Chiral lattice gauge theories from warped domain walls and Ginsparg-Wilson fermions

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& work in progress

strong chiral gauge dynamics remains largely mysterious

in non-SUSY case only tools are ‘t Hooft anomaly matching and MAC

- standard model chiral but perturbative (mostly)
- non-QCD-like chiral dynamical electroweak symmetry breaking ?

analytic methods, like large-N expansions, incl. recent “AdS/QCD dualities” do not apply to chiral case on a more basic level, one may ask:

do chiral gauge theories exist outside perturbation theory?

nonperturbative definitions:

constructive field theory

string theory

lattice field theory

lattice definition of the theory remains the only option for general chiral theories; however

numerical or analytic methods using the lattice face the difficulty of preserving chiral symmetries on the lattice

Nielsen-Ninomiya theorem

there has been significant progress in understanding lattice chiral symmetries in the last 10 years

Ginsparg and Wilson ‘82
D.B. Kaplan ‘92
Narayanan and Neuberger ‘94-5
Neuberger ‘97-8
Hasenfratz and Niedermayer ‘98
Luscher ‘99-’00

we will make use of these developments

plan - present a new **proposal:**

1 domain wall and “waveguide” models
& their failure to obtain chiral spectrum

2 the use of warped domain walls (Bhattacharya, Csaki, Martin, Shirman, Terning ‘05)

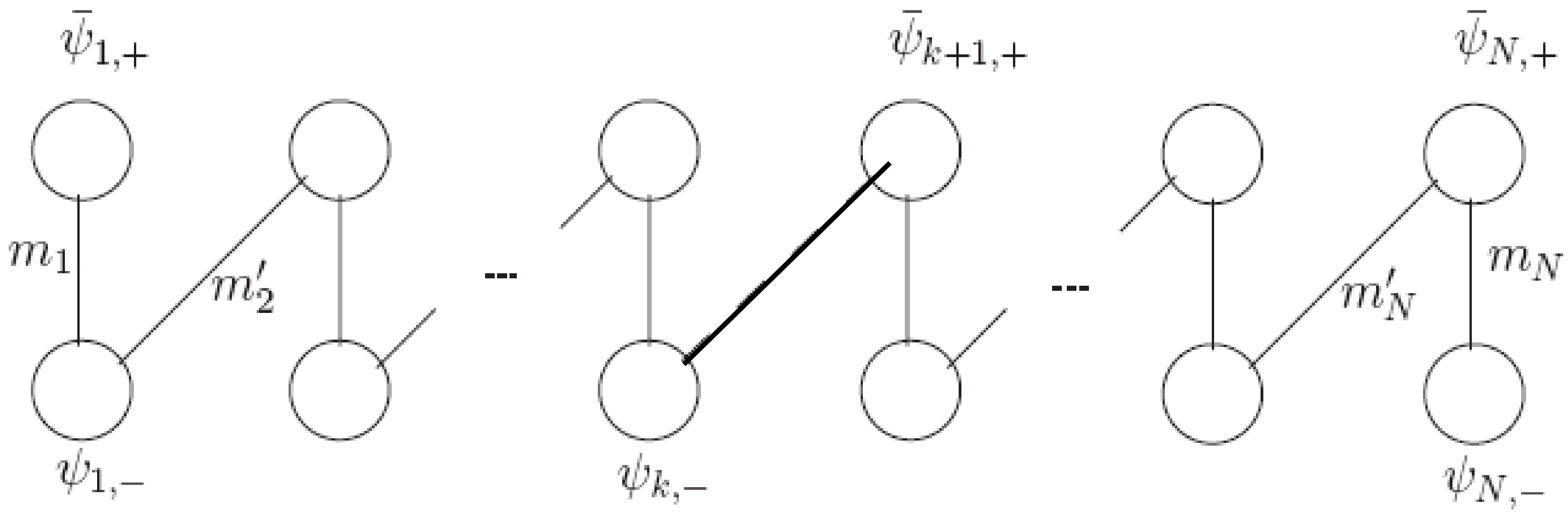
3 a proposal using Ginsparg-Wilson mechanism
to impose a modified exact lattice chiral symmetry

