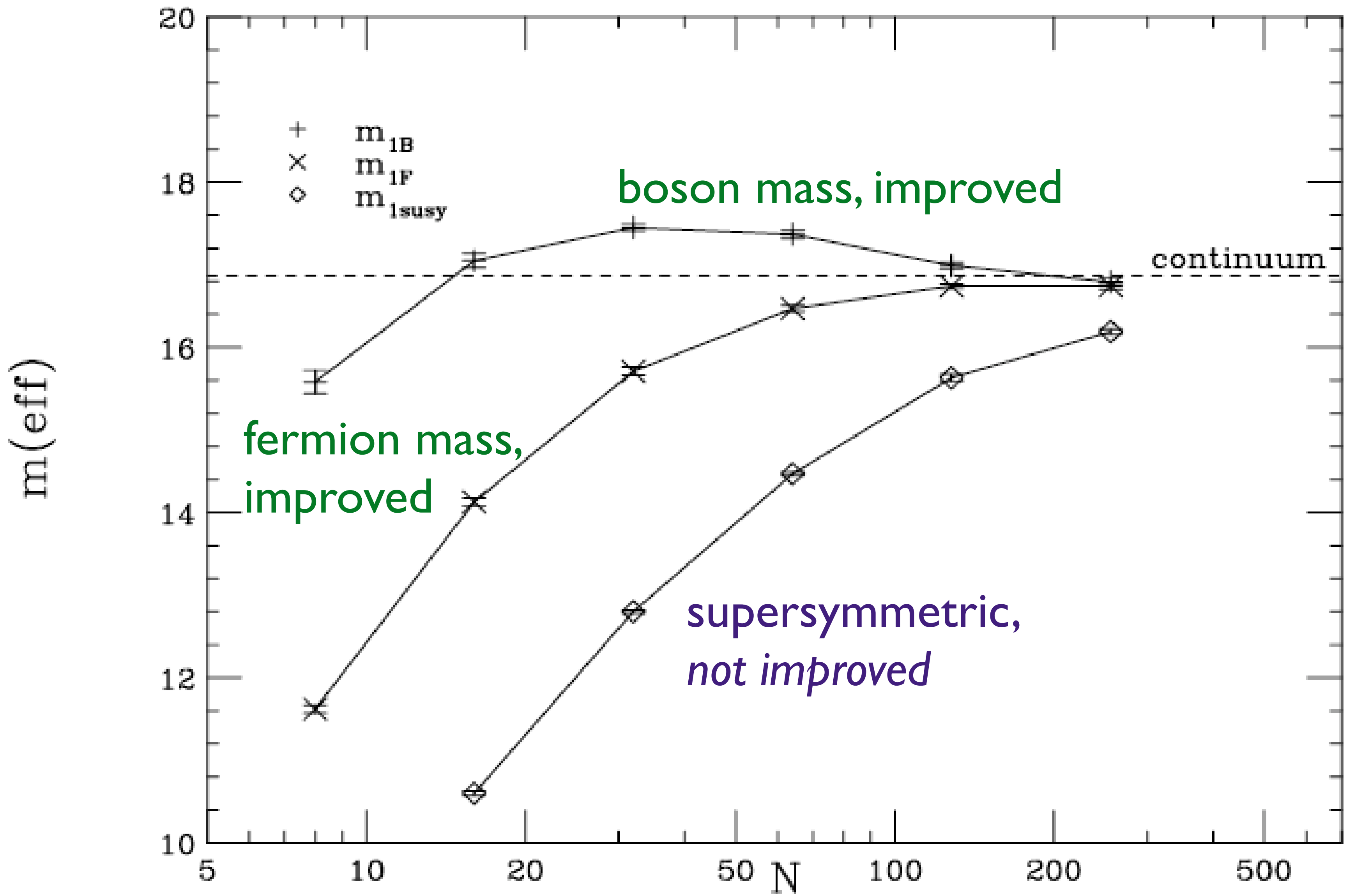
$$S = a \sum_{i,j} \frac{1}{2}(\Delta^- x_i)^2 + \frac{1}{2}\left(h'(x_i)\right)^2 + \bar{\psi}_i \left(\Delta^-_{ij} + h''(x_i)\delta_{ij}\right)\psi_j$$

IMPROVED "NAIVE"

$$S = a \sum_{i,j} \frac{1}{2}\left(\Delta^- x_i\right)^2 + \frac{1}{2}h'_i h'_i + \frac{1}{2}h''_i + \bar{\psi}_i \left(\Delta^-_{ij} + h''_i \delta_{ij} + \frac{1}{2}a\delta_{ij}h''_i\right)\psi_j$$

"SUPERSYMMETRIC" Catterall, *2001;*

$$S = \sum_{i=1}^{N} a \left(\frac{1}{2}\left(h'(x^i) + \frac{x^i - x^{i-1}}{a}\right)^2 + \chi^i h''(x^i)\psi^i + \chi^i \frac{\psi^i - \psi^{i-1}}{a}\right)$$



strong coupling: $g/m^2 = 1$

$g = 100, m = 10$

Conclusion: supersymmetric lattice action works!

