The mechanical impedance for sound waves (the acoustic impedance)

We define the acoustic impedance $Z$ as:

$$Z = \frac{\text{elastic cons tan t}}{\text{associated speed}}$$

$$Z = \frac{B}{v} = \sqrt{B \rho}$$

$B$ is the elastic modulus for longitudinal waves in a gas.

Connection between two media

To illustrate incomplete reflection, we consider two media with different densities $\rho_1$ and $\rho_2$ and sound velocities $v_1$ and $v_2$.

At the boundary between the two media:

The energy transfer rates associated with the incident, reflected and transmitted waves of frequency $\omega = 2\pi f$ are given by the general expression:

$$\left\langle \frac{dE}{dt} \right\rangle = \frac{1}{2} \nu \rho_i \omega^2 s_m^2$$

We assume that the incident wave has an amplitude $s_{mi}$, the reflected wave: $s_{mr}$, and the transmitted wave: $s_{mt}$, respectively. The energy flow and the direction (little arrow) associated with:

- the incident wave is: $\frac{1}{2} \nu_1 \rho_1 \omega^2 s_m^2 \rightarrow$

- the reflected wave: $\frac{1}{2} \nu_1 \rho_1 \omega^2 s_m^2 \leftarrow$

- the transmitted wave: $\frac{1}{2} \nu_2 \rho_2 \omega^2 s_m^2 \rightarrow$
From the principle of energy conservation, we must have:

\[
\frac{1}{2} v_1 \rho_1 \omega^2 s_m^2 - \frac{1}{2} v_1 \rho_1 \omega^2 s_{mr}^2 = \frac{1}{2} v_2 \rho_2 \omega^2 s_{mt}^2
\]

or:

\[
v_1 \rho_1 (s_m^2 - s_{mr}^2) = \rho_2 v_2 s_{mt}^2
\]

The displacement at the boundary has to be continuous which means:

\[
(s_m + s_{mr}) = s_{mt}
\]

The two simultaneous equations above allow \(s_{mr}\) and \(s_{mt}\) to be calculated:

Solving these, we find:

\[
s_{mr} = \frac{\sqrt{\rho_1 B_1} - \sqrt{\rho_2 B_2}}{\sqrt{\rho_1 B_1} + \sqrt{\rho_2 B_2}} s_m
\]

and:

\[
s_{mt} = \frac{2\sqrt{\rho_1 B_1}}{\sqrt{\rho_1 B_1} + \sqrt{\rho_2 B_2}} s_m
\]

which can be re-written by using the mechanical impedance of each medium:

\[
s_{mr} = \frac{Z_1 - Z_2}{Z_1 + Z_2} s_m
\]

and:

\[
s_{mt} = \frac{2Z_1}{Z_1 + Z_2} s_m
\]

**The condition for the absence of wave reflection at a boundary between two media is that the impedance of the media be the same**

We can calculate now, for instance, the fraction of reflected wave energy:

\[
\left( \frac{s_{mr}}{s_{mt}} \right)^2 = \left( \frac{Z_1 - Z_2}{Z_1 + Z_2} \right)^2
\]

where \([(Z_1 - Z_2)/(Z_1 + Z_2)]^2\) is the coefficient of reflection.

If \(Z_2 >> Z_1\), all the wave (energy) is reflected.
If \(Z_2 = Z_1\), all the wave (energy) is transmitted.