4 remaining issues and outlook

- domain wall and "waveguide" models
& their failure to obtain chiral spectrum

lattice domain wall fermions

(D.B. Kaplan '92)



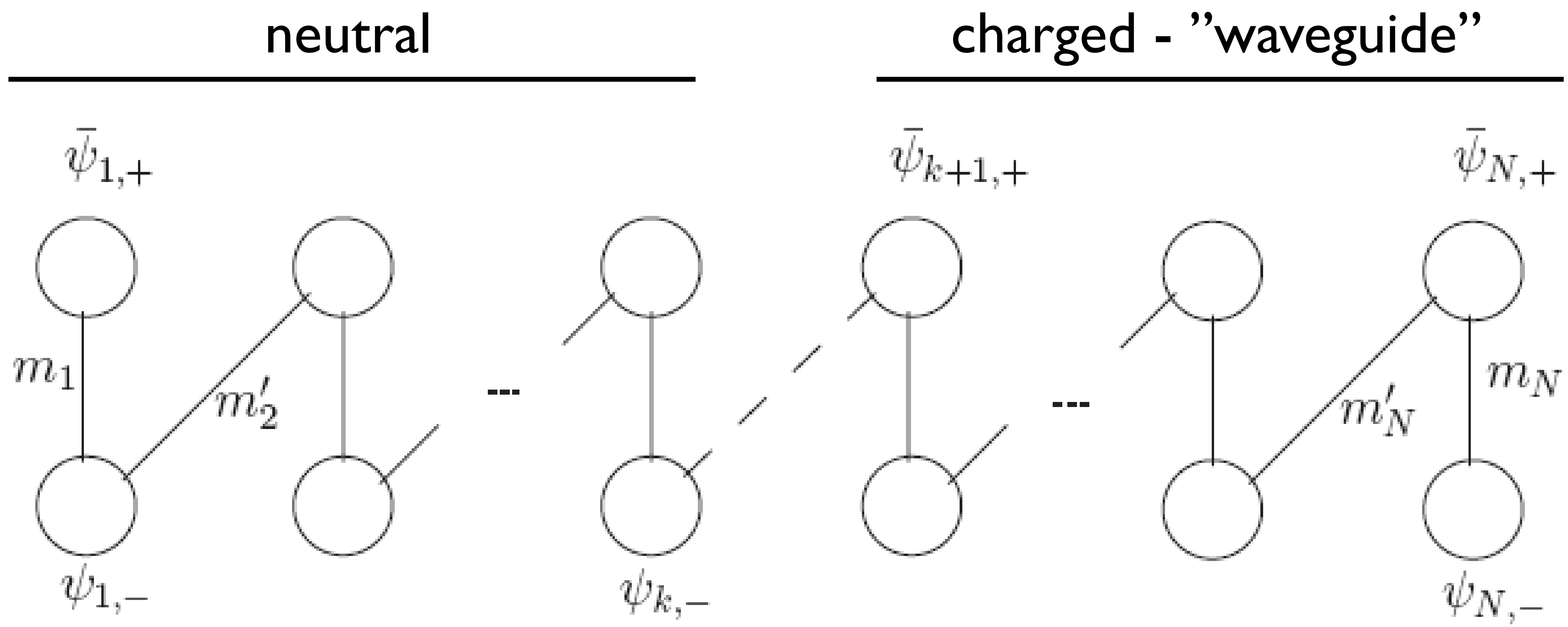
vectorlike gauge theory with exponentially light Dirac fermion;
becomes massless at infinite N, where chiral symmetry restored

domain wall and "waveguide" models
& their failure to obtain chiral spectrum

waveguide domain wall fermions

(D.B. Kaplan '92)