- exact lattice supersymmetry
  (I.) assures that D=0 diagrams cancel, counterterms not needed
  (II.) leads to degeneracy of spectrum already at finite N
  [1 exact Q suffices, in this model]

- hence perturbative series approaches continuum one;
  in the continuum limit all continuum Ward identities should be reproduced,
  at least in perturbation theory

- nonperturbative proof in supersymmetric quantum mechanics via transfer matrix (skipped; ”theory”) and simulations (as shown; ”experiment”)

What theories admit supersymmetric lattice actions?

formulated general criteria [Giedt, EP, hep-th/0407135]

1. Clearly, it is necessary that such nilpotent anticommuting* charges exist.
not enough!

2. If some interactions are given by integrals over restricted superspace (e.g., chiral), *there must exist a linear combination of nilpotent anticommuting Q's* **such that** *the Q-variation of these interactions is not a total derivative.*

* anticommuting is not a must; central charges are allowed on the r.h.s. of the anticommutator, but not derivatives [in gauge theories in WZ gauge: all up to gauge transforms]

important example of the limitations imposed by criterion 2:

4-supercharge 3d and 4d theories; nilpotent anticommuting charges exist:

$$Q_\alpha \ (\alpha = 1, 2 \text{ is the } SL(2, C) \text{ index})$$

However, "1/3" of the lattice action (W*) will not be supersymmetric - as criterion II. is violated:

nilpotent-Q variation of antiholomorphic superpotential is a total derivative,
so requires Leibnitz rule (this only happens for integrals over restricted superspace)

In 3d WZ models, one can combine super-renormalizability with supersymmetry of K and W, but not of W*, to argue that fine tuning of counterterms can either be avoided or is one-loop only... future work...
see also J. Elliott, G.D. Moore hep-lat/0509032 for 3d N=2 SYM

as opposed to 4d WZ, the 3d models have interesting infrared dynamics: 3d supersymmetric "Wilson-Fischer" fixed points, where some anomalous dimensions are predicted by the 3d R-symmetry/anomalous dimension correspondence

So, clearly, there are the above "kinematic" constraints...

But there are also "dynamic" constraints, as actions are, as a rule - in all cases studied - not generic:

I.
non-finite theories will likely require fine tuning... manageable, perhaps, if superrenormalizable

II.
if the lattice theory is finite [in a precise technical sense, if lattice-D<0 for all integrals] will not need tuning: thus, in 3d and 4d, this approach is more likely to succeed in finite theories...in any case, they are the ones with most nilpotent Qs:

3d 4-supercharge (N=2) and higher...
4d 16-supercharge (N=4), perhaps N=2 as well...

...nothing to say (yet?) about the most interesting case of 4d N=1 (incl. chiral) supersymmetric theories....

The "other" - extended supersymmetry - theories, though, are also of interest at least from a more formal point of view (string theory, AdS/CFT...), and so there have been

a number of recent proposals for lattice versions of theories with extended supersymmetry, preserving some exact nilpotent supersymmetry, similar in spirit to the (2,2) scalar-fermion theories discussed here:

Recent proposals:

Do they work?

(Cohen), Kaplan, Katz, Unsal (2002-):
4,8,16 supercharge SYM in 2-4 dimensions via deconstruction

difficult to simulate: complex fermion determinant (Giedt, 2003-4)

Sugino (2003-4):
4,8,16 supercharge SYM in 2-4 dimensions via "untwisted TFT"

unknown: renormalization, Euclidean invariance, chiral anomalies, positivity of determinant...?

Catterall (2004-):
4[16] supercharge SYM in 2[4] dimensions via Kaehler-Dirac fermions

unknown: renormalization, chiral anomalies, integration contour, positivity of determinant..?

Giedt, E.P. (2004-):
2d (2,2) sigma model actions via Wilson or twisted mass term fermions

for general non-flat Kaehler manifolds: likely to require fine tuning;
but work well for WZ:

For the (2,2) 2d WZ models (most detailed study):

A supersymmetric lattice action can be written (old and new) and its renormalization studied (new).