want: **A.) unbroken gauge theory**
B.) chiral light spectrum

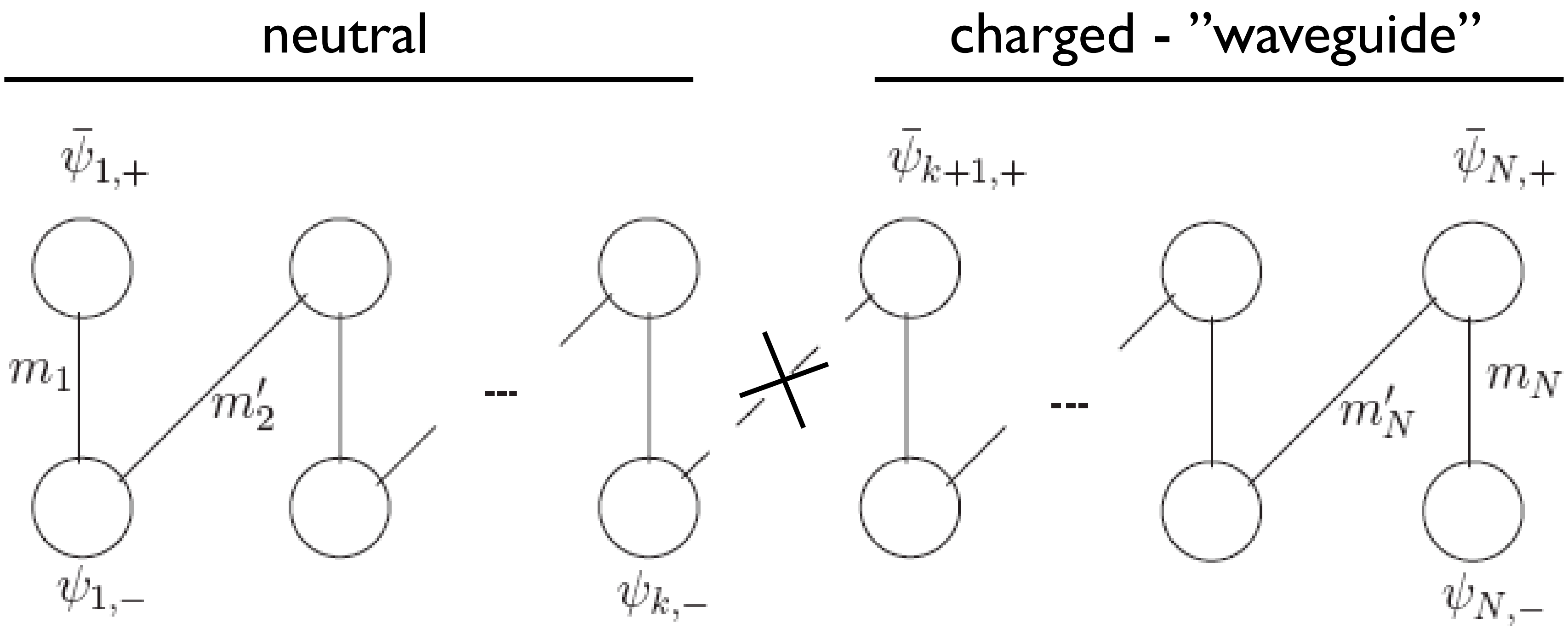


$$y\bar\psi_{k+1,+}\phi\psi_{k,-}$$

- domain wall and “waveguide” models
  & their failure to obtain chiral spectrum

waveguide at small Yukawa coupling (Golterman, Jansen, Petcher, Vink ‘93)

- vectorlike fermion spectrum in the symmetric phase



$$y\bar\psi_{k+1,+}\phi\psi_{k,-}$$

- domain wall and "waveguide" models
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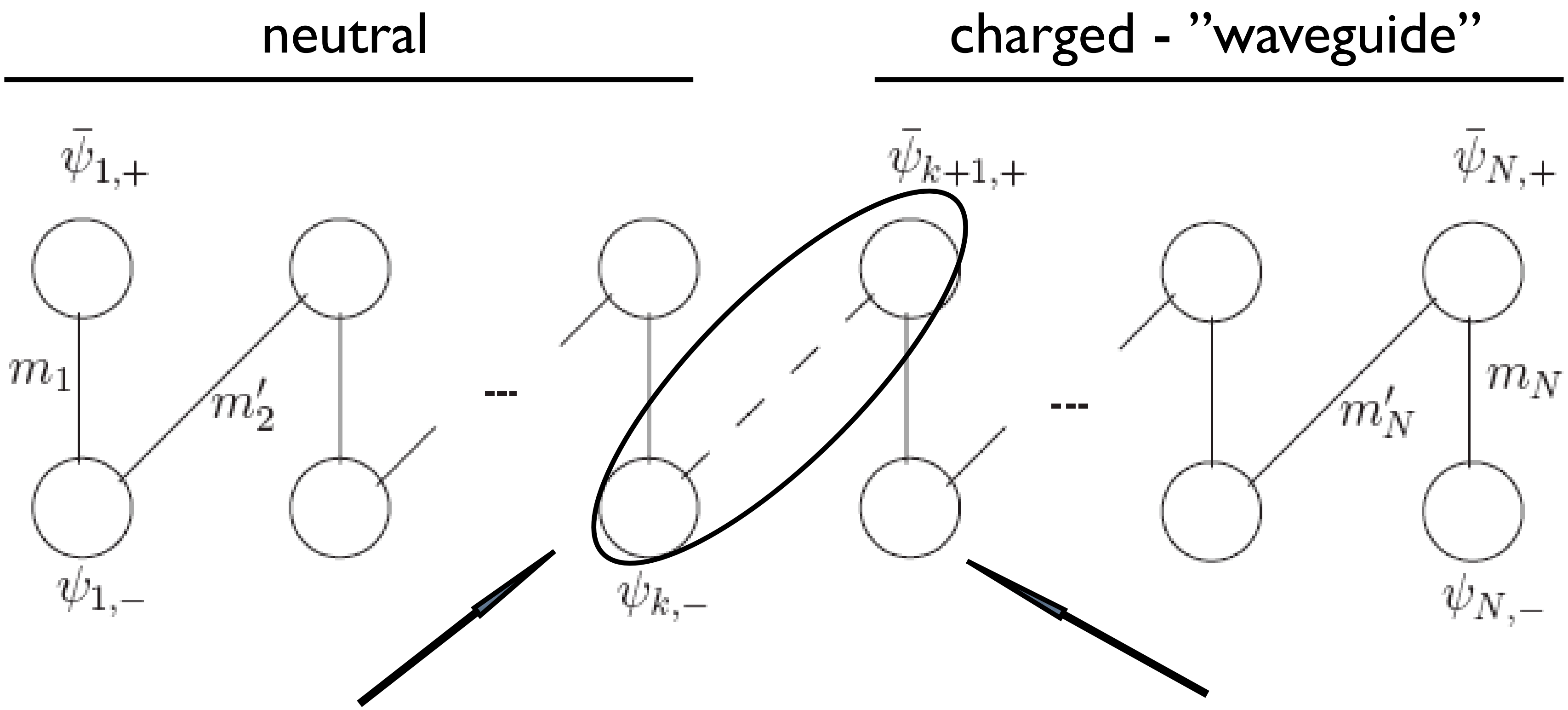
strong Yukawa symmetric phase:

(symmetric phase $\kappa < \kappa_c$, where $\frac{\kappa}{2}\sum_x\sum_{\hat{\mu}}[2-(\phi(x)^* U(x,\hat{\mu})\phi(x+\hat{\mu})+\text{h.c.})]$)

Fradkin, Shenker '79
Foerster, Nielsen, Ninomiya '80



$$y\bar{\psi}_{k+1,+}\phi\psi_{k,-}$$

(Golterman, Shamir '94)

vectorlike spectrum again!

1 domain wall and "waveguide" models
& their failure to obtain chiral spectrum

so far: waveguide doesn't work at both weak and strong Yukawa coupling

"mirror" fermion and gauge boson mass both determined by Higgs vev; in the symmetric phase "mirror" becomes massless:

weak Yukawa proposal:

2 the use of warped domain walls

extra "mirrors" appear near boundary because of loss of Wilson term of nearest neighbor in the bulk

strong Yukawa proposal

3 a proposal using Ginsparg-Wilson mechanism to impose a modified exact lattice chiral symmetry

so far, gauge field was purely 4d (not an extra dimension, rather 4d YM with N flavors)

introduce curvature and make gauge field 5d

in AdS fermion zero modes have similar localization properties

"mirror" fermion mass determined by Higgs, as in waveguide

(Bhattacharya, Csaki, Martin, Shirman, Terning '05)

if gauge field in curved space (AdS) gauge mass independent of Higgs vev

$$m_{A_0}^2 = \frac{2}{R'^2}\frac{1}{\ln(R'/R)}\left(1+\mathcal{O}\left(\frac{1}{\ln(R'/R)}\right)\right)$$

while

$$m_{KK} \equiv \frac{\pi}{R'}$$

thus can decouple gauge boson and "mirror" fermion masses

take a limit where $m_{KK} \gg \Lambda_{\chi GT} \gg m_{A_0}$ while keeping mirror fermion massive

get massless chiral spectrum as

$$m_{KK} \to \infty$$
$$m_{A_0} \to 0$$

does it work?

deconstructed AdS_5 version was studied in detail
found strong goldstone mode/fermion coupling (Bhattacharya, Csaki, Martin, Shirman, Terning '05)

- is analysis of spectrum valid, then?

then, clearly, 4-dim case requires a lattice simulation to settle

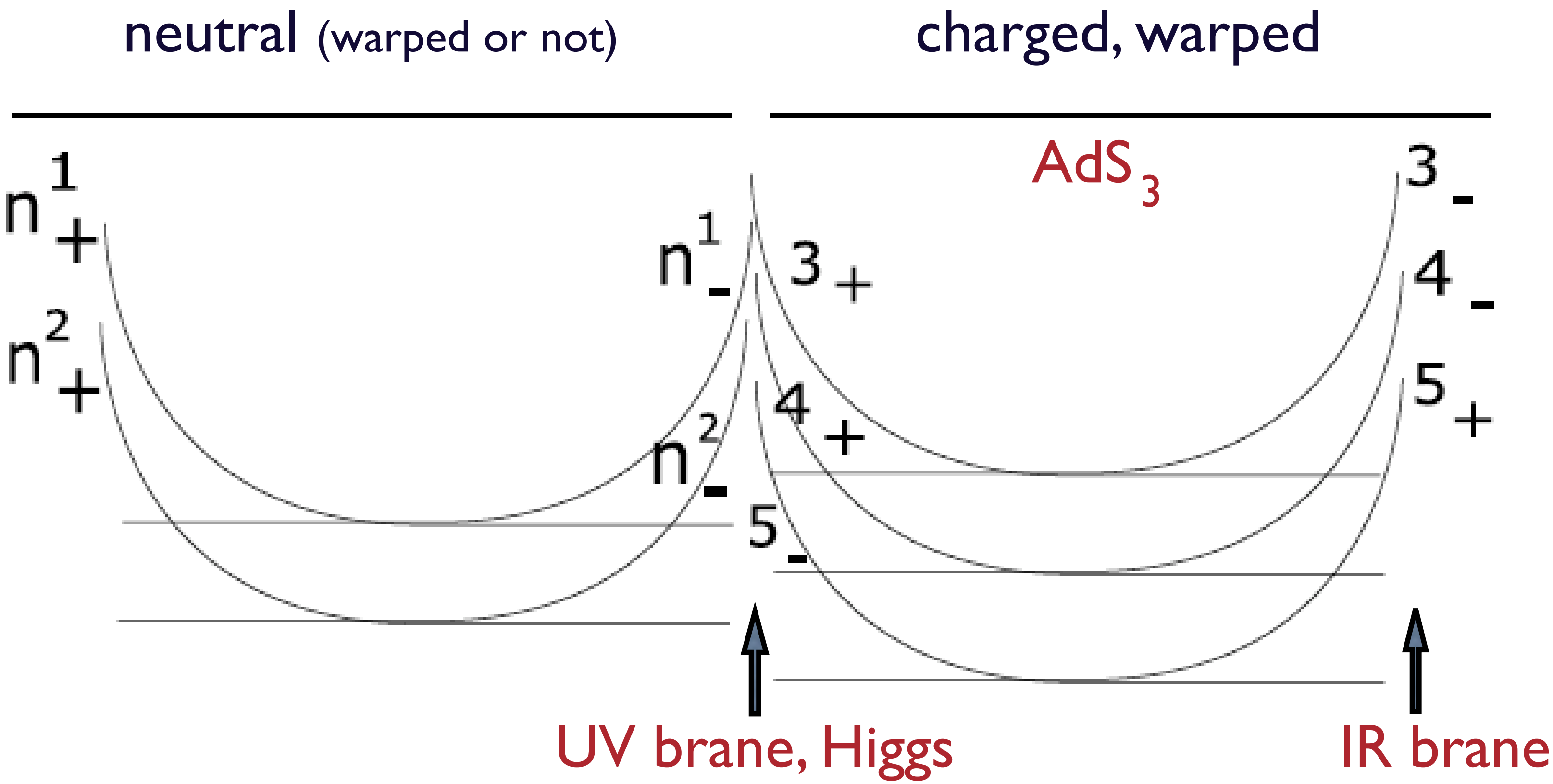
we took on a less ambitious (and easier) task: study the 2-dim case