[Giedt, E.P., 2004]

Restoration of continuum supersymmetry in WZ models works like in supersymmetric quantum mechanics:

- lattice perturbation theory reduces to continuum in small-a limit; all lattice (super-)graphs have lattice-D<0, so no need for counterterms

- nonperturbative "experiment" ongoing...

the most general (relevant and marginal) type-A supersymmetric lattice action consistent with the imposed lattice symmetries:

$$S_D = -a^2 \sum_{\vec{m}} \int d\bar{\theta}^+ d\theta^- K_{I\bar{J}}(U_{\vec{m}}, \bar{U}_{\vec{m}})\ \bar{\Xi}^{\bar{J}}_{\vec{m}}\ \Xi^{I}_{\vec{m}}$$

$$\begin{aligned} = -a^2 \sum_{\vec{m}} \quad & -K_{I\bar{J}}\ \Delta_z \phi^I_{\vec{m}} \cdot \Delta_{\bar{z}} \bar{\phi}^{\bar{J}}_{\vec{m}} + K_{I\bar{J}}\ F^I_{\vec{m}} \bar{F}^{\bar{J}}_{\vec{m}} \\ & + \ iK_{I\bar{J}}\ \bar{\psi}^{\bar{J}}{}_{-,\vec{m}} \left[\Delta_z \psi^I{}_{-,\vec{m}} + K^{I\bar{Q}} K_{ML\bar{Q}} \Delta_z \phi^M_{\vec{m}} \cdot \psi^L{}_{-,\vec{m}} \right] \\ & - \ iK_{I\bar{J}}\ \psi^I{}_{+,\vec{m}} \left[\Delta_{\bar{z}} \bar{\psi}^{\bar{J}}{}_{+,\vec{m}} + K^{\bar{J}L} K_{L\bar{M}\bar{Q}} \Delta_{\bar{z}} \bar{\phi}^{\bar{M}}_{\vec{m}} \cdot \bar{\psi}^{\bar{Q}}{}_{+,\vec{m}} \right] \\ & + \ K_{I\bar{J}\bar{L}} F^I_{\vec{m}} \bar{\psi}^{\bar{L}}{}_{+,\vec{m}} \bar{\psi}^{\bar{J}}{}_{-,\vec{m}} + K_{\bar{I}JL} \bar{F}^{\bar{I}}_{\vec{m}} \psi^J{}_{-,\vec{m}} \psi^L{}_{+,\vec{m}} \\ & + \ K_{IL\bar{J}\bar{M}} \bar{\psi}^{\bar{M}}{}_{+,\vec{m}} \psi^L{}_{-,\vec{m}} \psi^I{}_{+,\vec{m}} \bar{\psi}^{\bar{J}}{}_{-,\vec{m}}\ . \end{aligned}$$

...all may appear wonderful, but it is not:

U(1)$_A$ is exact in above lattice action...
clearly, there are doublers to be dealt with... will simply state results...

for flat Kaehler manifold:
(2,2) 2d WZ (aka Landau-Ginsburg) models; "experiment" is underway

[Giedt, 2004-...]

(2,2) WZ models are interesting:

- depending on superpotential, conjectured to flow to particular N=2 CFT "minimal models"

- some are mirror duals to particular U(1) gauge theories with matter

the goal of the ongoing "experiment" is to verify the predicted values of critical exponents via a lattice simulation:
first direct "proof" of flow to minimal models

arguments for flow to CFT similar to 4d Seiberg duality:
chiral rings, nonrenormalization and "NSVZ"- and "Leigh-Strassler"-like formulae...

$$W = X^3$$ hence, at a conformal fixed point dim X = 1/3:

$$\langle X^\dagger(x) X(0)\rangle \sim \frac{1}{|x|^{2/3}}$$

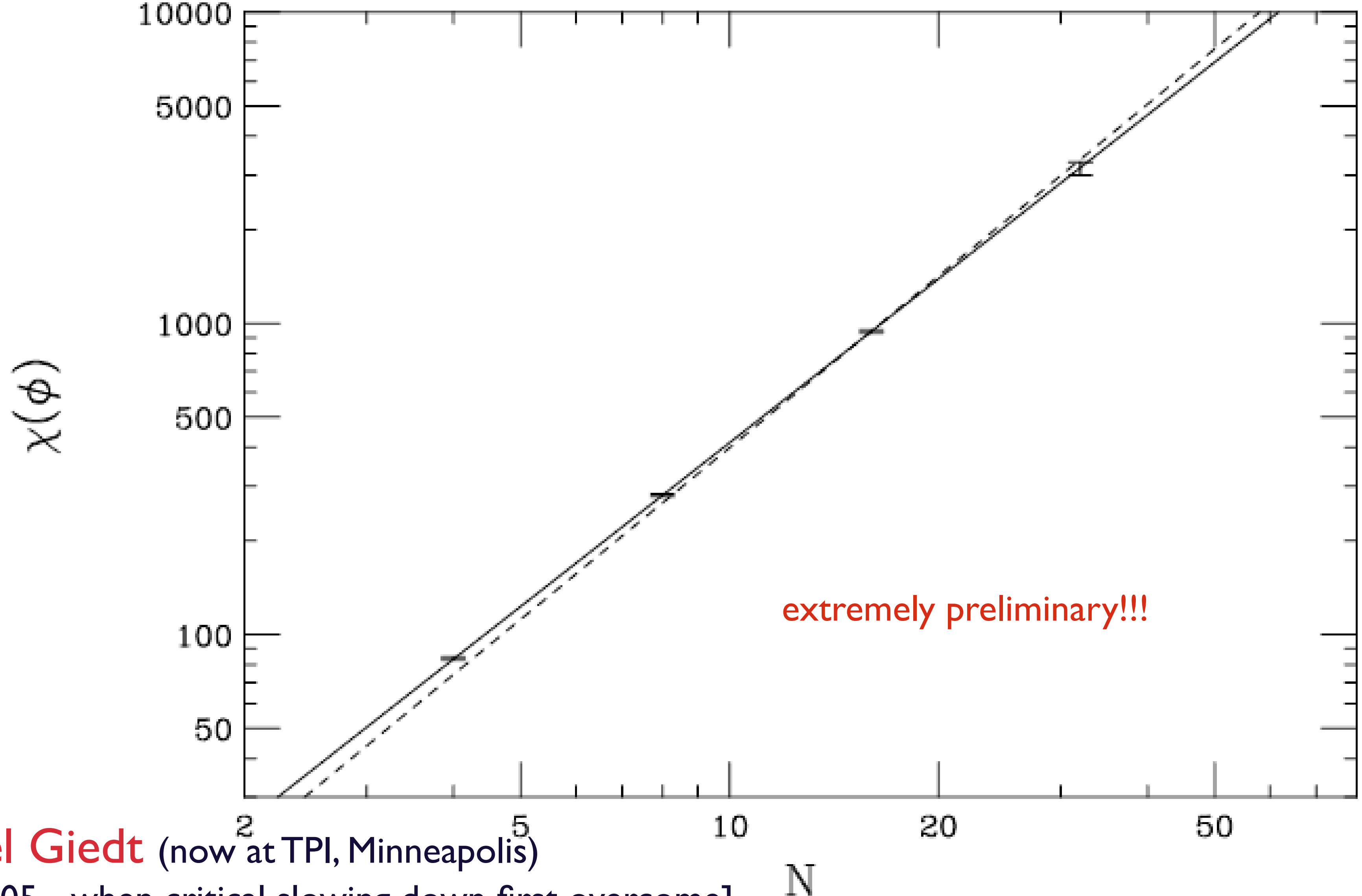
leading to expected finite-size scaling of susceptibility:

$$\chi(X) = \int_{|x|\leq\xi\sim L} d^2x \langle X^\dagger(x) X(0)\rangle \sim L^{\frac{4}{3}} \sim N^{\frac{4}{3}}$$

(2,2) 2d LG with $W=X^3$: finite size scaling of susceptibility, NxN lattice

dashed line: expected N=2 minimal model value

solid line: best fit...



courtesy of Joel Giedt (now at TPI, Minneapolis)

[as of August 2005 - when critical slowing down first overcome]

More new proposals are certainly welcome...

...without doubt, yet unwritten supersymmetric lattice actions of the sort considered here exist!

... careful analysis of the lattice renormalization, fermion det, etc., in each case needs to be performed

...SYM proposals via deconstruction, or via constructions similar to ours not studied in detail yet

...generalizations to "more interesting" chiral 4d models?

from a theoretical point of view:

it would be interesting if some analytical insight could be gained using the theories formulated on the lattice, into, e.g. U(1)/LG mirror symmetry?

on the “experimental” side:

we will know more about the results of simulations of 2d LG models soon (and what they teach us!)

This is the first d>1 field theory example where the “supersymmetric” lattice methods are tried in practice and shown to work.

...the results will guide further studies...