- 2-dim chiral gauge theories may be of interest on their own
- numerical simulation likely easier in 2 dimensions

- so it is of interest to have a formulation for AdS_3

2-dim chiral theory: U(1) "345" theory $3_-, 4_-, 5_+$ chiral matter

133 global U(1) anomaly free
111 global U(1) anomalous, 't Hooft vertex $(3_-)^3\, \partial_+ (4_-)^4\, (\bar{5}_+)^5$



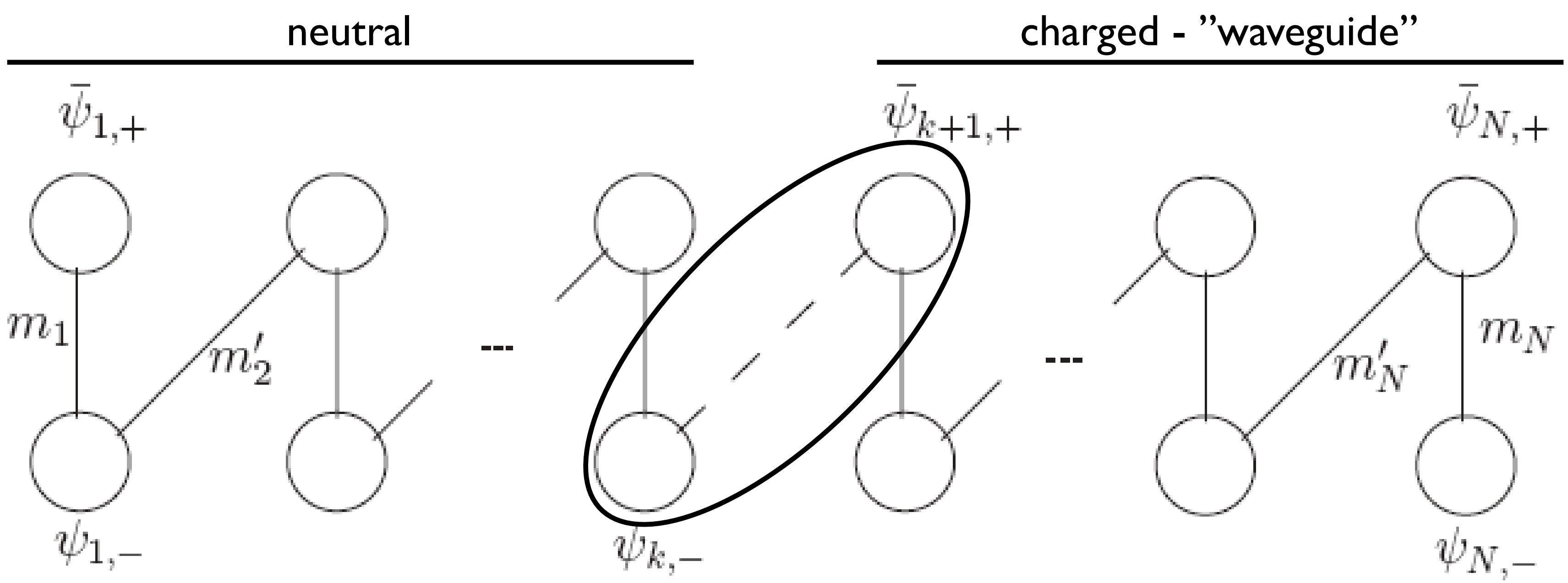
(exponentially light modes shown; 345 symmetry; 133 broken - IR restoration?)

all couplings are weak, so perturbative analysis is self-consistent

... simulation...?

3 a proposal using Ginsparg-Wilson mechanism to impose a modified exact lattice chiral symmetry

recall reason for failure to obtain vectorlike spectrum of strong coupling "waveguide"



$$y\bar\psi_{k+1,+}\phi\psi_{k,-}$$

at large y $\quad \psi_{k+1,+} \to \frac{1}{\sqrt{y}}\psi_{k+1,+}$

charged massless doublers due to lost Wilson term

$$\bar\psi_{k+1,-}\gamma\cdot D\psi_{k+1,-} + ra\bar\psi_{k+1,+}D^2\psi_{k+1,-} \to \bar\psi_{k+1,-}\gamma\cdot D\psi_{k+1,-} + \frac{ra}{\sqrt{y}}\bar\psi_{k+1,+}D^2\psi_{k+1,-}$$

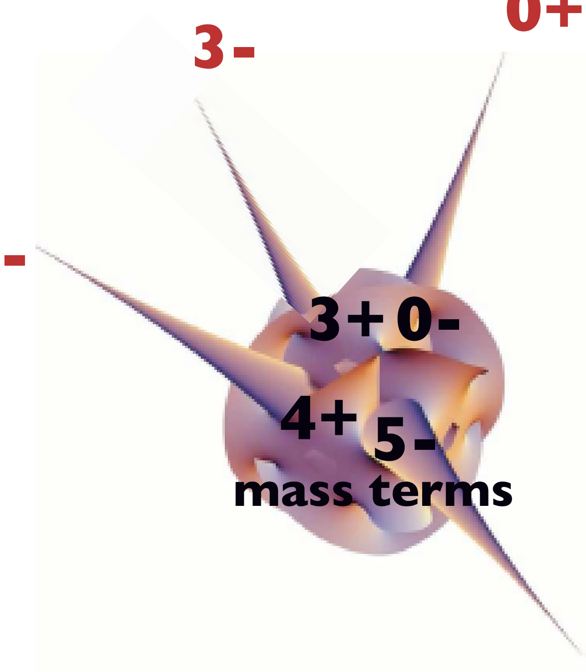
hence, +/- mixing in Wilson term is source of problem!

3 a proposal using Ginsparg-Wilson mechanism to impose a modified exact lattice chiral symmetry

what if we use fermions where +/- mixing does not happen?

Ginsparg-Wilson fermions obey $\bar{\psi} D^{GW} \psi = \bar{\psi}_+ D^{GW} \psi_+ + \bar{\psi}_- D^{GW} \psi_-$ while having no doublers

"345" theory fields: 3- 4- 5+ 0+
mirrors: 3+ 4+ 5- 0-



add Dirac and Majorana masses for mirrors break $U(1)^8$ exact lattice chiral symmetry to 345, 133, and 111 and $U(1)_0$

345, 133: anomaly free exact!
correct lattice anomalous WI

strong Yukawa symmetric phase
(to leading order) rescaling mirrors by $\frac{1}{\sqrt{y}}$ causes no appearance of doublers

does it work?

leading strong-coupling expansion indicates so...

but many issues need to be worked out: (work in progress)

- stability of next order of strong coupling, g=0, expansion
- order g corrections (what if anomalous light content?)
- fermion measure split into +/- chirality in nontrivial gauge backgrounds
- behavior in nontrivial topology backgrounds
- relation to Luscher proposal's fermion measure...
- if it all holds up, is there a sign problem?

4 remaining issues and outlook

weak Yukawa proposal:

2 the use of warped domain walls

2-dim case seems to work better than AdS_5 case

strong Yukawa proposal

3 a proposal using Ginsparg-Wilson mechanism to impose a modified exact lattice chiral symmetry

preliminary indication OK...

not an intrinsically 2-dim proposal!